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A Study of Parameters Important to
Soil-Structure Interaction in
Seismic Analyses of Nuclear Power Plants

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Abstract

The development of state-of-the-art techniques for analyzing the effects of soil-structure interaction (SSI) on structures during earthquakes is outlined. Emphasis is placed on methods to account for energy dissipation as a result of both wave propagation away from the structure's foundation and hysteretic soil response.

Solution techniques are grouped into two major types: substructure methods, which break the problem into a series of steps; and direct methods, which analyze the soil-structure model in one step. In addition to theoretical and historical development of SSI methodology, case studies are presented illustrating the application of these solution techniques.

The case studies are designed to quantify the effects of parameters important to the response of nuclear power plant containment structures. Studies in the following areas are presented:

- Effects of foundation embedment.
- Composite modal damping.
- Effects of radiation damping.
- *Fixed base assumption for foundation on rock.*
- Comparison of a substructure technique to a direct technique.

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1. INTRODUCTION

In order to evaluate the seismic resistance of structures, an accurate model of the system must be developed which can account for the major elements that contribute to the response. With the advent of nuclear power plants, for which massive, stiff structures are needed, it is generally agreed that including the compliance of the soil in the model is important. Early techniques, developed for analysis of machine foundation vibrations, modeled the foundation-soil system as a circular footing on the surface of an elastic half-space. One of the more difficult phenomena to accurately account for was the radiation of energy back into the soil.

One purpose of this study is to examine the unique nature of the soil-structure interaction (SSI) problem and the physical mechanisms which participate in the dissipation of energy in soil-structure systems. Mathematical techniques used to account for this energy dissipation in soil-structure interaction analysis will be described, critically evaluated, and compared.

In order to put in perspective the role of energy dissipation effects, the interrelationship among the various aspects of soil-structure interaction will be presented. This will be accomplished by means of review of state-of-the-art and historical literature and on the basis of the author's own experience in seismic analysis. Comparisons of methods will be based on their theoretical development and assumptions employed as well as their

predicted results for typical soil-structure systems. Because of the vast number of potential parameters involved in soil-structure systems, some of the parameters which are important to nuclear power plants will be studied. Case studies will be compared which employ actual power plant structures and various soil configurations.

SSI analysis methods have seen rapid development over the past decade, both in research performed to formulate new methods and in implementation into the design process. Sometimes the results of research was implemented so rapidly that the engineering community could not keep abreast of the broad scope and complexity of SSI. One of the objectives of this dissertation is to chronicle the development of SSI techniques and illustrate the application of these techniques.

1.1 The Nature of SSI and Resulting Structural Effects

The response of a structure during an earthquake is influenced by the nature of the seismic motion reaching the site, the response of the soil at the site, the influence of structure on the soil, and the characteristics of the soil and the structure itself. The first aspect involves the source mechanism, distance to the causative fault, and other seismological problems not covered here except to say that great uncertainty is inherent in predicting seismic input to a site. The second aspect is commonly referred to as the site response problem and is analyzed to determine the free field control

motion, usually at the surface of the site. The remaining aspects are what constitute the elements of SSI.

The structure can influence the wave propagation characteristics by virtue of the fact that a relatively rigid foundation is placed on, or in, the compliant soil medium. This is sometimes referred to as a scattering effect or kinematic interaction. The compliance of the soil affects the dynamic characteristics of the system. The feedback from the structure sends energy back into the soil. These three aspects of SSI affect structural response in the following ways:

- Change foundation driving force from that of the free field motion due to the presence of the foundation.
- Lower apparent modal frequencies of soil-structure system due to foundation compliance.
- Reduce amplitude in various modes by dissipating energy as a result of wave propagation away from the base of the foundation and hysteretic soil response.

1.2 Scope of Case Studies

In addition to theoretical and historical development of SSI methodology, case studies are presented illustrating the application of various methodologies and determining the effects of important parameters and analysis assumptions. Three important methodologies that treat energy dissipation in different ways are included:

- Direct methods--finite element
- Methods employing frequency-dependent impedances
- Frequency-independent half-space methods

These methods are employed to study:

- Effects of foundation embedment
- Composite modal damping
- Effects of radiation damping
- Fixed base assumption for foundation on rock

For completeness, a summary of a case study by others which compares a substructure technique (CLASSI) to a direct technique (FLUSH) is presented.

2. NATURE OF SSI COMPONENTS AND THEIR INTERRELATIONSHIP

2.1 Energy Dissipation Mechanisms

2.1.1 Wave Propagation from Base of Foundation

2.1.1.1 Physical Phenomenon

In order to describe the phenomenon whereby energy is transmitted away from a vibrating footing resting on soil, it is desirable to start with a rather idealized case before showing the effects imposed by the actual physical parameters. Consider a rigid circular footing on the surface of an ideal linear, isotropic, homogeneous, elastic half-space. If this footing is subjected to vertical oscillations, several wave forms will be induced in the half-space. Since there are no bounds to the half-space, the wave fronts will propagate away from the area of disturbance with no energy being reflected back. The wave fields which are generated away from the immediate area of disturbance ($r > 2.5$ wavelengths) are composed of body waves (shear and compression waves) and Rayleigh waves.

The components of the wave field are illustrated in Fig. 2.1, which represents the relative arrival times for the three waves in a medium with Poisson's ratio, $\nu=0.25$. The shaded regions illustrate the relative amplitudes along the wave front (Miller and Pursey, 1954). The body waves propagate radially away from the

source along hemispherical wave fronts. The Rayleigh wave has a cylindrical wave front which comprises two components that vary with depth, a vertical component and a horizontal component.

Since the wave fronts are expanding, the energy in each wave is spread over a larger area and the energy density, thus the amplitude at any point, decreases as the waves travel outward. This decrease in energy density or amplitude has been referred to as "geometrical damping", (Richart, Hall, and Woods, 1970, for example). The reason for employing this nomenclature will become more apparent in subsequent sections. It has been shown (Ewing, Jardetzky, and Press, 1967, for example) that the amplitude of the body waves

Fig. 2.1 Distribution of displacement waves from a circular footing on a homogeneous isotropic, elastic half-space, $\nu = 0.25$ (from Woods, 1968). Richart, Hall, Woods (1970)

decreases with distance in proportion to $1/r^2$ at the surface and in proportion to $1/r$ elsewhere. The amplitude of the Rayleigh wave decreases in proportion to $1/\sqrt{r'}$ (where r' is the radius of the cylindrical wave front).

The three waves described have different velocities and different particle motions associated with them. The particle motion associated with the shear wave is a transverse displacement perpendicular (or in this case, tangential) to the direction of the wave front which moves with a velocity $v_s = \sqrt{G/\rho}$, where G is the shear modulus and ρ is the mass density of the soil. The particle motion associated with a compression wave is a push-pull motion

Fig. 2.2 Rayleigh wave characteristics (a) particle motion (b) amplitude ratio vs. dimensionless depth. (Richart, Hall, Woods, 1970)

parallel to the direction of the wave front motion. Its velocity is given by $v_p = v_s/\alpha$, where $\alpha^2 = (1-2\nu)/(2-2\nu)$. The particle motion associated with a Rayleigh wave forms an elliptical path, as shown in Fig. 2.2(a), which can be resolved into two components, a horizontal and a vertical, which are 90 degrees out of phase and vary with depth as shown in Fig. 2.2(b).

The velocity of a Rayleigh wave can also be described in terms of shear wave velocity, $v_r = nv_s$, where n^2 can be found from the equation $n^6 - 8n^4 + (24 - 16\alpha^2)n^2 + 16(\alpha^2 - 1) = 0$. The relative velocities for the three wave types for various values of Poisson's ratio are shown in Fig. 2.3.

Fig. 2.3 Relation between Poisson's ratio, ν and velocities of propagation of compression (P), shear (S), and Rayleigh (R) waves in a semi-infinite elastic medium. Richart, Hall, Woods (1970)

For the rather idealized case discussed above, Miller and Pursey (1955) determined that the distribution of energy among the waves is as shown in Fig. 2.1(d). The fact that 67 percent of the total input energy is transmitted away from a vertically oscillating footing by Rayleigh waves indicates that Rayleigh waves are potentially a primary contributor to energy dissipation.

2.1.1.2 Mathematical Idealization of Energy Dissipation due to Wave Propagation

Steady State Oscillations of Simple Systems

Before complicating the problem by introducing all of the effects of a footing on a real soil, it is instructive to continue the investigation of vertical oscillations on an idealized elastic half space. An additional idealization, employed by Lysmer and Richart (1966), is that only normal stresses are transferred at the interface between the rigid circular footing and soil. These authors extended the work reported by E. Reissner who developed a method for finding dynamic surface displacements for steady-state oscillations for any given axisymmetric surface pressure distribution on an elastic half space. Developments in the 1950's and early 1960's made it possible to calculate the steady state response of a rigid footing-soil system in the low frequency range. Lysmer and Richart (1966) showed that the frequency range could be extended and that the complete steady state solution could be used

to calculate the transient response caused by an arbitrary pulse loading. Since Fourier analysis techniques are used to accomplish this, formulation of the problem in the frequency domain is most tractable.

The solution to the problems are presented in terms of the steady-state response of a system to a periodic vertical force

$$P = P_0 e^{i\omega t}. \quad (2.1)$$

The displacement response, δ , can be written in the form

$$\delta = \frac{P_0}{k} F e^{i\omega t} \quad (2.2)$$

where k is the "spring constant" which is usually equal to the static spring constant of the system. For dynamic problems, the exciting force can be resisted some by other mechanism, such as a dashpot, for which k will take on a different physical meaning. The dimensionless function F is frequency dependent and can be written as a complex function $F = F_1 + iF_2$. In Lysmer and Richart (1966) it is referred to as the "displacement function" for the system.

Fig 2.4 Typical linear system and analogous damped oscillator

It can be thought of as a complex dynamic load factor which includes phase information.

The next step performed by the authors was to determine if a simple damped oscillator, such as shown in Fig. 2.4b, exists which would reproduce the predicted displacement of the system. For the massless system shown, the equation of motion is,

$$C\dot{\delta} + K\delta = P_0 e^{i\omega t} \quad (2.3)$$

substituting for δ from 2.2 gives

$$i\omega CF + KF = k \quad (2.4)$$

By separating the real and imaginary parts and solving the resulting pair of linear equations, one obtains

$$K = \frac{F_1}{F_1^2 + F_2^2} k$$

$$C = \frac{-F_2/\omega}{F_1^2 + F_2^2} k \quad (2.5)$$

The important conclusion to be drawn from Eq. 2.5 is that the linear system can be modeled by a simple damped oscillator provided that the stiffness K and damping coefficient C are functions of the frequency ω of the driving force, i.e., they are frequency dependent. In the notation used herein, k and c are frequency-independent static spring and dashpot values, and K and C are frequency-dependent system values.

To further develop the concept, mass is added to the system, as shown in Fig. 2.5, and is excited by the harmonic force

Fig 2.5 Typical linear system and analogous damped oscillator with added mass

$Q = Q_0 e^{i\omega t}$. The new system has a displacement function $\bar{F} = \bar{F}_1 + i\bar{F}_2$ such that

$$\delta = \frac{Q_0}{k} \bar{F} e^{i\omega t} \quad (2.6)$$

The relationship between F and $\bar{F}$ can be found by solving the equation of motion

$$m\ddot{\delta} = (Q_0 - P_0) e^{i\omega t} \quad (2.7)$$

As shown in Fig. 2.5, $P_0 e^{i\omega t}$ is the reaction force due to the displacement, δ , of Eq. 2.6. By use of Eqs. 2.2, 2.6, and 2.7, the solution for $\bar{F}$ becomes

$$\bar{F} = \frac{F}{1 - m\omega^2 F/k} \quad (2.8)$$

Thus, the steady state response of the system, in terms of the massless system, is

$$\delta \approx \frac{Q_0}{k} \frac{F e^{i\omega t}}{1 - m\omega^2 F/k} \quad (2.9)$$

If the system is excited by the real force $Q = Q_0 \cos \omega t$, the real part of the solution yields the displacement

$$\delta = \frac{Q_0}{k} M \cos(\omega t + \phi) \quad (2.10)$$

in which M is equal to the magnitude of $\bar{F}$

$$M = |\bar{F}| = \sqrt{\frac{F_1^2 + F_2^2}{(1 - m\omega^2 F_1/k)^2 + (m\omega^2 F_2/k)^2}} \quad (2.11)$$

This represents the familiar dynamic load (or magnification) factor for a SDOF oscillator, and

$$\phi = \tan^{-1} \frac{F_2}{F_1 - (m\omega^2/k) (F_1^2 + F_2^2)} \quad (2.12)$$

is the phase angle.

In order to obtain a physical interpretation of the formulation derived above, the solution will be developed for a simple damped oscillator for which $K=k=\text{constant}$ and $C=c=\text{constant}$. Starting with the massless system, as in Fig. 2.4b, the constants F_1 and F_2 can be determined from Eq. 2.5, namely

$$F_1 = \frac{1}{1 + \bar{a}_0^2} \quad (2.13)$$

$$F_2 = \frac{-\bar{a}_0}{1 + \bar{a}_0^2}$$

in which $\bar{a}_0 = c\omega/k$ is a dimensionless "frequency ratio". The

form of F_1 and F_2 are shown graphically in Fig. 2.6. As before,

Fig. 2.6 Displacement function for a simple damped oscillator (mass = 0)

the solution for the system with mass added can now be found in terms of the massless system. A new dimensionless term is introduced at this point, the mass ratio

$$\bar{B} = \frac{km}{c^2} \quad (2.14)$$

It can be considered as a measure of the mass of the system. In terms of the damping ratio, ζ , of the simple system,

$$\bar{B} = \frac{1}{4\zeta^2} \quad (2.15)$$

It satisfies the relationship

$$\bar{B} \bar{\omega}_0^2 = \frac{m_w^2}{k} \quad (2.16)$$

which in combination with Eqs. 2.11 and 2.13 yields the magnification factor

$$M = \frac{1}{\sqrt{(1 - \bar{B} \bar{\omega}_0^2)^2 + \bar{\omega}_0^2}} \quad (2.17)$$

This is equivalent to the more standard form found in textbooks (see, for example, Clough and Penzien, 1975) in which

$$M = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \quad (2.18)$$

where ω_n is the undamped natural frequency of the oscillator.

The nature of M in terms of $\bar{B}$ and $\bar{a}_0$ is shown in Fig. 2.7. It is

Figure 2.7 Steady-state spectra for simple damped oscillator.

seen that these curves are identical in shape to those in terms of Eq. 2.18, which are shown in Fig. 2.8, but are shifted in frequency.

Fig. 2.8 Variation of dynamic magnification factor with damping and frequency.

For $\bar{B} > 1/2$ ($\xi < .707$) the magnitude of the peaks are

$$M = \frac{\bar{B}}{\sqrt{\bar{B} - 1/4}} = \frac{1}{2\xi\sqrt{1 - \xi^2}} \quad (2.19)$$

which occur at the frequency ratios

$$\bar{a}_0 = \frac{\sqrt{\bar{B} - 1/2}}{\bar{B}}, \text{ or } \frac{\omega}{\omega_n} = \sqrt{1 - 2\xi^2} \quad (2.20)$$

Note that these frequencies are not at what many refer to as the resonant frequencies, where $\omega = \omega_n$. For the latter case,

$$M = \sqrt{\bar{B}} = \frac{1}{2\xi} \quad (2.21)$$

For $B < 1/2$ ($\xi > .707$) no amplification occurs and the largest displacement results from static loading (zero frequency). As damping further increases, such that $\xi \geq 1.0$, the SDOF system will not exhibit oscillatory motion under free vibration.

In the conventional presentation of the dynamic response of SDOF systems, the dimensionless parameters $\xi = c/2\sqrt{km}$ and $\omega/\omega_n = \omega\sqrt{m/k}$ are usually used. These two terms separate the effects of damping and frequency. In the above presentation, the terms $\bar{a}_0 = \omega c/k$ and $\bar{B} = mk/c^2$ separate the effects of frequency and mass. This was done because for design of foundations, the latter two variables can be adjusted whereas damping is not so easily manipulated. In addition, as will be seen when the dynamic response of a footing on a soil mass is further investigated, the use of

similar ratios will allow some physical interpretation of the phenomena as related to the simple oscillator just described.

Development of Analog for Elastic Half Space With Uniform Periodic Loading.

Lysmer (Lysmer and Richart, 1966) is credited with the development of a crucial step in the physical interpretation for simplified modeling of an elastic half space. To accomplish this step, he studied a perfectly elastic half space with mass density, ρ and elastic properties G and ν . The surface of the half space is subjected to a uniform, vertical harmonic loading, $p = p_0 e^{i\omega t}$ per unit area. The quantity of interest is the displacement, δ , of the free surface.

Since the vertical load is applied over the entire surface of the half space, no horizontal displacements can occur. Thus, the motion of the soil can be described by a unit column of soil which will behave like a rod with zero lateral displacement, Fig. 2.9a. The constrained modulus of the soil column is defined as $E_p = G/\alpha^2$, where α^2 was defined earlier in this chapter, as was v_p . No reflected waves will occur because the rod is infinitely long. The displacement of the rod is given by

$$w = Ae^{i\omega(t-z/v_p)} \quad (2.22)$$

which represents a sinusoidal wave propagating downward with

velocity v_p . Evaluating the normal stress

$$\sigma(z) = E_p \frac{\partial w}{\partial z} \quad (2.23)$$

at the surface and equating it to the applied load yields

$$A = \frac{-ip_0}{\omega \sqrt{\rho E_p}} = \frac{-ip_0}{\omega \sqrt{\rho G}} \quad (2.24)$$

Thus the displacement δ at the surface, from Eq. 2.22 is

$$\delta = \frac{-ip_0 e^{i\omega t}}{\omega \sqrt{\rho G}} \quad (2.25)$$

This is exactly the steady state solution of the differential equation

$$\frac{\sqrt{\rho G}}{a} \dot{\delta} = p_0 e^{i\omega t} \quad (2.26)$$

which is also the equation of motion for the simple analog system shown in Fig. 2.9b. The analog is just a dashpot with damping coefficient

$$c = \frac{\sqrt{\rho G}}{a} \quad (2.27)$$

This important result establishes the fact that energy dissipation caused by wave propagation can be modeled by a viscous damper. It allows the study of an infinite system by considering a simple finite system.

Lysmer and Richart (1966) extend the analog principle to more complicated systems; namely to a rigid circular footing resting on

the surface of an elastic half space. The assumptions employed in this development for vertical oscillations, as well as by other authors for other degrees-of-freedom (Richart, Hall, Woods, 1970), will be discussed in Chapter 3.

Fig. 2.9 Uniformly loaded half space and its analog.

2.1.1.3. Applications of Mathematical Idealization

The formulations described above were a step in the development of so-called "half-space" theories. These were originally developed for predicting the dynamic response of machine foundations which tend to be governed by steady-state phenomena. To the point in this development that has been described above, the following important conclusions are drawn:

- Vertical oscillations of an elastic half space can be modeled by a simple oscillator with frequency-dependent stiffness and damping coefficients

- The total solution for harmonic motion of a mass-spring-dashpot system can be found from the simpler solution of a massless foundation
- Energy dissipation caused by wave propagation can be modeled by a viscous damper

In addition, once the solutions for harmonic motion at all frequencies are obtained, the solution for a transient motion can be found by Fourier analysis techniques.

When analyzing the dynamic response of a rigid machine foundation, the cited principles may be used. First, the response of the rigid massless foundation to harmonic motion at various frequencies is determined. The ratios between the steady state forces (or moments) and the resulting displacements (or rotations) at the base of the foundation are called dynamic impedance functions, $\kappa(\omega)$. Using the symmetrical rigid foundation as an example, the vertical impedance is

$$\kappa = \frac{p(t)}{\delta(t)} \quad (2.28)$$

where κ is the reciprocal of the displacement function F . As shown previously; the dynamic force and displacement are generally out of phase and, thus, can be written as the sum of two components: one in phase (or 180° out of phase) and one 90° out of phase with

the harmonic load. This is reflected in the impedance by expressing it in terms of complex notation as was F .

$$\kappa = \kappa_1 + i\kappa_2 \quad (2.29)$$

The real and imaginary components are functions of the forcing frequency, ω . The real component reflects the stiffness and inertia of the supporting soil. Since, in this theory, the soil properties are considered to be frequency independent, the frequency dependency of the real part is attributed solely to the influence which frequency has on inertial effects. This shows that the mass of the soil has an effect on the effective stiffness associated with the massless foundation. The magnitude of this effect is sometimes associated with a soil "virtual mass." The imaginary component reflects the energy dissipation of the system which is attributed to radiation and material damping of the system. As shown previously, the radiation damping, which is modeled by equivalent viscous damping to account for energy dissipation by wave propagation, is frequency dependent. As will be shown in subsequent sections, material damping, which models the energy losses from the cyclic hysteric behavior of the soil, is in fact, nearly frequency independent.

To further impart a physical-intuitive understanding of the half-space solution, consider the SDOF oscillator described earlier whose response was described by Eqs. 2.13 through 2.21. The classical equilibrium equation for this system is

$$m\ddot{x} + C\dot{x} + Kx = P(t) \quad (2.30)$$

Again, assuming a harmonic input $P(t) = P_0 e^{i\omega t}$, the steady state response $x(t) = x_0 e^{i\omega t}$. Substituting this into Eq. 2.9 yields

$$(K - m\omega^2) + iC\omega = \frac{P(t)}{x(t)} \quad (2.31)$$

which, when compared with Eq. 2.27 leads to the conclusion that

$$\kappa = (K - m\omega^2) + iC\omega \quad (2.32)$$

which is the dynamic impedance function for the SDOF system. The real and imaginary parts are then

$$\kappa_1 = K - m\omega^2 \quad (2.33)$$

$$\kappa_2 = C\omega$$

This provides further confirmation that the dynamic impedance can be expressed as a complex number with the frequency dependent real part representing the stiffness and inertia of the system and the frequency dependent imaginary part representing the energy dissipation in the system. Note that the coefficient of the imaginary part is composed of the viscous damping coefficient times ω . This, again, indicates that the analogous SDOF oscillator representing the soil half space emulates the energy losses due to radiation damping as a viscous damper.

As done with the displacement function, F , the impedance is frequently written in terms of the spring constant k , which represents the static stiffness of the system in this case,

$$\begin{aligned} \kappa &= k (1 - \omega^2/\omega_n^2) + i2\xi\omega/\omega_n \\ \text{or} \quad \kappa &= k (K_S + i\omega C_S) \end{aligned} \quad (2.34)$$

where $C_S = C/K$. The coefficients K_S and C_S represent the dynamic part of the impedance and are plotted as a function of frequency in Fig. 2.10. Note that at zero frequency, the dynamic part of the impedance becomes a real number, equal to one, and the impedance becomes equal to the static stiffness of the system. It

Fig. 2.10 Dynamic stiffness and damping coefficients of a SDOF simple oscillator.

is of interest to note that at a frequency ratio of one, the dynamic stiffness coefficient, K_S equals zero. This corresponds to the resonant condition in Fig. 2.8. Using the undamped case as an example, the dynamic magnification factor goes to infinity at this point. This is consistent since the deflection is equal to the load divided by κ (Eq. 2.28) and κ for this case would be zero.

When the frequency ratio is greater than one, K_S is negative. This means that the phase angle is 180° , or the displacement is 180° out of phase with the input forcing function. This is illustrated in the plot of phase angle for a SDOF system in Fig. 2.11. This occurs when $1 - \omega^2/\omega_n^2$ in Eq. 2.34 is negative or when the inertial term, $m\omega^2$ in Eq. 2.32 is larger than K . For the analogous soil system, Veletsos and Wei (1971), refer to this

Fig. 2.11 Variation of phase angle with damping and frequency for a SDOF system

condition as the point where a significant "virtual mass" of soil is vibrating in phase with the foundation.

As the frequency ratio increases, the magnification factor decreases, such that when ω^2/ω_n^2 reaches 2, the steady state response of the undamped system again becomes equal in magnitude to the static response. This is shown in Fig. 2.8 for $\omega/\omega_n = \sqrt{2}$. The effect that damping has on these phenomena can be seen in Fig's. 2.8 and 2.11.

A further visualization can be constructed to illustrate the relationship between the coefficient of the imaginary part of the complex impedance and the viscous damping coefficient. First, the steady state displacement can be interpreted by plotting its two component vectors on an Argand diagram as shown in Fig. 2.12. In this diagram, the real and imaginary contributions are rotating about the origin with an angular velocity of ω . The resultant vector is given by the magnification factor, M , derived previously,

impedance functions are then used directly to describe the dynamic response of a footing. In addition, only vertical response of symmetrically loaded areas have been discussed. The same principles apply to other directions and foundation shapes which will be discussed in more detail in the next chapter in the substructure approach section.

The concept of using viscous damping to model energy dissipation due to wave propagation is also used in the direct approach to SSL. As was shown, the continuum solution employed in the substructure approach leads to a solution which properly accounts for radiation of energy from the footing. However, for complicated geometry, it is not possible to find simple closed form solutions and, therefore, analysts frequently resort to numerical methods employing finite element or finite difference techniques. With these methods only a finite number of nodal points can be considered and thus are not directly applicable to problems of infinite extent. In order to accommodate this problem, the ideas developed above are used; namely, the artificial boundaries of the model are treated as interfaces with the remaining semi-infinite space. In order to simulate the uninterrupted propagation of energy through these interfaces, viscous boundary conditions can be imposed at the edges of the model such as those described by Lysmer and Kuhlemeyer (1969). The development of these boundary conditions will be described in more detail in subsequent sections.

2.1.2 Material Damping of Soil

2.1.2.1 Physical Phenomenon

When an oscillator constructed of any real material is set into free vibration, the amplitude of vibration will decrease and eventually disappear. If the oscillations are occurring in a continuum, the reduction in amplitude is caused by internal damping within the mass of material and is sometimes referred to as material damping. Free vibration tests of soils, such as that shown in Fig. 2.13 indicate that the amplitude decay is similar to that resulting from free vibration of a viscously damped system. However, the internal damping in soils is not considered to be the result of viscous behavior. Rather, it represents the energy dissipated as a result of nonviscous damping (often referred to as hysteretic damping).

Fig. 2.13 Typical free vibration-decay curves obtained from resonant-column tests of Ottawa sand (from Hall, 1962).
 (a) Amplitude-time decay curves.
 (b) Amplitude vs. cycle number plot.

The primary motion considered in soil structure interaction induces shear stresses in the soil beneath the structure. Tests conducted to determine dynamic soil properties are concerned primarily with cyclic shear modulus and the associated hysteretic damping. Richart (1975) summarizes several of the test programs conducted up to 1975. Shearing stress-shearing strain relations have been found to be strain-softening as shown in the postulated curve in Fig. 2.14.

Fig. 2.14 Postulated generalized simple shear stress-strain curve
(from Hardin and Drnevich, 1972a)

Tests by Hardin and Drnevich (1972a, 1972b) have been conducted using hollow cylinder simple shear tests. In these tests the soil sample is subjected to a confining pressure to simulate the static state of stress of the soil in-situ. Cyclic simple shear, simulating the horizontal shear caused by an earthquake, is induced by applying torsion at the ends of the cylinder. In their test

series they considered only cases where complete stress reversal occurred. To define the resulting closed hysteresis loop, two parameters were employed: the secant shear modulus, G , which represents the slope of the line through the end points of the loop and the hysteretic damping ratio, D . Figure 2.15 illustrates these parameters, where the damping ratio is defined as $D = A_1/4\pi A_t$. The area A_t is as illustrated and A_1 is the area enclosed within the hysteresis loop. This relationship will be derived in the subsequent section.

Fig. 2.15 Definition of modulus, damping and strain amplitude
(from Hardin and Drnevich, 1972a)

It is apparent from the above arguments that the internal soil damping is a material property of the soil and is tied together with the shear modulus of the soil. Both properties are obviously strain dependent; the damping increases with amplitude of vibration.

Parameters Affecting Damping

Many experimental studies have been performed to determine the effect that other parameters have on damping in soils. Hardin (1965) studied damping in dry sands subjected to small amplitude sinusoidal vibration over a large frequency range in order to evaluate the applicability of the Kelvin-Voigt model. He found that the viscosity in this model should vary with frequency such that the ratio, viscosity times frequency divided by shear modulus, is a constant with frequency. He also found that the damping was not affected much by density. However, damping in sands is affected by shear strain amplitude and confining pressure. It increases with decreasing confining pressure and increasing amplitude. Hardin and Drnevich (1972a) studied the effects of several parameters on modulus and damping which are summarized in Table 2.1. These relative effects are based on their findings as well as previously published data. The following conclusions were found regarding the damping ratio:

- It increases with increasing strain amplitude. The initial rates of increase are greater for 1.) lower effective mean principal stress, 2.) higher void ratio, 3.) lower number of cycles of loading.
- It decreases with increasing effective mean principal stress.
- It decreases with increasing void ratio in cohesive soils.

TABLE 2.1

Parameters Affecting Shear Modulus and Damping For
Complete Stress Reversal (from Hardin and Drnevich, 1972a)

Parameter	IMPORTANCE TO ^a			
	Modulus		Damping	
	Clean sands (2)	Cohesive soils (3)	Clean sands (4)	Cohesive soils (5)
Strain Amplitude	V	V	V	V
Effective Mean Principal Stress	V	V	V	V
Void Ratio	V	V	V	V
Number of Cycles of Loading	R ^b	R	V	V
Degree of Saturation	R	V	L	U
Overconsolidation Ratio	R	L	R	L
Effective Strength Envelope	L	L	L	L
Octahedral Shear Stress	L	L	L	L
Frequency of Loading (above 0.1 Hz)	R	R	R	L
Other Time Effects (Thixotropy)	R	L	R	L
Grain Characteristics, Size, Shape, Gradation, Mineralogy	R	R	R	R
Soil Structure	R	R	R	R
Volume Change Due to Shear Strain (for strains less than 0.5%)	U	R	U	R

^aV means Very Important, L means Less Important, and R means Relatively Unimportant except as it may affect another parameter; U means relative importance is not clearly known at this time.

^bExcept for saturated clean sand where the number of cycles of loading is a Less Important Parameter.

Seed and Idriss (1970) have summarized data from Hardin and Drnevich (later published in 1972) and other authors and have plotted the variation of damping with respect to these important parameters. Some of their conclusions are presented at the end of this chapter.

2.1.2.2 Mathematical Idealization of Hysteretic Damping

In nature, all real materials exhibit some inelastic behavior. When subjected to cyclic load, the stress-strain (or load-deformation) diagram will exhibit some hysteresis. That is, the loading path will not lie on top of the unloading path such that an area will be enclosed by the resulting lines. This area is proportional to the energy absorbed (and subsequently dissipated) by the material. Lazan (1964) defines this energy dissipation as damping. The mechanisms which cause this energy dissipation are many and varied, however, all of them can be described in terms of their resulting hysteretic loops, many of which have certain similarities. Lazan groups the various damping phenomena into two major types: rate-dependent and rate-independent. Both types may be amplitude dependent: however, the former depends on the rate of loading, whereas the latter is independent of the loading rate (and thus, frequency).

An example of rate-dependent damping is viscous damping characterized by the familiar dashpot. This is also referred to as linear damping or quadratic damping. It is linear since it is

characterized by linear viscosity; also linear differential equations and linear superposition are used to capture its effects. It is called quadratic because the energy dissipation varies as the square of the stress amplitude. As a result, the hysteretic loop is elliptical in form. As an example, when a viscous damped SDOF system is subjected to a harmonic vibration such that $x = x_0 e^{i\omega t}$ or considering the real part, $x = x_0 \cos \omega t$, the net force exerted on the mass by the damper is $C\dot{x}$. The net total force is

$$P = Kx_0 \cos \omega t - C\omega x_0 \sin \omega t \quad (2.35)$$

Fig. 2.16 Hysteresis curve for a spring and viscous damper in parallel

This is plotted in Fig. 2.16 which illustrates the elliptical nature of the hysteresis loop (Harris and Crede, 1961). Note that this loop is different from that for soils illustrated in Fig. 2.15. It should also be noted that in this discussion, the hysteretic behavior based on stress-strain and force-deflection are used

interchangeably. Computations based on energy expressions using either form are comparable in this context.

The energy dissipated by viscous damping per cycle is then

$$\Delta W_V = \int_{t_1}^{t_1+T} C \dot{x} dx = \int_{t_1}^{t_1+2\pi/\omega} C \dot{x}^2 dt = \int_0^{2\pi/\omega} C \omega^2 x_0^2 \sin^2 \omega t dt$$

$$\Delta W_V = \pi C \omega x_0^2 \quad (2.36)$$

The term ΔW_V is the total damping energy due to viscous damping and represents the area within the hysteresis loop for one cycle. Another important energy component is the total elastic strain energy, W , which represents the area under the elastic portion from zero to $P_{\max}$. In Fig. 2.16 this area is bounded by the dashed line and the lines $P = 0$ and $x = x_0$. In this figure,

$$W = kx_0^2/2 \quad (2.37)$$

There are many different parameters which can be used to describe the level of energy dissipated by a linear system. One is the loss coefficient, η , where

$$\eta = \frac{\Delta W}{2\pi W} \quad (2.38)$$

The quality factor, Q , is

$$Q = \frac{1}{\eta} \quad (2.39)$$

The logarithmic decrement

$$\delta = \ln \frac{x_t}{x_{t+T_d}} \quad (2.40)$$

where T_d is the damped natural period of the oscillator. In terms of the decay constant β , where $\beta = C/2m$,

$$T_d = \frac{2\pi}{\sqrt{\omega_n^2 - \beta^2}} = \frac{2\pi}{\omega_n \sqrt{1 - \xi^2}} \approx \frac{2\pi}{\omega_n} \quad (2.41)$$

The term ξ is the damping ratio, which defines the ratio of damping C to critical damping C_{cr} when $\beta = \omega_n$

$$C_{cr} = 2\sqrt{Km} = \frac{2K}{\omega_n} \quad (2.42)$$

Linear viscoelastic systems can be described in terms of complex shear modulus

$$G^* = G_1 + iG_2 = G_1(1 + i\eta) \quad (2.43)$$

where G_1 is the elastic component and G_2 is the viscous component. All of the above factors are interrelated. Lazan (1965) has summarized these and others in his paper, some of which are reproduced below

$$\eta = \frac{\Delta W}{2\pi W} = \frac{G_2}{G_1} = \frac{1}{Q} = \frac{\delta}{\pi} = 2\xi \frac{\omega}{\omega_n} \quad (2.44)$$

The last term shows that the damping ratio is frequency dependent. For a system at resonance or for a free vibration test, $\omega = \omega_n$ and $2\xi = \delta/\pi$. In addition, for multi-degree-of-freedom systems, the modal damping ratio is similarly defined at the modal frequencies for those systems where the modes may be uncoupled.

The above discussion involves the viscous damping case. As stated earlier, internal soil damping is not viscous in nature.

Rather, its hysteretic behavior is a result of nonlinear response caused in part by sliding between grains. This type of energy dissipation is known as structural damping or hysteretic damping. The resulting damping is actually a mechanism to replace a nonlinear, inelastic system with an equivalent linear system. The approach was pioneered by L.S. Jacobsen and is summarized in Jacobsen (1960). His work is related to structures but is applicable to soils as well. His idea is to calculate an equivalent viscous damping term such that the energy dissipated per cycle is equivalent to that of the nonlinear system. Since the viscous damping term is frequency dependent the energy loss per cycle is to be compared with that occurring at resonance in a viscous damped system. The energy loss per cycle in the hysteretic system is described by the area within the loop. However, as pointed out by Hudson (1965) the maximum strain energy is only easily defined for a linear system. This was also recognized by Jacobsen. If one uses an equivalent linear system to define the maximum strain energy, such as that defined by A_T in Fig. 2.15, the equivalent damping ratio is given by

$$\xi_{eq} = \frac{1}{4\pi} \frac{\Delta W}{W} \quad (2.45)$$

For nonlinear problems, the definition of W is ambiguous. Jacobsen suggested that the area under the skeleton (or backbone) curve be used. The skeleton curve is that described by the first loading and for many practical problems is unknown. As a first

approximation, the straight line between the origin and peak strain (which defines the secant modulus) has been commonly used (see, for example, Hardin and Drnevich, 1972a, and Seed and Idriss, 1970). Hudson (1965) proposed instead merely equating the energy loss per cycle for the viscous system (Eq. 1.35) and solving for the equivalent viscous damping. For a bilinear system he found that the equivalent maximum strain energy to be used was considerably larger than that proposed by Jacobsen, Fig. 2.17. The net result was that Jacobsen's method appeared to overestimate the equivalent viscous damping. For systems with both bilinear and curved hysteresis loops, the largest equivalent viscous damping coefficient would be

$$\xi_{eq} = \frac{1}{2\pi} = 0.159 \quad (2.46)$$

The variation of ξ_{eq} with yield ratio is illustrated in Fig. 2.18.

Fig. 2.17 Assumptions for calculating maximum strain energy in hysteretic systems

Fig. 2.18 Variation of equivalent viscous damping with yield ratio in a hysteretic system

For Hudson's equivalent damped systems, the force-deflection characteristic of the equivalent linear system had a slope equal to the initial slope of the first-loading force-deflection curve or, in other words, the initial tangent stiffness. For soil continua this would be equivalent to using the initial tangent modulus. However, in most cases a secant modulus is used in soil dynamics problems, as will be discussed in the next section. In a discussion of Hudson (1965), Rosenblueth (1965) indicated that the equivalent linear structure should be more flexible than that defined by the initial stiffness of the system under study. He stated, "By choosing as equivalent structure a sufficiently flexible one, almost as large as equivalent viscous damping ratios as desired can be justified", pointing out that the choice of equivalent modulus affects the resulting equivalent viscous damping. Thus, it appears that the

modulus and damping cannot be considered separately. Iwan and Gates (1979) have evaluated nine different methods available for defining equivalent linear systems to model simple nonlinear systems subjected to earthquake motions. They compared two fundamental parameters, effective period and effective damping. The effective period essentially defines the effective stiffness of the hysteretic system, assuming the mass stays constant.

In most structural applications, the energy dissipation is modeled by a viscous damper. However, for soils applications this does not appear to match experimental results (Hardin, 1965 and Hardin and Drnevich, 1972a) for which a frequency independent hysteretic damping is more appropriate. As pointed out by Dobry (1970), the complex modulus formulation is useful when dealing with hysteretic materials and it does not require any a priori assumption about the damping mechanism. It is convenient to discuss energy dissipation in terms of the loss angle, δ_L , where

$$\tan \delta_L = \frac{G_2}{G_1} = 2D \quad (2.47)$$

In this case $\tan \delta_L$ is taken as a constant (frequency-independent) and the frequency-independent hysteretic damping ratio is D . If we wish to simplify the problem by constructing a simple equivalent linear one (e.g., Voigt solid), the net effect is to replace the actual form of the hysteretic loop, Fig. 2.15, with an elliptical one of equivalent area Fig. 2.16. This corresponds to a Voigt solid with constant spring and variable dashpot. Veletsos and Verbic (1973) studied the vibration of viscoelastic foundations using both

the standard Voigt solid and the formulation using a variable dashpot to model a constant hysteretic solid. The authors employ the correspondence principle (Bland, 1960) in which the real valued shear modulus G is replaced by the complex modulus G^* in the expressions for impedances derived for the elastic half space. The form for the impedance of an elastic half space is shown in Eq.

2.34. An alternate form can be written in terms of a_0 , the dimensionless frequency, where

$$a_0 = \omega r / v_s = \omega r \sqrt{\rho / G} \quad (2.48)$$

In this expression, r is a critical foundation dimension, such as the radius of a circular foundation or the half width of a rectangular foundation. Then the impedance for an elastic half space can be written in the form

$$\kappa_0 = k[k_0(a_0) + ia_0 c_0(a_0)] \quad (2.49)$$

where the zero subscript is for the undamped case and $c_0 = c_s r / v_s$.

The solution for a footing on a viscous foundation can be expressed as

$$\kappa_D = k[k_D(a_0) + ia_0 c_D(a_0)] \quad (2.50)$$

Recognizing that k is a function of G and that the complex modulus, G^* , approximately equals $G(1+2iD)$ as implied by Eqs. 2.43 and 2.47, one can apply the correspondence principle to obtain

$$\kappa_D = k(1+2iD)[k_0(a_0) + ia_0 c_0(a_0)] \quad (2.51)$$

where the subscript, D , denotes the damped case. Neglecting terms of the order D^2 ,

$$a_D = \omega r \sqrt{\rho/G^*} = a_0 \sqrt{G/G^*} = \frac{a_0}{\sqrt{1+2iD}} \approx \frac{a_0}{1+iD}$$

$$a_D = a_0(1 - iD) \quad (2.52)$$

The functions k_0 and c_0 are those defined for the undamped case as in Eq. 2.49; however, as stated by Lysmer (1980) they are now complex-valued functions of a complex argument which no one has as yet evaluated.

Veletsos and Verbic (1973) used analytical functions for k_0 and c_0 which were empirically derived as approximations for simplified models (Meek and Veletsos, 1973). In order to obtain solutions for the damped case, Veletsos and Verbic assumed that the polynomial approximations were also valid for complex arguments which, in the opinion of Lysmer (1980) is questionable. Instead, Lysmer proposes that the following assumption be employed:

$$k_0(a_D) = k_0(a_0), \text{ real} \quad (2.53)$$

$$c_0(a_D) = c_0(a_0), \text{ real}$$

Substituting Eqs. 2.53 and 2.52 into Eq. 2.51, and neglecting terms of the order D^2 , one obtains

$$r_D \approx k[k_0(a_0)(1+2iD) + ia_0c_0(a_0)(1+iD)] \quad (2.54)$$

From this relationship, the behavior of a footing on a damped foundation can be predicted from a given solution for the undamped case. Lysmer presents results from the application of this relationship using solutions for the undamped case from both the solution of the elastic half-space and the approximations of Meek

and Veletsos (1973). He obtained essentially identical results as Veletsos and Verbic (1973) when he used their approximate undamped solutions as a base case. However, the use of approximate solutions is unnecessary with Eq. 2.54 because it can operate directly from the exact solution for the elastic half-space. Figure 2.19 illustrates an example of the effects of material damping and a comparison of the various methods used to compute impedance functions. In order to get comparable stiffness and damping functions, the real and imaginary parts of Eq. 2.54 are separated such that the coefficients are of the form in Eq. 2.50. Then

Fig. 2.19 Comparison of impedance functions for vertical oscillations of a circular footing. (1) Undamped half-space solution, (2) approximate undamped solution, (3) damped solution by Eqs. 2.55 applied to (1), (4) damped solution by Eqs. 2.55 applied to (2), (5) damped solution from (2) using method of Veletsos and Verbic (1973). (Lysmer, 1980)

$$k_D = k_0(a_0) - D a_0 c_0(a_0) \quad (2.55)$$

$$c_D = c_0(a_0) + 2Dk_0(a_0)/a_0$$

Equation 2.54 is different from the equation

$$k_D = k[k_0(a_0) + i a_0 c_0(a_0)] (1+2iD) \quad (2.56)$$

which has appeared in several publications, see for example Kausel, et.al. (1978). It appears that the change in a_0 was not considered during application of the correspondence principle when Eq. 2.56 was derived. The use of this expression will over estimate the effect of damping on the equivalent spring constant. As Eqs. 2.55 show, the effect of soil material damping is two fold: it decreases the effective stiffness coefficient and increases the damping coefficients.

Lysmer (1980) and Gazetas (1983) point out that some caution must be used with Eq. 2.54. It is assumed that the coefficients k_0 and a_0 are independent of material damping. For a half space this is reasonably true. However, for a shallow stratum on rigid rock, the coefficients are sensitive to the material damping ratio (see, for example Kausel and Ushijima, 1979). Thus the correspondence principle is not really applicable in this case.

One way of viewing the total energy dissipation is by summing the component parts such that

$$\frac{\Delta W}{W} = 4\pi \left(\xi \frac{\omega}{\omega_n} + D \right) \quad (2.57)$$

This expression suggests that the simple addition rule $D + \xi \omega/\omega_n$ may be used to obtain the "effective" damping ratio. Some analysts have interpreted this to mean that the damping ratios of each type

of damping could simply be added together to get a total damping ratio. This procedure would be equivalent to treating the material damping as viscous, which, as is shown in Fig. 2.20, exhibits significantly different behavior from constant hysteretic damping.

Figure 2.20 Values of k_z and c_z for viscoelastic medium with $\nu = 1/3$ (from Veletsos and Verbic, 1973)

2.1.2.3 Applications

In order to incorporate material damping in the analysis of a soil-structure system one must first determine the cyclic stress-strain behavior of the soil. Hardin and Drnevich (1972a&b)

have conducted an extensive series of tests on both cohesionless and cohesive soils. As a result of these tests and those available in the literature, they concluded that a modified hyperbolic curve satisfactorily represents the shearing stress-shearing strain behavior of the range of soils tested for strain amplitudes up to failure. Recognizing that a given strain doesn't have the same effect on all soils nor the same soil under different pressures, the authors utilized the concept of a reference strain. They found that soil behavior can be defined more simply in terms of strain normalized with respect to the reference strain. The parameters used to define a basic hyperbolic relationship are shown in Fig. 2.21a, where γ_r is the reference strain defined by $\gamma_r = \tau_{max}/G_{max}$. Methods to determine G_{max} will be described in the subsequent section.

Fig. 2.21 Hyperbolic Stress-Strain Relations

In actuality, there is a slight deviation from an idealized hyperbolic relationship. As shown in Fig. 2.21b, the stress-strain relation for sand lies slightly above a hyperbola while that for clay lies slightly below. Hardin and Drnevich (1972b) have used the concept of hyperbolic strain to provide the necessary correction factors. Figure 2.22 shows the usefulness of normalizing the shear strain in terms of the reference strain. Use of the normalized strain allows the use of a single curve to define the data from several states of ambient stress. When plotted against normalized strain or hyperbolic strain, the ratios $G/G_{\max}$ and $D/D_{\max}$ are related.

σ_m^i = mean principle effective stress

Fig. 2.22 Use of reference strain to normalize damping data (Hardin and Drnevich, 1972b)

In fact, when plotted against hyperbolic strain, as in Fig. 2.23, they may be represented by one curve.

Fig. 2.23 Normalized shear modulus and normalized damping ratio for all soils versus hyperbolic strain. (Hardin and Drnevich, 1972b)

Seed and Idriss (1970) have used the relationships developed by Hardin and Drnevich as well as data from others to plot curves for use in dynamic response analysis of soils. Their curves have been used extensively for site response and soil-structure interaction studies. Seed and Idriss (1970) used the unmodified hyperbolic stress-strain relation, such that

$$D = \frac{D_{\max} \gamma/\gamma_T}{1 + \gamma/\gamma_T} \quad (2.58)$$

The maximum damping ratio for clean sands corresponding to very large strains, is evaluated by

$$D_{\max} = D_0 - 1.5 \log_{10} N \quad (2.59)$$

where D_0 ranges from 28 percent for saturated sands to 33 percent for dry sands. Seed and Idriss suggest using an average value for

D_0 of 30 percent and that N be taken as 5 cycles. They also concluded that the main factor affecting the relationship between damping ratio and shear strain is the vertical confining pressure σ'_v . Except near the soil surface, where confining pressures are less than 500 psf, the effect of variations in pressure is small compared with the effect of shear strain. Seed and Idriss suggest an average damping ratio vs. shear strain relationship for an effective vertical stress of 2000 to 3000 psf would appear to be adequate for many practical purposes. Approximate relationships for mean and upper and lower bound damping together with test data for sands are shown in Fig. 2.24

Fig. 2.24 Damping ratios for sands. (Seed and Idriss, 1970)

For saturated cohesive soils, the relationship developed by Hardin and Drnevich is

$$D_{\max} = 31 - (3 + 0.03f)(\sigma'_m)^{1/2} + 1.5f^{1/2} - 1.5 \log N \quad (2.60)$$

where f = frequency of applied load in Hz, and σ'_m = mean principle effective stress in kg/cm^2 . Seed and Idriss (1970) have indicated that the relatively sparse data for damping in saturated clays show large variations. Therefore, they merely plotted average and bounding relationships through the available data and suggest that the average, as shown in Fig. 2.25, may be sufficiently accurate for practical purposes.

Fig. 2.25 Damping ratios for saturated clays.
(Seed and Idriss, 1970)

2.2 Soil Compliance

One of the most obvious effects of founding a structure on a non-rigid medium is that the compliance of the medium will affect the natural frequencies of the structure. This effect is especially significant for a stiff, massive structure founded on a relatively soft soil. A description of soil compliance can be divided into two aspects: determination and description of material properties and use of the properties to determine behavior of a soil structure such as a soil column or an entire half-space.

2.2.1 Material Properties

The term "soil" describes a diverse collection of materials composed of various grain sizes and shapes with various amounts of water. In general, it is a nonlinear, anisotropic, nonhomogeneous, inelastic material. Thus, it is necessary to look at it on a macro level. For seismic evaluation purposes, one of the important macro characteristics of soil is the way in which it propagates waves. The two important wave forms which impart loads on structures during earthquakes are p-waves and s-waves. The wave velocities for these two wave types are determined by the elastic modulus and shear modulus respectively (if isotropic), as well as the mass density of the soil. If one modulus, or wave velocity, is determined, the other one can be determined since they are related via Poisson's ratio.

One of the ways to determine soil moduli is by in-situ wave propagation studies such as seismic refraction surveys. One of the problems that presents itself in such surveys is the effect of water in saturated soils. Since water is essentially incompressible when compared with the compressibility of soils, seismic methods of evaluating propagation of the compression wave in saturated soils may be measuring only the wave transmission in water. On the other hand, there is no structural coupling between the soil structure and contained water for shear wave propagation since the fluid has no shearing stiffness. As pointed out by Richart, Hall, and Woods (1970), field measurements of shear waves in saturated soils determine the shear modulus of the soil structure. Similarly, by use of Poisson's ratio, the elastic modulus of the soil structure may be determined.

The preceding argument, plus the fact that the primary motion induced in a soil layer by earthquakes is simple shear, (See for example, Idriss and Seed, 1968) provides the motivation to concentrate mainly on the shear modulus of soil. As has already been alluded to in the previous section, the studies conducted for determining dynamic properties of soils have concentrated on the shear stress-shear strain behavior of soils.

As shown in Table 2.1, the shear modulus of soil is affected by many parameters. A generalized constitutive relationship which accounted for all of these parameters would be quite complex. In order to simplify the problem to a more tractable one, some of the

lesser important parameters have been removed from consideration. In order to further simplify the problem, the hysteresis loop resulting from cyclic loading of soil is described in terms of the extreme points on the loop and the area enclosed within the loop. These two parameters essentially define the secant modulus and hysteretic damping, respectively. The secant modulus then becomes a function of the shear strain induced in the soil. The hyperbolic relationship proposed by Hardin and Drnevich (1972b) has already been described in the previous section, where the secant modulus is described by the relation

$$G = \frac{G_{max}}{1 + \lambda/\lambda_T} \quad (2.61)$$

Hardin and Drnevich (1972b) express G_{max} for undisturbed cohesive soil, as well as sands, by the expression

$$G_{max} = 14760 \frac{(2.973 - e)^2}{1 + e} (ocr)^a (\sigma'_m)^{1/2} \quad (2.62)$$

where G_{max} = maximum (low strain) shear modulus in psf,

e = void ratio

ocr = overconsolidation ratio

σ'_m = mean principle effective stress in psf

a = a parameter dependent on the plasticity index (PI) as shown in Table 2.2.

Table 2.2 Value of parameter a

<u>PI</u>	<u>a</u>
0	0
20	0.18
40	0.30
60	0.41
80	0.48
>100	0.50

Seed and Idriss (1970) have presented a different formulation for shear modulus for sands which relates shear modulus and confining pressure by the equation

$$G = 1000K_2 (\sigma'_m)^{1/2} \quad (2.63)$$

where G and σ'_m are in psf.

In this equation, the influence of void ratio (or relative density) and strain amplitude are contained in the parameter K_2 . The authors found that the relationships plotted from the results of several authors agreed quite well with the curves from Hardin and Drnevich (1972b). Figure 2.26 shows the typical scatter of data. When the shear modulus is normalized with respect to G_{max} , in this case the modulus at a strain of 10^{-4} percent, the results fall within a fairly narrow band as shown in Fig. 2.27.

Fig. 2.26 Shear moduli of sands at relative density of about 40% (Seed and Idriss, 1970)

Fig. 2.27 Variation of shear modulus with shear strain for sands (Seed and Idriss, 1970)

As pointed out by Seed and Idriss (1970), the shear moduli of saturated clays are very difficult to determine accurately since clay modulus values are very sensitive to strain amplitude and sample disturbance. While in-situ measurements essentially eliminate the problems caused by sample disturbance, field techniques have been effectively limited to small strain levels. The authors maintain that Eq. 2.62 does not always provide reasonable values for shear moduli for in-situ conditions. Therefore, they propose normalizing shear modulus, G , with respect to undrained shear strength, S_u , and expressing the relationship G/S_u as a function of shear strain. To construct the plot in Fig. 2.28, much of the data was adjusted by a factor of 2.5 to account for sample disturbance. This was considered a reasonable average correction factor to apply to moduli obtained from unconsolidated-undrained laboratory tests. There is considerable scatter in the test data shown in Fig. 2.28; however, most of the data points fall within $\pm 50\%$ of the average values (the range within the dashed lines).

Because of the difficulty in correlating laboratory test results to in-situ values of shear modulus, the procedure frequently employed is to measure the small strain shear modulus in-situ. This is usually accomplished by shear wave velocity studies. A description of techniques for measuring dynamic soil properties can be found in Woods (1978). The next step is to adjust the modulus to coincide with the value at the expected strain as shown, for example, in Figs. 2.27 and 2.29. These two figures have been

Fig. 2.28 In-situ shear moduli for saturated clays.
(Seed and Idriss, 1970)

Fig. 2.29 Typical reduction of shear modulus with shear strain
for saturated clays. (Seed and Idriss, 1970.)

extensively employed in analyses for studying the effects of both local site amplification and soil-structure interaction. However, more recent studies have been published which come to different conclusions regarding the variations in shear modulus with strain. Arango, Moriwaki, and Brown (1978) have reported results from large strain in-situ tests. Typical results are shown in Fig. 2.30 which illustrates that Fig. 2.29 may represent a nearly lower bound for in-situ properties of clay. Further evidence that the modulus

Fig. 2.30 Shear modulus reduction with strain.
(Arango, et.al., 1978)

reduction with strain shown in Fig. 2.29 may be too great was presented by Stokoe and Lodde (1978). Their test results are shown in Fig. 2.31.

It is apparent that there is a great deal of uncertainty in the determination of soil modulus and its dependency on strain. The correlation between laboratory tests and in-situ properties is also suspect. This is especially true for shear modulus determined by cyclic triaxial compression tests. In fact, Woods (1978), states that there is little justification for using the cyclic triaxial compression test to obtain dynamic shear modulus.

Fig. 2.31 Variation in normalized shear modulus with shearing strain amplitude for San Francisco Bay mud. (Stokoe and Lodde, 1978)

As previously mentioned, the simplest formulation for the behavior of soil is that of a linear, isotropic, elastic medium. In

order to calculate the compliance of such a medium, a determination of Poisson's ratio is necessary. It is possible to calculate Poisson's ratio from measured values of compression wave and shear-wave velocities. These computations involve small differences of relatively large numbers and are subject to the uncertainty in measuring the compression wave velocities discussed earlier. Thus, significant errors are possible. However, Richart, Hall and Woods (1970) report that Poisson's ratio varies from about 0.25 to 0.35 for cohesionless soils and from about 0.35 to 0.45 for competent cohesive soils. They recommend the following values for Poisson's ratio: 0.33 for cohesionless soils and 0.40 for cohesive soils.

More recent work regarding dynamic Poisson's ratio for soils has been published by several Japanese authors, Ohsaki and Iwasaki (1973), Hara (1973), Yokota and Konno (1980). Some of their primary results are shown in Figs. 2.32 to 2.34. Their results suggest that for cohesive soil, Poisson's ratio is independent of shear strain and essentially independent of shear modulus. However, for saturated soft clay, Poisson's ratio approaches that of water, i.e., $\nu=0.5$. For cohesionless soil, Poisson's ratio appears dependent on shear modulus or, at least, is affected by some of the same parameters that affect shear modulus such as confining pressure and shear strain magnitude. However, there appears to be some discrepancy between the absolute magnitude as shown in Fig. 2.32 which was compiled by comparing shear wave and compression wave velocities measured in the field and Fig. 2.34 which shows results from laboratory triaxial compression tests.

Fig. 2.32 Dynamic Poisson's ratios vs shear moduli (Ohsaki and Iwasaki, 1973)

Fig. 2.33 Poisson's ratio of clay vs shear strain and frequency (Hara, 1973)

Fig. 2.34 Confining pressure, water content and Poisson's ratio (Yokota and Konno, 1980)

2.2.2 Application of Dynamic Soil Properties

Since there is such great uncertainty in the in-situ properties of soil, there is considerable motivation to avoid complicated constitutive relationships to describe soil behavior and to use instead a simplified approach. By far the most commonly used technique in soil dynamic analysis is the concept of an equivalent linear shear modulus. For the case of machine vibrations, the strain levels are generally small enough such that the initial tangent modulus may be satisfactorily employed. This method is not satisfactory, however, for strain levels induced by earthquakes. Some of the early applications of an equivalent linear modulus for earthquakes were site amplification studies using the SHAKE computer code (Schnabel, Lysmer, Seed, 1972). The concept of using an equivalent linear shear modulus to characterize a non-linear soil was discussed in some detail in section 2.1.2. In the SHAKE program, the modulus, and accompanying material damping, are determined by iteration. As stated by Seed and Idriss (1969), the response can be determined with sufficient accuracy provided that the equivalent moduli and damping factors are chosen to correspond to the "average" strain developed in each particular layer of a soil deposit. The iterative procedure can be briefly described in the following manner.

In a site response analysis, the equivalent linear method uses linear analysis techniques to analyze the response of a soil deposit

which has been modeled in one or two dimensions. In a one dimensional model, each soil layer is assumed to be homogeneous and is modeled with discrete vertical elements to represent each layer. A linear analysis is performed using estimated soil properties in each layer of the system which yields a time history of shear strain in each layer. The "average" or "effective" shear strain amplitude in each layer is usually taken as 65% of the maximum shear strain or as the RMS value of the shear strain time history (Chen, 1980). Using this average strain amplitude, a refined estimate of shear modulus and damping are obtained from relationships such as those shown in Figs. 2.24, 2.25, 2.27 and 2.29. Then the process is repeated with the new properties using another linear analysis. This process continues until the properties from two consecutive analyses differ by less than a specified tolerance, for example 5 percent. The results of the last iteration are used as the approximate solution to the nonlinear problem. For a two dimensional problem, the solution is similar except the properties are found by iteration for each soil element.

The equivalent linear approach has been extensively used in practice for both site amplification and soil-structure interaction analyses because of its efficiency and ease of implementation. Some recent analyses have been performed to evaluate the accuracy of this approach as compared to more rigorous nonlinear analysis techniques. Martin and Seed (1982) have used the method of characteristics and direct integration using the cubic inertia

method as a means of evaluating the equivalent linear method used in the SHAKE program. The agreement between the equivalent linear method and the nonlinear methods was good for the distribution of maximum accelerations and dynamic shear stresses for the cases studied. However, there were differences in the frequency content of the computed surface motions as illustrated by the resulting spectral shapes. The equivalent linear method tended to attenuate the motions with frequencies above 5 to 10 Hz. This is a characteristic inherent to the equivalent linear method. Öner, et.al., (1981) found a similar effect in their comparisons with nonlinear analyses. The use of a high-strain modulus in a soil layer reduces the natural frequency of the layer and increases the amount of damping for all frequencies. The substantial effects of hysteresis at low strains can cause filtering out of the small oscillations at high frequency. Conversely, there is practically no energy loss by hysteretic damping in high frequency oscillations in the nonlinear analyses.

The filtering out of high frequencies could have a detrimental effect on the design of stiff structures, such as those found at a nuclear power plant. The deficiencies could be even more pronounced for equipment mounted in the structures. The errors introduced by simplified methods lead to a dilemma on the part of the analyst. He may decide that a more sophisticated model is necessary to avoid the introduction of error. However, sophisticated analyses employing complex models can be quite expensive and time consuming. The

potential benefit to be gained should be weighed in light of these costs. For soil dynamics problems, the potential increase in accuracy gained by using nonlinear analysis techniques may be overshadowed by the uncertainties involved in predicting in-situ soil properties. These uncertainties are quite large even in one dimensional analyses as described above. As stated by Silver (1981), it is meaningless to use poorly measured soil properties in a sophisticated constitutive relationship. Dynamic soil constitutive relationships for three dimensional loading are just starting to be developed (Hardin, 1978), and are as yet unverified. Thus, it seems more productive to use simplified models and conduct multiple analyses employing parameter variations in order to obtain bounding solutions which will encompass the uncertainties. Such procedures have been advocated by many authors, such as Roesset (1980), Kausel (1975) and Newmark, et.al. (1981).

2.3 Input Motion

One constituent necessary for conducting a seismic response of a soil-structure system is the description of the seismic loading. The material properties described above are based on the assumption that the seismic input at a site consists primarily of vertically propagating waves. However, the motion generated by an earthquake source in the earth's crust are composed of many wave types at various angles of incidence. In addition, the presence of a massive structure in the soil can alter the local seismic environment near the structure.

2.3.1 Specification of Site Motion

Earthquake motion originates at a source in the earth's crust and travels to a site through complex mechanisms. When the wave fronts of the basic wave trains, shear and compressional waves, hit the ground surface or soil layer interfaces, Rayleigh waves and Love waves may be generated. Thus, the resulting motion at a site represents a combination of wave types. The relative contributions and arrival times for each wave type depend on epicentral distance, focal depth, and paths that the waves can traverse. The path that the various waves follow can involve multiple reflection, refraction, and dispersion. Very little is known about the exact composition of the resulting site motion. The typical recording at a site captures only three orthogonal translational motions which do not reveal the composition or angle of incidence of the wave forms. Even if such wave forms could be postulated for a given site, practical limitations with current analysis techniques only allow for linear analysis of layered systems (Chen, 1980). So these wave forms are subject to the restriction of equivalent linear analysis, as discussed.

Luco (1980) feels it is necessary to include non-vertically incident waves in the representation of free-field motions. By assuming that the wave field is composed of vertically incident SH- and P-waves, all points on the surface are assumed to experience the same motion. For non-vertically incident waves, the surface motion will exhibit spatial variation. The manifestation of this variation

due to SH waves is a reduction in translational response of a surface footing and an accompanying increase in torsional response; the effects becoming more pronounced at higher frequencies and as incident SH waves approach a *horizontal orientation*. Similarly, non-vertically incident P, SV, and Rayleigh waves result in reduced translational response of a footing while imparting a rocking component. Unfortunately, the application of these principles is difficult since there is so little data available. It would be prudent to recognize that significant rotational components may be imparted to a foundation. If the response of the structure is sensitive to these rotations, some provision should be made to account for them.

To the structural analyst, the exact composition of the resulting motion at the site is of lesser importance than the effects of the total motion. Chen (1980) has shown that

- horizontal motions within depths of interest to engineers are comparable for horizontally and vertically propagating waves
- at frequencies above 2 Hz, the Rayleigh wave contribution to the total motion at a soil site is small
- on soil sites the motions of structures are essentially independent of the nature of the waves which cause the control motion.

These conclusions provide some explanation for the success of a number of engineering applications employing the conventional

assumption of vertical wave propagation. Gazetas and Yegian (1979) confirm these findings.

In current practice, the motion for a given site is specified at a given point. The specified motion is called the control motion and the specified point is called the control point. Choosing the correct control motion and point is one of the crucial aspects of the SSI problem. Currently, parameters used for describing the control motion include the peak acceleration, frequency content, and sometimes duration of the earthquake. However, as discussed, the variation of motion over the depth and width of a foundation can be an important factor.

The determination of temporal and spatial variations of motions within a site is referred to as the site response problem (Seed and Lysmer, 1980). The first step is the determination of the control motion and the control point. According to Seed and Lysmer (1980), the preferable control point is located in the free field, that is, away from the influence of structures, at the ground surface or at an assumed rock outcrop. This is desirable since the data base from which the control motion must be estimated consist mainly of surface recordings of strong motion earthquakes. In addition, the frequency content of motions at depth is influenced by reflections at the free surface. Current design practice for nuclear power plants specifies a broad-band spectrum (Newmark, Blume, and Kapur, 1973) at the ground surface in the free field.

2.3.2 Foundation Motion

Since the motions induced in a structure are primarily influenced by near-surface motions within the depth of embedment of the structure, the next step is to determine the variation of amplitude and frequency content of free-field motions within this depth. In a perfectly elastic half-space, the horizontal amplitude of a vertically propagating shear wave of frequency, f (Hz), with respect to depth is

$$U = U_0 \cos 2\pi \frac{zf}{v_s} \quad (2.64)$$

Thus, the amplitude decreases monotonically to a depth of

$$z = \frac{v_s}{4f} \quad (2.65)$$

at which point the displacement becomes zero. Seed and Lysmer (1980) have shown that a similar phenomenon occurs for Rayleigh waves, inclined shear waves, and layered soil systems such that, in general, horizontal motions at depth z will be deficient in components of frequency

$$f = \frac{\bar{v}_s}{5z} \text{ to } \frac{\bar{v}_s}{4z} \quad (2.66)$$

where $\bar{v}_s$ is the average shear wave velocity above depth z . Thus, for a site which has a smooth response spectrum at the surface, the spectrum at depth z will exhibit a dip at the fixed base natural frequency of the soil column above depth z .

These variations with depth are for the free field in the absence of the influence of a structure. However, even with the structure present, there does seem to be a reduction in horizontal spectral accelerations with depth. Kausel, et.al. (1977 & 1978) have shown that the translation of an embedded massless foundation is similar to the free field translational motion at the foundation level except that the foundation motion did not show the pronounced absence of frequency components described in Eq. 2.66. In addition to the reduction of translation motion, a rotational motion is induced, Figure 2.35. These studies show that the presence of the foundation affects the motion that would otherwise occur in the soil.

Fig. 2.35 Kinematic interaction problem. (Kausel, et.al., 1978)

These effects are referred to as "kinematic interaction" (see, for example, Kausel, et.al., 1978). Whitman (1975) has pointed out that ignoring kinematic interaction will result in a conservative estimate of foundation motion. However, incorporating kinematic interaction may be unconservative if a smoothed standard spectrum is

specified as the control motion. Such smoothed spectra may be slightly deficient at high frequencies and may be further reduced at such frequencies by kinematic interaction. As an alternative, Whitman suggests using the smoothed spectral shape, scaled to an appropriate acceleration, as input to the soil-foundation model. Additional details and alternatives will be discussed in the section on substructure methods. Note that there is no kinematic interaction for a surface footing if the seismic waves are presumed to be vertically propagating only.

Luco (1980) solves for the response of the rigid massless foundation by means of a driving force vector. The problem of determining the driving force vector corresponds to a problem of "scattering" of seismic waves by a rigid, fixed intrusion in an elastic half-space. In principle, the foundation is held fixed under the action of the seismic excitation. The external forces needed to hold the foundation fixed are equal and opposite to the driving forces. If the driving force vector and compliance matrix for the foundation are known, the foundation input displacements can be calculated.

For direct methods of analysis that employ a finite element model of the structure and underlying soil, the motions at the base of the soil column model are needed. The common procedure used to obtain these motions is termed the "deconvolution" process which comprises:

- generation of a time history record compatible with the specified surface control motion
- analysis of the soil column, usually by use of transfer functions in the frequency domain, to obtain the corresponding motion at depth. This model usually employs vertically propagating waves.

In summary, predicting the resulting foundation motion at a site presents a difficult problem for which adequate data is not yet available. However, as stated by Roesset (1980), the following generalizations can be made:

- When considering translational motions, the normal assumption of vertically propagating waves generally produce conservative results, but this assumption may exaggerate the nonlinear soil effects.
- If other wave trains are considered, consistent analyses should not change the results by more than 20% in most cases.
- Rotational effects caused by traveling waves can be accounted for realistically by rather simple means. Application of accidental eccentricity induces torsional excitations and kinematic interaction induces rocking in embedded foundations.
- While research is proceeding in the area of predicting wave trains emanating from a seismic source, it is not ready for application in practice.

3. SSI AND ENERGY DISSIPATION SOLUTION TECHNIQUES

Solution techniques for SSI analysis can be categorized into two broad classifications: *direct methods*, in which both the soil and structure models are analyzed in the same step, and *substructure methods*, in which the analysis is broken into a series of steps. Each method accounts for the elements of SSI differently and, while one may handle one aspect of the problem more precisely than the other, both methods have uncertainties associated with them.

3.1 Substructure Methods

Early efforts in accounting for soil compliance in the study of foundation vibrations involved the representation of the soil as a subspace. Lysmer's analog (Lysmer and Richart, 1966) further reduced representation of the soil contribution to that of a simple oscillator which could be described in terms of its spring and dashpot coefficients. These methods uncouple the soil from the structure supported by it and were frequently applied to foundations which were assumed to be on the surface of an ideal elastic half-space and which were loaded by vibrating machinery. The same type of procedure was adopted when it was deemed essential to consider foundation compliance for earthquake problems as well. However, since the input motion for earthquake problems is transmitted through the soil, it became apparent that the presence

of the structure's foundation could affect the motion. The foundation effects are particularly important for embedded foundations. Thus, to develop a theoretically correct method of accounting for these effects, the substructure method was developed which employs the superposition principle (Kausel, et. al., 1978).

The Basic Superposition Theorem

If the soil-structure system is considered to be a single system, the equation of motion for the total system is

$$m\ddot{x} + C\dot{x}_T + Kx_T = 0 \quad (3.1)$$

where, x = absolute displacement vector

x_T = relative displacement vector (relative to some general ground reference system)

This equation can be broken into two parts:

$$m_1\ddot{x}_1 + C\dot{x}_{r1} + Kx_{r1} = 0 \quad (3.2)$$

$$m\ddot{x}_{r2} + C\dot{x}_{r2} + Kx_{r2} = -m_2\ddot{x}_1 \quad (3.3)$$

where Eq 3.1 is the sum of Eqs 3.2 and 3.3. The quantity m_1 represents the mass of the soil and m_2 the mass of the structure.

The displacements are related by the following:

$$x_1 = x_{r1} + x_g \quad (3.4)$$

$$x = x_1 + x_{r2} \quad (3.5)$$

$$x_r = x_{r1} + x_{r2} \quad (3.6)$$

The solution may be obtained in two steps. In the first step, the soil and massless structure are subjected to the generalized ground motion. The structure experiences a motion, x_{r1} , relative to the ground, which is a result of the presence of a rigid inclusion. This is the solution to Eq 3.2 and is called kinematic interaction. In the second step, the fictitious inertia forces due to the motion computed above are applied to the structure. The effective earthquake force in Eq 3.3, $-m_2 \ddot{x}_1$, is equivalent to a base acceleration on the structure of $\ddot{x}_1$. Note that this is a fictitious support excitation and never actually occurs in the physical system. The results of the second step are the additional motions x_{r2} which result from the inertia of the structure. Thus, step 2 is referred to as inertial interaction. Superposition is pictorially shown in Fig. 3.1. As shown in Eq 3.5, the total relative displacement is the sum of the displacements from each step.

Fig. 3.1. Superposition theorem. (Kausel, et. al., 1978)

Substructure Approaches

A number of formulations have been proposed which utilize superposition to break the total SSI analysis into manageable components. In general, these methods involve dividing the complete system into two substructures which can be analyzed separately. While the definition of the boundary between the substructures may be different for different formulations, the substructures comprise the two main portions, the soil system and the structural system.

The method employed by Luco (1980) is composed of three steps:

- (1) Determine the motion of the massless foundation subjected to the prescribed seismic motion. This represents the solution to Eq. 3.2.
- (2) Determine the subgrade stiffness, or impedance functions (which may be viewed as frequency-dependent soil "springs").
- (3) Analyze the coupled soil-structure system subjected to the motion computed in 1.).

This methodology is schematically represented in Fig. 3.2. In general, the input motion can comprise any combination of wave forms at any angle of incidence. Luco's method uses a representation of the structure which requires what may be thought of as a fourth step, analyzing the structure separately using a fixed base model. The modal properties from this analysis are then used to represent the structure in the coupled SSI analysis. In addition, the seismic motion employed by Luco is specified by the free-field surface motion.

Fig. 3.2. Schematic representation of the elements of the substructure approach.

The three step method described by Kausel, et.al. (1978) is illustrated in Fig. 3.3. When using either method, it is usually assumed that the foundation is rigid such that the displacements resulting from the first step (Eq. 3.2) will correspond to rigid body motion. Since the structure does not deform in this step, it can be replaced by an ideal rigid, massless foundation. In addition, the stiffness matrix may be replaced by a set of frequency dependent "springs" and "dashpots". With this assumption the solution can be obtained by performing the three steps described above. However, in the method shown in Fig. 3.3, the full

structural model is included in the third step in which the detailed model is supported on the springs calculated in step 2. This total system is then subjected at the base of those springs to the motions computed in step 1.

Fig. 3.3. The three-step solution. (Kausel, et.al., 1978)

This general methodology has been used for many analyses employing SSI for the study of nuclear power plant structures (Subudhi, et.al., 1980). However, for the majority of these structures, the foundation has been assumed to be on the surface of the soil. For a surface footing and vertically propagating waves in a horizontally layered system, there is no kinematic interaction and the input to the base of the springs is equal to the free-field

surface motion. However, for an embedded foundation, the equivalent motion is composed of both translational and rotational components.

The only approximation involved in the three steps involves the deformability of the foundation. If the foundation were rigid, the solution using the substructure approach would be identical to that from the direct approach (using consistent procedures). In fact, the limitation of a rigid foundation is not essential for application of the three-step method. Luco (1980) points out that application of kinematic boundary conditions along the soil-foundation interface can model the expected deformation of the foundation.

An alternative formulation of the substructure technique was presented by Gutierrez and Chopra (1978). The complete system is separated into two subsystems which correspond to, first, the structure and its foundation and, second, to the soil region after the cavity for the foundation has been excavated. Compatibility is imposed along the common boundary which results in equal and opposite external loads on each substructure. This formulation is addressed to the case of flexible foundations and uses displacement patterns to describe the displacement of the soil-structure interface. However, as pointed out by Luco (1980), this approach does have some shortcomings. It assumes the foundation motion is available but does not include a method to calculate modified motion for embedded foundations. In the calculation of elements of the impedance matrix, unit displacements of given nodes are imposed and a linear variation of displacement to the surrounding fixed nodes is

assumed. This leads to errors in the calculated impedances. Representing the displacement of the soil-structure interface relative to the input motion by use of a few displacement patterns may not be adequate. While the total displacement of the interface may be adequately described in this fashion for stiff foundations, it may not be adequate for relative displacement between nodes of a flexible foundation.

Several approaches (Roesset, 1980; Whitman and Richart, 1967; Richart, Hall, and Woods, 1970) have been developed for applying the substructure approach to the SSI problem. The simplest and most commonly applied approach to date has been the use of soil springs and dashpot coefficients based on an elastic half-space. Other approaches have evolved to account for embedment, layered sites, nonvertically incident waves, and flexible foundations. In the case of embedment, some means of determining kinematic interaction must be employed. This means could involve a finite element model of the foundation and soil, a continuum mechanics approach, or application of simplified rules such as those suggested in Kausel, et.al. (1978).

Soil Properties for Use in Substructure Methods

Since all of the substructure approaches utilize superposition, they are only applicable to linear problems. As discussed in Chapter 2, one way to approximate the nonlinear behavior of soils was to employ an equivalent linear technique for which the soil shear modulus at a characteristic strain is used. For a site study,

i.e., determination of the free field motion at a site without the presence of the structure, this shear modulus is determined by iteration as described earlier. The nonlinear strains experienced by the soil during this wave propagation are referred to as primary nonlinearities. When a structure is founded at the site, additional localized nonlinear soil strains will be induced by the SSI effects. These are referred to as secondary nonlinearities.

Kausel, et.al. (1976 and 1975a) conducted studies to determine the relative effects of primary and secondary nonlinearities. Figure 3.4 shows the ratio of the iterated shear moduli from the analysis of a 2-d finite element model, to the iterated moduli from a 1-d site analysis. In other words it shows the ratios of the final moduli including primary and secondary nonlinearities to those

Fig. 3.4. Relative reduction of shear modulus with secondary nonlinearities (Kausel, 1975a)

resulting from primary nonlinearity alone. The shaded areas illustrate these ratios for each element. Near the structure, secondary nonlinearities can cause significant reductions in shear modulus. Kausel, Roesset, and Christian (1976) suggest that the region influenced is small and the effects of the decreased moduli are somewhat offset by an increase in material damping. As a result, the effects of secondary nonlinearities are small within the structure.

Figure 3.5 shows in-structure response spectra at two locations for analyses both with and without secondary nonlinearities. The results are quite close for the two cases. The results of this study indicate that it is reasonable to neglect secondary nonlinearities. Thus, it has been recommended that the equivalent linear shear modulus and level of material damping for use in calculating impedances for SSI analysis be determined from a 1-d site analysis (such as with the SHAKE code, Schnabel, Lysmer, Seed, 1972). This modulus would account only for primary nonlinearities but should be sufficient considering the uncertainties inherent in

Fig. 3.5. Comparison of response spectra considering primary and secondary nonlinearities (1% damping) (Kausel, 1975a).

determining the in-situ soil properties. It is suggested that the savings accrued by employing this simplified technique would be put to better use by covering a range of values for soil properties and bounding the resulting responses.

3.1.1 Frequency Independent Half-space Methods

One of the developments which allowed practical analysis of footings subjected to dynamic loads was the ability to represent the footing-soil system by a simple damped oscillator. This was accomplished by Lysmer (Lysmer and Richart, 1966) for vertical footing oscillations and the resulting simple oscillator is often referred to as Lysmer's analog. In Chapter 2, it was shown that the energy dissipation caused by wave propagation from the surface of a uniformly loaded half-space could be modeled by an equivalent viscous damper. In this chapter, this concept will be extended to a system composed of a rigid circular footing resting on the surface of an elastic half-space. The effects of deviations from this idealized case will be discussed and applications presented.

Development of Surface Formulations

Using the same type of development as illustrated in Ch. 2, Lysmer first considers the case where the mass is zero. He defines the dimensionless frequency ratio

$$a_0 = \frac{\omega r_0}{v_s} \quad (3.7)$$

It is interesting to note that a_0 represents the number of cycles in a shear wave traveling from the center to the edge of the footing within the time of propagation for this distance, r_0/v_s .

As before, the expression for the displacement of the massless footing is

$$\delta = \frac{P_0}{k} F(a_0, \nu) e^{i\omega t} \quad (3.8)$$

where the static spring constant is

$$k = \frac{4Gr_0}{1-\nu} \quad (3.9)$$

The analytical determination of the complex function F involves the solution of the wave equation for an elastic solid with a mixed boundary condition. Since the footing is considered rigid, the boundary condition under the footing consists of a uniform displacement. For the region outside of the footing, there is a zero stress condition on the surface of the half space. Lysmer and Richart (1966) present results from an approximate solution in which the footing is divided into discrete concentric rings in order to simulate the correct pressure distribution under the footing. In addition, it is assumed that only normal stresses are transferred at the interface between the footing and soil.

The solution for this half-space model, for Poisson's ratio equal to 1/3, is shown in Fig. 3.6. Lysmer states that, for practical purposes, the variation of F with Poisson's ratio is insignificant. As evidence of this insensitivity, he compares low frequency solutions from Bycroft (1956) for various Poisson's ratios, Fig. 3.7.

It should be noted that this displacement function is similar to that for the simple damped oscillator as shown in Fig. 2.6. In view of this comparison, Lysmer introduces an analog of the type shown in Fig. 2.4 for the half-space model. The spring and dashpot coefficient for the analog become

$$K = \frac{F_1}{F_1^2 + F_2^2} \quad k = k_1 k \quad (3.10)$$

$$C = \frac{-F_2/a_0}{F_1^2 + F_2^2} \frac{k r_0}{v_s} = c_1 \frac{k r_0}{v_s}$$

The factors c_1 and k_1 are dimensionless functions of frequency and Poisson's ratio as shown in Fig. 3.8.

As with earlier derivations, Lysmer then adds the inertia term to the equation of motion, the steady state solution for which is

$$\delta = \frac{Q_0}{k} \bar{F} e^{i\omega t} \quad (3.11)$$

The function $\bar{F}$ then becomes

$$\bar{F} = \frac{1}{(k_1 - B\omega^2) + ic_1\omega} \quad (3.12)$$

where B, the mass ratio is defined by

$$B = \frac{m}{k} \left(\frac{v_s}{r_0} \right)^2 = \frac{1 - \nu}{4} \frac{m}{\rho r_0^3} \quad (3.13)$$

The magnification factor M is the magnitude of $\bar{F}$ and is plotted for various mass ratios in Fig. 3.9. This figure shows the response of the analog system incorporating frequency dependent stiffness and damping terms which represents the solution according to half-space theory. Lysmer represents that this solution, as represented by the displacement function $\bar{F}$, is practically independent of Poisson's ratio and thus, can be used for all values of ν .

Application of elastic half-space theories to practical problems has shown that only limited agreement can be obtained between observed and calculated amplitudes. The biggest unknowns in these problems are the properties of the soil underlying the footing. Lysmer makes the point that the mathematical difficulties involved in the use of the half-space model are not justified for such an ill-defined system. Thus, he suggests further simplification of the solution such that the stiffness and damping coefficients are assumed to be constant, i.e., frequency independent.

Fig. 3.6. Displacement function F for vertical vibration of a weightless rigid circular disk ($m = 0, \nu = 1/3$) (Lysmer and Richart, 1966).

Fig. 3.7. Variation of displacement function components with Poisson's Ratio. (Lysmer and Richart, 1966).

Fig. 3.8. Variation of functions k_1 and c_1 with (a) Frequency and (b) Poisson's ratio (Lysmer and Richart, 1966).

Fig. 3.9. Steady-state spectra for footing-soil system. (Lysmer and Richart, 1966)

Since c_1 and k_1 are bounded, the limiting values for $\bar{F}$ can be found by looking at very low and very high frequencies, which,

for nonzero B yields

$$\bar{F} = \begin{cases} 1/k_1 \approx 1, & \text{for } a_0 \ll 1 \\ -1/Ba_0^2, & \text{for } a_0 \gg 1 \end{cases} \quad (3.14)$$

The latter results in a displacement

$$\delta = -\frac{Q_0}{k} \left(\frac{1}{Ba_0^2} \right) e^{i\omega t} = -\frac{Q_0}{m\omega^2} e^{i\omega t} \quad (3.15)$$

which shows that at high frequency the response of the footing is independent of the soil properties but is governed by the footing inertia.

To have agreement for the static case, $k_1 = 1$ must be chosen.

For low frequencies $\bar{F}$ is independent of c_1 . Since the influence of c_1 is strongest at resonant peaks, that is in the range $0.3 < a_0 < 0.8$, it is deemed reasonable to choose c_1 as an average value over this range. Lysmer chooses 0.85 as the average based on values plotted in Fig. 3.8b. Thus, the simplified analog consists of a simple damped oscillator with the following properties:

m = mass of footing

$$C = 0.85 \frac{kr_0}{v_s} = \frac{3.4}{1-\nu} r_0^2 \sqrt{G\rho} \quad (3.16)$$

$$K = k = \frac{4Gr_c}{1-\nu} = \text{static spring constant}$$

The results from the simplified analog are quite close to those predicted by half-space theory as shown in Fig. 3.9.

Procedures similar to those of Lysmer and Richart (1966) have been used to derive properties for simplified analogs to model footing vibrations in directions other than vertical. These derivations are summarized in Richart, Hall, and Woods (1970) and will be briefly discussed herein. In all cases, the stiffness term represents the static stiffness of a rigid circular footing for the same reasons discussed above. The damping term is chosen to give the best approximate solution over the frequency range where resonant peaks occur.

For torsional vibration, it is assumed that the displacement varies linearly from the center of the footing to the edge in accordance with the rigid assumption. This results in infinite shearing stresses predicted at the periphery of the footing which would not physically occur. Thus, the predicted frequency should be higher and amplitude lower than those occurring for real footings. The solution for this case differs from the case for vertical oscillation in two significant areas: this oscillation is not influenced by Poisson's ratio and energy is dissipated by propagation of shear waves only. As a result, the geometric damping contributed by torsional oscillation is smaller than that for vertical oscillation.

Hall (1967) applied Lysmer's methodology to rocking and horizontal translational (sliding) motions. He found that the stiffness and damping terms for horizontal translation were nearly independent of both Poisson's ratio and frequency for values of a_0 up to 1.5. However, for rocking oscillations he found more

significant frequency dependence. Thus, in order to force the resonant frequency to agree with the value predicted from half-space theory, he developed a factor to adjust the mass moment of inertia, I_ψ , or inertia ratio, B_ψ .

$$B_{\psi \text{eff}} = n_\psi B_\psi \quad (3.17)$$

$$\text{where } B_\psi = \frac{3(1-\nu) I_\psi}{8 \rho r_o^5} \quad (3.18)$$

$$\text{and } n_\psi = \frac{B_\psi}{(-0.1125 + \sqrt{0.01265 + 1.0248 B_\psi})^2} \quad (3.19)$$

Table 3.1 Properties of analog systems for rigid circular footings.

<u>Motion</u>	<u>Spring Constant</u>	<u>Damping Coefficient</u>
Vert	$K_V = \frac{4Gr_o}{1-\nu}$	$C_V = \frac{3.4r_o^2}{(1-\nu)} \sqrt{\rho G} = 0.85K_V r_o \sqrt{\rho/G}$
Tors	$K_t = \frac{16}{3} Gr_o^3$	$C_t = \frac{\sqrt{K_t I_t}}{1+2I_t/\rho r_o^5}$
Horz	$K_h = \frac{32(1-\nu)Gr_o}{7-8\nu}$	$C_h = \frac{18.4(1-\nu)}{7-8\nu} r_o^2 \sqrt{\rho G} = 0.575K_h r_o \sqrt{\rho/G}$
Rock	$K_I = \frac{8Gr_o^3}{3(1-\nu)}$	$C_I = \frac{0.8r_o^4 \sqrt{\rho G}}{(1-\nu)(1+B_\psi)} = \frac{0.30}{1+B_\psi} K_I r_o \sqrt{\rho/G}$

The four solutions discussed above represent all six degrees of freedom for the circular footing. The geometric damping terms are equivalent frequency-independent, viscous damping coefficients (dashpot coefficients) which represent the energy dissipated by waves propagating away from the footing into the elastic medium. The equivalent spring constants and geometric damping coefficients for the four types of motion are summarized in Table 3.1. Since these parameters were developed for isolated footings which were modeled as single-degree-of-freedom systems in each vibration direction, it was convenient to express damping in terms of damping ratio. Richart, Hall, and Woods (1970) express the damping ratios in terms of mass ratios as shown in Table 3.2.

Table 3.2 Equivalent Damping Ratios for Rigid Circular Footings

Mode of Vibration	Mass (or Inertia) Ratio	Damping Ratio ξ
Vertical	$B_v = \frac{(1-\nu)}{4} \frac{m}{\rho r_o^3}$	$\xi_v = \frac{0.425}{\sqrt{B_v}}$
Horizontal	$B_h = \frac{(7-8\nu)}{32(1-\nu)} \frac{m}{\rho r_o^3}$	$\xi_h = \frac{0.288}{\sqrt{B_h}}$
Rocking	$B_\psi = \frac{3(1-\nu)}{8} \frac{I_\psi}{\rho r_o^5}$	$\xi_r = \frac{0.15}{(1+B_\psi) \sqrt{B_\psi}}$
Torsional	$B_t = \frac{I_t}{\rho r_o^5}$	$\xi_t = \frac{0.50}{1+2B_t}$

Figure 3.10 illustrates the relative energy loss for each mode of oscillation for a footing on an undamped half-space. The values of ξ in this figure were calculated by equating the peak amplitude from the half-space solution to that predicted by the simple analog and then solving for ξ . It can be seen that significantly more energy is lost in translational modes than is the case for

Fig. 3.10. Equivalent geometric damping ratios vs mass ratio for oscillation of a rigid circular footing on an elastic half-space. (Richart, Hall, and Woods, 1970)

rotational modes. In fact, for $B_\psi > 1$, material damping becomes more important than geometric damping for rocking oscillation.

Equivalent stiffness and damping coefficients have also been developed for rectangular foundations (see, for example, Whitman and Richart, 1967). The static stiffnesses for vertical and rocking degrees of freedom are found from elasticity solutions. The horizontal spring constant was obtained by assuming a uniform distribution of shear stress over the contact area and computing the average horizontal displacement. As an approximation, the torsional stiffness was obtained by using an equivalent circular footing

Table 3.3. Properties of rectangular footings.

<u>Motion</u>	<u>Spring Constant</u>	<u>Equivalent Damping Coefficient</u> ¹
Vertical	$K_V = \frac{G}{1-\nu} \beta_V \sqrt{BL}$	$R_V = \sqrt{BL/\pi}$
Horizontal	$K_H = 2(1+\nu)G\beta_H \sqrt{BL}$	$R_H = \sqrt{BL/\pi}$
Rocking	$K_R = \frac{G}{1-\nu} \beta_R B^2 L$	$R_R = \sqrt[4]{B^3 L / 3\pi}$
Torsion	Use Table 3.1 with $R_t = \sqrt[4]{BL(B^2 + L^2)/6\pi}$	

¹The equivalent radius, R_H , R_R or R_V is substituted for r_0 in Table 3.1 for computing radiation damping C_H , C_R or C_V , respectively.

having the same polar moment of inertia as the contact area of the rectangular footing. The equivalent properties for rectangular footings are shown in Table 3.3. The factors β are obtained from Fig. 3.11.

Fig. 3.11. Constants β_h , β_r and β_v for Rectangular Bases (Whitman and Richart, 1967).

The stiffnesses thus far presented have been static values and, as such, are frequency independent. In an attempt to account for the frequency dependency of the stiffness, a fictitious mass, referred to as "effective mass" (Whitman and Richart, 1967), is sometimes added to the footing mass. In other words, instead of decreasing K , the mass is increased so that the system has the correct resonant frequency. According to Whitman and Richart (1967) the effective mass is small and it usually is best to ignore it completely.

Modifications to Account for Layering and Embedment

As might be inferred by intuition the static stiffnesses for an embedded footing are larger than those for a surface footing. One

might also expect that proximity of a stiff layer, such as bedrock, would similarly affect the static foundation stiffnesses. In addition, other complexities are introduced into the response of footings when they aren't on the surface of a halfspace. As discussed in Section 2.3.2, embedment causes an alteration of the foundation motion from that of the free field. Other parameters affected by layering and embedment include damping and frequency dependency of the dynamic impedances.

Since frequency-independent half-space methods have found such large use in seismic analysis of large structures, Kausel, Whitman, Morray, and Elsabee, (1978), sought to derive simplified rules whereby the effects of embedment and layering could be incorporated. First, they evaluated the effect on the foundation motion. To do this, they constructed finite element models of a soil system incorporating rigid massless foundations which were cylindrical and had various embedment depths. A typical example is shown in Fig. 3.12. The plots show transfer functions which indicate the foundation translation and rotation amplitudes with respect to the free field surface motions. The approximate transfer function for horizontal translation is given by

$$F_U(n) = \begin{cases} \cos\left(\frac{\pi}{2} \frac{f}{f_n}\right) & \text{for } f \leq 0.7f_n \\ 0.454 & \text{for } f > 0.7f_n \end{cases} \quad (3.20)$$

Fig. 3.12 Absolute values of transfer functions for the motion of a cylindrical massless foundation; D is the soil material damping, and γ is the unit weight. (Kausel, et.al., 1973)

where f_n , as described in Eq. 2.66, is $v_s/4E$ and E is the depth of embedment. The transfer function for rotation is given by

$$F_{\phi}(\Omega) = \begin{cases} \frac{0.257}{R} \left(1 - \cos \left(\frac{\pi}{2} \frac{f}{f_n} \right) \right) & \text{for } f \leq f_n \\ \frac{0.257}{R} & \text{for } f > f_n \end{cases} \quad (3.21)$$

The studies conducted for determining these functions involved sites which had a soil stratum of finite depth overlying bedrock. It appears that layering affects the frequency dependence of the transfer function. Luco (1980) reports results of Day using similar finite element analyses as in Kausel, et.al. (1978), but with an elastic half-space model. The results of Day's study are shown in Fig. 3.13. The approximate transfer functions from Eqs. 3.20 and 3.21 for $h/a = 1$ are superposed and show fairly good agreement. The normalized displacements of Fig. 3.13 are essentially transfer functions between free-field surface motions and foundation motions.

These approximate transfer functions are of direct use if one conducts analyses in the frequency domain. However, they are not directly applicable for scaling response spectra. Roesset (1980b) suggests that until spectra ratios are obtained, Eqs. 3.20 and 3.21 be used as spectra ratios. It would seem best to use these ratios for zero-damped spectra since they bear the greatest similarity to the Fourier spectra for the surface motions. For broad-band spectra, such as those described in the Nuclear Regulatory Commission's R.G. 1.60, the scaled spectra at the foundation could then be adjusted for different damping. For other spectra, rules for adjusting spectra for different damping such as those described by McQuire (1977) could be employed.

Fig. 3.13 Input motion amplitudes for cylindrical foundations subjected to vertically incident, plane S waves: (a) rocking; and (b) horizontal translation, h/a is the ratio of depth to radius of the cylinder. (Luco, 1980)

Figure 3.14 illustrates the level of accuracy for the approximate method for the configuration shown in Fig. 3.12.

Several conclusions can be drawn from this figure:

- The approximate procedure does a reasonably good job of predicting the effects of kinematic interaction.

- The free field motion, at the foundation level, is not a good description of the foundation motion.
- There is a significant reduction in the effective horizontal motion of the foundation as a result of embedment.

Fig. 3.14 Translation of massless foundation, response spectra, 1% oscillator damping. (Kausel, et.al., 1978)

The last point may be misleading if considered on its own. The rotational component induced on the foundation could make up for the reduction in translation, depending on the structure. Roesset (1980a) suggests that if the base rotation is neglected, it may not be appropriate to consider any reduction in the horizontal motion. It should be noted that the rotational component is sensitive to the

lateral soil conditions and the flexibility of the side walls of the foundation. As the side walls become more flexible, the rotation decreases, and at the extreme case of no side walls, the rotation changes sign.

The next major effect of embedment and layering is on the values of static stiffness and damping coefficients. The effect of embedment on the static stiffness coefficient for vertical deflection was presented in Richart, Hall, and Woods (1970) as shown in Fig. 3.15. This figure shows that as the embedment ratio increases, the static stiffness increases. Other authors have presented ratios for the increase in static stiffness due to embedment, some of which are reproduced herein. Luco, et.al., (1975), cite work by Novak and Sachs (1973) as shown in Table 3.4. Note that these values are for the case of perfect adhesion between the foundation side wall and the soil. Since, after several oscillations, a gap may be created at this interface it might be prudent to use the lower values in Table 3.4.

Luco, et.al., (1975) also list ratios of equivalent damping coefficients as shown in Table 3.5, whereas Kausel, et.al., (1978) use the same ratios for both stiffness and damping.

Fig. 3.15 Effect of depth of embedment on the spring constant for vertically loaded circular footings (from Kaldjian, 1969, via Richart, Hall, Woods, 1970).

Table 3.4

Ratio of Embedded to Surface Stiffnesses
for Circular Footings with Embedment Ratio E/r_0

	Novak & Sachs (1973)	Kausel, et.al. (1978)	Johnson, ^a et.al. (1975)
Horiz	$1 + .80 E/r_0$ ^b	$1 + .67 E/r_0$ ^b	$1 + 1.3 E/r_0$ ^b
Rocking	$1 + E/r_0 + .55 (E/r_0)^3$	$1 + 2 E/r_0$	$1 + 3 E/r_0$
Vert	$1 + .36 E/r_0$	-----	$1 + .5 E/r_0$
Tors	$1 + 2.38 E/r_0$	-----	$1 + 3.8 E/r_0$

a) Read from graphs at $E/r_0 = 1$

b) These authors define the horizontal static stiffness for the surface case as $8Gr_0/(2-v)$.

Table 3.5

Ratio of Embedded to Surface Damping Coefficients

Horizontal	$1 + 3.36 E/r_0$
Rocking	$1 + 4.2 E/r_0 + 8.2 (E/r_0)^3$
Vertical	$1 + 0.99 E/r_0$
Torsional	$1 + 7.7 E/r_0$

The values in Tables 3.4 and 3.5 are based on the assumption that the ratios of surface to embedded impedances are constant over the frequency range. Kausel, et.al., (1978) suggest that constant ratios are fairly accurate for stiffnesses. However, only the lower ranges of dimensionless frequency were considered.

For coupling terms, which Johnson, et.al., (1975) state become more important for embedded than for surface foundations, the following are proposed:

$$K_{h\psi} = K_{hh} r_0 [2/5 (E/r_0) - 0.03] \quad \text{Kausel, et.al.}$$

$$K_{h\psi} = 1.2 E r_0^2 \text{ (for } E/r_0 = 1) \quad \text{Johnson, et.al.}$$

$$K_{h\psi} = K_{hh} r_0 [0.18 (E/r_0)^2] \quad \text{Luco, et.al.} \quad (3.22)$$

$$C_{h\psi} = C_{hh} [1.68 (E/r_0)^3] \quad \text{Luco, et.al.}$$

A greatly complicating factor in the analysis of soil-structure systems is the presence of a very stiff stratum overlain with a shallow stratum of soil. A first approximation to this shallow stratum over bedrock is presented in Richart, Hall, and Woods (1970) for the cases of torsional and vertical oscillations. In this method, bedrock is treated as perfectly rigid. The general trends noted are that stiffness increases and radiation damping decreases as the stratum becomes shallower. For the vertical case, the static stiffness becomes twice that of the half-space value when the depth of the stratum is reduced to one radius. It is also recognized that natural modes of the stratum will occur at various frequencies and, thus, the effective stiffness and radiation damping will vary considerably with frequency.

The problem becomes more complex when foundation embedment and a shallow stratum are combined. Then, at least three ratios are needed to describe the problem, the depth of embedment to radius, E/r_o , as given previously in Tables 3.4 and 3.5, radius to stratum depth, r_o/H ; and embedment to stratum depth, E/H . It is implied by the use of these factors that embedment depth and stratum depth are coupled quantities. Figure 3.16 shows the static stiffnesses for a circular footing embedded in a shallow stratum as calculated from finite element analyses by Johnson, Christiano and Epstein (1975). Kausel, et.al. (1978) recognize the frequency dependence of the damping terms but assume the stiffness terms have the same frequency relationship as the half-space values. The static horizontal and rocking stiffness coefficients are given in Kausel by

$$K_{hh} = \frac{8Gr}{2-v} \left(1 + \frac{1}{2} \frac{r}{H}\right) \left(1 + \frac{2}{3} \frac{E}{I_0}\right) \left(1 + \frac{5}{4} \frac{E}{H}\right) \quad (3.23)$$

$$K_{\psi\psi} = \frac{8Gr}{3(1-v)} \left(1 + \frac{1}{6} \frac{r}{H}\right) \left(1 + 2 \frac{E}{I_0}\right) \left(1 + 0.71 \frac{E}{H}\right)$$

Fig 3.16 - Stiffness Coefficients for Circular Footing
(From Johnson, et.al., 1975)

For damping, Kausel, et.al. gives the following:

$$C_h = \begin{cases} 0.65D (a_0/a_{01})/(1-(1-2D)(a_0/a_{01})^2) & \text{for } a_0 \leq a_{01} \\ \text{Half space solution} & \text{for } a_0 > a_{01} \end{cases} \quad (3.24)$$

$$C_\psi = \begin{cases} 0.5D (a_0/a_{02})/(1-(1-2D)(a_0/a_{02})^2) & \text{for } a_0 \leq a_{02} \\ \text{Half space solution} & \text{for } a_0 > a_{02} \end{cases}$$

where $a_{01} = (\pi/2)(r_0/H)$
 $a_{02} = v_p/v_s$

The damping factors shown are normalized in the form of Eq. 2.56, and a_{01} and a_{02} are non-dimensional fundamental shear and dilatation frequencies of the stratum. Figure 3.17 illustrates the fit of these approximate

Fig. 3.17 - Dynamic stiffness coefficients, $\nu = 1/3$, $D = 0.05$
 (From Kausel, et.al., 1978)

expressions to the frequency-dependent solutions for the cases shown. The damping coefficients are corrected only for layering in this case with no provision for embedment. In addition, no soil damping has been considered for this formulation which is reflected in the damping coefficients at frequencies below the fundamental frequencies of the stratum. Theoretically, in a medium with no material damping, no energy is dissipated by radiation below these frequencies (Richart, Hall, Woods, 1970). Since it is difficult to combine hysteretic and viscous damping in a frequency-independent formulation, it is suggested that, as an approximation, the larger damping ratio be used. For translational modes, radiation damping will control and for rocking, material damping will usually control. For torsion, it depends on the mass ratio of the problem (see Fig. 3.10).

Solution of the Soil-Structure Interaction Problem

Once the input motion and foundation impedances (stiffness and damping coefficients) are known, the structural system may be analyzed by subjecting the multidegree-of-freedom model to a dynamic analysis. If the solution were conducted in the frequency domain, the stiffness and damping coefficients could be added directly to the corresponding terms for the structure. However, such techniques have not been developed to the extent that time-domain solutions have. In addition, time domain solutions often offer simpler physical interpretation of results. However, within the framework

of the simple models considered, time domain solutions require frequency-independent stiffness and damping coefficients. Using such coefficients, the equations of motion can be solved rigorously by direct integration or by using a complex eigenvalue analysis method.

Because engineering judgment plays such an important role in verifying the validity of structural models and because of familiarity with elastic modal-superposition analysis, the engineer frequently likes to conduct a modal analysis of the soil-structure system. The problem which arises when this technique is used stems from the large value of apparent damping ratios associated with soil radiation damping.

Damping coefficients, as shown in Table 3.1, could be used directly in a time-history analysis using direct-integration of the equations of motion. However, the usual procedure is to convert the damping coefficients to damping ratios, as in Table 3.2, in order to predict the effect of damping on the structural response. This practice may have begun because engineers are familiar with damping ratios as applied to material damping in structures. As discussed by Tsai (1980), this practice may introduce further uncertainties in the seismic analysis. The common formulation used is that for a single-degree-of-freedom system, such that the damping ratio, $D_h = C_h / 2\sqrt{km}$. It is usually assumed that $k=K_h$ and m = the total structural mass. Note that a given soil with a given foundation geometry will have a unique damping coefficient but not a unique damping ratio. The ratio is dependent on the mass of the structure. Because this ratio doesn't have a well defined physical

meaning, its magnitude is sometimes misinterpreted. A structure with a large foundation area to weight ratio will exhibit large damping ratios, sometimes in excess of 100% of critical damping. This large value can cause concern for the analyst who wishes to conduct a modal superposition analysis. The first temptation is to reduce this value to a magnitude that is more familiar to a structural engineer. This obviously would introduce error.

The use of modal analysis introduces approximations since a non-proportional damping matrix cannot be incorporated into modal decomposition. The radiation damping ratio associated with the soil is not directly applicable to the modal superposition analysis which accepts modal damping specification only. Therefore, a method must be used which will combine damping from two different materials.

The accuracy of the resulting solution depends on the method used to select equivalent composite modal damping values. Three methods have commonly been used: 1) Bigg's method (commonly referred to as the strain-energy approach or the stiffness-proportional method), Hanson (1969) 2) a modified Bigg's method as proposed in Roesset, et.al. (1973) or 3) a method proposed by Tsai (1974). In Bigg's method, the portion of the soil radiation damping contributing to a given mode is determined by the ratio of strain energy in the soil springs in that mode relative to that in the remainder of the structural model. Roesset's method includes an additional scale factor for the soil damping contribution based on a ratio of structure frequency to soil frequency. Tsai's method uses transfer functions calculated in the frequency domain and modal

properties from an eigenvalue analysis. The modal damping values are back-calculated by matching responses predicted by the two methods at a point within the soil-structure system. Johnson, et.al. (1977) showed that the latter two methods give results similar to a direct integration technique. For the stiffness proportional method in that study, the damping was arbitrarily cut-off to 10% and, thus, was not directly comparable. However, in certain cases, Bigg's method gave similar results to the other methods. Because the stiffness proportional method has found such common use, it was used in two of the case studies reported in Chapter 4.

As pointed out by Kausel, et.al., (1978), the frequency dependence of the impedances could be approximately accounted for by using the values at a frequency corresponding to the fundamental frequency of the combined soil-structure system. This option would require a trial-and-error solution. The usual practice is to assume the constant values corresponding to the static (zero frequency) case.

Other uncertainties inherent in the use of modal analysis of SSI problems include:

- The eigensolution of a system which may have large modal damping ratios.
- The inability to account for complex foundation geometry.
- The use of a half-space approximation to represent a site which may be layered.

The eigensolution is determined from the undamped equations of

motion. The applicability of the resulting modes to a system with large damping ratios may be suspect. This problem is addressed further in case studies in Chapter 4.

3.1.2 Frequency-Dependent Impedance Methods

When the analytical problem of an elastic half space subjected to oscillatory loading was introduced in Chapter 2, it was pointed out that the solution involved frequency-dependent stiffness and damping coefficients. Lysmer and Richart (1966) proposed that as a first approximation frequency-independent coefficients would suffice. While this assumption appears reasonable for small values of dimensionless frequency, a_0 , and for surface footings on sites reasonably characterized by a half space, it has been shown that many factors result in increased frequency dependence of the stiffness and damping terms which comprise the impedance functions. For example, material damping (Figs. 2.19 and 2.20), Poisson's ratio (Fig. 3.8), and embedment and layering (Fig. 3.17), affect the frequency dependence of the foundation impedances. Thus, it became desirable to be able to properly characterize the impedances as a function of frequency.

As pointed out by Gazetas (1983), newly developed mathematical techniques to solve mixed boundary-value elastodynamic problems allowed the development of impedances for all modes of vibration of a footing on a half space for both low and high frequencies. Luco and Westmann (1971) and Veletsos and Wei (1971) presented frequency-

dependent impedances for circular foundations. Typical results for a rigid massless disk with "relaxed" boundary conditions on an elastic medium are shown in Fig. 3.18. The coefficients shown are the frequency dependent terms normalized by the static stiffness as shown in Eq. 2.56 where the material damping is zero. The authors concluded that the rocking stiffness is much more sensitive to variations in frequency than the horizontal stiffness, the latter

Fig. 3.18 Stiffness and damping coefficients for rocking and horizontal motion. (Veletsos and Wei, 1971)

being essentially independent of frequency. It should be noted at this point, however that the range of a_0 considered is relatively small for application to structures such as nuclear power plants. The authors also concluded that the coupling, i.e., off-diagonal, terms of the impedance matrix are considerably smaller than the other terms and, thus, may be ignored.

The effects of material damping are illustrated in Figs. 2.19 and 2.20 where the soil was considered to be a viscoelastic material with linear hysteretic damping. As shown in these figures, material damping increases the damping coefficients and decreases the stiffness coefficients. It also causes the stiffness coefficients to vary more significantly with frequency. It may be noted in Fig. 3.18 that the radiation damping decreases to zero at zero frequency. Therefore, it is important to include material damping for this degree of freedom which can be accomplished relatively easily in a frequency-dependent formulation as discussed in Section 2.1.2.2.

It has been shown in Chapters 2 and 3 that a substructure technique can be employed to analyze a soil-structure system. In order to accomplish this analysis, impedance functions must be calculated. While for simple systems, frequency-independent stiffness and damping may be adequate, it becomes apparent that many aspects of real structures and soil conditions require the calculation of frequency-dependent impedances. Thus, to complete the topic of substructure techniques, it remains only to discuss

procedures for determining impedance functions and, coincidentally, scattering terms where necessary.

Computation of Impedance Functions

The first solutions to determine the steady-state vibration characteristics of a foundation, and thereby its impedance functions, were applied to rigid circular foundations. Lysmer and Richart (1966) divided the foundation into discrete concentric rings and solved the vertical vibration problem. This analysis avoided the problem of imposing an assumed contact pressure distribution which had theretofore been employed. Both Luco and Westmann (1971) and Veletsos and Wei (1971) solved the mixed boundary value problem by numerical integration of Fredholm integral equations. Following these works, other authors presented solutions for standard foundation shapes, e.g., circular, strip, and rectangular, on the surface of a layered half space or layered stratum. An extensive reference list covering these efforts is included in Gazetas (1983). Some of the available procedures for computing impedances will be briefly discussed herein.

Impedances are dependent on several factors:

- Shape of the foundation, e.g., circular, strip, rectangular, irregular.
- Characteristics of the soil deposit, e.g., deep layered deposit, shallow layered stratum on bedrock.

- Embedment
- Nature of contact between the soil and foundation
- Flexibility of the foundation.

Regarding the last point, most of the work considered herein considers the foundation to be rigid. Two methods which do not have this restriction are presented by Chopra and Guitierrez (1973) and Whittaker and Christiano (1982). However, subsequent comments will be directed primarily to the rigid foundation case.

The methods used to date to compute impedances can be broadly grouped into two types: continuum approaches and discrete approaches. The continuum approaches may be further divided into analytical techniques and what Gazetas (1983) describes as semi-analytical techniques. As stated previously, analytical solutions use a relaxed boundary condition under the footing. That is, no frictional shear is developed during vertical or rocking vibration and no normal stress is developed during horizontal vibrations. The solution of this model usually involves a Fredholm integral equation which is solved numerically. Such solutions are available for relatively simple foundation shapes. An example was shown in Fig. 3.18.

For foundations which do not have simple geometry or which are embedded, semi-analytical techniques are based on the determination of influence coefficients which describe the displacement of a point at the soil-foundation interface caused by a harmonic force applied at another point on the interface. By dividing the contact surface

into discrete regions, a matrix of dynamic influence coefficients (Green's Functions) can be assembled. The impedances for the rigid footing are then found by imposing the rigid-body motion boundary conditions which force the displacements within each subregion to be compatible with the six rigid-body motions of the footing [see, for example, Luco (1974) and Wong and Luco (1976)].

Another approach which may be classed as semi-analytical is the boundary element method which applies boundary integral equations to the discretized contact surface, in the case of surface footings with relaxed boundary conditions, or both the contact surface and surrounding soil surfaces in the case of embedded footings or surface footings attached to the soil. The application to discrete elements results in algebraic equations in a fashion similar to the finite element method. An example of application of the boundary element method to an embedded foundation has recently been published by Muto, et.al., (1983).

These methods have two applications. For structures with footings of simple geometry founded on a uniform half-space, standard tables have been published. Since most impedances are presented in a normalized fashion, they may be scaled to fit the problem at hand. Several of the published works are listed in Gazeta (1983). Another application is to use the developed method directly on the individual footing-foundation problem to be analyzed. Wong and Luco (1980) have developed a set of computer programs to solve for impedances as well as the soil-structure

interaction problem called CLASSI, which stands for Continuum Linear Analysis for Soil-Structure Interaction. The first part calculates impedance and "scattering" matrices for a variety of conditions. To reiterate, the former matrix describes the harmonic force displacement relationship of the foundation while the latter describes the response of the massless, rigid foundation to the incident wave as discussed in Section 2.3.2. The conditions that can currently be analyzed by CLASSI include the following:

- Multiple surface foundations of arbitrary shape
- Embedded foundations of cylindrical shape
- Multilayered viscoelastic half-space
- Scattering of plane waves of arbitrary direction and amplitude (obtained by the same set of calculations as for impedances)
- Bonded boundary condition at foundation-soil interface

The second part of the program completes the substructure method of analysis by combining the force-displacement relationship of the structure at the base level, the impedances, and the input motion to determine the motion of the structural system.

The determination of scattering and impedance matrices may also be performed by discrete methods, i.e., by finite element analysis. Usually finite element analyses are associated with direct methods of analysis which will be discussed in the subsequent section. One method, introduced by Gupta, et.al. (1980), uses a hybrid model in which the near-field soil described by a hemispherical region is

modeled by a finite element mesh. The far field, consisting of the remaining semi-infinite soil, is modeled by an impedance matrix at the interface.

3.2 Direct Methods

In this section, a brief discussion will be presented to describe methods by which a soil-structure interaction analysis may be conducted in essentially one step. In practice, finite element techniques are almost exclusively used to accomplish direct analyses in which the soil and structure are included in one model. In structural engineering applications, the structural system is subdivided into finite elements of the proper type to capture the behavior of the system, appropriate boundary conditions are specified, and the system analyzed for the prescribed loading. However, in soil dynamics applications, several complications arise. The soil strata are of essentially infinite extent in horizontal directions and are often semi-infinite vertically as well. A model of finite extent creates a box effect, which is nonexistent in the physical system, that causes energy to be reflected from the boundaries and thus distort the predicted response of the system. Thus, one of the important developments in applying the finite element problem to SSI problems was to develop wave absorbing lateral boundaries to account for the radiation of energy into the soil region not occupied by the model.

Two main types of lateral energy absorbing boundaries have been developed. Lysmer and Kuhlemeyer (1969) proposed an approximate viscous boundary condition in which the boundary is supported on what is equivalent to dashpot elements oriented normal and tangential to the boundary. These form energy dissipation mechanisms similar to that which Lysmer and Richart (1966) developed to account for radiation damping from a surface-mounted circular footing. In the general form, Lysmer and Kuhlemeyer express the basic viscous terms times a coefficient. It turns out that for maximum energy absorption of shear and compression waves, 95% and 98.5% respectively, the coefficients are chosen to be one. This leaves a wave type similar to a Rayleigh wave which travels along the boundary with an amplitude that decreases exponentially with distance from the boundary. To efficiently absorb Rayleigh waves, the coefficients must vary with depth. Thus, the standard viscous boundary is suggested for use by the authors. To reduce the effect of the remaining wave, the boundary should be placed as far away from the structure as practicable. This is somewhat at odds with the author's requirement for accurate analysis in which the ratio of length of element to minimum wavelength of interest should be held to a value less than $1/12$. It can be seen that potentially a large number of elements might be needed to model the system accurately.

In order to overcome the problem of excessive size of some finite element soil models, a "consistent" boundary condition was developed (Lysmer and Waas, 1972). This method assumed that the loading was periodic and perpendicular to the plane of the

structure. The resulting wave forms in the free field would then be love waves, i.e., plane, horizontally-polarized shear waves. Kausel, et.al., (1975) generalized the technique to include axisymmetric systems. The greatest advantage of the method is that the boundary can be located directly at the edge of the footing. The consistent boundary provides a dynamic stiffness matrix for the medium surrounding the plane or cylindrical cavity occupying the projected volume under the footing. This matrix corresponds to the boundary stiffness matrix which would be obtained from a continuum formulation (Gazetas, 1983). Since the procedure is restricted to harmonic loading, the solution is obtained by linear analysis in the frequency domain.

To date, a problem common to the finite element methods is that the bottom boundary of the soil is specified as fixed. This is the location where the input motion is specified. When there is a clearly defined bedrock layer or when material damping of the soil is high, vertical radiation of energy doesn't have a large effect and the assumption of a rigid base is justified. Otherwise, there will be a spurious peak in surface motion at the frequency of the stratum.

For non-axisymmetric three-dimensional (3d) models, the viscous boundaries are the only currently available alternative. Thus, the boundaries must be placed a significant distance away from the structure such that the energy that is reflected from the boundary, as discussed above, can be dissipated by material damping in the soil before returning to the foundation region. As pointed out by

Gazetas (1983), the cost of such analyses are prohibitive and, subsequently, 3d analyses are rarely performed in practice. Consequently, two types of finite-element models are available: plane-strain 2d models appropriate for strip footings or elongated rectangular structures which are assumed to be symmetric (e.g., Lysmer, et.al., 1973), and 3d axisymmetric models appropriate for cylindrical foundations, which may also be applied to square foundations.

An approximate method has been developed to account for 3d energy dissipation in a 2d model by adding vertically and horizontally oriented dashpots as shown in Fig. 3.19 (Hwang, et.al., 1975). This model has been incorporated in the widely used program,

Fig. 3.19 - Schematic view of a simplified 3-D model.
(Hwang, et.al., 1975)

FLUSH (Lysmer, et.al., 1975). In essence, the addition of these boundary elements merely introduces an increase in damping. Luco and Hadjian (1974) indicated that in some cases, 2d models already overestimate the radiation damping associated with the 3d problem. For these cases, it appears that the addition of viscous damping may exacerbate rather than improve the situation.

The preceding description of finite-element methods apply whether they are being used to find impedance and scattering matrices or solve the direct SSI problem. With regard to the latter, there are some practical aspects to consider in addition to the important energy transmission phenomena. One of these aspects is the use of a control motion which, for a soil deposit is usually specified at the ground surface. For a finite-element analysis the motion is specified at the rigid boundary specified at some depth. Thus, a "deconvolution" analysis must be performed to calculate an equivalent motion at depth. Seed and Lysmer (1980) caution that the design motion at the surface should be consistent with the natural response of the site. A broad-band spectrum with significant high frequency content may not be consistent and a site specific motion should be used instead. Some other considerations regarding input motions are listed in ASCE (1979).

Since conceivably the number of elements needed to describe the foundation may be large, the level of detail included in the model of the structure may be sacrificed. This is especially true of the planar models such as that shown in Fig. 3.19. In order to get a more accurate representation of forces and accelerations in the

structure, the base motions computed from the SSI analysis can be input to the base of a more refined model. However, to be correct, both rotational and translational components and their proper phasing must be used. Even if the proper input is used, there is a danger that the second stage analysis may be subject to very large errors (Christian, et.al., 1980). These authors recommend that the detailed structural model be included with the total system model. As an alternative, they suggest that the impedance representation of the foundation behavior be incorporated with the detailed structural model.

It appears that with all of the restrictions placed on the analyst when attempting to conduct a direct analysis there are really few problems that can be competely solved in one step. For instance, neither axisymmetric nor planar models can acequately represent the coupled motion that occurs in an asymmetric structure. The finite-element method, rather than being thought of as a direct analysis method may serve better as a tool to determine impedances and input motions of foundations.

4. COMPARISON OF RESULTS FROM APPLICATION OF DIFFERENT SOLUTION TECHNIQUES

As the preceding discussions show, the two major effects of soil-structure interaction are the modification of the free-field surface motion by the presence of an embedded foundation and the change in the frequency and damping characteristics of the structural system caused by the compliance (or impedance) of the soil. It has been shown that one of the more difficult phenomena to capture in an SSI analysis is the dissipation of energy in the soil-structure system. The following discussion and case histories will show the applicability and limitations of assumptions made to account for energy dissipation, as well as stiffness, in different solution techniques.

4.1 Application of Frequency Independent Half-Space Methods

As indicated in Section 3.1.1, the applicability of a modal analysis using an eigensolution derived from the undamped equations of motion when large damping is present in the system may be suspect. In addition, some form of composite modal damping must be used when performing modal analysis. The following case studies provides a vehicle for evaluating these two approximations. Three power plants were analyzed for the U.S. Nuclear Regulatory Commission (NRC) for which soil-structure interaction was an important consideration. The first was the Palisades nuclear power

plant for which the containment building housing a pressurized water reactor (PWR) was analyzed (Nelson, et.al., 1981). The second was the Oyster Creek nuclear power plant (Murray, et.al, 1980). Both the reactor building and turbine building were analyzed for this plant which uses a boiling water reactor (BWR). Since the turbine building had the larger calculated damping ratio, it was chosen for comparison herein. These two nuclear plants were analyzed for an earthquake with a peak ground acceleration of about 0.2g. The third power plant structure analyzed for seismic loads was the El Centro Steam Plant, Unit 4 (Murray, Nelson, et.al., 1980). This plant is an oil-fired conventional plant that experienced the October 15, 1979, Imperial Valley Earthquake.

The two nuclear plants were analyzed as part of the NRC's Systematic Evaluation Program; a part of which is to investigate the seismic safety of older plants. The philosophy employed in this program was to perform a screening analysis in order to identify significant weaknesses in the structures or equipment housed in them. Therefore, it was not desirable to perform sophisticated analyses which might prove to be long and costly. Rather, the intent was to perform reasonably conservative simple analyses. In order to provide guidance for this effort, the NRC asked a Senior Seismic Review Team chaired by N.M. Newmark to formulate guidelines for soil-structure interaction analyses of these plants. The resulting guidelines are published as part of the Oyster Creek report. N.C. Tsai provided further guidance in Tsai, (1980). He showed that as the soil damping increased in a two-degree-of-freedom

system, the peak structure response to a series of harmonic input decreased. The peak remained at the primary frequency of the combined system. As the soil damping increased further, to a damping ratio of one or greater, the structure response started to increase and the frequency of the peak shifted to that of the fixed base structure. One lesson from his example is that at sufficiently high damping, the structural response does not continue to decrease and it may be unconservative to arbitrarily limit the radiation damping values.

The final guidelines allow the representation of soil impedances by formulas such as those derived for a rigid footing resting on the surface of an elastic half space given by Richart, Hall, & Woods, (1970). These parameters are frequency independent and have been used in many comparative studies which have demonstrated their suitability, see for example, Tsai, et.al., (1974) and Johnson, et.al., (1977). In addition, the guidelines specify that analyses incorporating a range of soil shear moduli should be used in order to account for the uncertainty in the soil properties. The final in-structure response spectra are taken as smooth envelopes of the resulting spectra from these analyses. By using this procedure, one should conservatively capture all the important frequencies which could be transmitted through the soil at various soil strain levels. For radiation damping, composite modal damping calculated by techniques as described in Chapter 3 are suggested. However, if a composite modal damping ratio exceeds 20%, the guidelines state that a confirmatory analysis must be performed.

For both of the nuclear plants studied, confirmatory analyses were performed and form the basis for the current discussion. The approach used was to construct a fairly detailed three-dimensional structural model which accounted for asymmetry and provided for many locations to generate in-structure spectra for subsequent equipment analysis. For confirmatory analyses, simplified models could then be used.

Palisades Containment Building

The containment building at the Palisades plant has a radius of 60 feet and rests on a foundation 150 feet above bedrock. Since the site is relatively shallow, spring constants and damping were calculated using methods that consider a layered site such as those in *Luco (1974)*, *Kausel et.al. (1979)*, and *Kausel et.al. (1978)*. The major effect in considering layering was a reduction in soil radiation damping in the vertical direction from 60%, as predicted by a half space method, to less than 10% for the layered methods. Composite modal damping values were calculated using the method in *Roesset et.al., (1973)*. The resulting modal damping ratios which include soil material damping are shown in Table 4.1. For this plant, the maximum modal damping ratio was limited to 20% so that the damping for mode 3 was reduced. The effect of this limit on damping is shown in Figure 4.1. The case 1 and case 2 curves result from a direct integration time history analysis, using a R.G. 1.60 compatible input, in which the soil damping is represented by

discrete dashpots attached to the base of the structure. In case 1, the full theoretical soil damping is employed and in case 2, 75% of the theoretical radiation damping is used, as suggested by the Senior Seismic Review Team. The in-structure spectrum generated by a direct modal method using a R. G. 1.60 spectrum, is labeled "Response-spectrum analysis." This spectrum resulted from modal analysis with composite modal damping limited to 20%. The response-spectrum analysis results gave conservative estimates for moments and shears in the structure, but, as shown in Figure 4.1, the in-structure spectra were not conservative at all frequencies. In the period range where the modal analysis was unconservative, between 0.06 and 0.12 seconds, the unconservatism may be due to limiting the modal damping to 20% or to the fact that the mode at 0.08 seconds comprises the deformation of the concrete internal structure, with little soil displacement.

Table 4.1 Modal damping ratios for the Palisades containment building (median soil case).

Mode	Frequency, Hz	Damping, % of critical
1	2.06	8
2	4.79	10
3	5.78	20 ^a
4	12.85	10
5	17.74	4
6	20.02	6

^aRA cutoff of 20% was used

Since composite modal damping tends to spread the damping throughout the soil-structure system, the apparent damping at this natural period may be greater than that resulting in the direct integration

analysis. Note that using only 75% of the theoretical damping did not appreciably affect the results. The final spectra, when made to envelop the results from the range of soil properties considered and having the peaks broadened, envelop the spectra resulting from the confirmatory analyses.

Fig. 4.1 - Comparison of in-structure response spectra (3% damping) for the Palisades containment building generated by modal analysis and direct time-history integration.

Oyster Creek Turbine Building

The turbine building at the Oyster Creek plant has base dimensions of 270' x 170'. It lies on a deep soil site with depth to bedrock of about 1700 feet. In this case, the half space formulations for stiffness and damping represent the site quite well. The shear wave velocity at the site is estimated to be 1250 feet per second. The modal properties of the building, the soil spring-mass system (assuming a rigid building), and the combined system are shown in Table 4.2. The composite modal damping values as calculated by the stiffness proportional method are shown in the last column.

Table 4.2 - Modal properties and damping for the
Oyster Creek turbine building.

	(1) Fixed-Base Bldg.		(2) Soil-Rigid Bldg.		(3) Combined Soil-Bldg.	
	T_i SEC	D_i DAMP	T_i SEC	D_i DAMP	T_i SEC	D_i DAMP
MODE 1 undamped	.0714	10%	.217	80%	.225	76.8%
damped *	.0718		.362		.352	
MODE 2 undamped	.027	10%	.083	100%	.079	84.0%
damped	.027		N/A		.146	
MODE 3 undamped					.040	28.9%
damped					.042	

* damped $T_i = \text{undamped } T_i / \sqrt{1 - D_i^2}$

The resulting in-structure response spectra are shown in Figures 4.2 and 4.3. Since the soil radiation damping values are large for this

structure, an essentially fixed base model (referred to as the high stiffness model in Figure 4.2) was analyzed to compare frequency content with the flexible base cases. As can be seen in Figure 4.2, the damping is not large enough to exhibit the effect described by Tsai, 1980, wherein the frequency of the soil-structure system shifts to the fixed-base frequency of the structure. Also illustrated in Figure 4.2 is the range of soil moduli considered. The confirmatory analyses (Dashpot analysis) conducted by direct integration methods show that the modal analysis gave results which were in the correct range of magnitude but slightly deficient in the higher frequency region. In addition, the modal analysis results at different elevations of the building were essentially the same since the primary contributing mode was a rigid-body lateral translation mode. The dashpot analysis results showed increased response at higher elevations.

Fig. 4.2 - Comparison of spectral curves (3% damping) for the Oyster Creek turbine building at the operating floor, El. 46'

Fig. 4.3 - Spectra comparison (3% damping) showing the effect of limiting modal damping to 20% for the Oyster Creek turbine building, El. 46'.

The spectrum obtained from a modal analysis with the modal damping limited to 20% is shown in Figure 4.3. This limitation had a dramatic effect in this case, producing a much higher peak at the lateral-rigid-body mode and broadening the response in the higher frequency region. Thus, reducing the soil damping contribution tends to increase the rigid-body lateral response and reduce portions of the medium frequency region which characterizes the properties of certain critical components in a nuclear power plant. For this plant, the final spectra used for equipment analysis were broadened to envelop the dashpot analysis results.

El Centro Steam Plant, Unit 4

The El Centro Steam Plant provided a unique opportunity for seismic study. It experienced a significant earthquake (0.5g peak horizontal acceleration) with very little damage and had several recording stations nearby which recorded the free field earthquake time history. The purpose in analyzing this plant was to estimate the response of the equipment contained therein in order to show the capacity of the equipment in a real earthquake environment. Unfortunately, there were no strong motion recorders in the plant at the time of the earthquake so an analysis of the building was required to provide an estimate of the in-structure response spectra.

The foundation of unit 4 measures approximately 204' x 96' and is composed of a honey comb-like foundation. The soil at the site consists of alluvial deposits composed primarily of stiff to hard clay layers.

Again, the stiffness proportional method was used to compute composite modal damping. The calculated soil damping is shown in Table 4.3 and the first ten frequencies and modal damping are shown in Table 4.4. Because the composite modal damping values were quite large, the concern regarding the applicability of modal analysis again was raised. By group consensus, it was decided that modal analysis should be applicable at least up to 40% modal damping, so a test case with 40% damping cutoff was analyzed. The resulting in-structure spectra comparison is shown in Figure 4.4.

The flexible base case, without any damping cutoff, looked quite reasonable. This case was very similar to the cutoff case with only

slightly reduced response. The use of high damping was considered appropriate because the purpose of the analysis was to estimate equipment capacity and, thus, overestimation of response was undesirable.

Based on the more comprehensive study in the report (Murray, Nelson, et.al., 1980), it was concluded that the dynamic model with the specified composite damping was a reasonable representation.

This conclusion was further evidenced by:

- The low level of damage observed at the plant.
- The close relationship of the design to predicted base shears.
- The small displacement of the turbine pedestal predicted by the analysis and evidenced in the earthquake.

Table 4.3 - Summary of soil damping for the
El Centro Steam Plant.

Direction	Damping ratio, % of critical
N-S translation	94.5
Rotation about E-W axis	77.8
E-W translation	89.2
Rotation about N-S axis	68.6
Vertical translation	152 ^a
Rotation about vertical axis	48.8

^a100% was used in the dynamic analysis.

Fig. 4.4 - Response spectra (5% damping) at El. 969' in the El Centro Steam Plant comparing three modal analyses results.

Table 4.4 - Natural frequencies and composite modal damping
for the El Centro Steam Plant

Mode No.	Natural frequencies, Hz	Composite modal damping %
1	1.92	43.4
2	2.02	54.4
3	2.69	32.0
4	3.69	32.7
5	4.43	79.6
6	4.75	50.3
7	4.90	78.7
8	5.01	42.7
9	5.47	61.4
10	6.16	12.3

Conclusion

Studies were performed for a class of soil-structure interaction problems involving stiff buildings on soils with moderate to soft stiffnesses where the radiation damping ratio ranged from 40% to greater than 100% of critical damping. The results indicate that elastic modal superposition analysis can provide reasonable and efficient prediction of seismic response of structures founded on soil, given that frequency-independent lumped parameters can adequately represent the soil. The variation in soil properties and enveloping spectra should conservatively account for frequency variation in soil stiffnesses. For the class of problems with no kinematic interaction and relatively simple geometry, readily available software may be conveniently applied to conservatively estimate response. Soil damping in these analyses can be accounted

for by using available composite modal damping calculation techniques. The results compared were in-structure response spectra which are most sensitive to variations in damping. Results from the referenced reports show that other response quantities such as story moments and shears were not appreciably affected by the approximations inherent in the modal analyses. One possible defect in the simplified techniques is that the composite modal damping techniques tend to overestimate damping in certain modes. However, it has been shown that arbitrarily limiting damping can result in artificially high response at certain frequencies and low response at others. Thus, if the analyst finds that modal analysis is not applicable for a problem with high soil damping, the answer is not to put artificial limits on the composite modal damping but rather to use a more rigorous method.

4.2 Comparison of Finite Element and Continuum Approaches Applied to Embedded Structures

The following description summarizes a case study conducted as part of the Nuclear Regulatory Commission's Seismic Safety Margins Research Program (SSMRP) by the Lawrence Livermore National Laboratory and is reported in Maslenikov, Chen and Johnson (1982). The object of the study is to indicate uncertainties within and between two SSI analysis techniques: the direct method, as represented by FLUSH (Lysmer, et.al., 1975), and the substructure approach, represented by CLASSI (Wong and Luco, 1980).

The case study was conducted using structures from the Zion Nuclear Power Plant in Zion, Illinois. The plant has two reactor containment buildings, each with its own foundation; and the auxiliary, fuel-handling, and turbine-generator (AFT) complex of buildings which is supported on a common foundation of various elevations and thicknesses. One of the reactor containment buildings will also be used for the case studies reported in Section 4.3.

4.2.1 Description of the Zion Plant

A plan view of the Zion plant is shown in Fig. 4.5. The structures are quite close together and thus, the effect on adjacent structures via through-soil coupling, referred to as structure-to-structure interaction, may be significant. The containment structure houses an independent internal structure which supports the nuclear steam supply system as shown in Fig. 4.6. This structure is embedded and has a relatively shallow soil layer between its basemat and bedrock. The AFT complex is a very complicated structure and weighs about five times as much as the containment building.

Fig. 4.5 - Plan view of the Zion Nuclear Power Plant.

4.2.2 Models Developed for SSI Analysis

The containment shell was modeled as a series of vertical beam elements with twelve lumped masses. This model was used for both direct and substructure methods. For more complex structures, i.e., the internal structure and NSSS, and the AFT structure, detailed three-dimensional models were developed for the SSMRP response calculations (Lo and Tsai, 1981) and were intended to represent

Fig. 4.6 - Simplified elevation view of Unit 1 containment building, facing west, showing two locations where responses are calculated.

state-of-the-art modeling. The CLASSI analysis could directly incorporate this level of detail in the form of the modal properties of the fixed base structure. A two-stage analysis was performed with FLUSH. In the second-stage analysis the detailed models were used. However, for the first stage, simplified models were necessary.

As discussed in Section 3.2, the simplified models should represent the more detailed ones as closely as possible or rather large errors can occur. In this case a modal equivalence principle was used which permits modeling more modes than does a simplified beam or element representation. The basic procedure is to develop a series of single-degree-of-freedom (DOF) models, each designed to represent one mode of the structure. The frequency, mass, and mass location of each single-DOF model are selected to match the modal frequency, mass, and moment about the foundation as determined from the detailed model. Judged on the basis of their modal participation factors, only the most important modes are included. Any residual mass or rotational inertia not represented in the dynamic models is added to the foundation to ensure proper modeling of rigid body behavior. A typical FLUSH model is shown in Fig. 4.7.

Since CLASSI uses the substructure technique, the foundations are analyzed separately from the structure. The CLASSI foundation models are illustrated in Fig. 4.8. From these models, impedances and scattering matrices were calculated. The impedance and scattering terms vary widely with frequency since the foundations are embedded and the site has different soil layers and a rather

Fig. 4.7 - Finite element model for FLUSH analyses showing a cross-section through the containment building and auxiliary building.

Fig. 4.8 - CLASSI foundation model of the coupled AFT complex and containment building foundations. The discretization used to compute impedance functions is shown.

shallow bedrock. The horizontal impedance terms are shown in Fig. 4.9 as an example. The current version of CLASSI can only analyze *embedded foundations of cylindrical shape*. Therefore, to obtain impedances for the AFT complex, the terms for the flat, T-shaped foundation were adjusted to account for embedment by evaluating embedment effects on an equivalent circular foundation. As an example of the variations between FLUSH and CLASSI, scattering terms calculated by both methods are shown in Fig. 4.10.

All of these analyses required simplifications. In the CLASSI analysis, a cylindrical shape was assumed. In the FLUSH case, plane strain analysis was performed, which does not duplicate three-dimensional behavior over the range of frequencies. In addition, attempts to simulate a rigid foundation in the FLUSH analyses met

Fig. 4.9 - Horizontal impedance terms for the containment building.

with only partial success. Hence, neither result can be considered exact, and differences are examples of uncertainty in SSI modeling.

One of the more difficult modeling issues was encountered while trying to construct finite-element models to account for structure-to-structure interaction, where the mass ratios between structures is an important parameter. When modeling only a 2d slice of a structure, such as the auxiliary building, including the entire mass would cause disproportionately high stress levels in the soil elements and distort structural response. To maintain reasonable levels of stress in the soil, the mass of the slice chosen should be proportional to the foundation area. The latter option was chosen for this analysis, which will have an influence on the structure-to-structure interaction results.

4.2.3 Results of Structural Analyses

The goal in this study was to compare results of SSI analyses using two different techniques. As in Section 4.1, the quantity used for comparison was in-structure response spectra, in this case at 2% damping. Two input motions were used: the El Centro 1940 record and a synthetic record with a narrow-band spectrum which might be representative of a site such as Zion.

To evaluate the effects of structure-to-structure interaction, the reactor buildings and AFT complex were analyzed first as isolated structures and then together as a coupled soil-structure system. Analysis of the reactor building as an isolated structure

Fig. 4.10 - Comparison of scattering matrix components for the AFT complex from the CLASSI embedded cylinder and FLUSH models. Shown are (a) horizontal component caused by horizontal free-field motion, (b) rocking component caused by horizontal free-field motion, and (c) vertical component caused by vertical free-field motion.

represented a benchmark comparison between FLUSH and CLASSI.

Results of this comparison showed excellent agreement between the two methods for horizontal response. At most frequencies, there was less than 10% difference in spectral ordinates but near the resonant frequencies of the coupled soil-structure system, the differences approached 35%. Vertical response did not compare as well as horizontal. Variations in peak acceleration ranged up to about 25%, and in the amplified frequency range ($<33\text{Hz}$) went as high as 50%, depending on structure location and the input motion. A typical comparison of spectra is shown in Fig. 4.11.

While the containment building and foundation were relatively straightforward to model, the AFT complex required significant simplification for both analysis types. In light of the subsequent approximations, the agreement between CLASSI and FLUSH results for the isolated AFT was surprisingly good. Peak horizontal accelerations varied by less than 25% (about 10% on the average) while spectral accelerations in the amplified range showed somewhat greater differences. Differences in the vertical spectra were less than those for the horizontal, typically less than 20%. CLASSI predicted a torsional response of a similar magnitude to N-S rocking while FLUSH could not capture torsion.

Additional conclusions were drawn in the report (Maslenikov, et.al., 1982) from results not shown herein. Structure-to-structure interaction as predicted by the two methods differed by 200% or more in some cases. Due to the modeling difficulties discussed, the FLUSH model could not model mass ratios correctly between the two

structures and still model the soil stress properly. The CLASSI results showed that the containment buildings have little effect on the AFT complex. However, the heavier AFT complex has a substantial effect on the containment buildings. The spectral ordinates on the foundation increase by as much as 50% and at higher elevations by up to 100% over those for the isolated containment.

Fig. 4.11 - Comparison of CLASSI and FLUSH analyses of the isolated reactor building using El Centro earthquake data. Response spectra shown are (a) N-S translation at the top of the containment shell, (b) vertical translation at the top of the containment shell, (c) N-S translation on the operating floor at the pressurizer, and (d) vertical translation on the operating floor at the pressurizer.

4.3 Comparison of State-of-the-Art SSI Techniques With Those Used for Design of Older Nuclear Power Plants

One of the objectives of the SSMP, described in the previous section, is to identify analysis procedures, assumptions, and parameter values used in design which introduced "conservatism" into the design calculations for nuclear power plants. Actually, it is not expected that procedures used in design were in every case conservative, but that they would result in deviations from state-of-the-art best estimates. The methodology used to describe these deviations is to select those procedures which are presumed to lead to the largest effects on seismic response and which are applicable to the largest number of plants. To quantify the deviation, results from an analysis which accounts for a given effect (e.g., embedment) are compared to those from an approximate analysis which doesn't explicitly include the effect.

A study was conducted by Subudhi, et.al., (1980) which summarized design practices for plants with operating licenses issued prior to April, 1980. Using this study, important SSI practices could be identified. Two factors which are believed to have a significant effect on predicted structural responses and which affected many older plants are embedment effects and radiation damping. Many design analyses for older plants ignored embedment for the SSI analysis and some ignored radiation damping or limited composite modal damping to 10%. Frequency independent half-space

methods were often used for SSI analyses of these plants, thus, frequency dependence of impedances also was not considered.

The Zion containment building, briefly described in the preceding section is used as representative structure to evaluate these effects. It is assumed this structure is founded on a half-space with various shear wave velocities.

4.3.1 Effects of Embedment on a Nuclear Containment Building

The purpose of the studies presented in this section is to quantify differences in responses calculated using procedures which ignore the effects of embedment to those which explicitly account for embedment. This shall be done by comparing response quantities obtained from state-of-the-art methods for a surface footing to quantities calculated for an embedded footing, both on a half space. The resulting factors will illustrate some of the physical effects of embedment.

The Zion containment structure, which was chosen for the studies, is founded on a circular mat, 157 ft. in diameter and about 13 ft. thick with an embedment depth, E , of 36 ft. This results in an embedment ratio, $E/r = 36/78.5 = 0.46$.

Parameters Studied

The subsurface foundation conditions included a range of soil sites with a uniform soil profile. The shear wave velocities (v_s) for the soil cases studied were 500 fps, 1000 fps, 2000 fps, and

3500 fps which covers the range from a relatively soft site to rock. The soil material damping ratio of 5% was chosen as representative of that which would occur during an SSE level event at Zion. Poisson's ratio was 0.40 and the mass density was 4 slugs/ft³ (129 lb/ft³).

A fixed base analysis was conducted to investigate the effects of base fixity, an assumption which has been frequently employed for the design of structures founded on rock.

Input Motion

Several test analyses with one set of earthquake records confirmed the fact that idiosyncrasies in results occurred because of the unique features of the records used. Therefore, two suites of time history records were used. The first suite consisted of 10 records, of three components each, which represented artificial records developed to match the spectra specified in R.G. 1.60 (USNRC, 1973). The second suite of 10 records were generated to represent site specific motion as might be applied at the Zion site.

SSI Analysis Techniques

The base case, i.e., that methodology deemed to give the best estimate of response, was determined by employing the CLASSI series of codes. This methodology employs essentially four steps:

- Determination of fixed base mode shapes and frequencies of the structure.
- Determination of the foundation input motion.

- Calculation of foundation impedances.
- Analysis of the coupled soil-foundation system.

The first step has been reported previously (Lo and Tsai, 1981).

For the determination of the foundation input motion, the foundation is assumed to be rigid and massless and the seismic waves are assumed to be propagating vertically. Employing these assumptions, the input motion for the surface-mounted massless footing is equal to the free-field surface motion. However, for the embedded case, a rigid inclusion is introduced into the soil medium which induces wave scattering, altering the motion which the massless foundation experiences.

The calculation of wave scattering and foundation impedances is accomplished in the same step in the CLASSI methodology, the solution of which is described below.

Foundation Impedances and Wave Scattering

Foundation impedances are complex-valued, frequency-dependent functions relating the dynamic forces at the foundation-soil interface to the resulting soil displacements. For a symmetric, rigid foundation, the impedances are defined at each frequency by five terms. Each complex term in the impedance matrix comprises a pair of normalized coefficients which represent the stiffness and damping associated with the soil.

The impedance values for both the surface and embedded cases are shown in Fig. 4.12. These terms were calculated for the case where $v_s = 500$ fps. For this case the dimensionless frequencies, $a_0 = \omega r/v_s$, are approximately equal to the frequencies in Hz. For

Horizontal terms K_{11} and K_{22}

Coupling terms K_{15} and K_{24}

Fig. 4.12 - Impedance functions for circular containment ($v_s = 500$ fps).

———— Embedded
 - - - - - Surface

Vertical term K_{33}

Rocking terms K_{44} and K_{55}

Frequency (Hz)

Fig. 4.12 - Continued - Impedance functions for circular containment.

———— Embedded
 - - - - - Surface

Fig. 4.12 - Continued - Impedance functions for circular containment.

————— Embedded
 - - - - - Surface

higher shear wave velocities, the abscissa is compressed, such that for $v_s = 3500$ fps, the abscissa value of 10 is equivalent to 70 Hz.

The left half of each figure shows the real part of the impedance which represents the stiffness of the soil-foundation system. The right half shows the imaginary part divided by frequency which represents the energy dissipation in the soil medium, including both radiation and material damping. The solid lines are for the embedded foundation and the dashed lines are for the surface foundation.

The amplitudes of the scattering terms are shown in Fig. 4.13. These curves are normalized to the surface motions in the case of the horizontal and vertical directions. For the rocking case, the curve represents rotation amplitude times the radius divided by the horizontal surface motion. The curves denote the amplitude of motion at each frequency that the hypothetical rigid massless foundation would experience. The dashed lines show the motion for the surface foundation, which is equal to the surface free-field motion, and the solid lines represent the motion of the embedded foundation. The two motions differ primarily for two reasons. First, the free field motion varies with depth in the soil. Second, the rigid inclusion causes scattering of the incoming waves because points on the soil-foundation interface are constrained to move according to the foundation's geometry and stiffness.

Fig. 4.13 - Amplitude of scattering matrices for the Zion containment. Shown is (a) horizontal component (Δ_H = horizontal displacement, U_0 = free-field horizontal displacement)

Fig. 4.13 - Continued - Amplitude of scattering matrices for the Zion containment. Shown are (b) rocking component caused by horizontal free-field motion (c) vertical component. (θ = rotation, Δ_v = vertical displacement, U_0 = free-field horizontal displacement)

The following general trends, as a result of embedment in a semi-infinite half-space, are shown in Fig. 4.12 and 4.13:

- Embedment increases the static (zero-frequency) values of stiffness.
- Damping is increased.
- Horizontal input motion is decreased, at some frequencies by a factor of 0.5.
- Rotational input is induced.
- Vertical input is reduced to a lesser extent than horizontal.

A comparison of suggested simplified methods to account for the effects on horizontal and rotational input is shown in Fig. 4.14. It appears that Roesset's (1980) suggested procedure for horizontal motion is more appropriate in this case than that suggested by Kausel, et.al., (1978). However, the factors from Kausel, et.al. for rotation provide a reasonable approximation.

One may deduce that the reduction of the horizontal input may be mitigated, to some extent, by the introduction of rotational input. However, as shown in Fig. 4.15, the rotational input appears to be out of phase with the horizontal and thus, may serve to further reduce the response. It may very well be that the phasing of rotational and translational motions may be different for different embedment depths or foundation geometries. This topic deserves further investigation.

(a) Horizontal input from CLASSI ———
 Approximate method - - - - -
 $F_U(\omega) = \cos(\pi/2 f/f_n)$ for $f < 0.59 f_n$
 0.6 for $f > 0.59 f_n$

(b) Rotational input from CLASSI ———
 Approximate method - - - - -
 $F_\phi(\omega)$ see Eq. (3.21)

Fig. 4.14 - Input motion amplitudes for cylindrical foundation with
 embedment ratio, $E/r = 0.46$

Fig. 4.15 - Real and imaginary terms of scattering matrix for the Zion containment.

— — — — — Rocking
 - - - - - Horizontal

Response Quantities Studied

The response quantities chosen for comparison for the containment structure are illustrated in Fig. 4.16. The force quantities, denoted by "F" in the figure, are moments, shears, and axial forces at the elevations indicated. The basis for comparing these quantities are the maximum values occurring during each time history analysis. Acceleration quantities, labeled "A" in Fig. 4.16, were calculated at each time step for various directions. At the base, all degrees of freedom were calculated. At the next three

nodes, the two horizontal translational accelerations were found, and at the two top nodes, the vertical components were determined. From the acceleration time histories at these locations, in-structure response spectra were calculated. These spectra provide at least two bases for comparison. First, the peak (zero-period) accelerations can be compared directly which indicates the effects on rigid equipment mounted at the named locations. Second, the spectra themselves can be compared. However, the latter comparison cannot be made on a scalar basis. Two comparisons are possible, spectral ordinates at each frequency, or frequency shift and amplitude changes of peak spectral values. These two methods will be discussed further in subsequent sections.

The locations where response quantities were calculated for the internal structure are shown in Fig. 4.17. For this structure, the primary quantities of interest are shear stresses and moments in wall elements and moments in floor elements. The elements were selected by subjecting the structural model to a static lateral force and picking those elements which had high stresses and that were important to the integrity of the structural system.

The acceleration quantities of interest associated with the internal structure are those which interface with the Nuclear Steam Supply System (NSSS). The points on the NSSS where acceleration response spectra were calculated are shown in Fig. 4.18.

Fig. 4.16 - The containment vessel stick model. "A" indicates nodes where acceleration responses were obtained and "F" indicates members where force responses were calculated.

Analyses Conducted

Four series of analyses were conducted for each of the four soil shear-wave velocities discussed, using a surface case for each of the time history sets and an embedded case for each set. In addition, a fixed base case was analyzed for each set of input. A comparison of the mean spectra from the free field records are shown in Fig. 4.19. Each analysis was run with three simultaneous components from each record.

Table 4.5 lists the cases run. A typical pair of runs would be cases 1 and 3. First the ten records were run for the surface foundation case then the embedded case. Ratios of the maximum force responses and zero-period accelerations (surface/embedded) were found for each earthquake, then a mean ratio and corresponding coefficient of variation were found for each response quantity. Lastly, the mean of all forces and the mean for all accelerations were calculated. For spectra comparisons, the mean spectra for each case were multi-plotted. Ratios of selected mean spectra were calculated at each frequency and plotted to provide additional comparisons.

In addition to surface vs embedded case comparisons, the fixed base cases were compared to the case for shear wave velocity corresponding to rock ($v_s = 3500$ fps). The same comparison procedure was followed for all of the pairs of cases.

Fig. 4.17a - Perspective view of steam generator and pressurizer compartments above El. 617 ft. "A" indicates acceleration output, "S" indicates shear stress output for shaded element.

Fig. 4.17b - Perspective view of operating floor (El. 617 ft).

M = moment output

A = acceleration output

Fig. 4.17c - Perspective view of ring wall and fuel handling pool wall between El. 590 ft. and El. 617 ft. S = shear output, M = moment output.

Fig. 4.17d - Perspective view of biological shield wall, ring wall, and fuel handling pool wall between El. 576 ft and El. 590 ft. M = moment output, S = shear output.

Fig. 4.17e - Perspective view of walls between El. 568 ft and El. 581 ft. M = moment output, S = shear output.

• = Locations where accelerations were calculated.

Fig 4.18 Isometric view of the reduced NSSS model.

Fig. 4.19 - Mean response spectra from free-field input motion.

TABLE 4.5 CASES ANALYZED

Case	v_s (fps)	Type	Record
1	1000	SURF	RG 1.60
2	1000	SURF	Site Specific
3	1000	EMBED	RG 1.60
4	1000	EMBED	Site Specific
5	2000	SURF	RG 1.60
6	2000	SURF	Site Specific
7	2000	EMBED	RG 1.60
8	2000	EMBED	Site Specific
9	500	SURF	RG 1.60
10	500	SURF	Site Specific
11	500	EMBED	RG 1.60
12	500	EMBED	Site Specific
13	3500	SURF	RG 1.60
14	3500	SURF	Site Specific
15	3500	EMBED	RG 1.60
16	3500	EMBED	Site Specific
17	FIXED	SURF	RG 1.60
18	FIXED	SURF	Site Specific

Results

First, the acceleration response spectra at the base of the structure will be discussed to illustrate the general effects of embedment. Figure 4.20 illustrates mean base horizontal spectra for a soft site ($v_s = 500$ fps) for both sets of input records. One of the major effects of embedment is to reduce the maximum spectral ordinate roughly in half. In addition, the ZPA is reduced by a

factor of 1.4 to 1.5. These reductions can be attributed to both reduction in input due to scattering and increase in stiffness and damping.

The vertical spectra for these cases are shown in Fig. 4.21. Note that the vertical spectra are not reduced at all frequencies as a result of embedment. The vertical stiffness is larger for the embedded case over the entire frequency range while the increase in damping is not as large as it was for the other degrees of freedom. Also the scattering effect is smaller for the vertical direction than for the horizontal. Thus, the increased peak spectral responses for the embedded case are caused by the increased stiffness in the absence of other mitigating factors.

The rocking spectra, as shown in Fig. 4.22 are also reduced, with a reduction smaller than that for the horizontal direction occurring near the spectral peak but with larger reductions at other frequencies. The rotational spectra are affected by the rocking response of the structure as a result of horizontal input and, in the embedded case, by the introduction of rotational input. Torsional spectra are not shown since they are insignificant for this nearly symmetric structure.

For the stiffest site, as shown in Fig. 4.23 and 4.24, embedment does not result in a large decrease in spectral ordinates for horizontal translation. As a result, the rocking response of the structure due to the horizontal input should also not be greatly reduced. However, the peak spectral values for rocking are reduced by a factor of two. One may infer that the increased stiffness due

Fig. 4.20 - Mean base horizontal response spectra ($v_s = 500$ fps) surface and embedded cases (a) x and (b) y spectra for site specific records, (c) x and (d) y spectra for R.G. 1.60.

Fig. 4.21 - Mean base vertical response spectra ($v_s = 500$ fps) for (a) R.G. 1.60 and (b) site specific records.

to embedment is not the major cause of the reduction of foundation response. Thus, it appears that the large reduction in rocking response for the stiff site may be caused by the induced rotational input being out of phase with the structure rocking caused by horizontal input motion.

The maximum force quantities and peak (ZPA) accelerations, being scalar quantities, are easiest to quantify and thus are easiest to use in calculating ratios for the purpose of quantifying differences. The mean values and coefficients of variation for all of the force and acceleration ratios calculated in each case are shown in Table 4.6. The ratios were calculated by dividing the quantities from the surface case by the corresponding quantities

Fig. 4.22 - Mean base rotational response spectra ($v_s = 500$ fps), for (a) x-axis and (b) y-axis rotation for site specific records and (c) x axis and (d) y axis rotation for R.G. 1.60 records.

Fig. 4.23 - Mean base translational response spectra ($v_s = 3500$ fps) for (a) x - axis (b) y - axis and (c) vertical translation for R.G. 1.60 records.

Fig. 4.24 - Mean base rotational spectra ($v_s \approx 3500$ fps) for R.G. 1.60 records.

Table 4.6 - Mean Ratios of Forces and Peak Accelerations for Surface vs Embedded Foundations and Fixed Base vs Surface Foundations

	Surface Vs Embedded ($E/r = 0.46$)								Fixed Base vs Surface	
	$v_s = 500$		$v_s = 1000$		$v_s = 2000$		$v_s = 3500$		$v_s = 3500$	
EQ Input	R.G. 1.60	SS	R.G. 1.60	SS	R.G. 1.60	SS	R.G. 1.60	SS	R.G. 1.60	SS
Cases	9/11	10/12	1/3	2/4	5/7	6/8	13/15	14/16	17/13	18/14
A										
C Mean	1.29	1.23	1.25	1.25	1.28	1.26	1.22	1.22	1.20	1.28
C										
E (CDV)	(.145)	(.160)	(.118)	(.152)	(.202)	(.195)	(.244)	(.277)	(.309)	(.298)
L										
F										
O Mean	1.28	1.20	1.22	1.23	1.25	1.22	1.19	1.18	1.16	1.24
R										
C (CDV)	(.120)	(.150)	(.107)	(.134)	(.101)	(.130)	(.175)	(.204)	(.204)	(.239)
E										

from the embedded case. There was not an easily discernable trend in the ratios based on location or direction. Therefore, the mean of all the ratios in a given case were computed.

Table 4.6 also lists the mean ratios for results from a fixed-base analysis versus an analysis on a rock site ($v_s = 3500$ fps) which considers soil-structure interaction. These mean ratios indicate that the best estimate forces in a structure at a site where R.C. 1.60 is representative of the expected earthquake are less than the forces determined from a fixed base analysis by a factor of 1.16. The uncertainty in this calculation is represented by the COV of 0.204. Similarly, the variation from best estimate values can be determined for those analyses of embedded structures where the effects of embedment were not included by selecting the appropriate factor from the table.

Specifying variations for in-structure spectra is a much more difficult task than providing simple ratios. Figures 4.25, 4.26, 4.28, and 4.31 show typical mean response spectra for the four soil cases. Figure 4.25 (a) shows a reduced spectral peak for the embedded case with no apparent frequency shift. At the operating floor level in the internal structure, Fig. 4.25 (b), there is a decrease in the spectral peaks and a shift in frequency for the embedded case. The same trend is exhibited higher on the containment structure, Fig. 4.25 (c). The vertical spectra do not show significant peak reductions or frequency shifts for this case. Figure 4.26 shows the same general trends except at a point low in the internal structure, Fig. 4.26 (a) and (e), where the first peak

is reduced and frequency shifted and the second peak is greatly reduced for the embedded case. In addition, the spectral peak for the vertical direction shows greater reduction than for the case with $v_s = 500$ fps.

As alluded to earlier, another way to represent differences between spectra from surface and embedded cases is by spectral ratios. Figures 4.27 and 4.29 show ratios for the spectra shown in Fig. 4.26 and 4.28 respectively. These ratios were calculated by dividing the ordinates of the spectra from the surface case by those from the embedded case for each of the ten time histories per set and plotting the mean of these values. The coefficients of variation associated with these ratios are unknown at this time since the ratios shown represent "typical" cases.

The remaining typical spectra show the same general trends as illustrated in Fig. 4.26 and 4.28. There are, however, cases which do not fit the "typical" type of spectra shown. Figures 4.30 (a) and (b) show cases of horizontal spectra where second peaks remain well defined and are reduced to various extents. Figure 4.30 (c) shows a vertical spectrum that shows an increase at certain frequencies due to embedment. This also occurs for the site specific spectra case equivalent to Fig. 4.30 (c) where the peak value is increased by about 7% for the embedded case instead of being decreased. Despite these anomalies, the trends are reasonably represented by the typical spectra shown.

Fig. 4.25 - Typical mean response spectra for $v_s = 500$ fps, R.G. 1.60 (a) low in the internal structure (RCP restraint) (b) operating floor level in internal structure (c) ring girder on containment (d) vertical at reactor coolant pump

Fig. 4.26 - Typical mean response spectra for $v_s = 1000$ fps, R.G. 1.60 (a) low in the internal structure (b) operating floor level in internal structure (c) ring girder on containment (d) vertical at reactor coolant pump

— — — — Embedded
————— Surface

Fig. 4.26 - Continued - (e) low in the internal structure for site specific spectra.

Fig. 4.27 - Spectral ratios for spectra shown in Fig. 4.26.

Fig. 4.28 - Typical mean response spectra for $v_s = 2000$ fps, R.G. 1.60 (a) low in the internal structure (b) operating floor level in internal structure (c) ring girder on containment (d) vertical at reactor coolant pump

Fig. 4.29 - Spectral ratios for spectra shown in Fig. 4.28.

Fig. 4.30 - Atypical mean response spectra for $v_s = 2000$ fps (a) S. S. spectra results for RCP restraint (b) R.G. 1.60, top of steam generator (c) R.G. 1.60, vertical at ring girder.

Fig. 4.31 - Typical mean response spectra for $v_s = 3500$ fps, R.G. 1.60 (a) low in the internal structure (b) operating floor level in internal structure (c) ring girder on containment (d) vertical at reactor coolant pump

It has been suggested that a fixed base analysis of a structure founded on rock will result in a conservative estimate of the structure's response. Figure 4.32 (a) shows what might be expected in terms of in-structure response spectra. The case which includes SSI shows an amplitude reduction and frequency shift to lower frequency than the fixed base case. The lower frequency is caused by the compliance of the rock and the amplitude reduction is attributable to radiation damping in the compliant medium. In some cases the reduction may be quite large such as for the vertical spectra shown in Fig. 4.32 (b). This reduction points out the effect of the rather large radiation damping associated with vertical motion. However, for those modes where damping is not large, such as for rocking, the shift in frequency caused by the compliant foundation may result in amplification of in-structure spectra at certain frequencies such as shown in Fig. 4.32 (c). Trends in these spectra are difficult to predict. However, it appears that amplitude reduction for the compliant foundation is most predominant at frequencies greater than 10 Hz while amplification may result below 10 Hz as shown in Fig. 4.32 (d).

Validation

In order to provide a check on the procedures employed in the parameter studies, the results of a similar recent study (Muto, et. al., 1983) were compared to those using CLASSI reported on herein. An analysis using the El Centro, 1940 record and a shear wave

Fig. 4.32 - Fixed base vs compliant foundation for $v_s = 3500$ fps, R.G. 1.60 (a) containment shell, op. floor elevation (b) vertical at ring girder (c) bottom of steam generator.

velocity of 1500 fps was performed which formed a basis of comparison with Muto's work. A comparison of the pertinent parameters is shown in Table 4.7.

First, peak acceleration ratios for surface versus embedded cases were compared. For an embedment ratio (E/b) of $1/3$, Muto got an acceleration ratio at the operating floor of 1.25 and for an E/b of $2/3$, he got a ratio of 1.70. The E/r for the CLASSI studies was 0.46, which is 0.4 of the way between $1/3$ and $2/3$. By interpolation, the comparable acceleration ratio from Muto is 1.40. For the CLASSI study the ratio on the containment shell at the operating floor elevation was 1.41 and on the operating floor it was 1.38. This comparison provides some confirmation that the CLASSI methodology is correct. The ratios of peak spectral amplitudes at the operating floor elevations do not compare as well with an interpolated value of 2.02 for Muto and 1.75 for CLASSI. However, considering the difference in structural types the agreement is still quite good.

Table 4.7 - Comparison with Muto (1983) Parameters

Muto et.al., (1983) BWR	Present Study PWR
$v = 0.4$	$v = 0.4$
$v_s = 500 \text{ m/s} = 1640 \text{ fps}$	$v_s = 1500 \text{ fps}$
$E/b = 1/3 \text{ square}$	$E/r = 0.46 \text{ circular}$
$= 2/3$	$(= 0.4 \text{ between } 1/3 \text{ \& } 2/3)$
$\xi_{\text{soil}} = 0?$	$\xi_{\text{soil}} = 5\%$
$\xi_{\text{Bldg}} = 5\%$	$\xi_{\text{Bldg}} = .01 \text{ to } .02$
$\rho = 2000 \text{ kg/m}^3 \text{ (3.9 slugs/ft}^3\text{)}$	$\rho = 4 \text{ slugs/ft}^3 \text{ (128.8 lb/ft}^3\text{)}$
$EQ = \text{El Centro 40 NS}$	$EQ = \text{El Centro 40 NS}$
$E_{\text{opt}} \xi = 5\%$	$E_{\text{opt}} \xi = 2\%$

4.3.2 Comparison of CLASSI with Frequency-Independent Half-space Method Without Damping

In order to evaluate the practice of conducting an SSI analysis in which radiation damping is ignored, the containment structural model described in Section 4.3.1 was analyzed by two methods. Results from CLASSI analyses of the surface founded structure were compared to those from a modal analysis which incorporated frequency-independent spring coefficients but no added damping to account for energy dissipation into the soil.

Since a set of eigenvectors for the fixed base model were known it is possible to determine an augmented set which incorporate the new degrees of freedom imparted by the foundation mass and soil springs. The program SOILST by Johnson (1979) determines the coupled soil-structure eigensystem by component mode synthesis described by the author.

The frequency-independent soil springs were calculated using the formulae given in Table 3.1. The resulting spring constants were nearly the same as the zero-frequency values calculated for the surface case by CLASSI as shown in Section 4.3.1. The damping was assumed to be equal to the structure damping, approximately 1.5% of critical.

The same response quantities were calculated as described in the previous section. Only the suite of ten R.G. 1.60 time history records were used since the previous analyses showed no significant difference in response ratios between the two types of records.

Response ratios were calculated for three soil conditions: $v_s = 500, 1000$ and 2000 fps. Early analyses using frequency-independent parameters would classically consider a shear wave velocity of 3500 fps to be rock and thus, a fixed-base analysis would be conducted.

The mean values and coefficients of variation for peak translational acceleration ratios are shown in Table 4.8. It can be seen that the effect of ignoring damping is quite large. Only acceleration levels are given in Table 4.8 because of difficulties in extracting reasonable forces using the component mode synthesis technique. Errors in forces were a result of modal truncation problems using this technique. A smaller test problem was used to quantify the accuracy of the results. In this test, translational accelerations from a 50 degree-of-freedom system characterized by the first 12 modes were within 11% (vertical) and 3% (horizontal) of respective accelerations calculated using all the modes in a direct analysis.

Table 4.8 - Mean Ratios of Peak Translational Accelerations from Frequency Independent Method Without Damping vs CLASSI

	$v_s = 500$	$v_s = 1000$	$v_s = 2000$
Mean	2.71	2.57	1.93
(COV)	(.39)	(.39)	(.37)

As shown in Table 4.6 the ratios of forces are quite close to those for accelerations. Thus, the values given in Table 4.8 may also be considered representative of force ratios for the cases shown.

Typical mean response spectra for the three soil cases studied are shown in Fig. 4.33 to 4.35. The spectra show some general

trends. The horizontal spectral peaks for the frequency-independent method (FI) are at a slightly higher frequency than those for CLASSI. This is reasonable since the zero-frequency impedances are the highest for CLASSI. For higher modes, the stiffnesses for CLASSI will be less. However, the amplitudes for CLASSI are considerably smaller than for the FI method. In fact, the second peak in Fig. 4.33 (c), 4.34 (c) and 4.35 (c) is virtually non-existent in the CLASSI case. Such amplitude reductions can be attributed to the rather high radiation damping which CLASSI properly includes. Tsai, et.al., (1974) found a similar spurious peak in the second mode when an arbitrary limit was placed on composite modal damping. The primary peak in these figures is reduced by factors of 5.4, 4.5 and 2.9 as v_s increases from 500 fps to 2000 fps. The vertical spectral peaks for the FI method increase in frequency as the shear wave velocities increase as shown in the (d) portions of Figs. 4.33 to 4.35. These frequencies apparently correspond to modes with the rigid structure on the soil spring. In CLASSI, these modes have very high radiation damping and are essentially damped out. The horizontal directions show two peaks because two structure-spring modes contribute to each: a horizontal mode and a rocking mode.

The results of this study show that radiation damping has a very significant effect in soil-structure interaction, e.g., a factor of 14 at the spectral peak in Fig. 4.33 (b). Ignoring this damping causes extreme overprediction of subsystem response and may cause spurious peaks in spectra that should not be present.

Fig. 4.33 - Typical mean response spectra for $v_s = 500$ fps, R.G. 1.60 (a) low in the internal structure (b) operating floor level in internal structure (c) ring girder on containment (d) vertical at reactor pressure vessel.

Fig. 4.34 - Typical mean response spectra for $v_s = 1000$ fps, R.G. 1.60 (a) low in the internal structure (b) operating floor level in internal structure (c) ring girder on containment (d) vertical at reactor pressure vessel.

Fig. 4.35 - Typical mean response spectra for $v_s = 2000$ fps, R.G. 1.60 (a) low in the internal structure (b) operating floor level in internal structure (c) ring girder on containment (d) vertical at reactor pressure vessel.

5. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

It has been shown, through theoretical development as well as case studies of actual soil-structure systems, that properly accounting for energy dissipation mechanisms is one of the key features of a reasonably accurate soil-structure interaction analysis. The work of Lysmer (Lysmer and Richart, 1966) which identified viscous damping as a means to represent the dissipation of energy into a half space sparked the development of modern SSI methodology. The other means of dissipating energy in the soil medium is by the inelastic response of the soil which is modeled by linear hysteretic damping in conjunction with an equivalent linear secant modulus.

It becomes obvious that, given the complexities of the problem, there are no methods currently available that can solve the SSI problem exactly. Some of the many uncertainties inherent in the analyses are as follows:

- The nature of the seismic input at a site cannot be precisely predicted.
- The in-situ soil properties and their strain-dependency cannot be determined accurately.
- The equivalent-linear modulus and hysteretic damping combination to account for inelastic material behavior introduces error.

- The level of adhesion between soil and foundation and possible separation during uplift for a surface footing and separation from sidewalls for an embedded footing is not explicitly considered by the methods studied.

Because of these and other uncertainties, an SSI study should include several analyses with variation of parameters. During the case studies, it was shown that the use of only one input record could give an erroneous picture of response because of idiosyncracies of a given record. Thus, a suite of records should be used.

Two general types of analysis methods have been described, applied to various aspects of the SSI problem, and compared. One is commonly referred to as the direct method which describes a method whereby soil and structure are analyzed in one step. However, for complex structures a two-step method is frequently employed. The other type of analysis is termed the substructure method which explicitly treats the problem in a sequence of steps which include: determination of the motion of a massless foundation, determination of the foundation impedances, and solution of the coupled soil-structure system. Each analysis type accounts for the elements of SSI differently. The success of each for various aspects of the problem are evaluated by means of case studies.

Early SSI methods used frequency-independent impedances to represent the soil which was assumed to be an elastic half space. By varying assumed soil moduli, conventional modal analysis procedures can accomodate virtually any range of stiffness (real

part of impedance) which is used to characterize the site. However, high radiation damping can be difficult to account for in a modal analysis, and large damping makes the use of undamped modal properties suspect.

The first set of case studies showed that composite modal damping, either by the strain-energy approach or that by Roesset, et.al. (1973) resulted in reasonable design values when compared to confirmatory direct time-history analyses. A possible defect in the composite modal damping techniques is that they tend to overestimate damping in certain modes. It was shown that arbitrarily limiting damping can result in artificially high response at certain frequencies and low response at others. If modal analysis is not deemed suitable for a problem with high soil damping, one should use a more rigorous analysis method rather than arbitrarily limiting composite modal damping.

One of the parameters which has a significant effect on soil-structure interaction is embedment. It has been shown that embedment increases static stiffness and damping values and changes the frequency dependence characteristics of the stiffness. Another effect of embedment illustrates one of the basic results of SSI, that is, the effect the structure has on the effective motion of the foundation. Kausel, et.al. (1978), suggested simplified methods to account for the decreased horizontal motion and inducement of rocking for an embedded foundation. As a result of case studies, it appears that Roesset's (1980) suggested procedure for horizontal motion is more appropriate for the structure studied herein.

The effects of an embedment ratio, E/r , of 0.46 were calculated by the substructure technique as performed by CLASSI. The peak forces and accelerations showed an average decrease from the surface case by factors of 1.18 to 1.29 depending on the stiffness of the site. The effects on in-structure response spectra are relatively large at some frequencies. For a soft site ($v_s = 500$ fps) the maximum spectral ordinate for the horizontal direction is reduced roughly by one-half due to embedment. The vertical spectra on the base are close for both cases and, at some frequencies, have larger ordinates for the embedded case. These increased spectral responses for the embedded case are caused by increased stiffness in the absence of other mitigating factors. On the other hand, the effect of increase in horizontal stiffness is overshadowed by the reduction in input due to scattering, the out-of-phase rotational input, and the increased damping as a result of embedment. In general, spectra at various locations in the structure showed a frequency shift and peak decrease for the embedded case.

Two early SSI analysis procedures were investigated by techniques similar to those used for the above comparisons: modal analysis without radiation damping and fixed-base analysis of structures founded on rock. Since frequency-independent (FI) methods were developed for use with machine foundations, a single-degree-of-freedom model was usually used in the analysis. For this type of model, the large radiation damping coefficient didn't present a problem. However, when FI techniques were applied to multi-degree-of-freedom structures, analysts didn't know how to

incorporate this damping into a modal analysis and, thus, ignored radiation damping. A comparison between this type of analysis and CLASSI indicated that average peak accelerations were overpredicted by the FI method by factors of 1.93 to 2.71. In-structure spectra were affected by even larger overpredictions, by increases in peak responses by up to a factor of 14 and addition of spurious peaks. Thus, it is shown that radiation damping plays a major role in SSI.

The evaluation of fixed-base analysis for a structure founded on rock ($v_s = 3500$ fps) showed that peak values of forces and accelerations were overpredicted by the fixed base analysis by factors of 1.16 to 1.28. There is a small shift in frequency of spectral peaks when the foundation compliance is considered. However, the major effect is a result of damping in the compliant medium. This is most dramatic in the vertical direction where high radiation damping occurs. The peak spectral ordinate in the vertical direction was reduced by a factor of about three when radiation damping was considered.

Most of the results involving comparisons with explicit means of accounting for various SSI phenomena showed levels of conservatism. Overprediction of structural forces usually results in a safer design and, in this context, reasonable conservatism is desirable. Obviously, gross overprediction can become an economic concern and can be wasteful. However, for risk assessment of nuclear power plants it is necessary to determine "best estimate" response and thus, it is desirable that the truest possible estimate of response be determined.

Spectral peaks show greater overprediction than peak forces. While it may seem desirable to have large overprediction of spectra for the design of equipment and subsystems; for piping systems, a less safe system may result. The addition of supports to piping to resist seismic loading, from an improbable event, may increase loads under normal service conditions such that the factor of safety under frequent loading conditions, e.g., thermal transients, would be reduced. The placement of constraints to resist seismic loading and the freedom of piping to move under thermal loading are a source of compromise in piping design. Gross overprediction of seismic load does not serve the overall goal of plant safety.

Finally, a summary was presented of a case study which compared the substructure approach, i.e., CLASSI, with a finite element "direct" approach, i.e., FLUSH. For complex models, the FLUSH approach isn't really direct since a two-stage analysis is usually performed. Each approach has its strengths and weaknesses. For a rather straightforward, axisymmetric structure, the agreement between the two methods was excellent. However, the variability in structural response due to the SSI analysis procedure employed increased with increasing complexity in the physical situation that was modeled.

Given the convenience and versatility of the substructure approach, it is suggested that the finite element method might be better suited to determination of impedance functions and scattering matrices of complex foundations for application within the substructure method rather than for a direct SSI analysis. While

this application would eliminate the consideration of secondary nonlinearities (which haven't been shown to have a large effect), it also eliminates the possible source of error in conducting two-stage analyses. In the substructure approach, a massless foundation is analyzed and there is no worry about correctly representing the modal properties of the structure with the simplified first-stage model. In addition, the variation of soil parameters is accommodated very easily with the substructure method since the fixed base structure need only be analyzed once and the massless foundation analysis is a separate step.

Some suggestions are offered for future research:

- Study similar cases to those reported herein with different embedment ratios to see if the same conclusions hold true.
- Investigate phasing between horizontal and rotational input due to embedment for different embedment ratios and foundation shapes.
- Investigate the effect of soil layering, including a shallow soil layer on bedrock.
- Investigate the combination of layering with different embedment depths.
- Study explicitly the applicability of simplified methods.
- Evaluate the effects of modal truncation and possible corrections when employing component mode synthesis.

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