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MASTER

Design and Fabrication of Binary-Valued

Alignment Reticle Patterns

by

Gallagher Associates, Inc.

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I. Introduction

A two year project to analyze and fabricate a binary diffractive elements to be used for precision alignment has been undertaken. Two different approaches have been studied: one involves an iterative technique, and the other uses results from the area of mathematics known as number theory. Either method could yield an alignment scheme substantially different from any being used at the present time.

This final report reviews the approaches we have considered for this alignment method. Previously submitted documents are included as part of this report as well as some discussion on our attempts at device fabrication. The effort we undertook proved too ambitious for the funding level. Although we did produce, in our opinion, a substantial amount of results. In the end, the cost of device fabrication proved to high for our budget.

The basic approach is simple:

- 1) Image a binary transparency pattern onto a copy of itself.
- 2) Collect all of the light from the second transparency with an integrating lens.
3. As one binary pattern moves with respect to the second, the integrated value scans through values of the autocorrelation function.
- 4) The peak value of the autocorrelation represents alignment of the two patterns.

The problem is to design and fabricate the best binary patterns for precision alignment. The iterative technique we first studied is similar to the technique we used to design the microwave holograms currently used for electron heating in TMX for MFE.

So, our task is to design a binary pattern that has a specified autocorrelation function. We briefly describe here the procedure. If the autocorrelation function is Fourier transformed, the result is simply to generate the magnitude squared value of

the Fourier transform of a function which has the specified autocorrelation. We now have two constraints that our unknown function must satisfy: 1) it must be binary valued, and 2) the magnitude of its Fourier transform is fixed by the fact that the autocorrelation function is given apriori. In general there may be no function which satisfies both of these constraints for a given autocorrelation function. In this case, our goal must be to find the binary function which best matches the given autocorrelation.

Given the magnitude of the Fourier, we first guess a random phase for the Fourier transform to form our first approximation of the unknown function's Fourier transform. We then form the inverse transform of this first guess function. Unless we have been extremely lucky the inverse transform is not binary valued. We then force it to be binary valued through a thresholding procedure, and Fourier transform this new binary valued function. In general this transform does not satisfy the original magnitude constraint. We then impose the magnitude constraint, leaving the phase values of this function unchanged. We then inverse transform this function and continue this procedure back and forth until a satisfactory solution is found, or we give up and start again. If we start again, we might choose a new random phase to start the iteration, or we might choose a different autocorrelation function. A more detailed discussion of the iteration approach is found in Appendix A.

Our opinion is that the iteration approach does not provide a satisfactory solution to the problem in many practical situations. A different approach, which seems very promising, is to use results from number theory to design the needed binary pattern. We found that it was possible to design both one dimensional and two dimensional functions which have a very peaked autocorrelation functions. If a two dimensional pattern is used, then a two dimensional search is required to find the peak of the autocorrelation function. We do not regard a two dimensional search as a particularly efficient procedure. However, a problem with one dimensional patterns, even

though the search procedure is greatly simplified, is that two patterns would be required for the two dimensional alignment, one pattern for the x-direction and one for the y-direction alignments. This difficulty can be addressed by using yet another technique from the design of the TMX microwave holograms. A grating structure, whose period is much smaller than the wavelength of light being used, can be made to appear either transparent or opaque depending on the polarization of the light. The plan then is to make two non-overlapping one dimensional grating structures on the same substrate material. One or the other of the patterns will be visible depending on the polarization of the illuminating wavefront. One pattern, when illuminated with vertically polarized light, will contain the information, for say, vertical alignment. The second pattern, visible with horizontally polarized light, will contain the information for horizontal alignment. Using this concept two simple one dimensional scans can find the peak of the autocorrelation function along each axis. This method is described in more detail in Appendix B.

In the next section we briefly describe the results of our attempts at fabrication of these polarization structures.

II. Fabrication

Purdue's Cambridge Systems E-beam machine was used in an effort to fabricate the polarization grating structures. Using standard E-beam technology the patterns were etched into chromium on a glass substrate. The pattern is actually a composition of a coarse pattern and a very high frequency pattern. The function of the coarse pattern is to provide the information for optical alignment. It is somewhat misleading to call this the coarse pattern because it would have the detail and resolution of say an ordinary cross-hair alignment pattern. It is this coarse pattern which would be imaged onto a copy of itself to produce the autocorrelation function used in alignment. An electron micrograph of a coarse pattern is found in Fig. 1. Imbedded

Fig. 1. Electron Micrograph of Reticle Pattern used for Horizontal Alignment.

Fig. 2. Electron Micrograph of Reticle Pattern's Polarizing Structure which Reflects Vertically Polarized Light and Passes Horizontally Polarized Light.

within the coarse pattern is the very high frequency polarizing pattern. So the coarse bar pattern has imbedded within it the polarizing pattern shown in Fig. 2.

The major problem we have had in fabricating these high frequency grating structures has been the resolution of the E-beam system. The system has a purported resolution of 0.1 micron. In practice we have been able to get only about 0.5 microns. Not to say that the machine isn't working properly, but that we have not learned all of the tricks and magic needed to get the best possible performance. As is often the case, another major limiting factor has been funds. In order for us to fabricate a large very high frequency pattern costs two to five thousand dollars. Although we actually received some E-beam machine time at minimal costs, it was not possible for us to have sufficient access to the machine to experiment on grating fabrication techniques. Appendix C contains a theoretical description of high frequency conducting patterns.

III. Summary

We have studied techniques for alignment reticle fabrication. Two design methods are discussed in this report: one uses an iterative procedure and the other uses results from number theory. A third method was also studied briefly; this being a binary chirp grating. Although this was described in a previous report, we did not find it particularly promising and do not discuss it in this document.

It is our opinion that either of the two methods discussed in this report can be made into a workable system if there is sufficient interest. It is also our opinion that the reticle design based on number theory, which uses the polarization encoding of two patterns onto a single substrate, is an powerful method which deserves further consideration.

Contained in Appendix C is a still very preliminary draft of a theoretical analysis describing the function of these very high frequency polarizing grating structures.

Appendix A
Iterative Design of Reticle Patterns

Recovery Of A Function From It's Autocorrelation

Function - An Iterative Approach

Jong-Waei Whang

and

Neal C. Gallagher

ABSTRACT

An iterative scheme for recovering a real binary signal from its autocorrelation function is described. The scheme is performed in the 1-D case first, then extended to the 2-D case. The triangular function function is used as the input autocorrelation function in 1-D case, while the pseudo-noise array pattern is chosen as the input autocorrelation function in 2-D case.. Several parameters, such as initial phase assignment and a binary threshold value are important when employing this technique. These parameters do affect the converging speed. The scheme is shown to be convergent in both 1-D and 2-D cases. Discussions of the underlying theory, the convergent criteria and the future extension, of this scheme are also investigated.

Introduction

The cross hair system (Fig. 1) has been used for traditional optical alignment problems. The alignment procedure is to position two rectangular cross sections which are formed by two pairs of cross hairs. Smaller cross sections lead to better alignment. The cross hair system described above provides a fair alignment pattern for humans being because of its simplicity. However, when the cross hair system is used, all the non-cross portion within the boresight area is not used for alignment. Therefore, the non-cross portion is totally wasted, and thus results in the poor signal to noise ratio. In order not to waste the non-cross portion, a new pattern reticle is used to tackle the optical alignment problems which promises a better signal to noise ratio. The binary pattern (Fig. 2) used in this study consists arrays of rectangular sections, which in fact is the pseudo-noise array or its variation. This pattern is imaged onto a second identical pattern. Then, the phase shifted version of the autocorrelation function is detected. The peak value of the autocorrelation function is obtained when two identical patterns are perfectly aligned. Similar arguments can be applied to the polar pattern (Fig. 3), in which function value changes with angle but not radius. When two polar patterns align perfectly, its autocorrelation function reaches its peak value. The design of the binary reticle for this alignment problem is in fact related to finding a signal pattern when its autocorrelation function is given. In optical alignment problems, narrow autocorrelation function assures a substantial change in the measured value of the autocorrelation even with relatively small translations between input signals. Thus, whatever patterns are used as input signals in optical alignment problems, it is always desirable to have narrow autocorrelation function. The iterative scheme used in this study is not guaranteed to converge to a signal which has the autocorrelation function as specified, because a real and binary signal which has the desired property may

not exist. However, based on the results of this study, the iterative scheme does converge to a function with a similar autocorrelation function, if not to the identically same one as designated.

The purpose of this study resides in designing the real and binary signal patterns which produce an autocorrelation (or crosscorrelation) function as narrower as possible because the great sensitivity to dislocation of two signal sources is a desirable property in optical alignment problems. The results of this study can be readily applied to those areas such as laser communication, boresight alignment and long range guided navigation systems, etc..

In the following sections, the method of applying this iterative scheme, a brief computational result, and future works are presented.

Methods

For an function $g(t)$, its autocorrelation function $R(\tau)$ and spectral density $S(f)$ are related by Fourier transformation : $F\{R(\tau)\} = S(f)$ or $F^{-1}\{S(f)\} = R(\tau)$, this can be represented as (1)

$$R(\tau) \leftrightarrow S(f) \quad (1)$$

Since $S(f) = |G(f)|^2$, where $G(f)$ is the Fourier transform of $g(t)$, equation (2) provides a way to recover $g(t)$ from it's autocorrelation $R(\tau)$ when $G(f)$ is nonnegative. That is :

$$\begin{aligned} g(t) &= F^{-1}\{G(f)\} \\ &= F^{-1}\{\sqrt{S(f)}\} \\ &= F^{-1}\{\sqrt{F\{R(\tau)\}}\} \end{aligned} \quad (2)$$

Thus, given a specific autocorrelation function $R(\tau)$, the study here utilizes an iterative scheme to obtain a function $g(t)$ which has the autocorrelation $R(\tau)$ similar or identical to $R(\tau)$.

The scheme is to iterate back and forth between time domain (or spatial domain) and frequency domain. This approach is based on the following facts :

[1] Let $G(f)$ be the Fourier transform of a real signal $g(t)$, then

$$g(-t) \leftarrow \rightarrow G(-f) \quad (3)$$

$$g^*(t) \leftarrow \rightarrow G^*(-f) \quad (4)$$

Where $*$ represents the complex conjugate. Thus, if $g(t)$ is a real function, then $G(-f) = G^*(f)$. And if $g(t)$ is even, then $G(f)$ must be real. Furthermore, if $g(t)$ is both real and even, then $G(f)$ is also both real and even.

[2] Because a real function $g(t)$ is desired, its companion $G(f)$ must satisfy $G(-f) = G(f)^*$. This fact has to be considered in initial phase assignment of the iterative scheme to speed up convergence.

[3] For optical signal processing problems, a 0 or 1 transmittance corresponds to binary signal, this binary condition must also be fulfilled in the time (or spatial) domain.

From the above reasons, the scheme is operated on the algorithms as follows :

$$R(\tau) \xrightarrow{F\{.\}} S(f) \xrightarrow{\sqrt{(.)}} |G(f)| \xrightarrow{x e^{j\phi}} G(f) \xrightarrow{F^{-1}\{.\}} g(t) \xrightarrow{\text{binarization}} g_b(t). \text{ Given } R(\tau),$$

a function $g(t)$ can be obtained which has $R(\tau)$ as its autocorrelation. Since $g(t)$ in general may not be binary, a binarization procedure needs to be performed on $g(t)$ to obtain $g_b(t)$. However, $g_b(t)$ may not have $R(\tau)$ as its autocorrelation. Keep this phase of $G_b(f)$, which is the Fourier transform of $g_b(t)$, unchanged; then, assign the phase to $|G(f)|$, to see whether $F^{-1}\{G(f)\} = \hat{g}(t)$ is binary or not. If not, keep iterating again and again. The iterating procedure can be terminated when there is not significance between consecutive functions $g_b(t)$.

There are two parameters which are essential to the scheme. The first one is the initial phase assigned : either zero phase or phase with $G(-f) = G^*(f)$ property. Zero phase assignment means $g(t)$ will not be real unless $G(f)$ is even. This is because the Fourier transformation of a real function does not in general yield a real function. But the Fourier transform of a real and even function is also real and even. This fact also explains the reason to assign the phase

satisfying $G(-f)=G^*(f)$, because this property assures $g(t)$, which is the inverse Fourier transform of $G(f)$, to be a real function, which is desired. The second parameter is the threshold value used in binarization; We either choose this value to be zero or the median value of $g(t)$. When the zero threshold is chosen, every sample value of $g(t)$ is examined to see whether it's positive or negative. If it is positive, a 1 is assigned to the corresponding sample value of $g_b(t)$, otherwise, a 0 is assigned. When the median of $g(t)$ is chosen as the threshold, each sample value of $g(t)$ which is greater than the threshold means a 1 will be assigned to the corresponding sample value of $g_b(t)$. On the other hand, a 0 will be assigned to $g_b(t)$ if the sample value of $g(t)$ is less than the threshold. The reason of choosing different threshold value is that it may help to "speed up" the iterative scheme depending on what the desired $g_b(t)$ looks like. If the desired $g_b(t)$ is symmetrical to 0 or has its median around 0, then zero thresholding might be a better choice. Otherwise, the median thresholding will be a reasonable alternative. In the next section we consider two examples. In a 1-D case the triangular function $\Delta(x)$, which is defined as (5), is used as the autocorrelation function.

$$\Delta(x) \equiv \begin{cases} 1 & \text{if } |x| \leq 1 \\ 0 & \text{if else} \end{cases} \quad (5)$$

While in a 2-D case, a pseudo-noise array pattern, which is defined in (6), or its variations, such as in (7), are used as the autocorrelation function.

$$R_A(i,j) \equiv \begin{cases} (pq+1)/2, & \text{if } i=0, j=0 \\ (pq+1)/4, & \text{if elsewhere} \end{cases} \quad (6)$$

$$R_A(i,j) \equiv \begin{cases} (pq+1)/2, & \text{if } (i,j) = \text{mod}(m,n) \\ & \text{where } m,n \text{ are integers} \\ (pq+1)/4, & \text{if elsewhere} \end{cases} \quad (7)$$

Computational Results

The triangular function (Fig. 4) is used as the autocorrelation function in

the 1-D case. The triangular function is a even function, and both Fourier transformation (for real signals) and binarization procedure preserve the even property during iteration. This fact illustrates a pitfall in the scheme : when zero phase and zero threshold (in binarization procedure) are chosen, it stops iteration right after the first step. This is because the function itself is real and even all the time during the iteration, thus keeping the zero phase unchanged. When this situation happens, each iteration step of the scheme is just like it starts from the very beginning, this means the scheme does not iterate. The similar situation seem to occurs when the phase is assigned according to $G^*(f)=G(-f)$ property. The $g_b(t)$ becomes real and even after the second iteration, thus for the same reason, terminates the iterating process. The best results occur when median thresholding and either when zero phase is assigned as initial phase or when $G^*(f)=G(-f)$ property be used(Fig.5 - 6). Both conditions converge to the rectangular function as expected, which has the triangular function as its autocorrelation.

For the 2-D case, the pseudo-noise array pattern and its minor modification (Fig.7, Fig.10 and Fig.13) are used as the input 2-D autocorrelation function. The convergent autocorrelation functions are shown in Figure 8, Figure 11 and Figure 14. The corresponding binary functions are shown also(Fig. 9, Fig. 12, Fig. 15). The results varies with different parameter values, such as the initial phase assignment as well as the threshold value. Although the results do not coincide indentially to the original functions selected, they do preserve the maximum peak values. The feature is noticed from the result graphs. Nevertheless, the scheme does converge under all conditions to a function with unimodal property, at least roughly. The general trend of the performance of the scheme in the 2-D domain needs further exploration.

Future Theoretical Works

- [1] Test some other patterns, such as a) two dimensional triangular function or, b) replicate a variation of a pseudo-noise pattern with a non-symmetrical property. scheme.
- [2] Translate this iterative scheme computer program, which is written in C language, to FORTRAN.
- [3] Provides the numerical and graphic data of the scheme under different conditions, and also include a well-documented computer program.

Implementation

Although we are doing research on both one and two dimensional patterns, for practical alignment problems we would like to work solely with one dimensional patterns. We want to use separate patterns, verticle and horizontal, for x and y alignment. In this way we may first align the pattern in the x-direction, then align the pattern in the y-direction. Consequently, simple one-dimensional search procedures can be used for each alignment. If we tried to align the x and y-directions simultaneously by use of a two dimensional verticle pattern, the two dimensional search would be much more complicated.

In order to have separate alignment for the x and y directions, it is important the verticle not interfere while horizontal alignment is being done. Similarly, the horizontal reticle should not interfere with vertical alignment. To accomplish this, we make the patterns polarization dependent. Say for example, that the horizontal pattern is totally transparent to vertical polarization. It only appears binary to horizontal polarization. Similarly, the verticle pattern only appears as binary to verticle polarization. These patterns might appear as in Figs.16 and 17. If these patterns are overlayed onto one another the result might be as shown in Fig. 8. If we illuminate the pattern of Fig. 18 with verticle

polarization one would only see the vertical reticle. The horizontal reticle would be seen by use of horizontal polarization. It is our intention to fabricate reticle patterns by use of Purdue University's e-beam facility.

Figure 4 1-D Autocorrelation : Triangular Function

Figure 5 1-D Estimate Autocorrelation Function During Iteraing Process

Figure 6 1-D Estimate Real & Binary Function During Iterating Process

initial phase assignment = 0 iteration no = 20

Figure 7 2-D Autocorrelation Function, code = 10

Figure 8 2-D Autocorrelation Function Estimate iter = 20th code = 10

Figure 9 1-D Projection Of Estimated Real & Binary Function : code = 10

initial phase assignment = 0 iteration no. = 20

Figure 10 2-D Autocorrelation Function : code = 12

Figure 11 2-D Autocorrelation Function Estimate : code = 12

Figure 12 1-D Projection Of Estimated Real & Binary Function : code = 12

initial phase assignment = 0 iteration no. = 20

Figure 13 2-D Autocorrelation Function : code = 13

Figure 14 2-D Autocorrelation Function Estimate iter = 20th code = 13

Figure 15 1-D Projection Of Estimated Real & Binary Function : code = 13

Fig. 16 **Vertical Pattern**

Fig. 17 **Horizontal Pattern**

Fig. 18 **Horizontal-Vertical Overlay**

Appendix B
Number Theoretic Design of Reticule Patterns

Pseudo-Noise Arrays and Optical Alignment

by

Neal C. Gallagher

Introduction

Traditionally a pair of cross hairs has been used for optical alignment problems. The cross hair system is particularly important for the human who may be doing the alignment. Consider the cross hair system found in Fig. 1. This figure shows two sets of non-zero width cross hairs. The intersection of two cross hairs is rectangular. The alignment procedure consists positioning one rectangular cross section over the second cross section. The precision of the alignment depends on the size of this cross sections. Smaller sections lead to better alignment.

For humans, a cross hair system provides a simple pattern that is easy to align. However, with the exception of the physical area occupied by the cross hair, the entire area of the boresight is wasted. The result is that in alignment systems with poor optical signal to noise ratio the performance of a cross hair scheme may be very poor. If, indeed, it were somehow possible to use the complete boresight area, system performance could be dramatically improved.

A superior alignment procedure arises from the application of number theory to the problem. It appears possible to design patterns which fill the entire boresight and allow alignment to be as precise as with cross hair at their best. Because these patterns use the entire boresight, their signal to noise performance is greatly superior to the cross hair design. In addition, a simple machine automated procedure may be employed to achieve alignment.

The patterns we consider are arrays of rectangular sections. Each rectangular section may have the same size as the rectangular center of a cross hair pattern. However,

Figure 1. Boresight of Cross-Hair Optical Alignment System.

each rectangular patch will contribute signal to the alignment system: consequently, the signal to noise ratio for such an array should be greater for an array than for a cross hair pattern. We are trying to design a system such as shown in Fig. 2. Here a binary pattern is imaged onto a second identical pattern. A second lens is used to integrate the total signal onto a photodetector. Therefore, the phase shifted autocorrelation of the pattern is detected. The second array is then adjusted to achieve the peaked detected value of the autocorrelation. In a similar fashion, one can envision a polar format binary pattern to be used for angular alignment. Here, the pattern is a constant function of radius and changes value only with angle; see Fig. 3 for an example of such a pattern. In the next section we present a summary of the basic results from number theory that may be used for the design of such binary arrays.

Number Theory and Uniformly Redundant Arrays

the basic results from number theory were first presented by Calabro and Wolf in 1968 [1]. Their work deals with the design of what since has been called a uniformly redundant array [2]. We begin with a few definitions. Let $A(x,y)$ be a basic block of p by q binary elements; a doubly periodic array is formed by repeating this basic block on each of its sides and continuing the process until the entire plane is covered. The x and y period of the array, p and q , are defined as the least positive integers such that

$$\begin{aligned} A(x + p, y) &= A(x, y) \\ A(x, y + q) &= A(x, y) \\ A(x + p, y + q) &= A(x, y) \end{aligned}$$

The area of the array is defined to be pq . The periodic autocorrelation of the array is defined as

$$R_A(i,j) = \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} A(x,y) A(x+i, y+j).$$

The array basic array $A(x,y)$ is formed as a quadratic residue array. It can be shown

Figure 2. Array 1 is Imaged onto Array 2 to form Autocorrelation on Photo-Detector. Array 2 is Adjusted in Position to Determine Maximum Value of the Autocorrelation.

Figure 3. Polar Binary Array to Be Used for Angular Alignment

that such arrays of area pq have the value -1 for all out-of-phase correlations (binary here means $A(x,y)$ takes values of either +1 or -1).

The basis of these arrays is the algebraic structure of the Legendre operator (x/p) , defined below, where p is always considered a prime:

$$(x/p) = \begin{cases} 0 & \text{if } x \equiv 0 \pmod{p} \\ 1 & \text{if } x \text{ is a square mod } (p) \\ -1 & \text{if } x \text{ is a non-square mod } (p). \end{cases}$$

Those values of x for which $(x/p) = 1$ are called quadratic residues. The Legendre operator has the following properties which we state without proof:

1. $(x/p) = (y/p)$ if $x \equiv y \pmod{p}$
2. $(xy/p) = (x/p)(y/p)$
3. $(x/p) \equiv x^{(p-1)/2} \pmod{p}$
4. $(1/p) = 1$
5. $(-1/p) \equiv -1$ if $p \equiv 3 \pmod{4}$, $(-1/p) = 1$ if $p \equiv 1 \pmod{4}$.
6. There are exactly $(p-1)/2$ quadratic residues and nonresidues in the set $(1, 2, 3, \dots, p-1)$ hence,

$$\sum_{x=1}^{p-1} (x/p) = 0$$

7. From the above property, the following can be shown true:

$$\sum_{x=1}^{p-1} ((x+i)/p) = -(i/p)$$

$$\sum_{\substack{x=1 \\ x \neq p-i}}^{p-1} (x/p) = -(-i/p)$$

$$\sum_{i=1}^{p-1} (x/p)((x+i)/p) = -1.$$

We define the array $A(x,y)$ as follows:

$$A(x,y) = \begin{cases} (x/p)(y/q) & x \not\equiv 0 \pmod{p}; y \not\equiv 0 \pmod{q} \\ -1 & x \not\equiv 0 \pmod{p}; y \equiv 0 \pmod{q} \\ +1 & x \equiv 0 \pmod{p}; y \not\equiv 0 \pmod{q} \\ -1 & (x,y) \equiv (0,0) \pmod{p,q}, \end{cases}$$

where p and q are prime numbers with $q-p=2$. The array $A(x,y)$ takes only two values $+1$ and -1 . The autocorrelation of $A(x,y)$ is

$$R_A(x,y) = \begin{cases} pq & (i,j) \equiv (0,0) \pmod{p,q} \\ -1 & (i,j) \not\equiv (0,0) \pmod{p,q}. \end{cases}$$

Arrays such as $A(x,y)$ are sometimes called pseudo-noise arrays. In our application; however, we will not be making binary ± 1 array, but we will be making arrays with the binary values of $+1$ and 0 . We accomplish this by changing all values in $A(x,y)$ of -1 to the value of zero. We call the new array $\hat{A}(x,y)$; consequently,

$$\hat{A}(x,y) = \frac{1}{2} (A(x,y) + 1).$$

Consider

$$\begin{aligned} D_A &= \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} \hat{A}(x,y) \\ &= \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} \frac{1}{2} (A(x,y) + 1) \\ &= \frac{1}{2} \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} A(x,y) + pq/2 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \left[-1 + \sum_{x=1}^{p-1} (-1) + \sum_{y=1}^{q-1} (1) + \sum_{x=1}^{p-1} (x/p) \sum_{y=1}^{q-1} (y/p) \right] + pq/2 \\
 &= \frac{1}{2} \left[-1 - (p-1) + (q-1) \right] + pq/2 \\
 &= \frac{1}{2} (q-p-1) + pq/2
 \end{aligned}$$

Recall that $q-p=2$, so

$$D_A = (pq + 1)/2.$$

The term D_A is the autocorrelation of $\hat{A}(x,y)$ evaluated at the origin.

How about the other value of the autocorrelation? Consider

$$\begin{aligned}
 R_{\hat{A}}(i,j) &= \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} \hat{A}(x,y) \hat{A}(x+i, y+j) \\
 &= \frac{1}{4} \left[\sum_{x=0}^{p-1} \sum_{y=0}^{q-1} A(x,y) A(x+i, y+j) + \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} A(x,y) \right. \\
 &\quad \left. + \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} A(x+i, y+j) + \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} 1 \right].
 \end{aligned}$$

According to Calabro and Wolf

$$\sum_{x=0}^{p-1} \sum_{y=0}^{q-1} A(x,y) A(x+i, y+j) = -1$$

We have just shown that

$$\sum_{x=0}^{p-1} \sum_{y=0}^{q-1} A(x,y) = \sum_{x=0}^{p-1} \sum_{y=0}^{q-1} A(x+i, y+j) = 1.$$

So,

$$R_A(i,j) = \frac{1}{4} \left[-1 + 1 + 1 + pq \right]$$
$$= (pq + 1)/4.$$

Combining results

$$R_A(i,j) = \begin{cases} (pq + 1)/2, & i = 0, j = 0 \\ (pq + 1)/4, & \text{elsewhere} \end{cases}$$

We are still investigating the best system implementation for these alignment arrays. Our future work will deal with this topic. In addition we are still looking at the iteration design technique described in my last report.

References

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Appendix C

Theoretical Description of Very High Frequency Conducting Gratings

**Diffraction of Electromagnetic Waves by a Perfectly Conducting
Grating with Periodic Groove Structure**

Yon-Lin Kok and Neal C. Gallagher

School of Electrical Engineering

Purdue University

West Lafayette, IN 47907

Abstract

A complete solution of plane wave scattering from a corrugated surface of infinite extent for arbitrary incidence is presented. The electromagnetic wave is decomposed into Fast- and Slow- modal representation and the solution is accomplished through the use of mode matching method. We solved for the zeroth order diffraction efficiency and the phase of each polarization component for arbitrary incident angles. Our results have both verified special cases previously published by others and have illustrated the phase shift between polarizations associated with the diffraction process as a function of the incident angles.

1. Introduction :

Unidirectional periodic metallic gratings are often employed to rotate or twist the polarization of an incident plane wave and in general convert from one polarization state to another. Suppose that linearly polarized light is incident on a periodic metallic groove grating (see fig. 1). If the phase difference introduced, by the groove structure, between the electric field components parallel and perpendicular to the grooves equals 180 degree, then the incident linear polarization will be twisted to another linear polarization rotated by twice the angle between the orientation of the grooves and the incident plane of polarization. The rotation effect is depicted in figure 1. The polarization twisting concept has been described previously in the literature by Mittra [1] for scanned reflector antenna systems. Although many designs were carried out over the years neither rigorous solutions nor general rules of twist reflector design for arbitrary plane wave incidence exist in the literature. The motivation of this paper emerges in the design of holographic microwave elements which combine principles of digital holography and twist polarizer in a unique fashion [2]. In this paper we analyze the scattering problem of electromagnetic waves by a perfectly conducting grating with periodic groove structure.

This diffraction problem has been intensively studied over the past few years with great interest in the diffraction efficiency of different spectral orders of grating structures [2]. Veldkamp and Swanson observed that high diffraction efficiency can be achieved by operating the holographic grating deep in the electromagnetic optics regime [3]. A number of asymptotic numerical methods have been proposed to investigate the problem of different grating structures. The dielectric problem has been studied by Moharam and Gaylord by using the state-variables method of solution for the rigorous coupled-wave equations [4]. On the other hand, the spectral-iteration approach proposed by Tsao and Mittra [5] leads to the resolution of plane wave scattering from a free standing periodically perforated conducting screen where the thickness of the

conducting screen is negligible. The iterative technique is further applied to the problem of a step discontinuity geometry. These geometries are characterized by the features that they comprise a junction of two or more regions of dissimilar dimension. The solution of a closed-region type problem - a step discontinuity in a parallel-plate wave guide - can be found in the literature.[5] However, for the open-region type problem - the infinite half space and a periodic array of short-circuited rectangular waveguides - the spectral iteration method (Kastner and Mittra) [1], though capable of handling a large number of unknowns and requires no matrix inversion, was found by us to be divergent as the height of the groove approaches the region where symmetric twisting of the incident polarization with respect to groove direction occurred.

Let us consider a rectangular coordinate system (see figure 2) and rectangular periodic groove surface with period d and groove width c . This perfect conducting grating is illuminated by a plane monochromatic wave with time dependence $\exp(-i\omega t)$ and a wave vector $\vec{k}$. The wave incidence angles are described by θ and ϕ shown in figure 3. We propose to study the properties of the steady state diffracted field with a special regard to the phase differences between the incident and the reflected wave components at the zeroth order. The groove period d is taken to be one quarter wavelength. The major references we used to formulate the problem have been the book by R. Petit [6] and the classic textbook by Born and Wolf [7]. Since the grating is periodic in x -direction and uniform in z -direction we are able to reduce the problem into a two dimensional grating problem and represent the wave function in terms of Rayleigh expansion. We successfully solve the problem by matching the Fourier coefficients of field functions at the boundary between the open region and inside the waveguide. In our calculations this method proves to be convergent for relatively deep grooves (one wavelength) as well as at the low frequency region where groove width size is one eighth of the wavelength. A special effort is made in this paper to give a complete set of numerical data on the relative phase changes between two orthogonal propagating

modes at the zeroth order (discussed later).

When one studies reflection from planar non-grooved conducting surfaces, it is sufficient to consider two special cases. One case occurs when the E-vector is parallel to the plane of incidence; the second case does when the E-vector is perpendicular to the plane of incidence. These two solutions are sometimes referred to as the TM and TE polarizations. With these solutions it is possible to solve the case of arbitrary angle of incidence. However, the problem we have here is a plane conducting surface with periodic infinitely long groove structure. In this situation it is no longer useful to only consider the relationship between the incident E-vector and the plane of incidence. The relationship of the E-vector and groove orientation is also an important consideration. The incident electromagnetic wave must be decomposed into two different cases; 1) Slow polarization : the E-field **does not** have a component in the groove direction. 2) Fast polarization : the H-field **does not** have a component in the groove direction. We can write

$$\vec{E}_f = (E_x, E_y, E_z),$$

$$\vec{H}_f = (H_x, H_y, 0),$$

$$\vec{E}_s = (E_x, E_y, 0),$$

$$\vec{H}_s = (H_x, H_y, H_z).$$

Noted that in general $\vec{E}_f$ and $\vec{E}_s$ are orthogonal. In section 2 we derive the coupled wave equation for each of the two cases, where a detailed analysis is presented.

Section 3 contains the results of our computer analysis. We found that the phase difference between the two reflected polarizations of the zeroth order is sensitive to both the incident angle θ and ϕ . Therefore, 180° phase difference between the two orthogonal polarized waves would not necessarily occur when the groove depth equals a quarter wavelength.

2. Problem Formulation :

We represent the incident E-field as a plane wave of the form $\exp[i(\alpha_0 x - \beta_0 y + \gamma z)]$

where

$$\alpha_0 = k \cos \theta \cos \phi,$$

$$\beta_0 = k \sin \theta \cos \phi,$$

$$\gamma = k \sin \phi,$$

$$k = \omega \sqrt{\mu \epsilon}.$$

See fig. 3 for the definition of parameters. So, as found in Appendix A

$$E_x = \frac{i\gamma}{\bar{h}^2} \frac{\partial E_z}{\partial x} + \frac{i\omega\mu}{\bar{h}^2} \frac{\partial H_z}{\partial y}, \quad H_x = \frac{i\gamma}{\bar{h}^2} \frac{\partial H_z}{\partial x} - \frac{i\omega\epsilon}{\bar{h}^2} \frac{\partial E_z}{\partial y},$$

$$E_y = \frac{i\gamma}{\bar{h}^2} \frac{\partial E_z}{\partial y} - \frac{i\omega\mu}{\bar{h}^2} \frac{\partial H_z}{\partial x}, \quad H_y = \frac{i\gamma}{\bar{h}^2} \frac{\partial H_z}{\partial y} + \frac{i\omega\epsilon}{\bar{h}^2} \frac{\partial E_z}{\partial x},$$

where

$$\bar{h}^2 = \omega^2 \mu \epsilon - \gamma^2 = k^2 \cos^2 \phi.$$

As might be expected [8] $\vec{E}$ and $\vec{H}$ can be entirely characterized by the z-components E_z and H_z . We divide the electromagnetic field into two orthogonal components :

1) Fast components where $H_z = 0$ ($\vec{E} = \vec{E}_f$).

2) Slow components where $E_z = 0$ ($\vec{E} = \vec{E}_s$).

For each of these two orthogonal cases we have the field solution exterior to the grooves ($y \geq h$) and the solution interior to the grooves ($0 \leq y \leq h$).

2.1. Field solutions for $y \geq h$

1) Fast polarization case ($\vec{E} = \vec{E}_f$)

By Maxwell's equation we have

$$E_x = \frac{i\gamma}{\bar{h}^2} \frac{\partial E_z}{\partial x}, \quad H_x = \frac{-i\omega\epsilon}{\bar{h}^2} \frac{\partial E_z}{\partial y},$$

$$E_y = \frac{i\gamma}{\bar{h}^2} \frac{\partial E_z}{\partial y}, \quad H_y = \frac{i\omega\epsilon}{\bar{h}^2} \frac{\partial E_z}{\partial x},$$

and

$$\nabla^2 E_z + k^2 E_z = 0,$$

where recall

$$\bar{h}^2 = k^2 \cos^2 \phi, \quad k^2 = \omega^2 \mu \epsilon.$$

The solution form given by Born and Wolf [7] is

$$\vec{E} = \left[\frac{i \sin \phi}{k} \frac{\partial u}{\partial x}, \frac{i \sin \phi}{k} \frac{\partial u}{\partial y}, u \cos^2 \phi \right],$$

$$\vec{H} = \left[\frac{-i}{\omega\mu} \frac{\partial u}{\partial y}, \frac{i}{\omega\mu} \frac{\partial u}{\partial x}, 0 \right],$$

where $u(x,y,z)$ must satisfy the scalar wave equation

$$\nabla^2 u + k^2 u = 0.$$

Referring to Petit [6], the solution exterior to the grooves must be of the form

$$u(x,y,z) = \exp[i(\alpha_0 x - \beta_0 y + \gamma z)] + \sum_{n=-\infty}^{\infty} r_n \exp[i(\alpha_n x + \beta_n y + \gamma z)],$$

where

$$\alpha_n = \alpha_0 + \frac{n2\pi}{d}, \quad \beta_n = \sqrt{k^2 - \alpha_n^2 - \gamma^2}.$$

consequently,

$$E_x(x,y,z) = \cos^2 \phi \left\{ \exp[i(\alpha_0 x - \beta_0 y + \gamma z)] + \sum_{n=-\infty}^{\infty} r_n \exp[i(\alpha_n x + \beta_n y + \gamma z)] \right\},$$

$$H_x(x,y,z) = \frac{(-i)}{\omega\mu} \left\{ (-i\beta_0) \exp[i(\alpha_0 x - \beta_0 y + \gamma z)] + \sum_{n=-\infty}^{\infty} r_n (i\beta_n) \exp[i(\alpha_n x + \beta_n y + \gamma z)] \right\}.$$

2) Slow polarization case ($\vec{E} = \vec{E}_s$)

By Maxwell's equations we have

$$E_x = \frac{i\omega\mu}{h^2} \frac{\partial H_z}{\partial y}, \quad H_x = \frac{i\gamma}{h^2} \frac{\partial H_z}{\partial x},$$

$$E_y = \frac{-i\omega\mu}{h^2} \frac{\partial H_z}{\partial x}, \quad H_y = \frac{i\gamma}{h^2} \frac{\partial H_z}{\partial y},$$

and

$$\nabla^2 H_z + k^2 H_z = 0.$$

The solution form given by Born and Wolf[7] is

$$\vec{E} = \left[\frac{i}{\omega\epsilon} \frac{\partial u}{\partial y}, \frac{-i}{\omega\epsilon} \frac{\partial u}{\partial x}, 0 \right],$$

$$\vec{H} = \left[\frac{i\sin\phi}{k} \frac{\partial u}{\partial x}, \frac{i\sin\phi}{k} \frac{\partial u}{\partial y}, u\cos^2\phi \right],$$

where for our problem

$$u(x,y,z) = \exp[i(\alpha_0 x - \beta_0 y + \gamma z)] + \sum_{n=-\infty}^{\infty} s_n \exp[i(\alpha_n x + \beta_n y + \gamma z)].$$

This last expression for $u(x,y,z)$ is after Petit [6]. Consequently,

$$H_z(x,y,z) = \cos^2\phi \left\{ \exp[i(\alpha_0 x - \beta_0 y + \gamma z)] + \sum_{n=-\infty}^{\infty} s_n \exp[i(\alpha_n x + \beta_n y + \gamma z)] \right\},$$

$$E_x(x,y,z) = \frac{i}{\omega\epsilon} \left\{ (-i\beta_0) \exp[i(\alpha_0 x - \beta_0 y + \gamma z)] + \sum_{n=-\infty}^{\infty} s_n (i\beta_n) \exp[i(\alpha_n x + \beta_n y + \gamma z)] \right\}$$

2.2. Field solution for $0 \leq y \leq h$

1) Fast polarization case ($\vec{E} = \vec{E}_t$)

Within the groove the field $E_z(x, y, z)$ must satisfy the equation

$$\nabla^2 E_z + k^2 E_z = 0,$$

subject to the boundary conditions at the conducting walls; ie.

$$E_z(x, y, z) = 0 \quad \text{at } x = 0, x = c, \quad \text{and} \quad y = 0.$$

Also the field must be zero within the conducting material. So, the solution of the scalar wave equation subject to the boundary condition is

$$E_z(x, y, z) = \begin{cases} \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{c}\right) \sin(A_n y) \exp(iz\gamma), & 0 \leq x \leq c. \\ 0, & c < x \leq d. \end{cases}$$

The term $\sin(A_n y)$ is replaced by $\sinh(\bar{A}_n y)$ if $A_n^2 < 0$ where

$$A_n^2 = h^2 - \left(\frac{n\pi}{c}\right)^2, \quad n=1, 2, \dots$$

$$\bar{A}_n = \text{Im}(A_n) \quad \text{when } A_n^2 < 0.$$

2) Slow polarisation case ($\vec{E} = \vec{E}_z$)

In this case we must solve the following differential equation

$$\nabla^2 H_z + k^2 H_z = 0,$$

with the boundary conditions

$$\frac{\partial H_z}{\partial x} = 0 \quad \text{at } x = 0, x = c, \quad \text{and} \quad \frac{\partial H_z}{\partial y} = 0 \quad \text{at } y = 0.$$

The solution is

$$H_z(x, y, z) = \sum_{n=0}^{\infty} b_n \cos\left(\frac{n\pi x}{c}\right) \cos(B_n y) \exp(iz\gamma), \quad 0 \leq x \leq c.$$

where

$$B_n^2 = b^2 - \left(\frac{n\pi}{c}\right)^2, \quad n=0, 1, 2, \dots$$

$$\bar{B}_n = \text{Im}(B_n) \quad \text{when } B_n^2 < 0.$$

The term $\cos(B_n y)$ is replaced by $\cosh(\bar{B}_n y)$ if $B_n^2 < 0$.

2.3. Matching tangential fields at the interface ($y=h$)

Consider the fields in one period of the groove structure ie. ($0 \leq x \leq d$). At the interface ($y=h$) between those two regions previously discussed in section 2.1 ($y \geq h$) and in section 2.2 ($0 \leq y \leq h$) the electromagnetic boundary conditions require that tangential E-field be continuous for all x and tangential H-field be continuous within the aperture (for $0 \leq x \leq c$).

1) Fast polarization case :

We impose the following conditions for the fast polarization case

$$a) E_z(x,y,z) \Big|_{y=h} \text{ is continuous for } 0 \leq x \leq d.$$

$$b) H_x(x,y,z) \Big|_{y=h} \text{ is continuous for } 0 \leq x \leq c.$$

By forcing the continuity of the above two fields we obtain the following coupled equations that allow us to solve for the unknown Rayleigh and Fourier coefficients; i.e. r_n and a_m . Referring to Appendix B for the derivations ;

$$\cos^2 \phi [\delta_{n0} \exp(-i2\beta_0 h) + r_n] = \sum_{m=1}^{\infty} \frac{a_m}{d} \sin(A_m h) \exp(-i\beta_m h) J_{nm}, \quad (1)$$

$$\sum_{m=-\infty}^{\infty} \cos^2 \phi \left[(-i\beta_0) \delta_{m0} \exp(-i\beta_0 h) + r_m (i\beta_m) \exp(i\beta_m h) \right] c_{n-m}$$

$$= \frac{1}{d} \sum_{m=1}^{\infty} s_m A_m \cos(A_m h) I_{nm} \quad (2)$$

2) Slow polarization case:

In this case we use the solutions for H_x and E_x ; we impose continuity where

a) $H_x(x, y, z) \Big|_{y=h}$ is continuous for $0 \leq x \leq c$.

b) $E_x(x, y, z) \Big|_{y=h}$ is continuous for $0 \leq x \leq d$.

Similarly, we obtain another set of two coupled equations involving the unknown Rayleigh and Fourier coefficients s_m and b_m . (See Appendix C for derivations)

$$\sum_{m=-\infty}^{\infty} \cos^2 \phi \left[\delta_{m0} \exp(-i\beta_0 h) + s_m \exp(i\beta_m h) \right] c_{n-m} = \frac{1}{d} \sum_{m=0}^{\infty} b_m \cos(B_m h) J_{nm}, \quad (3)$$

$$\begin{aligned} & \cos^2 \phi \left[(-i\beta_0) \delta_{n0} \exp(-i2\beta_0 h) + s_n (i\beta_n) \right] \\ &= \frac{1}{d} \sum_{m=0}^{\infty} (-b_m B_m) \sin(B_m h) \exp(-i\beta_n h) J_{nm} \quad (4) \end{aligned}$$

2.4. Numerical method

We have four equations (1), (2), (3) and (4) and we solve for four set of unknowns r_n , a_n , s_n , and b_n . In this fashion we can solve for

$$E_z = \cos^2 \phi \left\{ \exp[i(\alpha_0 x - \beta_0 y + \gamma z)] + \sum_{n=-\infty}^{\infty} r_n \exp[i(\alpha_n x + \beta_n y + \gamma z)] \right\},$$

for the fast modes and

$$H_z = \cos^2 \phi \left\{ \exp[i(\alpha_0 x - \beta_0 y + \gamma z)] + \sum_{n=-\infty}^{\infty} s_n \exp[i(\alpha_n x + \beta_n y + \gamma z)] \right\}.$$

for the slow modes.

1) Fast polarization case :

Rewrite equations (1) and (2) as

$$r_n + \delta_{n0} \exp(-i2\beta_0 h) = \sum_{m=1}^{\infty} \left[\frac{\sin(A_m h) \exp(-i\beta_n h) I_{nm}}{d \cos^2 \phi} \right] a_m, \quad (5)$$

$$\begin{aligned} \sum_{m=-\infty}^{\infty} \left[c_{n-m}(i\beta_m) \exp(i\beta_m h) \right] r_m - c_n(i\beta_0) \exp(-i\beta_0 h) \\ = \sum_{m=1}^{\infty} \left[\frac{A_m \cos(A_m h) I_{nm}}{d \cos^2 \phi} \right] a_m. \end{aligned} \quad (6)$$

Let

$$m1_{nm} = \frac{\sin(A_m h) \exp(-i\beta_n h) l_{nm}}{d \cos^2 \phi}, \quad m2_{nm} = \frac{A_m \cos(A_m h) l_{nm}}{d \cos^2 \phi},$$

$$c2_{nm} = c_{n-m} i \beta_m \exp(i\beta_m h).$$

Assume that the field is sufficiently well described by $(2k+1)$ coefficients r_n ($n = -k, -(k-1), \dots, 0, \dots, (k-1), k$) and $(2k+1)$ coefficients a_m ($m = 1, 2, \dots, (2k+1)$). Then the summations over infinite terms in equations (5) and (6) are truncated and represented by matrix arithmetic as

$$\bar{r} + \bar{v} = M_1 \bar{a}, \quad (5.1)$$

$$C_2 \bar{r} - \bar{t} = M_2 \bar{a}, \quad (6.1)$$

where

$$\bar{r} = \left[r_{-k}, r_{-(k-1)} \dots r_0 \dots r_{(k-1)}, r_k \right]^k,$$

$$\bar{a} = \left[a_1, a_2 \dots a_{2k}, a_{(2k+1)} \right]^k,$$

$$\bar{v} = \left[0 \dots 0, \exp(-i2\beta_0 h), 0 \dots 0 \right]^k,$$

$$\bar{t} = (i\beta_0) \exp(-i\beta_0 b) \left[c_{-k}, c_{-(k-1)} \dots c_0 \dots c_{(k-1)}, c_k \right]^T,$$

$$M_1 = \begin{bmatrix} m1_{-k,1} & m1_{-k,2} & m1_{-k,(2k+1)} \\ m1_{-(k-1),1} & m1_{-(k-1),2} & m1_{-(k-1),(2k+1)} \\ \vdots & \vdots & \vdots \\ m1_{k,1} & m1_{k,2} & m1_{k,(2k+1)} \end{bmatrix},$$

$$M_2 = \begin{bmatrix} m2_{-k,1} & m2_{-k,2} & m2_{-k,(2k+1)} \\ m2_{-(k-1),1} & m2_{-(k-1),2} & m2_{-(k-1),(2k+1)} \\ \vdots & \vdots & \vdots \\ m2_{k,1} & m2_{k,2} & m2_{k,(2k+1)} \end{bmatrix},$$

$$C_2 = \begin{bmatrix} c2_{-k,-k} & c2_{-k,-(k-1)} & c2_{-k,k} \\ c2_{-(k-1),-k} & c2_{-(k-1),-(k-1)} & c2_{-(k-1),k} \\ \vdots & \vdots & \vdots \\ c2_{k,-k} & c2_{k,-(k-1)} & c2_{k,k} \end{bmatrix}.$$

Substitute $\bar{t}$ in (5.1) into (6.1) and solve for $\bar{a}$

$$(C_2 M_1 - M_2) \bar{a} = 2\bar{t}, \quad (7)$$

then substitute $\bar{a}$ into (5.1) to get $\bar{r}$

$$\bar{r} = M_1 \bar{a} - \bar{v}. \quad (8)$$

From (8) we obtain the column vector $\bar{r}$ whose elements are the Rayleigh coefficients

r_n ($n = -k, -(k-1), \dots, 0, \dots, (k-1), k$) for the Fast polarization electromagnetic field.

2) Slow polarization case:

Rewrite equations (3) and (4) as

$$\sum_{m=-\infty}^{\infty} c_{n-m} \exp(i\beta_m h) s_m + c_n \exp(-i\beta_0 h) = \sum_{m=0}^{\infty} \left[\frac{\cos(B_m h) J_{nm}}{d \cos^2 \phi} \right] b_m, \quad (9)$$

$$(i\beta_n) s_n - \delta_{n0} (i\beta_0) \exp(-i2\beta_0 h) = \sum_{m=0}^{\infty} \left[\frac{-B_m \sin(B_m h) \exp(-i\beta_n h) J_{nm}}{d \cos^2 \phi} \right] b_m, \quad (10)$$

Let

$$n1_{nm} = \frac{-B_m \sin(B_m h) \exp(-i\beta_n h) J_{nm}}{d \cos^2 \phi}, \quad n2_{nm} = \frac{\cos(B_m h) J_{nm}}{d \cos^2 \phi},$$

$$d2_{nm} = c_{n-m} \exp(i\beta_m h), \quad b1_n = i\beta_n.$$

Similar to part (1) we can represent equations (9) and (10) as

$$D_2 \bar{s} + \bar{w} = N_2 \bar{b}, \quad (9.1)$$

$$B_1 \bar{s} - \bar{u} = N_1 \bar{b}, \quad (10.1)$$

where

$$\bar{s} = \left[s_{-k}, s_{-(k-1)}, \dots, s_0, \dots, s_{(k-1)}, s_k \right]^T,$$

$$\bar{b} = \left[b_0, b_1, \dots, b_{2k-1}, b_{2k} \right]^t,$$

$$\bar{u} = \left[0 \dots 0, i\beta_0 \exp(-i2\beta_0 h), 0 \dots 0 \right]^t,$$

$$\bar{w} = \exp(-i\beta_0 h) \left[c_{-k}, c_{-(k-1)} \dots c_0 \dots c_{(k-1)}, c_k \right]^t,$$

$$N_1 = \begin{bmatrix} n1_{-k,0} & n1_{-k,1} & \dots & n1_{-k,2k} \\ n1_{-(k-1),0} & n1_{-(k-1),1} & \dots & n1_{-(k-1),2k} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ n1_{k,0} & n1_{k,1} & \dots & n1_{k,2k} \end{bmatrix},$$

$$N_2 = \begin{bmatrix} n2_{-k,0} & n2_{-k,1} & \dots & n2_{-k,2k} \\ n2_{-(k-1),0} & n2_{-(k-1),1} & \dots & n2_{-(k-1),2k} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ n2_{k,0} & n2_{k,1} & \dots & n2_{k,2k} \end{bmatrix},$$

$$D_2 = \begin{bmatrix} d2_{-k,-k} & d2_{-k,-(k-1)} & \dots & d2_{-k,k} \\ d2_{-(k-1),-k} & d2_{-(k-1),-(k-1)} & \dots & d2_{-(k-1),k} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ d2_{k,k} & d2_{k,-(k-1)} & \dots & d2_{k,k} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} b1_{-k,0} & \dots & 0 \\ 0 & b1_{-(k-1),0} & \dots & 0 \\ . & 0 & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ 0 & 0 & \dots & b1_k \end{bmatrix},$$

Substitute $\bar{s}$ in (9.1) into (10.1) and solve for $\bar{b}$

$$(B_1 D_2^{-1} N_2 - N_1) \bar{b} = \bar{u} + B_1 D_2^{-1} \bar{w}, \quad (11)$$

Substitute $\bar{b}$ into (9.1) and get $\bar{s}$

$$\bar{s} = D_2^{-1} (N_2 \bar{b} - \bar{w}). \quad (12)$$

From (12) we obtain the column vector $\bar{s}$ whose elements are the Rayleigh coefficients for the Slow polarization electromagnetic field.

3. Numerical results and Conclusions

The theoretical results developed in previous section have been implemented as FORTRAN programs on CYBER 205 computer. We tested the program against previously published results and were able to duplicate the results found in Petit [6] (See Fig. 6.2a,b, p.162). Our test results are shown in figure 4 and figure 5.

With this program we simulate the diffraction processes for arbitrary incident angles i.e. $0^\circ < \theta \leq 90^\circ$ and $0^\circ \leq \phi < 90^\circ$. The groove period d is taken to be $\lambda/4$. The groove width is $c = d/2$. The groove depth varies over $0 \leq h \leq 1.5\lambda$. As expected the zeroth order efficiency is 100% for both Fast and Slow polarization at all incident angles. Interestingly we observed that the zeroth order phase angle of reflected Slow-polarized wave varies nonlinearly as the groove depth ranges over $0 \leq h \leq 1.5\lambda$ while the reflected Fast-polarized wave stays 180° out of phase with the incident wave regardless of the groove depth. Our major concern is the relative phase difference between these two different polarizations. Figure 6 to figure 9 show the phase differences between the two reflected waves at the zeroth order. Refer to table 1 to table 4 for numerical values. There are a few comments we would like to make on these figures:

1) For groove depth of about $\lambda/4$ and the incident angle $\phi \leq 30^\circ$ a phase difference of 180° is induced between the reflected Fast- and Slow-polarized waves at the zeroth order. If the incident angle ϕ is greater than 30° , the phase difference of 180° will occur at the groove depth beyond $\lambda/4$.

2) With the incident angles ϕ fixed and θ decreasing from 90° to 10° , 180° phase difference described in 1) is almost unaffected and keeps occurring with the same value of groove depth.

3) With the incident angles θ fixed and ϕ increasing from 0° to 80° , 180° phase difference described in 1) occurs with increasing value of groove depth which appears to relate to ϕ as the following function ;

$$h \cos \phi = \frac{\lambda}{4}$$

4) The dependence of phase differences on changing incident angles θ and ϕ is highly nonlinear. The effect due to varying incident angle ϕ dominates that of θ .

Consequently, at the groove depth of $\lambda/4$ it is not exactly 180° phase angle difference between these two orthogonal polarized waves but the incident angles would introduce a phase shift to the diffraction process. To illustrate this phenomenon we plot the groove depth(h) versus the incident angles(θ and ϕ) for fixed 180° phase angle difference (See figure 10). Figure 11 shows that groove depth changes linearly with $\frac{1}{\cos \phi}$ and we observed the function relating h to ϕ as represented in comment 3). The deviation of groove height becomes serious in the region where $\phi \geq 30^\circ$ and it plays an important role if one were to have an accurate phase measure in designing such a diffractive element.

As a final note, the numerical computations in solving the matrix equations appear to be stable. At present we are taking 23 terms in our sum calculations. However, we believe that in this frequency range relative to the groove dimension the fields are sufficiently well described by one propagating mode (there exists only one) and 22 evanescent modes. The rigorous mode matching method of analysis is therefore accurate and efficient.

$$F(x, y, z) = H_x(x, y, z) c(x)$$

where

$$c(x) = \begin{cases} 1, & 0 \leq x \leq c. \\ 0, & c < x \leq d. \end{cases}$$

Noted that both $E_i(x, y, z)$ and $F(x, y, z)$ are continuous almost everywhere in the interval $[0, d]$.

a) Matching E_i at $y=h$.

$$\begin{aligned} & \cos^2 \phi \left\{ \exp[i(\alpha_0 x - \beta_0 h + \gamma z)] + \sum_{n=-\infty}^{\infty} r_n \exp[i(\alpha_n x + \beta_n h + \gamma z)] \right\} \\ &= \begin{cases} \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{c}\right) \sin(A_n h) \exp(i\gamma z), & 0 \leq x \leq c. \\ 0, & c < x \leq d. \end{cases} \end{aligned}$$

The Fourier coefficients for both sides must be equal. So,

$$\begin{aligned} & \cos^2 \phi \left[\delta_{n0} \exp(-i\beta_0 h) + r_n \exp(i\beta_n h) \right] \\ &= \frac{1}{d} \int_0^c \sum_{m=1}^{\infty} a_m \sin(a_m h) \sin\left(\frac{m\pi x}{c}\right) \exp(-i\alpha_n x) dx. \end{aligned}$$

Consequently,

$$\cos^2 \phi \left[\delta_{n0} \exp(-i2\beta_0 h) + r_n \right] = \sum_{m=1}^{\infty} \frac{a_m}{d} \sin(A_m h) e^{-i\beta_n h} I_{nm},$$

where

$$I_{nm} = \int_0^c \sin\left(\frac{m\pi x}{c}\right) \exp(-i\alpha_n x) dx$$

$$= \begin{cases} \frac{m\pi c}{(c\alpha_n)^2 - (m\pi)^2} \left[(-1)^m \exp(-i\alpha_n c) - 1 \right] & \left(\frac{m\pi}{c}\right)^2 \neq \alpha_n^2 \\ \frac{-ic}{2}, & \frac{m\pi}{c} = \alpha_n \\ \frac{ic}{2}, & \frac{-m\pi}{c} = \alpha_n \end{cases}$$

b) Matching $F(x, y, z)$ at $y=h$

$$\frac{(-i)}{\omega\mu} \left\{ (-i\beta_0) \exp[i(\alpha_0 x - \beta_0 h + \gamma z)] + \sum_{n=-\infty}^{\infty} r_n(i\beta_n) \exp[i(\alpha_n x + \beta_n h + \gamma z)] \right\} c(x)$$

$$= \begin{cases} \left(\frac{-i\omega\epsilon}{h^2} \right) \sum_{n=1}^{\infty} a_n A_n \sin\left(\frac{n\pi x}{c}\right) \cos(A_n h) \exp(i\gamma z), & 0 \leq x \leq c \\ 0, & c \leq x \leq d \end{cases}$$

The Fourier coefficients of both sides must be equal. So,

$$\sum_{m=-\infty}^{\infty} \cos^2 \phi \left\{ (-i\beta_0) \delta_{m0} \exp(-i\beta_0 h) + r_m(i\beta_m) \exp(i\beta_m h) \right\} c_{n-m}$$

$$= \frac{1}{d} \sum_{m=1}^{\infty} a_m A_m \cos(A_m h) I_{nm}$$

where c_n is the Fourier coefficient for $c(x)$; i.e.

$$c_n = \begin{cases} \frac{1}{i2\pi n} \left[1 - \exp(-in2\pi \frac{c}{d}) \right], & n \neq 0 \\ \frac{c}{d}, & n = 0 \end{cases}$$

Appendix C :

As before we expand these functions in a Fourier series over $0 \leq x \leq d$. To begin, we define the new function $G(x, y, z)$ as

$$G(x, y, z) = H_1(x, y, z)c(x).$$

Note that both $G(x, y, z)$ and $E_1(x, y, z)$ are continuous almost everywhere in the interval $0 \leq x \leq d$.

a) Matching $G(x, y, z)$ at $y=h$

$$\cos^2 \phi \left[\exp(-i\beta_0 h) + \sum_{n=-\infty}^{\infty} s_n \exp[i(\frac{2\pi n}{d} + \beta_n h)] \right] c(x)$$

$$= \begin{cases} \sum_{n=0}^{\infty} b_n \cos(B_n h) \cos(\frac{n\pi x}{c}) \exp(-i\alpha_n x), & 0 \leq x \leq c. \\ 0, & c \leq x \leq d. \end{cases}$$

The Fourier coefficients for both sides must be equal. So,

$$\sum_{m=-\infty}^{\infty} \cos^2 \phi \left[\delta_{m0} \exp(-i\beta_0 h) + s_m \exp(i\beta_m h) \right] c_{n-m} = \frac{1}{d} \sum_{m=0}^{\infty} b_m \cos(B_m h) J_{nm},$$

where

$$J_{nm} = \int_0^c \cos(\frac{m\pi x}{c}) \exp(-i\alpha_n x) dx$$

$$= \begin{cases} \frac{ic^2 \alpha_n}{(c\alpha_n)^2 - (m\pi)^2} \left[(-1)^m \exp(-i\alpha_n c) - 1 \right], & (\frac{m\pi}{c})^2 \neq \alpha_n^2. \\ \frac{c}{2}, & \text{otherwise.} \end{cases}$$

b) Matching $E_x(x, y, z)$ at $y=h$

$$\left(\frac{i}{\omega \epsilon} \right) \left\{ (-i\beta_0) \exp[i(\alpha_0 x - \beta_0 h + \gamma z)] + \sum_{n=-\infty}^{\infty} s_n(i\beta_n) \exp[i(\alpha_n x + \beta_n h + \gamma z)] \right\}$$

$$= \begin{cases} \frac{i\omega\mu}{h^2} \sum_{n=0}^{\infty} (-b_n B_n) \cos\left(\frac{n\pi x}{c}\right) \sin(B_n h) \exp(-i\gamma z), & 0 \leq x \leq c. \\ 0, & c \leq x \leq d. \end{cases}$$

The Fourier coefficients of both sides must be equal. So,

$$\cos^2 \phi \left\{ (-i\beta_0) \delta_{n0} \exp(-i2\beta_0 h) + s_n(i\beta_n) \right\} = \frac{1}{d} \sum_{m=0}^{\infty} (-b_m B_m) \sin(B_m h) \exp(-i\beta_n h) J_{nm}.$$

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Figure 1. Relationship among incident field, reflected field, and groove orientation.

Figure 2. The perfectly conducting groove surface in rectangular coordinate system. The groove structure is periodic with period d , width c in x -direction and infinitely uniform in z -direction. The groove depth extending in y -direction is b .

Figure 3. $\vec{k}$ is the incident wave vector. ϕ is the angle between $\vec{k}$ and its projection on x-y plane. θ is the angle between the projection and x-axis.

Figure 4. The zeroth order efficiencies of two orthogonally polarized waves versus the inverse of the groove period (d cm.) where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 0$. degree, $\theta = 60$. degree, is diffracted by the periodic groove structure with $c=d/2$ and $h=d/4$.

Figure 5. The zeroth order phases of two orthogonally polarized waves versus the inverse of the groove period d (cm.) where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 0$. degree, $\theta = 60$. degree, is diffracted by the periodic groove structure with $c = d/2$ and $h = d/4$.

Figure 6. The phase angle differences between two orthogonal polarized (fast- and slow- modes) waves at the zeroth order versus the groove depth h (cm.) where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 0$. degree, θ ranging from 10 to 90 degree is diffracted by the periodic groove structure with $c=0.125$ cm. and $d=0.25$ cm.

Figure 7. The phase angle differences between two orthogonal polarized (fast- and slow- modes) waves at the zeroth order versus the groove depth h (cm.) where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 10$. degree, θ ranging from 10 to 90 degree is diffracted by the periodic groove structure with $c = 0.125$ cm. and $d = 0.25$ cm.

Figure 8. The phase angle differences between two orthogonal polarized (fast- and slow- modes) waves at the zeroth order versus the groove depth h (cm.) where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 20$. degree, θ ranging from 10 to 90 degree is diffracted by the periodic groove structure with $c=0.125$ cm. and $d=0.25$ cm.

Figure 9. The phase angle differences between two orthogonal polarized (fast- and slow- modes) waves at the zeroth order versus the groove depth h (cm.) where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 30$. degree, θ ranging from 10 to 90 degree is diffracted by the periodic groove structure with $c=0.125$ cm. and $d=0.25$ cm.

Figure 10. For fixed 180 degree phase angle difference between the fast- and slow-polarized waves at the zeroth order, the groove depth h nonlinearly varies as a function of the incident angles ϕ and θ where the groove period $d=0.25$ cm, and groove width $c=0.125$ cm. The wavelength of the incident unit plane wave is 1 cm.

Figure 11. For fixed 180 degree phase angle difference between the fast- and slow-polarized waves at the zeroth order, the groove depth h linearly varies as a function of $1/\cos\phi$ while we can hardly observe the variation of h with respect to incident angle θ . The groove period and width are $d=0.25$ cm. and $c=0.125$ cm respectively, and the wavelength of the incident unit plane wave is 1 cm.

θ		90(deg.)	80(deg.)	70(deg.)	60(deg.)	50(deg.)	40(deg.)	30(deg.)	20(deg.)	10(deg.)
ϕ	$h(\text{cm.})$									
0(deg.)	0.01	360.0	359.2	359.2	359.1	359.0	358.8	358.4	357.7	355.4
	0.02	356.6	357.3	357.2	356.9	356.5	355.8	354.6	352.0	344.3
	0.03	354.5	354.7	354.4	353.9	353.0	351.6	349.2	344.2	329.3
	0.04	355.3	361.5	351.0	350.2	348.9	346.7	342.8	334.0	312.6
	0.05	352.6	347.8	347.2	340.0	344.2	341.0	335.6	324.8	295.9
	0.06	348.8	343.8	343.0	341.5	339.0	335.0	328.0	314.3	280.4
	0.07	344.0	339.5	338.5	336.6	333.5	328.5	319.9	303.6	266.7
	0.08	335.0	334.8	333.6	331.3	327.6	321.6	311.5	293.1	254.9
	0.09	332.9	329.8	328.4	325.7	321.4	314.4	303.0	282.8	244.8
	0.10	328.9	324.5	322.8	319.7	314.7	306.9	294.2	273.0	236.1
	0.11	321.7	318.7	316.7	313.3	307.7	299.0	285.4	263.6	228.7
	0.12	314.9	312.4	310.2	306.4	300.2	290.8	276.5	254.7	222.2
	0.13	231.9	305.5	303.2	298.9	292.3	282.3	267.7	246.3	216.6
	0.14	300.1	298.0	295.5	290.9	283.8	273.5	258.8	238.4	211.6
	0.15	290.5	289.8	287.1	282.3	274.9	264.4	250.0	230.9	207.1
	0.16	282.3	280.8	277.9	272.9	265.5	255.1	241.3	223.9	203.0
	0.17	277.6	270.9	268.0	262.9	255.5	245.5	232.8	217.2	199.3
	0.18	262.9	260.1	257.2	252.2	245.1	235.8	224.4	210.9	195.9
	0.19	251.7	248.3	245.5	240.8	234.3	226.0	216.1	204.9	192.7
	0.20	235.6	235.6	233.1	228.9	223.2	216.2	208.1	199.2	189.7
	0.21	227.3	222.1	220.0	216.5	211.9	206.4	200.3	193.7	186.9
	0.22	180.4	208.1	206.4	203.9	200.6	196.7	192.6	188.4	184.2
	0.23	198.2	193.7	192.7	191.2	189.3	187.2	185.2	183.3	181.6
	0.24	172.4	179.4	179.1	178.6	178.1	177.9	177.9	178.3	179.1
	0.25	170.0	165.5	165.8	166.4	167.3	168.8	170.9	173.5	176.6
	0.26	156.7	152.2	153.1	154.6	156.9	160.0	164.0	168.8	174.2
	0.27	143.6	139.8	141.1	143.5	146.9	151.5	157.3	164.1	171.8
	0.28	131.5	128.2	130.0	133.1	137.5	143.4	150.7	159.5	169.5
	0.29	121.7	117.6	119.7	123.3	128.5	135.5	144.3	155.0	167.1
	0.30	112.0	108.0	110.3	114.2	120.1	127.9	138.1	150.4	164.7

Table 1. The phase angle differences(degree) between two orthogonal polarized (fast- and slow- modes) waves at the zeroth order for the groove depth: $0.01\text{cm.} \leq h \leq 0.30\text{cm.}$, where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 0$. degree, θ ranging from 10 to 90 degree is diffracted by the periodic groove structure with $c = 0.125$ cm. and $d = 0.25$ cm.

θ		90(deg.)	80(deg.)	70(deg.)	60(deg.)	50(deg.)	40(deg.)	30(deg.)	20(deg.)	10(deg.)
ϕ	b(cm.)									
10(deg.)	0.01	359.8	359.2	359.2	359.1	359.0	358.8	358.4	357.7	355.6
	0.02	356.4	357.4	357.2	357.0	356.6	355.9	354.7	352.1	344.5
	0.03	355.4	354.8	354.5	354.0	353.2	351.8	349.4	344.5	329.8
	0.04	354.9	351.6	351.2	350.4	349.1	346.9	343.1	335.4	313.3
	0.05	352.7	348.0	347.4	346.3	344.4	341.4	336.1	325.4	295.9
	0.06	348.9	344.1	343.3	341.8	339.4	335.4	328.0	315.1	281.5
	0.07	344.2	339.9	338.9	337.0	334.0	329.1	320.0	304.6	267.9
	0.08	342.8	335.3	334.1	331.9	328.2	322.4	312.5	294.2	256.1
	0.09	333.1	330.5	329.0	326.4	322.2	315.3	304.1	284.1	246.0
	0.10	328.1	325.2	323.6	320.6	315.7	308.0	295.5	274.4	237.3
	0.11	323.0	319.0	317.7	314.3	308.8	300.3	286.9	266.1	229.0
	0.12	317.3	313.5	311.4	307.6	301.6	292.3	278.2	256.3	223.4
	0.13	316.1	306.9	304.6	300.4	293.9	284.1	269.5	248.0	217.7
	0.14	311.5	299.7	297.2	292.7	285.7	275.5	260.8	240.1	212.6
	0.15	292.4	291.8	289.1	284.3	277.1	266.6	252.1	232.7	208.1
	0.16	289.4	283.1	280.3	275.3	267.9	257.5	243.6	225.7	204.0
	0.17	280.5	273.6	270.7	265.7	258.3	248.2	235.1	219.0	200.3
	0.18	263.0	253.3	250.3	245.3	238.7	228.7	216.8	202.7	186.9
	0.19	245.6	251.9	249.1	244.4	237.7	229.1	218.7	206.7	193.7
	0.20	238.8	239.7	237.1	232.8	226.8	219.4	210.7	201.0	190.7
	0.21	218.6	226.7	224.4	220.7	215.8	209.8	202.9	195.6	187.9
	0.22	223.9	213.0	211.2	208.4	204.6	200.2	195.4	190.3	185.2
	0.23	201.3	199.0	197.8	195.9	193.4	190.8	188.0	185.2	182.6
	0.24	192.9	184.9	184.3	183.4	182.4	181.5	180.8	180.3	180.1
	0.25	175.0	171.0	171.0	171.2	171.6	172.5	173.7	175.5	177.6
	0.26	162.0	157.6	158.3	159.4	161.2	163.7	166.9	170.8	175.2
	0.27	149.7	145.0	146.1	148.2	151.2	155.2	160.2	166.2	172.9
	0.28	136.0	133.2	134.8	137.6	141.6	147.0	153.7	161.6	170.5
	0.29	118.4	122.3	124.3	127.7	132.6	139.1	147.3	157.1	168.2
	0.30	112.1	112.4	114.6	118.4	124.0	131.5	141.0	152.6	165.8

Table 2. The phase angle differences(degree) between two orthogonal polarized (fast- and slow- modes) waves at the zeroth order for the groove depth: $0.01\text{cm.} \leq b \leq 0.30\text{cm.}$, where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 10$. degree, θ ranging from 10 to 90 degree is diffracted by the periodic groove structure with $c=0.125$ cm. and $d=0.25$ cm.

θ		90(deg.)	80(deg.)	70(deg.)	60(deg.)	50(deg.)	40(deg.)	30(deg.)	20(deg.)	10(deg.)
ϕ	$h(\text{cm.})$									
20(deg.)	0.01	360.7	359.3	359.2	359.2	359.0	358.9	358.5	357.8	355.7
	0.02	356.4	357.5	357.4	357.1	356.7	356.1	354.9	352.6	345.3
	0.03	355.2	355.0	354.8	354.3	353.5	352.2	349.0	345.3	331.4
	0.04	354.9	352.0	351.6	350.9	349.6	347.6	344.0	336.7	315.7
	0.05	351.9	348.7	348.1	347.0	345.3	342.4	337.4	327.3	299.8
	0.06	349.6	345.0	344.2	342.8	340.5	336.8	330.3	317.5	284.9
	0.07	345.8	341.0	340.1	338.3	335.5	330.8	322.9	307.5	271.5
	0.08	340.8	336.8	335.6	333.6	330.1	324.6	315.2	297.6	259.8
	0.09	336.1	332.3	330.9	328.5	324.5	318.0	307.3	288.0	249.7
	0.10	330.7	327.5	325.9	323.1	318.5	311.2	299.2	278.6	241.0
	0.11	323.2	322.3	320.5	317.3	312.2	304.1	291.1	269.6	233.4
	0.12	319.9	316.8	314.8	311.2	305.5	296.7	282.9	261.1	226.8
	0.13	313.2	310.8	308.6	304.7	298.5	289.0	274.7	253.0	221.1
	0.14	300.3	304.3	301.9	297.7	291.0	281.1	266.5	245.3	215.9
	0.15	299.9	297.3	294.7	290.2	283.2	272.9	258.3	238.0	211.3
	0.16	292.9	289.6	286.9	282.1	274.9	264.5	250.1	231.1	207.2
	0.17	282.3	281.2	278.4	273.5	266.1	255.8	242.1	224.5	203.4
	0.18	275.6	272.1	269.2	264.2	256.9	246.9	234.1	218.3	199.9
	0.19	266.1	262.2	259.3	254.4	247.3	237.9	226.3	212.4	196.7
	0.20	252.4	251.4	248.6	243.9	237.3	228.8	218.6	206.7	193.7
	0.21	242.4	239.8	237.2	232.9	227.1	219.7	211.0	201.3	190.8
	0.22	231.5	227.4	225.2	221.5	216.6	210.5	203.6	196.1	188.1
	0.23	218.5	214.4	212.6	209.8	205.9	201.4	196.4	191.0	185.5
	0.24	206.6	201.1	199.8	197.8	195.3	192.4	189.3	186.2	183.1
	0.25	192.1	187.6	187.0	186.0	184.8	183.5	182.4	181.4	180.7
	0.26	179.0	174.3	174.2	174.2	174.4	174.9	175.7	170.8	178.3
	0.27	165.7	161.4	161.9	162.9	164.3	160.4	169.1	172.3	170.0
	0.28	153.1	149.0	150.1	151.9	154.6	158.2	162.6	167.9	173.8
	0.29	141.1	137.5	139.0	141.6	145.3	150.2	156.3	163.5	171.5
	0.30	129.8	120.7	128.6	131.8	136.4	142.5	150.1	159.1	169.3

Table 3. The phase angle differences(degree) between two orthogonal polarized (fast- and slow- modes) waves at the zeroth order for the groove depth: $0.01\text{cm.} \leq h \leq 0.30\text{cm.}$, where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 20$. degree, θ ranging from 10 to 90 degree is diffracted by the periodic groove structure with $c=0.125$ cm. and $d=0.25$ cm.

θ	$h(\text{cm.})$	90(deg.)	80(deg.)	70(deg.)	60(deg.)	50(deg.)	40(deg.)	30(deg.)	20(deg.)	10(deg.)
30(deg.)	0.01	359.6	359.3	359.3	359.2	359.1	358.9	358.6	358.0	356.1
	0.02	356.7	357.7	357.6	357.4	357.0	356.4	355.4	353.2	346.6
	0.03	355.7	355.4	355.2	354.8	354.1	352.9	350.8	346.6	333.8
	0.04	355.4	352.7	352.3	351.7	350.5	348.7	345.4	338.8	319.4
	0.05	353.6	349.7	349.1	348.2	346.6	344.0	339.4	330.2	304.6
	0.06	350.4	346.3	345.7	344.4	342.3	338.9	333.0	321.3	290.5
	0.07	346.8	342.8	342.0	340.4	337.8	333.6	326.3	312.1	277.6
	0.08	342.2	339.0	338.0	336.1	333.0	328.0	319.4	303.1	266.1
	0.09	338.0	335.1	333.9	331.7	328.0	322.2	312.3	294.1	256.0
	0.10	333.6	330.9	329.5	326.9	322.8	316.1	305.1	285.4	247.3
	0.11	326.4	326.4	324.8	321.9	317.3	309.8	297.8	277.0	239.6
	0.12	323.6	321.6	319.9	316.7	311.5	303.4	290.4	268.9	232.9
	0.13	318.4	316.6	314.6	311.1	305.4	296.6	283.0	261.2	226.9
	0.14	316.1	311.2	309.0	305.2	299.1	289.7	275.5	253.6	221.7
	0.15	308.3	305.4	303.0	298.9	292.4	282.6	268.0	246.8	216.9
	0.16	301.7	299.1	296.6	292.2	285.3	275.2	260.6	240.1	212.7
	0.17	295.1	292.3	289.7	285.0	277.9	267.6	253.2	233.7	208.8
	0.18	287.2	284.9	282.2	277.4	270.1	259.8	245.8	227.6	205.2
	0.19	280.0	277.0	274.1	269.2	261.9	251.9	238.6	221.8	201.9
	0.20	271.4	268.3	265.5	260.6	253.4	243.8	231.4	216.3	198.8
	0.21	262.3	259.0	256.2	251.4	244.6	235.5	224.3	210.9	195.9
	0.22	251.9	248.9	246.3	241.7	235.4	227.2	217.3	205.8	193.2
	0.23	242.4	238.2	235.7	231.6	226.0	218.8	210.4	200.9	190.6
	0.24	230.7	226.8	224.7	221.2	216.4	210.5	203.6	196.1	188.2
	0.25	219.8	215.0	213.2	210.4	206.6	202.1	197.0	191.5	185.8
	0.26	207.1	202.7	201.5	199.5	196.9	193.8	190.6	187.0	183.5
	0.27	195.7	190.3	189.6	188.5	187.1	185.6	184.1	182.7	181.3
	0.28	182.5	178.0	177.9	177.7	177.6	177.6	177.9	178.4	179.1
	0.29	170.1	165.9	166.3	167.1	168.2	169.7	171.7	174.2	177.0
	0.30	158.2	154.3	155.2	156.8	159.0	162.0	165.7	170.1	174.9

Table 4. The phase angle differences(degree) between two orthogonal polarized (fast- and slow- modes) waves at the zeroth order for the groove depth: $0.01\text{cm.} \leq h \leq 0.30\text{cm.}$, where the incident unit plane wave with wavelength 1 cm. and incident angles $\phi = 30$. degree, θ ranging from 10 to 90 degree is diffracted by the periodic groove structure with $c=0.125$ cm. and $d=0.25$ cm.