STRONG INTRABEAM SCATTERING IN HEAVY ION AND PROTON BEAMS*

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Introduction

Intrabeam scattering is the scattering of the particles in the beam from each other through the coulomb forces that act between each pair of particles. This causes the beam dimensions to grow both longitudinally and transversely. In strong intrabeam scattering, the beam dimensions may grow by several fold, and the accelerator aperture is large enough to contain the beam as it grows. The growth rates may be very large initially, but they quickly decrease as the beam increases in size.

The growth of a beam of particles has been studied over long periods of time of the order of many hours, for a beam of gold ions and for a beam of protons, and as function of the beam energy. These studies revealed certain features of strong intrabeam scattering which are likely to have a general validity. Some simple general results were found to hold in the high energy limit which hold for γ sufficiently above the transition energy, γ_t . One result is the time invariant.

$$(X_p \sigma_p)^2 - \sigma_x^2 = \text{constant},$$

where σ_x is the rms betatron oscillation amplitude, σ_p is the rms relative momentum, $\Delta p/p$, and X_p is the horizontal dispersion.

High γ -Limit

For sufficiently high γ , some simple results have been found for the growth rates and the final state of the beam. These results hold for $\gamma > \gamma_t$, where γ_t may be taken as the transition energy of the accelerator. More exact criteria for γ_t will be given in a future paper where the results given below will be derived. In this paper, the results are summarized without derivation. The beam dimensions are described by σ_x , σ_y and σ_p . σ_x is the rms horizontal betatron oscillation amplitude and σ_y is the rms vertical betatron oscillation amplitude, and σ_p is the rms relative momentum, $\Delta p/p$. Because of intrabeam scattering, σ_x and σ_p will grow with time, and they will grow as

$$\sigma_E^2 - \sigma_x^2 = \text{constant}, \quad (2.1)$$

where $\sigma_E = X_p \sigma_p$, X_p is the horizontal dispersion, and σ_E is the rms horizontal size of the beam due to the rms energy spread, σ_p .

If one plots σ_E versus σ_x as the beam grows with time, one obtains a hyperbola which approaches the asymptote $\sigma_E = \sigma_x$. If at $t = 0$, one starts with $\sigma_E \gg \sigma_x$, then the hyperbola will first rise steeply perpendicular to the σ_x axis and then curve over towards the asymptote $\sigma_E = \sigma_x$; this shows that initially σ_E will grow rapidly while σ_x remains relatively constant and this growth will continue until $\sigma_E \approx \sigma_x$, where σ_E and σ_x will grow at about equal rates.

The derivation of Eq. (2.1) will be given in a future paper. It was derived for a lattice consisting of cells only with no insertions. Computer studies with lattices having long insertions indicate that Eq. (2.1) holds fairly well in this case too, when applied at location in the regular cells. A more complicated invariant has been formed for the

case when insertions are present, but the invariant given by Eq. (2.1) seems more convenient. In the regular cells, where $X_p/\beta_x^{1/2}$ is constant, the σ_x and σ_p growth rates are related by

$$\frac{1}{\sigma_p} \frac{d\sigma_p}{dt} = \left(\frac{\sigma_x}{\sigma_E} \right)^2 \frac{1}{\sigma_x} \frac{d\sigma_x}{dt} \quad (2.2)$$

Eq. (2.2) follows from Eq. (2.1).

The vertical growth in the high γ limit is small, and sometimes it is weakly damped. In the regular cells of the lattice one finds that

$$\left(\frac{1}{\sigma_y} \frac{d\sigma_y}{dt} \right) / \left(\frac{1}{\sigma_x} \frac{d\sigma_x}{dt} \right) = - \left(\frac{\gamma_t}{\gamma} \right)^2 r^2 \frac{2-r}{1+r}, \quad (2.3)$$

$$r = (\sigma_x/\beta_x)/(\sigma_y/\beta_y)$$

The vertical growth rate is smaller than the horizontal growth rate by the factor $(\gamma_t/\gamma)^2$ and will be weakly damped as long as the vertical size σ_y is not too small ($r < 2$).

For the type of strong intrabeam scattering being considered here, the final state of the beam after some time interval t , has been found to be independent of γ when the final state is described by the $\epsilon_{x,N}$, normalized horizontal emittance, and $\gamma\sigma_p^2$. For the above to hold one allows the beam to grow until the large initial growth rates have subsided and the growth rates have become much smaller and of the same order for σ_x and σ_p . It has then been found that the final normalized emittance $\epsilon_{x,N}$ is independent of γ for a definite growth time. This is a semi-empirical result which appears to hold very well. It is closely connected with the observation that the final state is not sensitive to the initial beam state. One can change the initial state by relatively large amounts, which changes the final state by relatively small amounts.

It follows from the above, that at the final beam state after some time interval t , that $\sigma_x \propto 1/\gamma^{1/2}$, $\sigma_p \propto 1/\gamma^{1/2}$, and the final beam size decreases like $1/\gamma^{1/2}$.

In the high γ limit, one can find a simple expression for the growth rates. One finds for a bunched beam

$$\frac{1}{\sigma_x} \frac{d\sigma_x}{dt} = 10 \frac{N_b c r_o^2}{\epsilon_x \epsilon_y \sigma_p \sigma_L (\beta\gamma)^3} \frac{X_p \sigma_p^2}{(\sigma_x^2 + X_p^2 \sigma_p^2)^{1/2}} \frac{X_p}{(\beta_x \beta_{x,av})^{1/2}} \left(1 + (\epsilon_y \beta_{x,av} / \epsilon_x \beta_{y,av})^{1/2} \right) \quad (2.4)$$

where N_b is the number of ions/bunch, ϵ_x , ϵ_y are the 90% emittances,

$$\epsilon_x = 4 \sigma_x^2 / \beta_x,$$

$$\epsilon_y = 4 \sigma_y^2 / \beta_y,$$

σ_L is the rms bunch length, r_o is the classical radius of the ion,

$$r_o = q^2 / Mc^2,$$

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$\beta_{x,av}$, $\beta_{y,av}$ are the average values of β_x and β_y over a regular cell and may be taken as

$$\beta_{x,av} = L/\sin \psi_x$$

$$\beta_{y,av} = L/\sin \psi_y,$$

where L is the cell length and ψ_x , ψ_y are the betatron phase changes over the cell. Equation (2.4) was derived for a lattice with only regular cells and no insertions, and should be roughly correct when insertions are included. The energy growth rate, and the vertical growth rate can be computed from Eq. (2.4), using Eqs. (2.2) and (2.3).

The result for unbunched beams can be found from Eq. (24) by replacing N_b/σ_L by the average current according to the substitution

$$\frac{N_b e \beta_c}{(2\pi)^{1/2} \sigma_L} \rightarrow 2^{1/2} I,$$

where I is the particle current, current/ Z , for the unbunched beam.

In the low γ limit, for γ smaller than the transition γ_t , for strong intrabeam scattering σ_x grows to be much larger than $\sigma_E = X_p \sigma_p$. An empirical relation found in computer studies is that in the final state

$$\sigma_x/\sigma_E = \bar{\gamma}_t/\gamma, \quad (2.5a)$$

where $\bar{\gamma}_t$ is roughly the transition energy and is given by

$$\bar{\gamma}_t = (\beta_x \beta_{x,av})^{1/2}/X_p. \quad (2.5b)$$

In the low γ limit σ_y is not damped, and an empirical relationship for the final state is

$$\sigma_x/\beta_x^{1/2} = \sigma_y/\beta_y^{1/2}. \quad (2.5c)$$

Equation (2.5) applies as locations in the normal cells, and not in the insertion where one can have $X_p = 0$ over considerable distances. In the example studied in this report σ_x grows to be about 4 times σ_E at the lower γ .

High γ -limit results have been found in References 1, 2, 3 in various special situations.

Intrabeam Scattering Results for a Beam of Gold Ions

The intrabeam scattering theory used in these calculations is the generalization of the original Piwinski theory⁴ provided by Möhl, Piwinski, Sacherer and Martini⁵ to include the variations of β_x , β_y and X_p around the accelerator.

The lattice used was the lattice proposed by J. Clauss in the Relativistic Heavy Ion Collider proposal.⁶ This lattice has 6 crossing points, and at the crossing points $\beta_x = .9$ m and $\beta_y = 6.3$ m. In the focussing quadrupoles of the regular cells, $X_p = 1.39$ m, $\beta_x = 51.4$ m. The actual variations of X_p , β_x , β_y around the accelerator for this lattice were used in the calculations. The gold ions are assumed to be completely ionized with a charge, $Z = 79$, and mass, $A = 200$.

The Initial Beam State

For the initial beam dimensions, the normalized horizontal and vertical emittances were assumed to be ϵ_x and $\epsilon_y = 10\pi \times 10^{-6}$ m.r. For the initial longitudinal dimensions, it was assumed that the bunch area was .3 ev-sec/amu for γ smaller than the transition γ_t , $\gamma = \gamma_t = 26.4$; and for $\gamma > \gamma_t$ the bunch area was assumed to be 1 ev-sec/amu. The rms energy spread, σ_p , and the rms bunch length σ_L , are then computed

assuming an RF voltage of $V = 1.2 \times 10^6$ volts and the harmonic number $h = 6 \times 57$.

The final beam state after 2 to 10 hours is not sensitive to the initial beam state; fairly large changes in the initial beam state tend to change the final state by a comparatively small amount.

Transverse Beam Growth

Figures 1 and 2 show the growth in the transverse dimensions of the beam. In this study, the horizontal and vertical betatron oscillations were assumed to be fully coupled and the vertical betatron oscillations grow as the horizontal oscillations grow.

Figure 1 shows the growth of ϵ_N , the normalized horizontal emittance divided by π , as a function of γ . ϵ_N starts out at 10×10^{-6} at all γ at $t = 0$. One sees that the final ϵ_N for a fixed time interval is independent of γ for γ above the transition energy, $\gamma_t = 26.4$. The injection γ for RHIC is about $\gamma = 12$. At high γ , ϵ_N grows by about a factor of 3 for $t = 10$ hours.

Fig. 1. Emittance growth. Gold ions.

Figure 2 shows the growth in the horizontal beam size, as defined by $\pm 2.5 (X_p \sigma_p + \sigma_x)$, as a function of γ . For high enough γ , the beam size decreases like $1/\gamma^{1/2}$.

Fig. 2. Beam half-width growth. Gold ions.

Longitudinal Beam Growth

The growth in the longitudinal dimensions are shown in Figures 3 and 4. Figure 3 shows the growth in the energy spread in the beam, $\pm 2.5 \sigma_p$, as a function of γ . Note that $\sigma_p = \delta E$. Also shown, as a dashed line, is the maximum momentum spread, $\pm \Delta p/p$ bucket, contained by the RF bucket. After $t = 10$ hours, the beam is barely inside the RF bucket at $\gamma = 12$ and $\gamma = 100$. The dimensions of the RF bucket correspond to an RF voltage of $V = 1.2 \times 10^6$ volts and the harmonic number $h = 6 \times 57$.

Fig. 3. Beam bunch height growth. Gold ions.

Figure 4 shows the growth in the bunch length as a function of γ . It was assumed that the bunch length would grow as the same rate as the energy spread. Thus the results shown are somewhat in error when the bunch size approaches the boundaries of the RF bucket, and the bunched energy spread and bunch length are no longer proportional to each other.

Fig. 4. Bunch length growth. Gold ions.

Intrabeam Scattering Results for a Proton Beam

In this study, the same lattice and RF system is used as was used for the gold beam study. The initial normalized transverse emittance was assumed to be $\epsilon_x = \epsilon_y = 20\pi \times 10^{-6}$ m.r. for 95% of the beam. The initial bunch area was assumed to be $A = .3$ eV-sec at all γ , and 1×10^{11} protons/bunch.

Figure 5 shows the growth in the normalized horizontal emittance as a function of γ for $t = 10$ hours

Fig. 5. Beam emittance growth for protons.

and for $t = 50$ hours. The horizontal and vertical motions are again assumed to be fully coupled and the vertical and horizontal emittances are assumed to remain equal at all times. After 10 hours, the emittance grows by only about 25%. For protons, beam times of the order of 50 hours appears possible as far as intrabeam scattering is concerned.

Figure 6 shows the growth in the energy spread of the beam, $\pm 2.5 \sigma_p$, as a function of γ . Note that $\sigma_p = \delta$. For protons, σ_p grows considerably more than σ_x , but the beam remains well within the limits of the RF bucket.

Fig. 6. Beam bunch height growth for protons.

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