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STATISTICAL AND SIGNAL-PROCESSING CONCEPTS IN SURFACE METROLOGY*

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Statistical and signal processing concepts in surface metrology

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Abstract

This paper proposes the use of a simple two-scale model of surface roughness for testing and specifying the topographic figure and finish of synchrotron-radiation mirrors. In this approach the effects of figure and finish are described in terms of their slope distribution and power spectrum, respectively, which are then combined with the system point spread function to produce a composite image. The result can be used to predict mirror performance or to translate design requirements into manufacturing specifications. Pacing problems in this approach are the development of a practical long-trace slope-profiling instrument and realistic statistical models for figure and finish errors.

Introduction

Brookhaven National Laboratory has established a laboratory for the metrology and testing of x-ray and UV optics in support of the National Synchrotron Light Source and the synchrotron-optics community. One of the critical steps in the testing and specification of these optics is the ability to relate component performance to laboratory measurements of surface topography. This paper reviews some of the statistical and signal-processing concepts that enter into this two-way relationship.

The components of interest are reflecting optics -- mirrors and gratings -- that have a number of unusual properties:

- o To achieve high reflectivity they must be used at glancing angles of incidence, which means that they are frequently large and asymmetric.
- o Since they operate at short radiation wavelengths their surfaces must be very smooth; with a finish of 10 Angstroms or less.
- o In order to focus the reflected radiation the surfaces are often curved; sometimes steep cylinders or aspheres.
- o They come in a wide variety of "one-of-a-kind" shapes and sizes.
- o They are expensive, take a long time to make, and are often made of exotic materials.

Figure 1. Three spherical silicon-carbide mirrors.

Figure 2. A toroidal silicon-carbide mirror.

Figure 1 is a photograph of three such mirrors, made of silicon carbide; each about 70 cm long, 10 cm wide and 3 cm thick. Their surfaces are spheres with a radius of 6 km, which corresponds to a sagittal depth of 10 μ m at the center and a maximum slope of 58 μ rad at the ends of the long dimension.

Figure 2 shows a second silicon-carbide mirror, but with a very different shape. This one is a toroid (bicycle tire) with a long radius of 1.2 km and a short radius of only 1 cm. It's about 50 cm long and 8 cm wide at its base.

These are examples of some of the extreme types of mirrors that we have to measure. But what do we measure?

To decide this we make use of two guiding principles: Whatever quantities we choose should have some rational connection with functional performance, and they should be easy to measure. On the one hand we don't want to make purely comparative measurements, and on the other, we can't afford to make a research project out of each mirror. A turn-around time of an hour per measurement would be ideal.

To get an idea of what surface properties are important in performance we look at the optical finish-function relationship; that is, diffraction theory.

Finish-function relationship

At the present state of the art the leading source of performance limitations of our mirrors lies in topographic shape errors. These do their dirty work by impressing unwanted phase modulation on the reflected wavefront which upsets the delicate balance required for rectilinear propagation and sharp focus. These errors are described by the two-dimensional error function

$$Z(\vec{x}) = Z_{\text{meas}}(\vec{x}) - Z_{\text{design}}(\vec{x}) - Z_{\text{adj}}(\vec{x}) \quad (1)$$

where Z_{meas} is the measured surface height, Z_{design} is its design value, and Z_{adj} is the effect of rigid body adjustments of the part, such as tilt and focus.

The simplest general expression that relates the error function to performance is textbook Fresnel - Kirchhoff diffraction theory, which involves the integral

$$I(\vec{f}) = \left| \int e^{-i2\pi\vec{f}\cdot\vec{x}} A(\vec{x}) e^{i\beta Z(\vec{x})} d\vec{x} \right|^2 \quad (2)$$

where A is the aperture function (including the surface reflectivity), $\exp(-i2\pi\vec{f}\cdot\vec{x})$ is the diffraction kernel, and $\exp(i\beta Z)$ is the phase modulation resulting from the height error Z . The connection between the surface frequency $\vec{f}$, the factor β , and the system geometry is given in Appendix 1.

Equation 2 is a deterministic relationship between the error function and the image intensity. If Z is known it can be used to compute that intensity. This direct method can be used to determine the effects of figure errors on image quality when enough data are available; for example, from full-part interferometry. But this is impractical for large steeply-figured optics, and in any case gives only a black-box connection between the measured figure errors and the computed performance. We would like a more direct way of treating the inverse problem — of translating image requirements into shape specifications.

To do this we make approximations. A first step is to separate the total error into additive figure and finish components

$$Z(\vec{x}) = Z_{\text{fig}}(\vec{x}) + Z_{\text{fin}}(\vec{x}) \quad (3)$$

where the figure errors are the large-amplitude long-wavelength errors of shape left by the figuring process, and the finish errors are the small-amplitude short-wavelength errors left by the polishing process. There's obviously a grey line between the two; small flats, for example, can be all finish with no figure. However, the two-scale model appears to have merit for large figured optics.

Since figure and finish add in the exponent in Equation 2, the diffraction integral can be viewed as the Fourier transform of the product of three factors — the aperture function, the figure phase error, and the finish phase error — which can then be written as the triple convolution of the transforms of each factor separately. This allows us to consider the factors one at a time, with the understanding that their effects must eventually be recombined to get a composite image intensity.

The transform of the aperture function leads to the unaberrated system point spread function. The figure and finish factors lead to the results described below.

Figure effects

The natural limit for evaluating figure effects is the rough-surface limit, where the Rayleigh-Strehl number g is large:

$$g_{\text{fig}} = \left(4\pi \frac{\sigma_{\text{fig}}}{\lambda} \sin \theta_i \right)^2 \gg 2 \quad (4)$$

Here σ_{fig} is the rms figure error, λ is the operating radiation wavelength, and θ_i is the glancing angle of incidence. The Fresnel-Kirchhoff integral can be evaluated in this limit using the stationary-phase method and leads to an intensity distribution which is a mapping of the slope-distribution function of the figure error. This is also known as the geometrical-optics, glint, or facet model. The vector-theory result is

$$\frac{1}{I_i} \left(\frac{dI}{d\omega} \right)_s = \frac{(1+m_x^2+m_y^2)^2}{4 \sin \theta_i} \cdot R(I) \cdot P(m_x, m_y) \quad (5)$$

where m_x, m_y are the surface slopes, $R(I)$ is the appropriate Fresnel intensity reflection coefficient at the local angle of incidence, I , and P is the joint slope-distribution function of the figure error. The function P contains all of the information about the figure error necessary to predict the scattering distribution in the rough-surface limit. Expressions relating the slopes and I to the scattering geometry are given in Appendix 1.

The physics of Equation 5 is simple: it says that scattering comes from that fraction of the surface that's tipped in the right direction to reflect radiation from the incoming beam to the observer according to the law of reflection. As befits a geometrical-optics result, it is independent of λ .

Equation 5 is identical with the corresponding rough-surface limit of the Beckmann-Spizzichino theory, considering the fact that their results are limited to the special case of a perfect reflector with a Gaussian slope distribution function. The present result is more general.

The present result is also essentially deterministic, although in practice it may be impractical to measure the value of P over the whole illuminated area of the surface. Instead, it may be possible to estimate it from limited measurements and to make up for the loss of information by fitting the data to a parametric model involving one or more adjustable slope parameters. A Gaussian slope distribution is only one of many possibilities. Performance could then be related to those parameters.

Figure errors are traditionally measured by interferometry, although as already noted, that method is impractical for the variety of large highly-curved mirrors we have to deal with. An ideal instrument for our purposes would measure surface slopes to microradian accuracies for spatial wavelengths lying between 1 nm and 1 meter. We favor a slope-profiling instrument; preferably a non-contacting optical type. Several designs are being considered and we hope to have such an instrument in our laboratory within a year.

At this time we have no in-house data to illustrate the use of geometrical optics for treating figure errors for x-ray optics. Figures 3 and 4, however, show data taken from the excellent review article by

Figure 3. Axial profile of a cylindrical x-ray telescope mirror. (Aschenbach, 1985.)

Figure 4. Point spread function of the mirror measured in Figure 3. (Aschenbach, 1985.)

Aschenbach on x-ray telescopes.⁴ The first shows an axial profile of the inside of a cylindrical mirror taken with a mechanical stylus gauge. The trace is about 22 cm long with an apparent precision of about 0.1 μm . The dashed line shows the raw measurements and the solid line is the same profile after the finish errors with wavelengths of less than about 1 cm have been removed by digital filtering. The rms roughness of the filtered profile is judged to be about 0.7 μm with an rms slope of about 45 μrad .

Figure 4 is the point spread function of this mirror plotted in a log-linear format. The solid line has been calculated from the measured mirror profiles using geometrical optics, and the crosses are experimental values measured at a radiation wavelength of 13.3 Angstroms. Measurements made at 44.8 Angstroms are essentially the same. The agreement between predictions and experiment is excellent.

Finish effects

The natural limit for evaluating finish effects is the smooth-surface limit, where the Rayleigh-Strehl number g is small:

$$g_{\text{sin}} = \left(4\pi \frac{\sigma_{\text{fin}}}{\lambda} \sin \theta_i \right)^2 < 0.2 \quad (6)$$

where σ_{fin} is the rms finish error. In this limit the quantity βz_{fin} in the exponent in Equation 2 is small and the exponential may be approximated by the first three terms in its power-series expansion. The first term is unity and corresponds to the ideal specular intensity, the second gives rise to large-angle scattering, and the third to the scattering correction to the specular term required by the conservation of energy. This is the famous Strehl factor $(1 - g_{\text{fin}})$ that multiplies the specular intensity.

The Fresnel-Kirchhoff theory leads to a particular form for the scattered intensity distribution in its smooth-surface limit. For our purposes, however, a more useful form may be derived using Rayleigh-Rice perturbation theory, which avoids the limitations of the tangent-plane approximation inherent in the F-K approach. The Rayleigh-Rice vector-theory result is^{5,6}

$$\frac{1}{I_i} \left(\frac{dI}{d\omega} \right)_s = \frac{16\pi^2}{\lambda^4} \sin \theta_i \sin^2 \theta_s \cdot Q \cdot S(\vec{f}_x, \vec{f}_y) \quad (7)$$

Here λ^{-4} is the Rayleigh "blue-sky" factor, θ_i and θ_s are the glancing angles of incidence and scattering, and $S(\vec{f})$ is the power spectral density of the finish error, which is given by

$$S(\vec{f}) = \left| \frac{1}{A} \int_A e^{-i2\pi \vec{f} \cdot \vec{x}} z(\vec{x}) d\vec{x} \right|^2 \quad (8)$$

This function contains all of the information about the finish error necessary to predict the scattering distribution in the smooth-surface limit.

Q is a materials-obliquity factor which equals the smooth-surface reflectivity in the forward-scattering direction but deviates from that value in important ways in other directions. For example, for s- to s-polarized scattering

$$Q_{ss} = \sqrt{R(\theta_i) R(\theta_s)} \cos^2 \varphi_s \quad (9)$$

where the R 's are the Fresnel intensity reflection coefficients and φ_s is the azimuthal scattering angle. Expressions for the Q 's and R 's are given in Appendix 1.

The physics of smooth-surface scattering is contained in the grating equation: The power spectrum gives the squares of the amplitudes of the Fourier components of the finish error, each of which acts as a weak sinusoidal grating that diffracts radiation from the incident beam into the direction of observation.

The corresponding Beckmann-Spizzichino result has the same form as Equation 7, but with a different obliquity factor which arises from their evaluation of the Fresnel-Kirchhoff integral. Their results are also limited to perfect reflectors and are stated in terms of particular forms of the finish power spectra. The restriction to perfect reflectors is particularly inappropriate for x-ray mirrors operating near the critical angle where the reflectivity is a strong function of the local angle of incidence. The Rayleigh-Rice theory is more realistic and leads to interesting physical effects in this case which do not appear in the Fresnel-Kirchhoff treatment.

Equation 8 may be viewed as a deterministic result, but it is always treated statistically when applied to finish measurements since it gives speckle-scale information about the scattered field that's of no interest in our application, and finish measurements are invariably sampling measurements which require a statistical leap of faith to apply to real problems.

We measure finish with a WYKO optical profiling microscope, which measures surface profiles to Angstrom precision over a wide range of surface wavelengths — from microns to millimeters — by using different microscope objectives. The raw profiles are first restored by passing them through an inverse digital filter to compensate for the varying transfer function of the measuring instrument, and their power spectra are estimated using standard statistical techniques. These spectra are then fitted to parametric models which are used to extrapolate the spectra outside the measurement bandwidth and to parameterize the predicted scattering distribution. Particular models are discussed in the following sections.

Figure 5. Profile of a gold-coated float-glass flat.

Figure 6. Comparison between measured x-ray scattered intensity (dotted line) and that predicted from the periodogram of the profile shown in Figure 5.

Figure 5 shows a WYKO profile of a piece of gold-coated float glass taken with a 10X objective. The length of the profile is 1.33 mm and its rms roughness over the bandwidth of the measurement is only 3.5 Angstroms.

Figure 6 shows a comparison between the measured and predicted scattering of 1-Angstrom x-rays from the same surface when illuminated at a glancing angle of 3 mr. The dotted line is the measured specular and scattered intensity distribution and the jagged line is the periodogram of the profile in Figure 5 that has been translated into scattering coordinates using Equation 7.

The periodogram is a non-parametric estimate of the finish spectrum and shows extreme statistical fluctuations or speckle. Parametric estimates which have fewer degrees of freedom give smoother spectra. A handy form suggested by the results in Figure 6 is a smooth linear falloff corresponding to a $1/f$ profile spectrum, which is discussed further below.

The comparison in Figure 6 is preliminary, but it does indicate how laboratory finish measurements may be related to scattering effects using the finish-function relationship. Further discussion of the interpretation of glancing-incidence scattering measurements is given elsewhere.

Finish models

Laboratory measurements give information about the surface profile over a limited range of surface wavelengths. Equation 7, however, involves the two-dimensional power spectrum of the surface — which is different than the profile spectrum — and over a wider range of wavelengths than are included in usual measurements.

One method of handling this is to inject a-priori information into the problem by using finish models; either empirical models based on systematics or analytic models based on microscopic or phenomenological considerations, such as spatial isotropy or scale invariance.

Experimentally, the profile spectra of polished surfaces frequently appear to be inverse power laws, $S_p(f) \sim f^{-n}$, where n is a number lying between 1 and 3. In some cases the spectra appear to round off at low frequencies although it is difficult sometimes to determine this unambiguously.

In the good old days it was common to fit profile data to an exponential autocovariance function, which corresponds to a $1/(a^2 + f^2)$ spectrum; that is, flat at low frequencies with a $1/f^2$ high-frequency tail. An obvious generalization of this that gives a flat spectrum at low frequencies but with an arbitrary power-law high-frequency tail is the so-called K-correlation family of functions. These have a correlation function of the form $T^* K_\nu(2\sqrt{a}T)$ where K_ν is a modified Bessel function, T is the lag and a is an inverse correlation length. The profile power spectrum, S_1 , and the corresponding two-dimensional spectrum for an

isotropically rough surface, S_2 , are given by the common expression

$$S_d(\xi) = \frac{\pi^{-d/2}}{2} \left(\frac{a}{\pi} \right)^\nu \frac{\Gamma(\nu+d/2)}{(a^2 + \xi^2)^{\nu+d/2}} \quad (10)$$

where $f = f_x$ in the first case and $f = (f_x^2 + f_y^2)^{1/2}$ in the second. The general relationship between S_1 and S_2 is given in Appendix 1.

This family of spectra has the nice property of reducing to the familiar $1/f^2$ falloff for $\nu = 1/2$ and having the arbitrary power-law falloff of $S_1(f) = 1/f^n$ where $n = 2\nu + 1$ for other values of ν . "Pure" power-law spectra appear in this model when the correlation length, $1/a$, is larger than the trace length of the profile measurement.

Scale invariance

Inverse-power-law spectra are directly related to the scale invariance of the surfaces from which they are derived. If a profile has a $1/f^n$ spectrum it has the same appearance (statistics) when its horizontal scale is magnified by the factor M and its vertical scale by the factor M raised to the power $(n-1)/2$. This means that a $1/f^3$ profile looks the same under isotropic magnification, while a $1/f$ profile becomes linearly rougher with increasing magnification. Figure 3 in reference 6 illustrates this.

Mandelbrot, in his elegant book, has pointed out that scale invariance or self similarity is a very common phenomenon in nature. Entities having this behavior over even a limited range of scaling have come to be called fractals. True fractals are characterized by fractal dimensions, and the Hausdorff-Besicovitch dimension corresponding to a $1/f^n$ profile is $D = (5-n)/2$.

Figure 7 is a set of photographs of a household item that shows scale invariance — a cauliflower. It looks the same at different magnifications, while the penny, which does not have this property, changes drastically. The value of n for the cauliflower has not been determined.

There appear to be few microscopic theories of the polishing process, and none that predict the second-order statistics (spectra or correlation functions) of the resulting finish. However, $1/f$ profile spectra are predicted for other surface phenomena: capillary waves on liquids and the spontaneous roughening of crystal surfaces. These suggest a surface-tension model of the finish of polished surfaces in which the residual roughness is the result of a polishing mechanism which smooths according to a minimum-area principle.¹⁰

Phenomenologically, inverse power law spectra are expected when the characteristic lengths of the finishing process fall outside the measurement bandwidth, with $1/f$ -like spectra favored since they give a constrained minimum roughness.

Figure and finish specification

Equations 5 and 7 give simple relationships between figure and finish errors and their corresponding scattering distributions. The composite image intensity depends on the square magnitude of the convolution of the amplitudes of these effects and the system function. To a first approximation, however, it may be sufficient to write the composite image in the form

$$\left(\frac{dI}{dw} \right)_{\text{comp}} \sim (1 - g_{\text{fin}}) \text{PSF} * \left(\frac{dI}{dw} \right)_{\text{fig}} + \left(\frac{dI}{dw} \right)_{\text{fin}} \quad (11)$$

where PSF is the system spread function and $*$ means convolution. The first term is the figure-aberrated image core and the second is the finish-generated image tail.

Figure 7. A household illustration of scale invariance or dilational symmetry.

Whether or not this approximation is adequate for our purposes has not yet been determined. If not, more refined approximations are waiting in the wings. However, the simplicity of Equation 11 recommends it for a first try.

Figure specifications may be given directly in terms of the shape of the slope-distribution function. Figure 4 shows that this comparison can be made in significant detail. Specifications can also be stated in terms of more general properties such as fractional peak widths or encircled-energy widths. Rms values are not good measures of image widths since they weight the image tail too strongly. A third method is to specify figure in terms of model parameters.

Finish specifications may be given in terms of the intensity of radiation scattered in a given direction or in terms of the finish Strehl factor. The first translates to a limit on the magnitude of the finish spectrum over a narrow frequency range, and the second to the value of the rms finish roughness obtained by integrating the spectrum over the full frequency range spanned by finish effects. Since that range is larger than that included in conventional finish measurements, values of σ_{fin}^2 calculated from such measurements are less than the value appearing in the Strehl factor. As a result, specifications expressed in terms of off-the-machine values of rms roughnesses are necessary but not necessarily sufficient.

How sufficient are they? The answer lies in the magnitudes of the contributions to the total finish spectrum that fall outside the measurement window, especially at long surface wavelengths. Model calculations suggest that WYKO measurements of surfaces with K-correlation spectra are reasonably sufficient when the finish correlation length falls within the measurement bandwidth. Wide-band measurements of 1/f surfaces are highly sufficient since the total finish roughness depends only logarithmically on contributions lying outside the measurement window. Further comments are given in Appendix 2.

Summary

The two-scale model of surface roughness has been applied to the problem of testing and specifying the topographic errors of large reflective optics. In this model the total error is written as the sum of a large-amplitude figure error and a small-amplitude finish error, the first of which is characterized by its slope-distribution function and the second by its power spectral density. In practice these functions have to be estimated from laboratory measurements with the aid of parametric models. At the moment there appears to be little information on the figure statistics of mirrors of interest to us. In the case of finish, however, data suggest K-correlation or simple inverse power-law spectra characteristic of scale-invariant finishing processes. The pacing problems in this area are the development of a long-trace slope-profiling instrument and realistic statistical models for figure and finish errors.

Disclaimer

Commercial names are used in this paper to identify experimental procedures. In no case does such identification imply recognition or endorsement by the Department of Defense or the Department of Energy.

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We thank Dr. B. Aschenbach for permission to reproduce Figures 3 and 4.

Appendix 1

Figure 8 is a sketch of the scattering geometry used in discussing glancing-incidence mirror surfaces.

The surface frequencies appearing in Equations 2 and 7 are related to the radiation wavelength and the scattering angles by the grating equations:

$$\lambda f_x = -\cos \theta_s \cos \varphi_s + \cos \theta_i \quad (11)$$

$$\lambda f_y = -\cos \theta_s \sin \varphi_s$$

The factor β in Equation 2 is

$$\beta = 2\pi (\sin \theta_i + \sin \theta_s) / \lambda \quad (12)$$

The surface slopes appearing in Equation 5 are related to the scattering geometry according to

Figure 8. Sketch of the scattering geometry.

$$\beta m_j = 2\pi \frac{f_j}{\lambda} \quad (13)$$

and the local angle of incidence is given by

$$2 \sin^2 I = 1 + \sin \theta_i \sin \theta_s - \cos \theta_i \cos \theta_s \cos \varphi_s \quad (14)$$

The Q factors in Equation 7 are

$$\begin{aligned} Q_{ss} &= \left| \frac{\epsilon - 1}{A_i A_s} \right|^2 \cos^2 \varphi_s & Q_{sp} &= \left| \frac{\epsilon - 1}{A_i} \cdot \frac{C_s}{B_s} \right|^2 \sin^2 \varphi_s \\ Q_{pp} &= \left| \frac{\epsilon - 1}{B_i B_s} (C_i C_s \cos \varphi_s - \epsilon \cos \theta_i \cos \theta_s) \right|^2 & Q_{ps} &= \left| \frac{C_i}{B_i} \cdot \frac{\epsilon - 1}{A_s} \right|^2 \sin^2 \varphi_s \end{aligned} \quad (15)$$

where

$$A_j = \sin \theta_j + C_j \quad B_j = \epsilon \sin \theta_j + C_j \quad C_j = \sqrt{\epsilon - \cos^2 \theta_j} \quad (16)$$

and ϵ is the surface permittivity. The first and second indices on Q are the initial and final states of linear polarization. The corresponding Fresnel intensity reflection coefficients are

$$R_s = |1 - 2C/A|^2 \quad R_p = |1 - 2C/B|^2 \quad (17)$$

Equations 5 and 7 involve the two-dimensional distribution functions P and S. Let $G(g_x, g_y)$ represent their generic form. It is possible to derive a number of different forms from this which are "one dimensional" in that they are functions of a single g variable. Three examples are:

$$G_1(g_x) = \int_{-\infty}^{+\infty} G(g_x, g_y) dg_y; \quad G_2(g) = G(g_x, g_y) \text{ where } g = (g_x^2 + g_y^2)^{1/2}; \text{ and } G(g_x, g_y) = G_3(g_x) \delta(g_y) \quad (18)$$

G_1 is the ideal profile distribution function taken along the x axis. G_2 is the two-dimensional distribution for an isotropic surface, and G_3 is the distribution function for the "corrugated" surface $Z(x, y) = Z(x, 0)$.

For an isotropic surface G_1 is the Abel or half-integral transform of G_2

$$G_1(g) = 2 \int_g^{\infty} \frac{g' dg'}{\sqrt{g'^2 - g^2}} G_2(g') \quad (19)$$

and G_2 is the inverse-Abel or half-derivative transform of G_1 ⁶

$$G_2(g) = -\frac{1}{\pi} \int_g^{\infty} \frac{dg'}{\sqrt{g'^2 - g^2}} \cdot \frac{d}{dg'} G_1(g') \quad (20)$$

The spectra S_1 and S_2 in Equation 10 satisfy these relationships.

Case 3 leads to the following expressions for the scattering from an arbitrarily corrugated surface:

$$\frac{1}{I_i} \left(\frac{dI}{d\theta} \right)_s = \frac{1}{2} (1 + m_x^2) (1 + m_x \cot \theta_i) \cdot R(I) \cdot P_3(m_x) \quad (21)$$

where $I = (\theta_s + \theta_i)/2$ and $m_x = \tan((\theta_s - \theta_i)/2)$; and

$$\frac{1}{I_i} \left(\frac{dI}{d\theta} \right)_s = \frac{16\pi^2}{\lambda^3} \sin \theta_i \sin^2 \theta_s \cdot \varphi \cdot S_3(f_x) \quad (22)$$

where $S_x(f_x)$ is the two-sided spectrum of the surface profile and Q and f_x are evaluated at $\varphi = 0$. These forms also appear in the interpretation of glancing-incidence scattering measurements involving a slit detector parallel to the surface.^{7,10}

Appendix 2

The minimum and maximum surface frequencies included in profile measurements are $F_x = 1/L$ and $N/2L$ where L is the trace length and N is the number of sampling points. The periodogram estimate of the profile spectrum between these limits is proportional to the square magnitude of the discrete Fourier transform of the measured profile times an appropriate data window. Explicit expressions are given elsewhere.⁸

The total range of surface wavelengths contributing to functional effects lies in an "annular" region in frequency space whose inner dimension is characterized by the surface wavelength d which separates figure and finish, and whose outer dimension is characterized by the radiation wavelength λ .

Since finish spectra are "red", d is generally the more important parameter. Unfortunately, there is no precise definition of this quantity, although it is easy to write constraints on its value in terms of a fraction of the part size, the rms figure or finish roughnesses, and the rms surface curvature; each of which may be expressed most simply in terms of the error power spectrum. In early work d was determined, in effect, by the limited bandwidths of the available measurement techniques. In the last analysis, the selection and significance of d are creatures of the two-scale model, and must await further studies of its applicability to real surfaces. Does it work or doesn't it?

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