A Ring Model of the Lasertron

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A RING MODEL OF THE LASERTRON

by

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ABSTRACT

A large-signal, 2-1/2 dimensional relativistic particle dynamics code has been written to simulate the electrical behavior of the lasertron. The theory and the approximations involved in this code are discussed in detail. Numerical results are presented to show the comparison with previous work and to show the general variation in lasertron performance with frequency, beam voltage, and beam current. The code has also been used to design a prototype 6-GHz lasertron for construction at the Laboratoire de l'Accélérateur Linéaire in Orsay, France. The calculations show that this prototype can produce 4.7-MW peak-power output with a conversion efficiency of 65%. At 3 GHz, the power would be increased by a factor approaching 10, and the conversion efficiency would be over 70%.

I. INTRODUCTION

The lasertron is a device to transform high-voltage power to microwaves, and it is much like a klystron with a photocathode. The beam is bunched in the lasertron by the laser light that is modulated at the microwave frequency. The photoelectrons are produced at a cathode (see Fig. 1) and are accelerated to a fairly high voltage in the gun region by an electric field. These electrons produce microwaves in the output section, which is one or more cavities that decelerate the beam with the microwave fields. With the present availability of powerful lasers with picosecond light pulses, there is little doubt that this device can be an efficient microwave generator at frequencies below 1 or 2 GHz, but the debunching electron dynamics are quite complicated; thus, the upper frequency limits are not well defined. This report discusses a computer model of the lasertron. This model

can be used to predict general trends of lasertron performance, dependent on the device parameters, and the model can also be used to design lasertrons for particular applications.

Fig. 1. Schematic diagram of a lasertron.

Codes for the ring-and-disk model of the electrodynamics in the klystron have recently been written.^{1,2,3} These codes have been tested with two types of 1.25-MW klystrons used at Los Alamos and with the Stanford Linear Accelerator Center (SLAC) 50-MW klystron; good agreement was obtained between predicted and actual operation of the electronic efficiency and the power-transfer characteristics. These klystrons had dc to rf conversion efficiencies of between 50 and 60%. A similar program has been written to study the lasertron interaction. The main features of the ring model of the lasertron are described below.

The relativistic electron dynamics of a set of particles are followed from the photocathode through the gun region and through the output region, with time as the independent variable. The fields are summed by superposition to find the total field at each electron's position. The model is 2-1/2 dimensional, because r and z are tracked for each particle, but v_ϕ is obtained from the relativistic form of Busch's theorem. Wake fields and the transient filling of the output cavity are ignored. A single bunch of electrons is followed through the lasertron, but the code can easily be modified to calculate the dynamics of a small number of bunches at the expense of computer time.

The program is modular and flexible; therefore, many of the fields can be approximated in several ways to alter the accuracy and time requirements to perform the simulation. In several instances, the work is not complete in that the various approximations have not all been carried out. In each field calculation, the equations that have been programmed

are described, and other alternatives will be described that could be used to speed up the execution time or to improve the overall accuracy.

For the examples in this report, the particles all have the same charge; thus, the laser light pulse is assumed to be rectangular. This restriction has recently been removed to allow trapezoidal, Gaussian, or arbitrary light pulses to be simulated. All processes in the photocathode are assumed to be instantaneous. The beam model is a central disk surrounded by rings. This combination starts at the cathode in a plane and is repeated many times over the length of the laser pulse. Typically, between 20 and 200 particles are followed, and between 0.5 and 50 minutes of VAX 11/785 time are needed to do a simulation. A separate graphical output program may be run to obtain the pictures of the interaction. The static electric fields in the gun region have both E_r and E_z components, and the static magnetic fields have B_r and B_z components. Both the space-charge and the output-cavity fields have E_r , E_z , and B_ϕ components.

II. EQUATIONS OF MOTION

Starting with the Lorentz force and the energy conservation equations⁴

$$\frac{d}{dt}(m_0\gamma\vec{v}) = e(\vec{E} + \vec{v} \times \vec{B}) \quad (1)$$

and

$$\frac{d}{dt}(m_0\gamma c^2) = e\vec{v} \cdot \vec{E} \quad (2)$$

(with the following normalizations:

$$\begin{aligned} \tau &= \omega t, \\ \vec{V} &= \vec{v}/u_0, \\ u_n &= u_0/c, \\ \eta &= |e|/m_0, \\ R &= \omega r/u_0, \text{ and} \\ Z &= \omega z/u_0, \end{aligned} \quad (3)$$

one finds that the force equations in cylindrical coordinates are

$$\frac{dV_z}{d\tau} = \frac{-\eta}{\omega\gamma} \left\{ \frac{E_z}{u_0} [1 - (u_n V_z)^2] - V_\phi B_r + V_r (B_\phi - u_n V_z E_r/c) \right\} \quad (4)$$

and

$$\frac{dV_r}{d\tau} = \frac{-\eta}{\omega\gamma} \left\{ \frac{E_r}{u_0} [1 - (u_n V_r)^2] + V_\phi B_z - V_z (B_\phi + u_n V_r E_z/c) \right\} + V_\phi^2/R, \quad (5)$$

where the velocities have been normalized to u_0 , the velocity of an electron on the axis and at the anode voltage. The equation for the conservation of angular momentum is

$$V_\phi = \frac{\eta R B_z}{2\omega\gamma} \left(1 - \frac{R_c^2 B_{zc}}{R^2 B_z} \right), \quad (6)$$

and the energy equation is

$$\frac{d\gamma}{d\tau} = -\frac{\eta u_n}{c\omega}(E_r V_r + E_z V_z) \quad . \quad (7)$$

Of course, γ is also given by

$$\gamma = [1 - u_n^2(V_r^2 + V_\phi^2 + V_z^2)]^{-1/2} \quad . \quad (8)$$

The γ 's are calculated by Eqs. (7) and (8), and the difference is used to estimate the integration errors. The new positions are found from the velocities

$$Z = \int V_z d\tau \quad (9)$$

and

$$R = \int V_r d\tau \quad . \quad (10)$$

The equations of motion, Eqs. (4) to (10), are numerically solved by the Euler method with τ as the independent variable.

There are a variable number of particles in the program. After the beam is injected, there are N_z axial slices, with N_r rings in each slice. At each time step within the laser pulse, the subroutine TESTNEW determines if a new set of N_r particles should be emitted at the cathode. When the laser pulse is over, a total of $N_r N_z$ particles will have been emitted. These electrons all start with zero velocity and are positioned on the cathode so that each particle represents equal charge. Thus, thermal velocity effects are neglected, and the beam initially has zero emittance.

III. ELECTROMAGNETIC FIELDS

The calculation of the field components is the most difficult task in the simulation. The philosophy is to integrate the other programs of the electron dynamics community and to use their output as input data for the lasertron program. Thus, the gun's static electric fields may be calculated with Herrmannsfeldt's gun program,⁵ the cavity fields calculated with SUPERFISH,⁶ and the magnetic focus fields calculated with POISSON.⁷

IV. SPACE-CHARGE FIELDS

Because the bunch changes its shape so quickly in the lasertron, the space-charge forces must be evaluated at every time step. The first action taken by the space-charge routine CHADESPAC is to set the five field components E_r , E_z , B_r , B_z , and B_ϕ to zero. At present there are three methods that may be used to calculate the space-charge fields: the ring model, a one-dimensional disk method, and a two-dimensional approximate model. All these methods are Green function methods; hence, the computation time varies as the square of the number of particles. One expects that the use of a finite-difference or finite-element computation and a fast Fourier transform method would significantly

reduce the space-charge computation time for large number of particles because this type of computation takes a time proportional to $n \log n$.

The ring Green function method has been tested by varying the number of rings and slices in a beam and checking the space-charge fields for a beam of constant velocity. The essence of the method⁸ is to solve Laplace's equation for a ring (or disk) of charge that is surrounded by a perfectly conducting drift tube. The potential in the rest frame of the ring is

$$\tilde{\Phi}(r, \tilde{z}; r', \tilde{z}') = \sum_{n=1}^{\infty} \frac{J_0(\nu_n r) J_0(\nu_n r')}{\nu_n a^2 J_1(\nu_n a)^2} \exp(-\nu_n |\tilde{z} - \tilde{z}'|) \quad (11)$$

and

$$\tilde{A}(r, \tilde{z}; r', \tilde{z}') = 0 \quad , \quad (12)$$

where $r, \tilde{z}$ are the coordinates of the observation point, and $r', \tilde{z}'$ are the coordinates of the source point of the ring of charge q inside the drift tunnel of radius a . The tildes denote quantities that will change under a Lorentz transformation. The eigenvalues are the solutions of

$$J_0(\nu_n a) = 0, \quad (13)$$

where a is the radius of the drift tube. The fields in the laboratory frame are found by a Lorentz transformation using the source particle's z velocity. The covariant four vectors

$$\Phi^i = (c\tilde{A}, \tilde{\Phi}) \quad (14)$$

and

$$x^i = (x, y, z, t) \quad (15)$$

both transform as $\Phi^j = Q_i^j \phi^i$ and $x^j = Q_i^j x^i$, where

$$Q_i^j = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & \beta\gamma \\ 0 & 0 & -\beta\gamma & \gamma \end{pmatrix} \quad (16)$$

and Q_i^j is the transformation matrix from the beam frame to the laboratory frame, v_z is the axial velocity of the source electron, $\beta = v_z/c$, and $\gamma = (1 - \beta^2)^{-1/2}$. The potentials in the laboratory frame are then

$$\Phi(r, z; r', z') = \frac{\gamma}{2\pi\epsilon_0} \sum_{n=1}^{\infty} \frac{J_0(\nu_n r) J_0(\nu_n r')}{\nu_n a^2 J_1^2(\nu_n a)} \exp(-\nu_n \gamma |z - z'|) \quad (17)$$

and

$$\tilde{A}(r, z; r', z') = \frac{\hat{z} v_z \gamma}{2\pi\epsilon_0 c^2} \sum_{n=1}^{\infty} \frac{J_0(\nu_n r) J_0(\nu_n r')}{\nu_n a^2 J_1^2(\nu_n a)} \exp(-\nu_n \gamma |z - z'|) \quad . \quad (18)$$

The laboratory frame fields are found from the potentials as

$$E_r(r, z; r', z') = \frac{\gamma}{2\pi\epsilon_0 a^2} \sum_{n=1}^{\infty} \frac{J_1(\nu_n r) J_0(\nu_n r')}{J_1^2(\nu_n a)} \exp(-\nu_n \gamma |z - z'|) , \quad (19)$$

$$E_z(r, z; r', z') = \frac{\text{sgn}(z - z')}{2\pi\epsilon_0 a^2} \sum_{n=1}^{\infty} \frac{J_0(\nu_n r) J_0(\nu_n r')}{J_1^2(\nu_n a)} \exp(-\nu_n \gamma |z - z'|) , \quad (20)$$

and

$$B_\phi(r, z; r', z') = \frac{v_z}{c^2} \mathcal{G}_r(r, z; r', z') . \quad (21)$$

The total space-charge fields are found by integrating the Green functions times the charge density over the volume occupied by the beam.

$$\vec{E}_{sc}(r, z) = \int \int \int \rho'(V') \vec{E}(r, z; r', z') dV' ,$$

where $\rho'(V')$ is the charge density in the beam. The Green functions for the zero-volume sources become infinite as one approaches the singularity, so it is prudent to use a finite volume source as the Green function. Consider the beam when all of it is moving with the axial velocity u_0 , which corresponds to V_0 , the voltage on the axis after the gun region. Each ring is given an axial extent of L_0/N_z and a radial extent dr so that the beam volume is exactly filled. The term L_0 is the distance the beam travels in one rf cycle when the beam velocity is u_0 . It is more convenient to define the normalized Green functions by the relations

$$E_r(r, z; r', z') = \frac{\gamma}{2\pi\epsilon_0 a^2} F_{2r}(r, z; r', z') \quad (22)$$

and

$$E_z(r, z; r', z') = \frac{\text{sgn}(z - z')}{2\pi\epsilon_0 a^2} F_{2z}(r, z; r', z') . \quad (23)$$

By integrating the normalized functions as described above, one finds the expressions³

$$\bar{F}_{2r}(r, z; r', z') =$$

$$\sum_{n=1}^{\infty} \frac{2}{(r' + dr)^2 - (r' - dr)^2} \{ (r' + dr) J_1[\nu_n(r' + dr)] - (r' - dr) J_1[\nu_n(r' - dr)] \}$$

$$\{ \exp[-\gamma \nu_n(|z - z'| - L_0/2N_z)] - \exp[-\gamma \nu_n(|z - z'| + L_0/2N_z)] \} \frac{N_z(\nu_n r)}{\gamma L_0 \nu_n^2 J_1^2(\nu_n a)} , \quad (24)$$

and

$$\bar{F}_{2z}(r, z; r', z') =$$

$$\sum_{n=1}^{\infty} \frac{2}{(r' + dr)^2 - (r' - dr)^2} \{ (r' + dr) J_1[\nu_n(r' + dr)] - (r' - dr) J_1[\nu_n(r' - dr)] \} \\ \{ \exp[-\gamma \nu_n(|z - z'| - L_0/2N_z)] - \exp[-\gamma \nu_n(|z - z'| + L_0/2N_z)] \} \frac{N_z J_0(\nu_n r)}{\gamma L_0 \nu_n^2 J_1^2(\nu_n a)} \quad (25)$$

for $|z - z'| > L_0/2N_z$; likewise for the other region,

$$\bar{F}_{2r}(r, z; r', z') =$$

$$\sum_{n=1}^{\infty} \frac{2}{(r' + dr)^2 - (r' - dr)^2} \{ (r' + dr) J_1[\nu_n(r' + dr)] - (r' - dr) J_1[\nu_n(r' - dr)] \} \\ \{ 2 - \exp[-\gamma \nu_n(|z - z'| + L_0/2N_z)] - \exp[\gamma \nu_n(|z - z'| - L_0/2N_z)] \} \frac{N_z J_1(\nu_n r)}{\gamma L_0 \nu_n^2 J_1^2(\nu_n a)} \quad (26)$$

and

$$\bar{F}_{2z}(r, z; r', z') =$$

$$\sum_{n=1}^{\infty} \frac{2}{(r' + dr)^2 - (r' - dr)^2} \{ (r' + dr) J_1[\nu_n(r' + dr)] - (r' - dr) J_1[\nu_n(r' - dr)] \} \\ \{ \exp[\gamma \nu_n(|z - z'| - L_0/2N_z)] - \exp[-\gamma \nu_n(|z - z'| + L_0/2N_z)] \} \frac{N_z J_0(\nu_n r)}{\gamma L_0 \nu_n^2 J_1^2(\nu_n a)} \quad (27)$$

for $0 < |z - z'| < L_0/2N_z$. The finite-volume normalized Green functions, Eqs. (24)-(27), are the ones used in the program for the Bessel function space-charge option. The second method is the one-dimensional disk method, which has the advantage of being very quick, but it is the least accurate space-charge field method. The potential on the axis of a disk of radius r' and position $\tilde{z}'$ is known as

$$\bar{\Phi}(0, \tilde{z}; r', \tilde{z}') = \frac{q}{2\pi\epsilon_0 r'^2} \left[\sqrt{(\tilde{z} - \tilde{z}')^2 + r'^2} - |\tilde{z} - \tilde{z}'| \right] \quad (28)$$

in the beam frame. The axial field in the beam frame is therefore

$$\bar{E}_z(0, \bar{z}; r', \bar{z}') = \frac{q \text{sgn}(\bar{z} - \bar{z}')}{\pi \epsilon_0 r'^2} \left[1 - \frac{|\bar{z} - \bar{z}'|}{\sqrt{(\bar{z} - \bar{z}')^2 + r'^2}} \right], \quad (29)$$

and this transforms into the laboratory frame as

$$E_z(0, z; r', z') = \frac{q \text{sgn}(z - z')}{\pi \epsilon_0 r'^2} \left[1 - \frac{\gamma |z - z'|}{\sqrt{\gamma^2 (z - z')^2 + r'^2}} \right]. \quad (30)$$

One can also calculate the transverse field components in the laboratory frame, but these are neglected in this one-dimensional approximation.

The third method for the space-charge fields is called the economic two-dimensional method, so called because it also is approximate but is economical with computer time. This method is relativistically correct to first order because the fields are calculated electrostatically in the beam frame of reference. Each source ring is treated as a ring for the innermost observer rings, as a charge ribbon for intermediate cases, and as a point charge for the outermost observation rings. Different approximations are used in the output region to approximately include the radial boundary condition in the output region: E_z is neglected and E_r is assumed to follow Gauss's law. It is the only method programmed to date that has the images of the charges in the cathode plane. This is the method that has been used for most of the calculations with large numbers of particles as it is about 10 times faster than the Bessel function method. In the laboratory frame, the axial field in the gun region is

$$E_z(r, z; r', z') = \frac{q(z - z')}{4\pi\epsilon_0[\gamma^2(z - z')^2 + r'^2]^{3/2}} \quad (31 - a)$$

for $r < r'$,

$$E_z(r, z; r', z') = \frac{q}{A_r \pi \epsilon_0} \arctan \frac{dr}{2\gamma(z - z')} \quad (31 - b)$$

for $r \simeq r'$, and

$$E_z(r, z; r', z') = \frac{q(z - z')}{4\pi\epsilon_0[\gamma^2(z - z')^2 + r^2]^{3/2}} \quad (31 - c)$$

for $r > r'$, where A_r is the area of the washer. The corresponding radial electric field is

$$E_r(r, z; r', z') = 0 \quad (32 - a)$$

for r an inner ring,

$$E_r(r, z; r', z') = \frac{q}{4\pi\epsilon_0} \frac{r}{[r_c^2 + \gamma^2(z - z')^2]^{3/2}} \quad (32 - b)$$

for all other r , where r_c is the maximum of r, r' , and the magnetic field is given by Eq. (20).

There are at least three major improvements that are being considered for the space-charge fields. The first is to use the Green function for a ring of charge in a cylindrical box⁹ in the gun region

$$\tilde{\Phi}(r, \tilde{z}; r', \tilde{z}') = \frac{1}{\pi \epsilon_0 a^2} \sum_{n=1}^{\infty} \frac{\sinh(\nu_n \tilde{z}) \sinh[\nu_n (\tilde{z}_a - \tilde{z})]}{\sinh(\nu_n \tilde{z}_a)} \frac{J_0(\nu_n r') J_0(\nu_n r)}{\nu_n J_1^2(\nu_n a)} \quad (33 - a)$$

for $\tilde{z} < \tilde{z}'$, and

$$\tilde{\Phi}(r, \tilde{z}; r', \tilde{z}') = \frac{1}{\pi \epsilon_0 a^2} \sum_{n=1}^{\infty} \frac{\sinh[\nu_n (\tilde{z}_a - \tilde{z})] \sinh(\nu_n \tilde{z}')}{\sinh(\nu_n \tilde{z}_a)} \frac{J_0(\nu_n r') J_0(\nu_n r)}{\nu_n J_1^2(\nu_n a)} \quad (33 - b)$$

for $\tilde{z} > \tilde{z}'$, where the eigenvalues are still given by Eq. (13). This expression puts an image charge both at the plane $\tilde{z} = 0$ and at the plane $\tilde{z} = \tilde{z}_a$, an anode plane. It should be possible to combine these Green functions for the gun region with the simpler open drift-tube expression of Eq. (11). One could also use different radii in the two regions for the radial boundary conditions. The Green function for a cylindrical tube, closed at only one end, is¹⁰

$$\begin{aligned} \tilde{\Phi}(r, \tilde{z}; r', \tilde{z}') &= \frac{1}{\pi \epsilon_0 a^2} \sum_{n=1}^{\infty} \frac{J_0(\nu_n r') J_0(\nu_n r)}{\nu_n J_1^2(\nu_n a)} \\ &\times \begin{cases} \exp(-\nu_n \tilde{z}) \sinh(\nu_n \tilde{z}') & \text{for } \tilde{z}' < \tilde{z} \\ \exp(-\nu_n \tilde{z}') \sinh(\nu_n \tilde{z}) & \text{for } \tilde{z}' > \tilde{z} \end{cases} . \end{aligned} \quad (34)$$

The radial boundary conditions cause the infinite series in the potentials. Thus, another method that would certainly reduce the amount of computer time would be to ignore the radial boundaries and use the free-space expression for the potential of a charge ring¹¹

$$\tilde{\Phi}(r, \tilde{z}; r', \tilde{z}') = \frac{1}{2\pi^2 \epsilon_0 \tilde{r}_1} K(\tilde{k}) , \quad (35)$$

where

$$\tilde{r}_1^2 = (\tilde{z} - \tilde{z}')^2 + (r' + r)^2 , \quad (36)$$

$$\tilde{k} = \frac{2\sqrt{rr'}}{\tilde{r}_1} , \quad (37)$$

and $K(k)$ is the complete elliptic integral of the first kind. The advantage of this method is that the power series¹² for the elliptic integrals is as short as the power series¹² for a single Bessel function. The cathode boundary condition may be satisfied with an image ring. It is not too difficult to calculate the following fields from the potential¹¹

$$\tilde{E}_r(r, \tilde{z}; r', \tilde{z}') = \frac{1}{4\pi^2\epsilon_0 r \tilde{r}_1} \left\{ K(\tilde{k}) - \frac{r'^2 - r^2 + (\tilde{z} - \tilde{z}')^2}{\tilde{r}_1^2} \frac{E(\tilde{k})}{1 - \tilde{k}^2} \right\} \quad (38)$$

$$\tilde{E}_z(r, \tilde{z}; r', \tilde{z}') = \frac{1}{2\pi^2\epsilon_0} \frac{\tilde{z} - \tilde{z}'}{\tilde{r}_1^3} \frac{E(\tilde{k})}{1 - \tilde{k}^2}, \quad (39)$$

where $E(k)$ is the complete elliptic integral of the second kind. As before, these Green functions may be Lorentz transformed from a static frame to the laboratory frame and integrated over a finite volume to remove the singularity at the source point, but this must be done numerically.

Another method that has an excellent chance of drastically reducing the computation time required for the space-charge fields is to use the Buneman fast Fourier transform method that has been modified by Vaughan¹³ for cylindrical coordinates. This method uses a moving grid that is one wavelength long to calculate the potential from all the charges at the grid points. The grid extends to a radial boundary and moves with the beam, so a rather small grid is used. The space-charge fields are assumed to be periodic in the axial direction, but this assumption is good in the lasertron as the effect of one bunch on another is not very important. The computation time is expected to be much shorter than for the Green function method because of the $n \log n$, rather than n^2 dependence of the computation time on the number of particles. It is also worth noting that Vaughan has developed a fast simultaneous differentiation and interpolation routine INTRA¹⁴ that should save time in all the Green function calculations. The interpolation procedure in the Bessel function case is complicated and time consuming. With the introduction of the Buneman-Vaughan subroutine INTRA, one could reduce the computer time by at least half because the potentials are simpler expressions than the fields, and there is only a single potential rather than two fields to calculate. The infinite series for the potential also converges faster than the fields because there is a factor of ν_n in the former denominator.

The most accurate method for calculating the space-charge fields is to solve the Maxwell equations with the moving bunch as a source as is done in MASK,¹⁵ Shintake's FCI,¹⁶ or Weiland's BCI.¹⁷ All of these programs can take into account the real, complicated boundary conditions. Because FCI solves only for a single component of the vector potential, one expects that the FCI program would be smaller and faster than the three-field programs.

V. ELECTROSTATIC FIELDS

The electrostatic fields in the gun region are approximated in several ways. The simplest is to take the one-dimensional expression

$$E_z(z) = \begin{cases} -V_a/z_a & \text{for } 0 \leq z \leq z_a \\ 0 & \text{for } z > z_a \text{ and } z < 0 \end{cases} . \quad (40)$$

The most accurate approximation is to run Herrmannsfeldt's electron gun code⁵ for the actual lasertron geometry, but for a very small beam current, and to write the potential to a disk file. This data file is then reformatted with a simple program and used as input data in the lasertron program.

Another method that has been programmed is to use a spherical static field. This method is useful to determine a new geometry before using the Herrmannsfeldt program. In this case the axial static field is

$$E_z(R, Z) = \frac{Q_v}{4\pi\epsilon_0} \frac{(Z - Z_v)}{[R^2 + (Z - Z_v)^2]^{3/2}} , \quad (41)$$

and the radial static field is

$$E_r(R, Z) = \frac{Q_v}{4\pi\epsilon_0} \frac{R}{[R^2 + (Z - Z_v)^2]^{3/2}} , \quad (42)$$

where Q_v and Z_v are parameters that are chosen to match the real field distribution.

Another possible method is to approximate the static potential in the gun region by a few terms of the series

$$\Phi(R, Z) = \sum_j^{ISTAT} A_j J_0(T_j R) \sinh(T_j Z) , \quad (43)$$

where Φ is normalized such that $\Phi(0, Z_a) = 1$. The electric fields are then

$$E_r(R, Z) = \sum_{j=1}^{ISTAT} A_j J_1(T_j R) \sinh T_j Z \quad (44)$$

and

$$E_z(R, Z) = \sum_{j=1}^{ISTAT} A_j J_0(T_j R) \cosh T_j Z , \quad (45)$$

where $ISTAT$ is the maximum number of terms used in the approximation. These last two equations have not yet been programmed because it will be difficult to relate the A_j 's and T_j 's to the electron gun geometry.

VI. STATIC MAGNETIC FIELDS

The static focus field has been assumed to be piecewise linear, and it may have 10 breakpoints from the cathode to the end of the output cavity. The axial field is constant before the first breakpoint and after the last breakpoint. Between breakpoints, the fields on the i th electron that is to the j th breakpoint are

$$B_{zi}(R, Z) = B_{zj} + (Z_i - Z_{bj}) SLOPE_j \quad (46)$$

and

$$B_{ri}(R, Z) = -0.5 SLOPE_j \quad (47)$$

where $SLOPE_j$ is dB_z/dZ . To use this option, a small table of the breakpoint fields and positions are read as input data.

A second option, which has not yet been incorporated into the code, is to read a data file of B_r, B_z fields that have been computed with POISSON, a magnetic field solver program. With this method, ferromagnetic components are allowable, but one has to make POISSON runs whenever the ferromagnetic components are changed.

A third option that has been used in the past is to evaluate the fields of j ideal current loops over a magnetic grid and then to interpolate the grid fields to find the fields at the particle positions. The magnetic fields are¹⁸

$$B_r(r, z) = \sum_j \frac{\mu_0 i_j (z - z_j)}{2\pi r r_{1j}} \left\{ \left[\frac{r_j^2 + r^2 + (z_j - z)^2}{r_{1j}^2} \right] E(k_j) - K(k_j) \right\} \quad (48)$$

and

$$B_z(r, z) = \sum_j \frac{\mu_0 i_j}{2\pi r_{1j}} \left\{ \left[\frac{r_j^2 - r^2 - (z_j - z)^2}{r_{1j}} \right] E(k_j) + K(k_j) \right\} , \quad (49)$$

where

$$r_{1j} = \sqrt{(r_j + r)^2 + (z_j - z)^2} \quad (50)$$

and

$$k_j^2 = 4rr_j/r_{1j} . \quad (51)$$

VII. RF OUTPUT CAVITY FIELDS

There are three methods of calculating the output cavity fields that may be used in the program. The most realistic is to use output from the SUPERFISH code. The B_ϕ component for a single eigenmode of a given cavity is calculated with SUPERFISH for the empty cavity, and this field is written as a data file that is read by the lasertron program. These data are interpolated and the appropriate derivatives are taken to find the fields for each electron in the output section for each time step. The amplitude and phase of these fields may be set by either of the methods that are discussed below.

The simplest method of estimating the output fields is to use the planar gap approximation

$$E_z(r, z) = V_{rf}/D_{rf} \cos(\omega t + \phi_{rf}) , \quad (52)$$

where V_{rf} , D_{rf} , and ϕ_{rf} are the gap voltage on the axis, the length of the output region in which there is field, and the phase angle of the output fields, respectively.

The third option for the output fields is to use a few terms of the series

$$E_z(r, z) = \frac{V_{rf}}{D_{rf}} \sum_{m=1}^{MRF} A_{rf}(m) \frac{I_0(\gamma_m r)}{I_0(\gamma_m a)} \cos[k_m(z - z_c)] \cos(\omega t + \phi_{rf}) . \quad (53)$$

The other field components are

$$E_r(r, z) = -\frac{V_{rf}}{D_{rf}} \sum_{m=1}^{MRF} \frac{A_{rf}(m) k_m I_1(\gamma_m r)}{\gamma_m I_0(\gamma_m a)} \sin[k_m(z - z_c)] \cos(\omega t + \phi_{rf}) \quad (54)$$

and

$$B_\phi(r, z) = -\frac{V_{rf}}{D_{rf}} \sum_{m=1}^{MRF} \frac{\omega A_{rf}(m) I_1(\gamma_m r)}{c^2 \gamma_m I_0(\gamma_m a)} \cos[k_m(z - z_c)] \sin(\omega t + \phi_{rf}) , \quad (55)$$

where

$$k_m = \pi(2m - 1)/D_{rf},$$

$$\gamma_m^2 = k_m^2 - (\omega/c)^2,$$

and z_c is the axial location of the gap center.

With all three methods of calculating the output-gap fields, the amplitude and the phase of these fields remain to be chosen. The simplest method is to adjust these values to obtain the maximum energy conversion efficiency, but the corresponding cavity impedance may not be physically realizable. The self-consistent method of calculating the gap voltage and phase is as follows.² The cavity voltage is the voltage on the axis,

$$V_{rf}(t) = \int_{-\infty}^{\infty} \vec{E}_{cavity} \cdot d\vec{z}, \quad (56)$$

but this voltage is also a solution of

$$V_{rf} = i_{ind1} Z_{cav} , \quad (57)$$

where Z_{cav} is the cold-cavity impedance

$$1/Z_{cav} = 1/R_{sh} + j(f/f_0 - f_0/f)/(R_{sh}/Q) , \quad (58)$$

R_{sh} is the cavity shunt impedance, Q is the loaded Q of the empty cavity, and f_0 is the resonant frequency of the output cavity. The current induced in the output circuit by the beam is given by the Ramo theorem,

$$i_{ind}(t)V_{rf}(t) = \int_V \vec{J} \cdot \vec{E}_{cavity} dv , \quad (59)$$

where the integral is over the output cavity volume. The Fourier component of the induced current is given by

$$i_{ind_n} = 1/f \int_0^{1/f} i_{ind}(t) \exp(j2\pi f n t) dt . \quad (60)$$

The higher components of Eq. (60) are not used at present. They are only required for harmonic cavities or cavities for which the impedance is important at more than one harmonic.

With the self-consistent method of the output voltages and phases, the calculations begin with a cavity impedance, based on realizable cavities. Next, a first approximation to the gap phase and amplitude is made, and the particles are tracked across the output region. When all the particles leave the gap, the induced current integral is complete, and the gap phase and voltage are recalculated. A relaxation parameter is used to estimate the next pair of voltages and phases, and the output cavity dynamics are redone until convergence criterion is satisfied. Three to six iterations are usually required for one part in 1000 self-consistency.

VIII. DC TO RF CONVERSION EFFICIENCY

The simplest procedure to estimate the dc to rf conversion efficiency is to ignore the output gap and calculate the fundamental harmonic current and the axial momentum spread in the beam at the location of the center of the output gap. If a gap were correctly placed and coupled to the bunched beam, Mihran¹⁹ has shown that the quantity

$$MF = \frac{i_1 (\gamma v_z)_{slow}}{2I_0 (\gamma v_z)_{avg}} \quad (61)$$

is a good estimate of the interaction efficiency. In Eq. (61), i_1/I_0 is the first Fourier component of the beam current, and the axial momentum is evaluated both for the slowest electron and for the average electron.

Rather than use the merit-figure approximation, it is more accurate to actually calculate the electron dynamics through the output gap with realistic, self-consistent gap fields and to estimate the efficiency as

$$\eta = \frac{1}{N_z N_r (\gamma_0 - 1)} \sum_i (\gamma_i - 1) , \quad (62)$$

where γ_0 is the relativistic factor for the dc anode voltage. This efficiency is the fraction of the kinetic energy that is transferred to the microwave fields. Some of this field energy is then lost as ohmic heating of the output cavity, but this is a very small fraction for high output power.

IX. THE COMPUTER PROGRAM

A computer program has been written in FORTRAN to solve the equations presented above. With the Euler method, the velocities are stepped forward in time, and the trapezoidal method is used to increment the positions. In the gun region, a 1.0-ps time step usually is a good value. The rf time step must generally be about 1° of rf phase, so it depends on the frequency. The program is written in a modular fashion, so it can be altered as the requirements change. The program is interactive, but it requires about 10 data files to produce a good simulation with accurate dc, rf, magnetic, and space-charge fields. Output data files are written for the code DESSIN,²⁰ which produces the graphical output.

X. MERIT-FIGURE RESULTS

The electron dynamics of the lasertron is very similar to that of the microwave triode, in that the bunching is very good initially and it grows worse as time goes on. The general conclusion is that the lasertron is an excellent microwave device at the lower frequencies and that it is more difficult to obtain good efficiency as the frequency is increased. High acceleration gradients improve the conversion efficiency at any frequency. In Fig. 2, the merit figure is plotted versus the fraction of the maximum charge extracted per pulse, q_{max} , where

$$q_{max} = \pi r_c^2 \epsilon_0 E_a / d, \quad (63)$$

E_a is the acceleration field at the cathode, r_c is the cathode radius, and d is the anode-cathode spacing. When a charge of q_{max} is emitted in a bunch, the acceleration field at the cathode is completely cancelled by the simplest approximation to the space-charge field. The calculations of Fig. 2 were made with one-dimensional acceleration fields, no rf output-gap fields, and the Bessel-function space-charge fields. The cathode radius is 15 mm, the anode is 30 mm from the cathode, the drift tunnel radius is 20 mm, and the voltage is 300 kV. The average current is 187.76 A at $q_{max} = 1$, and it is proportional to q_{max} . The other curve on the figure is the B_z field (in gauss) that was required to constrain the beam within 20.0 mm. The axial magnetic field starts out at the plotted value at the cathode and increases to twice the cathode value at 65 mm, where the center of the output gap is located. The frequency is 3 GHz, and 3 radial and 16 axial subdivisions are used in the calculations. The electron pulse length at the cathode is 60 ps. The predictions are very encouraging because the case for a fraction of 0.6, for example, indicates an efficiency of 86% and an output power of 29 MW with only a 900-G peak magnetic confining field. The magnetic fields may be doubled to account for the output gap fields, but they remain low, partly because of the relatively low current for the voltage.

Fig. 2. Merit-figure estimate of efficiency and magnetic focus field versus fraction extracted of q_{max} at 3 GHz. Planar acceleration fields, Bessel function space-charge fields, $r_c = 15$ mm, $V_0 = 300$ kV, anode cathode distance = 30 mm.

To show the effect of frequency, the above calculations were repeated for 6 GHz, with only the cathode pulse length changed to 30 ps. The average and the peak currents are now twice as much for the same emitted charge fraction, and the required focus fields are much higher. The results are shown in Fig. 3. Note that the slope of the merit figure versus emitted charge fraction is about twice as much at 6 GHz as at 3 GHz. The current for the critical charge is now 375.5 A. The results are still very encouraging because, at a fraction of 0.6, the efficiency is 71.5% with an output power of 48.4 MW and a peak magnetic field of 2400 G.

Fig. 3. Merit-figure estimate of efficiency and magnetic focus field versus fraction extracted of q_{max} at 6 GHz. Planar acceleration fields, Bessel function space-charge fields, $r_c = 15$ mm, $V_0 = 300$ kV, anode cathode distance = 30 mm.

XI. COMPARISON OF OUTPUT WITH PREVIOUS PROGRAMS

The first case studied is the one-dimensional diode region, which was also studied by Shintake.¹⁶ The diode has a 30-mm anode/cathode spacing, and the beam has a 30-mm radius. The anode voltage is 300 kV, and the laser light pulse is 35 ps long. Shintake presents two cases, 10 and 250 nC, with the latter being the charge limit for this geometry. The 10-nC case corresponds to only 0.04 of the critical charge; hence the space-charge forces are very low, and this code gives the same results as Shintake's. The high space-charge case has the emitted charge equal to the critical charge, and it is FUNCell beyond the maximum efficiency case. The results with the Bessel function space charge are shown in Fig. 4-a, and the results with the two-dimensional simplified space charge are shown in Fig. 4-b. The Bessel function case shows more radial and axial space-charge effects than do Shintake's results, but there is a general agreement. The major differences are due to the inclusion of the cathode image fields in Shintake's work. The two-dimensional simplified

space charge do include the cathode image fields, and the agreement is much better with Shintake in this case. It seems clear that the present code would be much improved if the two-dimensional simplified space-charge forces were used in the gun region and the Bessel function space charge were used in the drift and output regions. This modification has been made and is being tested now.

Fig. 4. Snapshots of the KEK lasertron at 300 kV, 250 nC, scale in mm.

- (a) Bessel function space charge with times of 50, 100, 150, 200, and 250 ps, 128 particles.
- (b) Two-dimensional space charge with times of 50, 150, and 250 ps, 256 particles.

The single output cavity SLAC prototype lasertron has also been studied with this code. The results are not complete at the time of this report, but the preliminary results are in general agreement; however, the present code shows larger radial space-charge forces than MASK does. The differences can be explained by the methods of truncating the space-charge forces when particles overlap. In the present work, the axial forces are truncated to zero as the charges approach one another, but the radial forces approach a constant, rather large value for zero charge separation. The usual plasma-physics approximation is to truncate both of these forces to zero for small particle separations. This approximation can be easily put into the present code if the experiments indicate that it is desirable to do so.

XII. CALCULATIONS ON THE 6-GHz ORSAY PROTOTYPE

The first detailed calculation made with this code is for the 6-GHz lasertron designed at Orsay for linear collider applications. A sketch of this lasertron is shown in Fig. 5. The electron gun was designed with Herrmannsfeldt's code; then the current was reduced to

Fig. 5. Sketch of the microwave geometry for the 6-GHz Orsay prototype lasertron.

almost zero and the static fields calculated. The SUPERFISH code was used to design an output cavity, and the H_ϕ table in r, z was calculated and used as input data. The first results are that the dc fields in this design are not correct for good beam transmission in the pulsed mode. The gun produces good trajectories for a continuous beam but very poor trajectories in the lasertron mode, with 20 to 40% interception at the anode. When the emission surface is made smaller, the same electrodes produce a much more uniform field over the cathode surface, but the current is smaller. With the smaller cathodes, excellent efficiencies are calculated. Figure 6-a shows the overall trajectories for the prototype lasertron with a 7-mm cathode simulated by 2 rings and only 16 total electrons. The anode voltage is 300 kV, and the beam clearly is too close to the drift tube, but only one electron is intercepted and the efficiency is 70.4%. An axial magnetic field of 250 G at the cathode rises to 800 G 80 mm away. The output cavity voltage is 350 kV, and the phase is 2.5 radians. The self-consistent method was not used in this calculation. The beam current is only 6.0 A on the average, but this lasertron will produce 1.27 MW at 6 GHz with a very respectable efficiency. The output cavity dynamics are shown in Fig. 6-b, where the radial scale is enlarged, and the electrons are shown every 100 ps.

Note that the beam initially spreads out because the magnetic focusing is ineffective near the cathode. The same simulation has been made with 4 rings and a total of 32 electrons, and the results are shown in Fig. 7. The calculated efficiency is still 70%, essentially the same as with 16 electrons. The CPU time for simulation of the 16-electron case is only 28 s; thus, many changes can be made in a few hours of time with the computer. The CPU time for the 32-electron simulation is 130 s because the computation time varies with the square of the number of electrons. A third set of calculations on the prototype was made with 12-A peak current and the beam voltage at 300 kV. A peak efficiency of 68.3% was found with all the same parameters as the 6-A case, except that the cathode focus field was increased to 350 G, and the focus field at 80 mm was increased to 1600 G. This efficiency corresponds to an output power of 2.46 MW, and still only 20% of the maximum available charge is taken from the cathode. When the current was doubled again, to 24 A, an optimum efficiency of 65.5% was found with a cathode field of 700 G, and an 80-mm field of 2300 G. The output power is 4.72 MW, and all the remaining parameters are the same as in the previous calculations. This output power is quite interesting, and the required shunt impedance in the output cavity is a factor of 4 lower than in the 6-A case; therefore, the modified design is easier to realize. The trajectories for the 24-A case are shown in Fig. 8. The space-charge forces are very large in the output cavity, and the beam suddenly goes from converging to diverging right at the center of the output gap.

More simulations with the self-consistent output field method and more particles have to be made, but the results so far indicate that the prototype should be capable of high efficiency at 6 GHz, and that it would work even better at lower frequencies. These calculations also show that the optimal gun design must have a flatter cathode field-forming electrode than the classical Pierce gun, and it seems very difficult to design any convergence into the beam. With some cathode materials, it will be possible to use a curved cathode; this should help the field uniformity problems at the cathode. The code has now been modified to handle curved cathodes, and the initial calculations are encouraging.

TIME (ps) = 100 200 300 400 500

(a)

(b)

Fig. 6. Electron trajectories for the prototype with $r_c = 15$ mm, 6 GHz $V_0 = 300$ kV, $V_{rf} = 350$ kV, and $I_{avg} = 6.0$ A. Sixteen particles.

Fig. 7. Electron trajectories for the prototype with $r_c = 15$ mm, 6 GHz, $V_0 = 300$ kV, $V_{rf} = 350$ kV, and $I_{avg} = 6.0$ A. Thirty-two particles.

Fig. 8. Electron trajectories for the prototype with $r_c = 15$ mm, 6 GHz, $V_0 = 300$ kV, $V_{rf} = 350$ kV, and $I_{avg} = 24$ A. Thirty-two particles.

XIII. CONCLUSIONS

A ring model theory for the lasertron has been written and programmed in FORTRAN. The code is self-consistent and relativistically correct to first order in v/c . The particle motion is followed from a photocathode, through an acceleration region, and through an output gap to a collector. Preliminary checks with previous work show good agreement, and calculations on the prototype lasertron to be constructed at Orsay indicate that efficiencies of over 65% should be available even at 6 GHz. At lower frequencies, high powers of over 50 MW should be possible, and the efficiency should rise above 80%, although it is likely that double or even triple output-gap cavities will be necessary to obtain the highest efficiencies. The code has recently been modified to include curved cathodes, the ability to model the time structure of the laser pulse, and to use different space-charge approximations in the different regions of the device. The next improvements will be the use of multiple bunches to follow the evolution to steady state and to include multiple output gaps. More comparisons with the past codes must be made, and comparisons with experimental data will be made when they become available.

The calculations already made show that the lasertron is capable of generating high output powers at very good efficiency up to at least 6 GHz. Both the peak output power and the conversion efficiency increase as the operating frequency is decreased.

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