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FUEL ENERGY BALANCE STUDIES OF PELLET IGNITION
REQUIREMENTS

MASTER

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ABSTRACT

A simple power balance model is used to study DT fuel ignition requirements in the parameter regime appropriate for many ICF target applications. The model includes fuel energy loss due to thermal conduction and radiation, gain due to PdV work and α particle deposition, and pusher boundary conditions. It can be shown analytically that a minimum pusher velocity of $\sim 10^7$ cm/sec is necessary to achieve ignition, with corresponding fuel $\rho R \approx 1$ gm/cm² and $T \approx 4$ keV. If thermal conduction losses are reduced, then the minimum necessary velocity required for ignition is shown to be substantially reduced. Various formulas and scaling laws are presented which should aid target design calculations which use large hydrocode models.

I. INTRODUCTION

A simple rate balance model has been constructed to model ICF target fuel behavior to provide guidance for target design and optimization. A similar diagrammatic approach was used by Kirkpatrick¹ to study minimal ignition requirements. A somewhat idealized model is used here which includes fuel energy loss and gain for conditions appropriate for most simple particle beam fusion targets. The advantages of reducing thermal conduction losses² are also studied and minimum velocity, temperature, and ρR conditions for ignition are given. As expected, reduction of thermal conductivity leads to a substantially reduced pusher velocity requirement, which, in turn, translates into a reduced particle beam intensity requirement. The results presented here should give guidance to target design and suggest changes in fuel parameters, pusher velocity, etc. during the course of design studies which utilize a large hydrocode models.

II. Model and Results

A simple description of fusion fuel behavior is given by the following rate balance model, normalized by the fuel internal energy density, ϵ_f ,

$$\bar{G} = \frac{(P_\alpha - P_R - P_T + P_W) \tau}{\epsilon_f} \quad (1)$$

where $\bar{G}$ = net fuel gain, P_α = α particle power deposition/volume, P_R = radiative power loss/volume, P_T = thermal conduction power loss/volume, P_W = PdV work rate/volume, and τ = containment time.

A spherical fuel volume of radius $R(\text{cm})$ is assumed with an equal mix of DT of uniform fuel density $\rho(\text{gm/cm}^3)$ and average temperature $T(\text{eV}) = T_e = T_i$. A constant fuel boundary velocity $V(\text{cm/sec})$ is assumed, and the boundary temperature is assumed to be much less than the fuel temperature T .

The various terms in Eq. (1) are then

$$P_\alpha = 8.01 \times 10^{33} \langle \sigma v \rangle \rho^2 F \quad (W\text{-cm}^{-3}) \quad (2a)$$

$$P_R = 9.70 \times 10^{14} \rho^2 T^{1/2} \quad (W\text{-cm}^{-3}) \quad (2b)$$

$$P_T = -8.91 \times 10^2 \frac{T^{5/2} \nabla T}{f \ln \Lambda R} \quad (W\text{-cm}^{-3}) \quad (2c)$$

$$P_W = -2.30 \times 10^5 \frac{\rho T V}{R} \quad (W\text{-cm}^{-3}) \quad (2d)$$

$$\bar{E}_f = 1.15 \times 10^5 \rho T \quad (J\text{-cm}^{-3}) \quad (2e)$$

where $\langle \sigma v \rangle = \text{DT Maxwell averaged fusion cross-section (cm}^3/\text{sec)}$, $F = \text{fraction of alpha particles deposited in fuel}$, $f = \text{thermal conductivity reduction factor}$, $\nabla = -(\rho R)^{-1}$, and $\alpha = \text{spatial gradient factor}$. The fuel is assumed optically thin with only free-free bremsstrahlung radiative losses.

Combining Eq. (2) with (1) gives the following expression.

$$\bar{G} = \frac{\left[8.01 \times 10^{33} \langle \sigma v \rangle F - 9.70 \times 10^{14} T^{1/2} - \frac{8.91 \times 10^2 T^{7/2}}{\alpha f (\rho R)^2 \ln \Lambda} + \frac{2.30 \times 10^5 T V}{\rho R} \right] \rho T}{1.15 \times 10^5 T} \quad (3)$$

Note that the terms inside the brackets depend explicitly on T , ρR , and V with only very weak explicit ρ dependence thru $\ln \Lambda$. Thus, for the $\bar{G} = 0$ boundary in parameter space, there is strong dependence on only three variables

and graphic projections of $\bar{G} = 0$ boundaries in $\rho R, T$ space for fixed V are quite useful.

Such plots were constructed numerically. An example is shown in Figure 1 with the regions where $\bar{G} > 0$ shaded. Note that the boundary locations do not depend on or ϵ_f , but only on the terms within the brackets in Eq. (3). For this case there are two separate regions where $\bar{G} > 0$: (1) the region in the upper right hand corner where the α deposition from fusion reactions exceeds the losses and (2) the region in the lower left portion where the PdV work rate exceeds the losses. For a range of fuel densities from .01 to 100 gm/cm³, only very small changes to the boundaries shown were seen to occur.

For this illustrative example, local α deposition was assumed, i.e., $F = 1$. A more realistic α deposition is used in later discussions. Note that even with local deposition, the $\bar{G} > 0$ region in the upper right quadrant is limited to $\rho R > .1$ due to thermal conduction losses. The region is bounded below by radiative losses and the $T \cong 4$ keV ignition temperature for $.1 < \rho R \leq 1$. This latter condition can be derived by setting $P_\alpha = P_R$ with $F = 1$.³

The lower left $\bar{G} > 0$ region is bounded above and to the left by thermal conduction losses given approximately by $P_T = P_w$. The condition for $P_T > P_w$, referred to as thermal conduction collapse, is given as

$$\rho R < \frac{3.88 \times 10^{-3}}{\alpha_{\text{fznA}}} \frac{T^{5/2}}{V} \quad (4)$$

This really represents a thermal conduction barrier, since in practice it is usually approached from below. The lower right boundary is given by $P_R = P_w$. The condition $P_R > P_w$, referred to as radiation collapse, is given as

$$\rho R > 2.37 \times 10^{-10} V T^{1/2} \quad (5)$$

Figure 1. Plots of $\bar{G} > 0$ Regions (Shaded) in $T - \rho R$ Space.

An additional useful boundary is that given by $P_T = P_R$ and is the dashed diagonal line in the figure. Above and to the left of this line thermal conduction is the dominate loss, below and to the right radiation loss dominates. The condition for ρR for $P_T = P_R$ is $\rho R = (\rho R)_c$, where

$$(\rho R)_c = 9.58 \times 10^{-7} \frac{T^{3/2}}{(\alpha f \ln \Lambda)^{1/2}} \quad (6)$$

If one neglects $|p_\alpha| \ll |p_w|, |p_R|, |p_T|$, then the maximum temperature T_x for the lower left region can be determined as

$$T_x = 1.24 \times 10^{-4} (\alpha f \ln \Lambda)^{1/2} V \quad (7a)$$

and

$$(\rho R)_x = 1.32 \times 10^{-12} (\alpha f \ln \Lambda)^{1/4} V^{3/2} \quad (7b)$$

It is interesting to note that this maximum occurs for $P_T = P_R$ as is apparent in Fig. 1. Equations (7a) and (7b) then represent an optimum of sorts for a given V , α , f , and $\ln \Lambda$, provided rate balance determines the final fuel conditions. It would also be possible to be within the lower left $\bar{G} > 0$ region or to the right of the radiation collapse boundary, although the temperature would be decreasing with time for the latter case. It would not be possible to exceed T_x provided P_α is negligible, i.e., $T < 4$ keV. Values of T_x and $(\rho R)_x$, neglecting P_α , are plotted in Fig. 2(a) ($f = 1$) and in Fig. 2(b) ($f = 36$) for different fuel densities and pusher velocities.

Ignition is possible then when the two shaded regions merge as V is

increased. The merging is seen to occur first, to a very good approximation, along the line where $P_R = P_T$.

Before extensive studies of such "ignition" conditions were undertaken, an improved α deposition model was used. The model for slowing down is that given by Fraley, et al.⁴ We assume here a spatially uniform T and ρ profile and calculate the fractional energy deposition of α particles born at radius $\hat{R}$ as

$$\hat{F}(R) = \frac{1}{2U_0} \int_0^\pi \left[\int_0^{X(\hat{R}, \theta)} \frac{du}{dr} dr \right] \sin \theta d\theta \quad (8)$$

where $\frac{du}{dr}$ is the α particle stopping power, r the radial path length, and X the path length from the origin point of the production to either the end of the range λ , or to the edge of the fuel volume, whichever is smaller. The production is then spatially weighted to obtain F as

$$F = \frac{3}{R^3} \int_0^R \hat{F}(R) R^2 dR \quad (9)$$

Plots of numerical solutions of F vs. ρR are shown in Fig. 3 for several values of T and ρ . The range is indicated by the respective arrow near the top of each curve. Note that for $\lambda \gg \rho R$, $F \approx \rho R / \lambda$.

Diagrams similar to that shown in Fig. 1 were constructed using this α deposition model. As the velocity is increased, for fixed α , f , and ρ , the two regions merge and ignition is possible. For example, for $\alpha = f = \rho = 1$, this occurs at $T = 3.7$ keV, $(\rho R) = .082$ gm/cm², and $V = 9.2 \times 10^6$ cm/sec. The regions merge to a very good approximation along the $P_R = P_T$ line. Additional calculations were performed varying ρ and f ,

Figure 2(a). Values of T_x and $(\rho R)_x$ for various pusher velocities and fuel densities. The cases $f = 36$ (a) and $f = 1$ (b) are shown.

Figure 2(b)

Figure 3. Plots of F (α particle energy deposited/ α particle energy produced) vs fuel ρR for several ρ and T values. The arrows on the upper portion of the curve represent the respective α particle range.

and determining the minimal necessary velocity for ignition, and the corresponding T and ρR .

The location where the two regions merge can be calculated analytically in the following way. First, define the normalized sum of the rates S as given in Eq. 3 as

$$S = (P_{\alpha} - P_R - P_T + P_w)\rho^{-2} \quad (10)$$

In ρR space there is a minimum of S along which the regions merge given by

$$\frac{\partial S}{\partial(\rho R)} = 0 \quad \text{or}$$

$$\rho R = \frac{7.74 \times 10^{-3} T^{5/2}}{\alpha f \ln \Lambda v} \quad (11)$$

Another condition is that $S = 0$ at the point the regions merge, which when combined with Eq. (11) gives a condition for the velocity as

$$v^2 = \frac{-8.01 \times 10^{33} \langle \sigma v \rangle F + 9.7 \times 10^{14} T^{1/2}}{1.49 \times 10^7 \alpha f \ln \Lambda T^{-3/2}} \quad (12)$$

The velocity where the two regions merge is the maximum of Eq. (12) given by $\partial v^2 / \partial T = 0$.

These results are plotted in Fig. 4(a) and 4(b). Note the minimum velocity v_M required for ignition and corresponding $(\rho R)_M$ and T_M can be approximated over the range of $1 \leq f \leq 100$ as

$$v_M \cong 9.2 \times 10^6 f^{-.38} \rho^{-.048} \quad (13a)$$

$$T_M \cong 2.7 \times 10^3 f^{.16} \quad (13b)$$

$$(\rho R)_H \cong .082 f^{-.27} \rho^{.042} \quad (13c)$$

It is unlikely that the above conditions will lead to significant burn since they represent marginal ignition and minor excursions in parameter space lead to regions of $\bar{G} < 0$. These conditions should, however, provide a useful guide to hydrodynamic calculations when it is desired to minimize pusher velocity requirements.

III. Summary

The described rate balance model is useful for defining ignition requirements for conditions appropriate for many ICF target applications. It has been shown that a minimum necessary pusher velocity of $\sim 10^7$ cm/sec is required near peak compression for DT fuel ignition. For this to occur, however, the fuel must be on a sufficiently high isentrope which has not been taken into account explicitly. Also, the pusher may likely slow down during the implosion and it may be necessary to start with a higher implosion velocity than that needed at ignition. Another feature not considered is the effect of including spatial ρ and T profiles in the fuel. Some preliminary analysis indicates that this may lead to somewhat higher velocity requirements. Inclusion of these profiles will be the subject of further research.

It was also shown that if one arbitrarily reduces thermal conductivity by some factor f , then the minimum necessary velocity needed for ignition decreases as $V \propto f^{-.38}$. The corresponding fuel ρR is also reduced while the temperature is increased. This indicates that a substantial reduction in beam driver requirements may be possible for magnetic thermoinsulation targets.

Figure 4(a). Values of V_m vs f for several ρ values (a) and corresponding $(\rho R)_m$ and T_m values vs f (b).

Figure 4(b)

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