MASTER

VACUUM VESSEL EDDY CURRENT MODELING FOR TFTR
ADIABATIC COMPRESSION EXPERIMENTS

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ABSTRACT

A relatively simple current filament model of the TFTR vacuum vessel is described. It is used to estimate the three-dimensional structure of magnetic field perturbations in the vicinity of the plasma that arise from vacuum vessel eddy currents induced during adiabatic compression. Eddy currents are calculated self-consistently with the plasma motion. The Shafranov formula and adiabatic scaling laws are used to model the plasma. Although the specific application is to TFTR, the present model is of general applicability.

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I. INTRODUCTION

One of the important considerations in tokamak design is the effect of eddy currents induced on the vacuum vessel. In present day tokamaks, such as the Tokamak Fusion Test Reactor (TFTR) and the Joint European Tokamak (JET), the vessel is designed in part to minimize these currents. However, it is impractical to construct a vessel on which the eddy current flow is negligible. Mechanical engineers, electrical engineers, and plasma physicists all need to know the amplitude and spatial structure of these currents. Vacuum vessel eddy currents produce potentially damaging mechanical forces at the vessel surface during disruptions. They also inhibit vertical magnetic field ramp-up and radially inward plasma motion during adiabatic compression. This paper describes a current filament model for approximating magnetic field perturbations in the plasma vicinity due to eddy currents induced on the TFTR vacuum vessel during adiabatic compression.

Section II describes features of the TFTR vacuum vessel pertaining to the eddy current problem. Section III describes a relatively simple current filament model used to calculate vacuum vessel eddy currents and Sec. IV discusses the resulting circuit equations. The plasma model and plasma coupling to the circuit equations are discussed in Sec. V. The usefulness of this model is demonstrated in Sec. VI where it is applied to TFTR adiabatic compression experiments. Finally, Sec. VII summarizes and describes some limitations and extensions to this work.

II. TFTR VACUUM VESSEL

This section discusses the features of the TFTR vacuum vessel that are pertinent to the eddy current problem. The vacuum vessel is constructed of twenty similar stainless steel sections joined into a torus by six stainless

steel reinforcing rings and fourteen Inconel bellows sections. The reinforcing rings are mounted adjacent to those sections containing the large ports through which neutral beams will be injected to heat the plasma. The Inconel bellows sections were formed from thin material and are actually cylindrical in shape. These bellows dominate the total toroidal resistance of the vacuum vessel. After the initial period of operation of TFTR, it became necessary to weld a stainless steel sealing ring over one of the Inconel bellows. For the purposes of this analysis, the reinforcing rings and the sealing ring were treated essentially as continuations of the neighboring solid stainless steel sections. This results in the equivalent overall electrical layout of the vacuum vessel shown in Fig. 1.

The vacuum vessel has a major radius $R_v = 265$ cm, and a minor radius $a_v \approx 114$ cm (see Fig. 3). The stainless steel structures have for the most part a thickness $d_v \approx 1.27$ cm. However, as is illustrated in Fig. 2, at each interface between a stainless steel section and a bellows the thickness of the stainless steel changes abruptly from d_v to $d_{rib} \approx 8$ cm. These thicker stainless steel junctions are denoted "ribs" and play an important role in the vacuum vessel model.

Eddy currents are induced on the vacuum vessel by time-varying currents in the external axisymmetric coils. Also, during adiabatic compression, the radially inward plasma motion and the increasing toroidal plasma current induce significant vacuum vessel currents. By construction, they are mostly confined to the stainless steel sections. In the next section is described a filament model which estimates the effects of vacuum vessel eddy currents during compression.

III. VACUUM VESSEL MODEL

Sophisticated electrical engineering codes exist which accurately calculate eddy currents on conducting surfaces in three dimensions.^{1,2} However, such codes require far too many elements to model fully the TFTR vacuum vessel. To make progress using these codes, it becomes necessary to model the vacuum vessel by an appropriate number of conducting sections of equal size. A translational symmetry results and eddy currents are calculated on only one of these structures.

The intention here is to model the three-dimensional aspects of the vacuum vessel eddy currents as realistically as possible and to couple self-consistently to a plasma dynamics model. To make these goals feasible, the vacuum vessel is modeled by a few rectangular cross-sectional, current-carrying bars, and physical intuition is used to prescribe the dominant eddy current paths.

The first step is to replace the cylindrically shaped bellows (darker regions in Fig. 1) by wedges (darker regions in Fig. 4). Doing this allows for an easier parameterization of the eddy current paths. The resulting approximate shape and positions of the stainless steel structures in TFTR are illustrated in Fig. 4. Moving counterclockwise around the vacuum vessel, seen from a top view and starting from the largest segment, there is one 68° segment (referring to toroidal degrees), followed by five 14° segments, followed by two 32° segments, followed by three 14° segments, followed by two 32° segments. All steel segments are separated by bellows subtending four toroidal degrees.

The next step is to choose the eddy current paths which are expected to dominate during compression experiments. Bellows sections are treated as perfect insulators so that no paths connect adjacent stainless steel

sections. This is a reasonable assumption since toroidally symmetric vacuum vessel eddy currents decay on a time scale which is much shorter than the compression time scale. On conducting sections it is assumed that eddy current flow is entirely in the toroidal direction except at the rib sections where all flow returns entirely in the poloidal direction. This is a reasonable assumption since, as seen in Fig. 2, the rib sections are relatively low resistance, poloidally directed paths. Currents flowing in the rib sections are commonly referred to as "saddle currents."

The simplest set of paths consistent with the above assumption is as seen in Fig. 5a. Each stainless steel section is replaced by two single-loop current filaments symmetrically placed about the midplane. The poloidal legs of each loop are located at the rib sections and the toroidal legs are placed near the midplane. These paths trace the centerline of rectangular cross-sectional bars through which current flows uniformly.

In practice, the vacuum vessel is further subdivided by adding two toroidal legs as indicated in Fig. 5b. This figure is a vertical projection onto the midplane of the upper half of one stainless steel section. The additional toroidal paths allow for a poloidal variation in the toroidally directed current and will more accurately reproduce the magnetic field shielding which occurs during compression experiments. Therefore, each stainless steel section is modeled as two triple-loop conducting bus-bar structures symmetrically placed about the midplane. Each of these structures is electrically insulated from the others. The majority of the stainless steel surface, indicated by the cross-hatched areas in Fig. 5, is treated as magnetically transparent. The TFTR vacuum vessel filament model is this collection of 26 triple-loop conducting structures.

Several physical dimensions are needed to specify each of the

structures. Figure 6 shows a typical triple-loop section in (θ, ϕ) space where $\theta(\phi)$ is the poloidal (toroidal) angle shown in Fig. 2. It is necessary to specify the angles ϕ_0 , $\Delta\phi$, δ , α , β , γ , the depth of each leg, and the material resistivity. The beginning toroidal position, ϕ_0 , and the total toroidal angle subtended, $\Delta\phi$, are given in Table I for the 13 triple-loop structures making up the top half of the vessel. The toroidal angular width of the rib sections, δ , is the same for all structures. It is given by $\delta = W_{rib}/R_v$ where W_{rib} is the width of the rib sections (See Fig. 3). The poloidal angles α , β , γ are adjustable parameters and are the same for all sections. The width of the two toroidal legs closest to the midplane are $W_M = [(a_v + d_v/2)\alpha]$. The width of the toroidal legs closest to the top, or bottom, of the vessel are given by $W_T = [(a_v + d_v/2)\beta]$. The dimensions of the three magnetically transparent areas are known once the angle γ is specified. The depth of each poloidal(toroidal) leg is $\delta_{rib}(d_v)$. Finally, the material resistivity is $\eta_{ss} = 75 \mu\Omega\text{-cm}$ the value for stainless steel.

The quantities W_{rib} , d_{rib} , d_v , a_v , R_v , ϕ_0 , $\Delta\phi$, and η_{ss} are all fixed by the vacuum vessel geometry and material type. The angles α , β , and γ are adjusted to best fit experimental measurements of the perturbed magnetic field due to vacuum vessel eddy currents. Such a fit will be discussed in Sec. V.

IV. CIRCUIT EQUATIONS

In this section is described a set of circuit equations for the vacuum vessel eddy currents. Induced current flow in a triple-loop section is given by the three Maxwellian-loop currents³ flowing along the three eddy current paths seen in Fig. 7. Paths #1, #2, and #3 lie on the inboard, top (or bottom), and outboard sides of the vacuum vessel, respectively. Two indices $\{k, j\}$ are used to indicate a single loop where k identifies a particular

triple-loop section and j identifies a loop within section k . For the present model of the TFTR vacuum vessel $k = 1, 2, \dots, K_V = 26$ and $j = 1, 2, 3$.

The inductance between vacuum vessel loops (k, j) and (k', j') is denoted $M^{VV}(k, j | k', j')$ and is calculated using the Neumann Equation⁴ for circuit inductances. In SI units:

$$M^{VV}(k, j | k', j') = \begin{cases} \frac{\mu_0}{4\pi} \oint_{d(k, j)} \oint_{d(k', j')} \frac{d\vec{l} \cdot d\vec{l}'}{|\vec{x} - \vec{x}'|} \dots k \neq k' & (1a) \\ \frac{\mu_0}{4\pi} \oint_{d(k, j)} \oint_{g(k', j')} \frac{d\vec{l} \cdot d\vec{l}'}{|\vec{x} - \vec{x}'|} \dots k = k', j = j' & (1b) \\ \frac{\mu_0}{4\pi} \oint_{g(k, j)} \oint_{g(k', j')} \frac{d\vec{l} \cdot d\vec{l}'}{|\vec{x} - \vec{x}'|} \dots k = k', j \neq j' & (1c) \end{cases}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m. Here $d(k, j)$ denotes the eddy current path (k, j) and $g(k, j)$ denotes the edge path (k, j) indicated in Fig. 7. The internal contribution to the self-inductance of the triple-loop structure is ignored. This approximation is valid when the surface area of the magnetically transparent regions is much greater than the surface area of the conducting bars, thereby restricting possible choices for the angles α, β, γ . Using edge paths for both line integrals in Eq. (1c) ensures that the inductance matrix is symmetric.

Next is evaluated the voltage around each vacuum vessel eddy current path, i.e.,

$$V(k, j) = \oint_{d(k, j)} \vec{E} \cdot d\vec{l} \quad (2)$$

where $\vec{E} = \eta_{ss} \vec{J}$ is the electric field and $\vec{J}$ is the induced current density. At this point it is necessary to identify the four branches, two poloidal and two toroidal, making up each vacuum vessel eddy current loop (k,j) . Denote branch " l " by the notation $(k,j|l)$ where, referring to Fig. 7, $l = 1, 2, 3, 4$ corresponds to the south, east, north, and west sides of path (k,j) , respectively. The arc length of branch $(k,j|l)$ is $L(k,j|l)$ and the cross-sectional area is $\sigma(k,j|l)$. Also, define $I^V(k,j)$ as the Maxwellian-loop current flowing along eddy current path (k,j) .

Evaluating Eq. (2) for arbitrary k and $j = 1, 2, 3$ gives:

$$\underline{V}(k) = \underline{S}(k) \cdot \underline{I}^V(k), \quad (3)$$

where

$$\underline{V}(k) \equiv [V(k,1) \ V(k,2) \ V(k,3)]^T,$$

$$\underline{I}^V(k) \equiv [I^V(k,1) \ I^V(k,2) \ I^V(k,3)]^T,$$

and $\underline{S}(k)$ is a 3×3 resistance matrix for section k . The components of $\underline{S}(k)$ are as follows:

$$\underline{S}(k) = \begin{bmatrix} S(k|1,1) & S(k|1,2) & 0 \\ S(k|2,1) & S(k|2,2) & S(k|2,3) \\ 0 & S(k|3,2) & S(k|3,3) \end{bmatrix}$$

where

$$S(k|j,j) = \eta_{ss} \sum_{l=1}^4 \frac{L(k,j|l)}{\sigma(k,j|l)} \dots j = 1, 2, 3,$$

$$S(k|1,2) = -\eta_{ss} \frac{L(k,1|3)}{\sigma(k,1|3)} = S(k|2,1) ,$$

and

$$S(k|2,3) = -\eta_{ss} \frac{L(k,2|3)}{\sigma(k,2|3)} = S(k|3,2) .$$

Finally, it is necessary to calculate the mutual inductance between vacuum vessel eddy current paths and external axisymmetric conducting coils (see Fig. 8). External coils are divided into groups of coils connected in series. They are identified by the index "k" where $k = 1, 2, \dots, K_x$ and K_x is the total number of groups. In Fig. 8 is indicated the positions of two groups of coils in TFTR. Group #1 consists of the twelve coils making up the Equilibrium Field (EF) coils and Group #2 consists of the eight coils making up the Variable Curvature (VC) coils. Individual coils within a group are identified by the index j where $j = 1, 2, \dots, J(k)$. Referring again to Fig. 8, $J(1) = 12$, $J(2) = 8$, and $K_x = 2$.

Each external coil (k,j) is made of $N(k,j)$ single-turn, axisymmetric filamentary loops of radius $R(k,j)$ and height $Z(k,j)$ above the midplane. $N(k,j)$ positive means that positive current flows through external coil (k,j) in the $+\hat{\phi}$ direction. In Table II is listed $N(k,j)$, $R(k,j)$, and $Z(k,j)$ for each of the EF and VC coils in TFTR. The current flow through a single turn of each coil in group k is denoted $I^x(k)$.

The mutual inductance between vacuum vessel loop (k,j) and external coil (k',j') is given by:

$$M^{vx}(k,j|k',j') = \frac{\mu_0}{4\pi} N(k',j') \oint_{d(k,j)} \oint_{x(k',j')} \frac{d\vec{r} \cdot d\vec{r}'}{|\vec{r} - \vec{r}'|} \quad (4)$$

where $X(k,j)$ denotes the axisymmetric path

$$\{R = R(k,j), Z = Z(k,j), 0 \leq \phi \leq 2\pi\}.$$

It is convenient to define the mutual inductance between vacuum vessel loop (k,j) and the entire external coil group k' , i.e.,

$$M^{vx}(k,j|k') \equiv \sum_{j'=1}^{J(k')} M^{vx}(k,j|k',j'). \quad (5)$$

The circuit equations for the vacuum vessel eddy currents become

$$\sum_{k'=1}^{K_v} M^{vv}(k|k') \cdot \frac{d}{dt} \tilde{I}^v(k') + S(k) \cdot \tilde{I}^v(k) = -M^{vx}(k) \cdot \frac{d}{dt} \tilde{I}^x - \frac{d}{dt} \tilde{\Psi}(k) \quad (6)$$

for $k = 1, 2, \dots, K_v$,

where

$$\tilde{I}^x \equiv [I^x(1) \ I^x(2) \ \dots \ I^x(K_x)]^T,$$

$$\tilde{\Psi}(k) \equiv [\Psi(k,1) \ \Psi(k,2) \ \Psi(k,3)]^T,$$

$$M^{vv}(k|k') \equiv \begin{bmatrix} M^{vv}(k,1|k',1) & M^{vv}(k,1|k',2) & M^{vv}(k,1|k',3) \\ M^{vv}(k,2|k',1) & M^{vv}(k,2|k',2) & M^{vv}(k,2|k',3) \\ M^{vv}(k,3|k',1) & M^{vv}(k,3|k',2) & M^{vv}(k,3|k',3) \end{bmatrix},$$

and

$$M^{vx}(k) \equiv \begin{bmatrix} M^{vx}(k,1|1) & M^{vx}(k,1|2) & \dots & M^{vx}(k,1|K_x) \\ M^{vx}(k,2|1) & M^{vx}(k,2|2) & \dots & M^{vx}(k,2|K_x) \\ M^{vx}(k,3|1) & M^{vx}(k,3|2) & \dots & M^{vx}(k,3|K_x) \end{bmatrix}.$$

Here, $\Psi(k,j)$ is the magnetic flux contribution from the plasma through vacuum vessel loop (k,j) . The plasma model and means of calculating $\Psi(k)$ self-consistently with the plasma motion are discussed in Sec. V.

In order to write Eq. (6) in a more compact form, it is necessary to further define the inductance matrices

$$M^{VV} \equiv \begin{bmatrix} M^{VV}(1|1) & M^{VV}(1|2) & . & . & . & M^{VV}(1|K_V) \\ M^{VV}(2|1) & M^{VV}(2|2) & . & . & . & M^{VV}(2|K_V) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ M^{VV}(K_V|1) & M^{VV}(K_V|2) & . & . & . & M^{VV}(K_V|K_V) \end{bmatrix}$$

$$M^{VX} \equiv [M^{VX}(1) \ M^{VX}(2) \ . \ . \ . \ M^{VX}(K_V)]^T$$

the resistance matrix,

$$R \equiv \begin{bmatrix} S(1) & 0 & 0 & . & . & . & 0 \\ 0 & S(2) & 0 & . & . & . & 0 \\ 0 & 0 & S(3) & . & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & S(K_V) \end{bmatrix}$$

the current vector,

$$I^V \equiv [I^V(1) \ I^V(2) \ . \ . \ . \ I^V(K_V)]^T ,$$

and the magnetic flux vector,

$$\underline{\Psi} \equiv [\Psi(1) \quad \Psi(2) \quad \dots \quad \Psi(K_V)]^T.$$

Eq. (6) becomes:

$$\frac{d}{dt} \underline{I}^V + \underline{Q} \cdot \underline{I}^V = - \underline{P} \cdot \frac{d}{dt} \underline{I}^X - (\underline{M}^{VV})^{-1} \cdot \frac{d}{dt} \underline{\Psi}, \quad (7)$$

where

$$\underline{Q} \equiv (\underline{M}^{VV})^{-1} \cdot \underline{S},$$

and

$$\underline{P} \equiv (\underline{M}^{VV})^{-1} \cdot \underline{M}^{VX}.$$

The eigenvalues of $\underline{Q}$ are the characteristic L/R decay rates of the vacuum vessel filament model. They are denoted $(\gamma_m: m = 1, 2, \dots, 3K_V)$ and are ordered such that $\gamma_{m+1} \geq \gamma_m$. Equation (7) is numerically integrated in time using the Euler scheme,⁵ i.e.,

$$(\underline{I}^V)^{n+1} = (\underline{I}^V)^n - \Delta t [\underline{Q} \cdot \underline{I}^V + \underline{P} \cdot \frac{d}{dt} \underline{I}^X - (\underline{M}^{VV})^{-1} \cdot \frac{d}{dt} \underline{\Psi}]^n, \quad (8)$$

where $\underline{I}^X$ is a prescribed function of time, n is the time level index, and Δt is time step. For numerical stability Δt must satisfy $\Delta t \leq 2(\alpha_{3K_V})^{-1}$.

V. PLASMA COUPLING

Assuming a filamentary plasma, the magnetic flux through vacuum vessel loop (k,j) due to the plasma is given by

$$\Psi(k,j) = M^{VP}(k,j|R_p(t)) I_p(t)$$

where $R_p(t)$ and $I_p(t)$ are the instantaneous plasma position and toroidal current, respectively, and $M^{VP}(k,j|R_p)$ is the mutual inductance between vessel loop (k,j) and the plasma filament. In the present study the plasma is constrained to lie on the midplane, i.e., $z_p(t) \equiv 0$. Again, the Neumann Equation is used to calculate mutual inductances.

The vessel-plasma inductance vector is defined as

$$M^{VP}(R_p) \equiv [M^{VP}(1|R_p) \quad M^{VP}(2|R_p) \quad \dots \quad M^{VP}(K_v|R_p)]^T,$$

where

$$M^{VP}(k|R_p) \equiv [M^{VP}(k,1|R_p) \quad M^{VP}(k,2|R_p) \quad M^{VP}(k,3|R_p)]^T,$$

so that

$$\Psi = M^{VP}(R_p) I_p.$$

Taking the time derivative of Ψ and assuming the adiabatic scaling law for the plasma current,⁶ $I_p \sim R_p^{-1}$, gives the equation

$$\frac{d}{dt} \Psi = I_{p0} \left\{ \frac{\partial M^{VP}}{\partial R_p} - \frac{M^{VP}}{R_p} \right\} \left(\frac{R_{p0}}{R_p} \right) \frac{dR_p}{dt},$$

where R_{po} and I_{po} are the initial plasma radial position and current, respectively. The vacuum vessel circuit equation, Eq. (7), becomes

$$\frac{d}{dt} I^V + \frac{\rho}{\mu} \cdot I^V = -R \cdot \frac{d}{dt} I^X - W[R_p] \frac{d}{dt} R_p, \quad (9)$$

where the following vector functional of position R_p has been defined:

$$W[R_p] \equiv I_{po} \frac{R_{po}}{R_p} (M^{VV})^{-1} \cdot \left\{ \frac{\partial M^{VP}}{\partial R_p} - \frac{M^{VP}}{R_p} \right\}. \quad (10)$$

On the time scale of adiabatic compression experiments the plasma evolves through a series of near equilibrium states. For a large aspect ratio plasma the equilibrium radial position R_p is related to the toroidally averaged external vertical magnetic field along the midplane $B_z(R, t)$, the plasma minor radius a_p , and current I_p , by the Shafranov formula⁷:

$$B_z(R_p, t) = -\frac{\mu_0 I_p}{4\pi R_p} \left\{ \ln\left(\frac{8R_p}{a_p}\right) - \frac{3}{2} + \frac{\ell_1}{2} + \beta_0 \right\}. \quad (11)$$

Here ℓ_1 is the plasma internal self-inductance and β_0 is the volume-averaged poloidal beta value. Given the adiabatic scaling laws,⁶ $I_p \sim R_p^{-1}$ and $a_p \sim R_p^{1/2}$, constant values for ℓ_1 and β_0 , and $B_z(R, t)$, Eq. (11) is a transcendental equation determining $R_p(t)$. The relatively weak scaling $\beta_0 \sim R_p^{-1/3}$ is not accounted for in the present calculation; however, it can be easily implemented. In the present calculations the combination $\ell_1/2 + \beta_0$ is assumed to be of order unity.

The vertical field is obtained from the external currents and the vacuum vessel eddy currents using the Biot-Savart Law.⁴ The eddy currents in turn depend upon the instantaneous plasma position and its time rate of change

through Eq. (9). It therefore becomes necessary to iterate on integrating Eq. (9) in time with a fixed $R_p(t)$ to obtain $I^v(t)$ and solving Eq. (11) for $R_p(t)$ given $B_z(R, t)$. The exact procedure is indicated below.

$$\frac{d}{dt} I^v(l) + \frac{\omega}{\pi} \cdot I^v(l) = - \frac{1}{\pi} \cdot \frac{d}{dt} I^x - W[R_p^{(l)}] \frac{d}{dt} R_p^{(l)} \quad , \quad (12a)$$

$$B_z^{(l)}(R_p^{(l)}, t) = - \alpha_0 \left(\frac{R_{po}}{R_p^{(l)}} \right)^2 \left[\frac{1}{2} \ln \left[\frac{R_p^{(l)}}{R_{po}} \right] + \beta_0 \right] \quad , \quad (12b)$$

where $R_p^{(l)} \equiv \omega R_p^{(l-1)} + (1-\omega) R_p^{(l-2)}$ and $l = 2, 3, \dots$, l is the iteration level. The quantities α_0 and β_0 have been defined as

$$\alpha_0 \equiv \mu_0 \frac{I_{po}}{4\pi R_{po}} \quad ,$$

and

$$\beta_0 \equiv \left\{ \ln \left(\frac{8R_{po}}{a_{po}} \right) - \frac{3}{2} + \frac{l_1}{2} + \beta_0 \right\} \quad ,$$

where a_{po} is the initial plasma minor radius. Also $R_p^{(1)}(t) = R_p^{(2)}(t) = R_{po}$. For $\omega = 1/2$ it is found that $L < 10$ is sufficient for $R_p(t)$ to converge within a sufficiently small tolerance.

The initial plasma minor radius, a_{po} , is obtained for desired initial values of I^x , I_p , and R_p by solving the Shafranov formula, Eq. (11), for a_p . The vector $W[R_p]$ is calculated once and stored at a discrete number of equally spaced radial grid points. Quadratic interpolation is used to define W between grid points. Similarly, $B_z^{(l)}(R, t)$ in Eq. (12b) is calculated each iteration level at the same radial grid points with quadratic interpolation used to define B_z between grid points.

VI. APPLICATIONS

The vacuum vessel and plasma filament models described in the previous sections are applied to demonstrate that the mutual inductive coupling between the vacuum vessel and either the external field coils or the plasma cannot be ignored during adiabatic compression experiments in TFTR. It is clearly shown that eddy current amplitudes are significant. On the compression time scale they partially prevent the external vertical magnetic field from penetrating through the vacuum vessel and they hinder the radially inward plasma motion necessary for compression. The usefulness of this model as one which can estimate the time rate of increase of EF current required to compress the plasma in a desired amount of time and which can estimate the amplitudes of three-dimensional magnetic field perturbations in the vicinity of the plasma arising from vacuum vessel eddy currents is illustrated.

The criteria used to optimize the vacuum vessel model with respect to the distances W_M and W_T and the angle γ (see Sec. III) is that calculated magnetic field perturbations agree as much as possible with those measured experimentally. Vertical magnetic field measurements have been taken under one of the bellows sections with no plasma present at four radial locations along the midplane, at $R = 2.15, 2.65, 3.15,$ and 3.65 m. In addition, toroidally averaged values are obtained at $R = 2.65$ m. These fields are measured as a function of the amplitude and frequency of a sinusoidally varying applied current in the various external coil systems. By employing the method of Fourier Transforms, these data are used to reconstruct measurements in the time domain for an arbitrary applied current waveform. An applied EF current is considered here.

The quantities W_M , W_T , and γ are chosen to minimize the root-mean-square difference between calculated and measured magnetic field perturbations.

Optimized values are $W_M = W_T = 30$ cm and $\gamma \approx 60^\circ$. The average width of the rib section is taken to be $W_{rib} \approx 8$ cm. The characteristic L/R decay times for the optimized triple-loop vacuum vessel model are shown in Fig. 9. Eigenvalues range from 1.0 to 7.5 msec so that this model will be most accurate for compression time scales ≤ 7.5 msec.

Magnetic field perturbations with no plasma present are examined first. The EF current waveform considered is shown in Fig. 10, i.e., a linear ramp-up from 3.77 kA to 7.72 kA in 10 msec. Figure 11(a) shows toroidally averaged vertical magnetic field profiles calculated along the vacuum vessel midplane at various times during the ramp-up. Dashed curves represent fields arising from external currents only and solid curves include the vacuum vessel eddy currents. Clearly, a significant amount of shielding occurs. The obvious dip in the solid curves is again due to the highly discrete nature of the vessel model. In Fig. 11(b) is shown a comparison between the calculated and measured toroidally averaged vertical field at midplane position $R = R_V \approx 2.65$ m. Modes with characteristic L/R decay times longer than 7.5 msec are seen to be excited in the actual vacuum vessel. For this particular EF waveform, 75% or more of the observed shielding at the vacuum vessel center is reproduced during the initial compression stage by the triple-loop vacuum vessel model.

Corresponding loop currents for stainless steel sections #1, #2, and #7 (see Table I) are seen in Fig. 12. Section #1 is physically largest, #2 is one of the smaller, and #7 is one of the intermediate sized sections. This figure shows that the amplitude of any given loop current is proportional to the physical size of the parent structure. Also, the currents in the toroidal legs nearest the midplane are nearly equal, with the inboard current slightly larger than the outboard current. The eddy current amplitudes in the top two

toroidal legs are always less than that in the midplane legs and are given by the difference between adjacent loop currents.

The effects of the mutual inductive coupling between the plasma and vacuum vessel are examined next. The EF current is again prescribed as shown in Fig. 10. The initial plasma position is $R_p(0) = 3.0$ m with minor radius $a_p(0) = 0.6$ m.

As discussed in Sec. V the plasma moves radially inward through a succession of equilibrium states in response to the increasing vertical field. In addition to vacuum vessel eddy currents being induced by time-varying externally applied currents, they are induced by the moving plasma. The direction of induced current flow is such that the radially inward plasma motion is hindered.

In Figs. 13 and 14 are results analogous to those illustrated in Figs. 11 and 12, respectively, only with the plasma motion calculated self-consistently with the vessel eddy currents. Figure 13 shows that the effective shielding of the vertical field is increased by nearly a factor of two. Figure 14 shows the overall increase in the loop currents. It is interesting to examine the detailed differences between loop currents #1 and #3 in Figs. 12 and 14. With no plasma present loop currents #1 and #3 are equal, up to a correction due to the finite aspect ratio of the vacuum vessel. However, when the plasma is present, the outboard eddy current is larger initially, when the plasma is positioned to the right of the vacuum vessel center, and vice-versa when the plasma is closest to the inboard side of the vacuum vessel.

In Fig. 15 is illustrated the plasma position versus time, i.e., $R_p(t)$. The dotted-dash curve is $R_p(t)$ if no eddy currents were to form on the vacuum vessel, i.e., if the vessel resistivity were effectively infinite. The dashed curve is $R_p(t)$ if vacuum vessel eddy currents were induced by externally

applied currents only, i.e., no plasma-vessel coupling. This situation is, of course, physically unrealizable. The solid curve is $R_p(t)$ with the plasma coupling fully accounted for. This would be the actual plasma motion. Clearly, vacuum vessel eddy currents adversely affect the capability to compress a TFTR plasma. Also, the mutual inductive coupling of the vacuum vessel to the plasma is as strong an effect as the coupling to external coils.

As a final application of the present vacuum vessel model, the $\sin(m\theta)$ $\cos(n\phi)$ Fourier component of the perturbed radial magnetic field, δB_r , is calculated on the compressing toroidal surface with major radius $R_p(t)$ and prescribed minor radius $a_f(t) \equiv a_{f0}[R_p(t)/R_{p0}]^{1/2}$. Because of mirror symmetry about the $z = 0$ plane, the corresponding $\cos(m\theta)$ component is identically zero. The perturbed magnetic field is defined to be the contribution from vacuum vessel eddy currents only. In Fig. 16 is shown $(\delta B_r)_{mn}$ at various points in time for $m = n = 1$, $a_{f0} = 18.6$ cm, and $R_p(t)$ as seen in Fig. 15. The amplitude of the perturbed radial field is as large as 50 G at some point during compression. Radial fields of this amplitude at rational flux surfaces may have a large affect on electron confinement times.

VII. SUMMARY AND CONCLUSIONS

A current filament model of the TFTR vacuum vessel has been described which is useful for estimating the amplitude of three-dimensional magnetic field perturbations in the vicinity of the plasma that arise from eddy currents induced on the vacuum vessel during adiabatic compression experiments. The model consists of 26 triple-Maxwellian-loop conducting structures. The effects of the mutual inductive coupling between the vacuum vessel and external axisymmetric coils and between the vacuum vessel and plasma are included in the model. A plasma filament model which employs the

Shafranov formula and the adiabatic scaling laws is used in order calculate the plasma motion self-consistently with the vacuum vessel eddy currents.

Comparisons were made between magnetic field perturbations measured experimentally and those calculated using the present model. These measurements were made with no plasma present. More than 75% of the observed shielding of the vertical field by vacuum vessel eddy currents was reproduced. The difference is partly due to the highly discrete nature of the triple-loop vacuum vessel model. However, other nearby metal structures are not accounted for here, such as the toroidal field coil casings, and these will also be a contributing factor.

It was demonstrated that the mutual inductive coupling of the vacuum vessel to the plasma is as important as the coupling to the external driving circuits during adiabatic compression experiments. Also, the radially inward plasma motion is significantly hindered by the induced vessel currents. Finally, it was seen that an error field of up to 50 G can occur across the $q = 1$ rational flux surface due to the vacuum vessel eddy currents. Again because of the highly discrete nature of the vessel model, this value may be an underestimate. For the same reason this model is unreliable for calculating error fields across higher order mode rational surfaces. These studies serve as an illustration of the usefulness of this model for estimating the effects of the vacuum vessel eddy currents.

The most serious limitation of the triple-loop vacuum vessel eddy current model presented here is its highly discrete nature. As it now stands, the model is only useful for estimating the eddy current effects. In order to calculate more accurately the resulting magnetic field perturbations, it is necessary to add more toroidal paths. To this end, the triple-loop model is in the process of being generalized to an arbitrary number of loops. The

results will be reported in a future publication.

Although the applications discussed in Sec. VI had mirror symmetry about the $z = 0$ plane, the present vacuum vessel model does not assume up-down symmetry. Hence, the magnitude of vacuum vessel currents induced during a plasma disruption can be easily estimated. Also, the parameterization of eddy current paths is developed in such a way to be applicable to other than the TFTR experiment.

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REFERENCES

- ¹U. R. Christensen, Princeton Plasma Physics Laboratory Report No. PPPL-1516 (1979).
- ²D. W. Weissenburger and U. R. Christensen, IEEE Trans. Magn., MAG-18, 422 (1982).
- ³L. Weinberg, Network Analysis and Synthesis, McGraw-Hill Inc., New York (1962).
- ⁴P. Lorrain and D. Corson, Electromagnetic Fields and Waves, 2nd Edition, W. H. Freeman and Co., San Francisco (1962).
- ⁵D. Potter, Computational Physics, Wiley-Interscience Publications, New York (1973).
- ⁶H. P. Furth and S. Yoshikawa, Phys. Fluids 13, 2593 (1970).
- ⁷V. D. Shafranov, Reviews of Plasma Physics, Vol. 2, p. 103, Plenum Publishing Corp., New York (1966).

TABLE I
VACUUM VESSEL STAINLESS STEEL SECTIONS

<u>Section #</u>	<u>ϕ (degrees)</u>	<u>$\Delta\phi$ (degrees)</u>
1	0.	68.
2	72.	14.
3	90.	14.
4	108.	14.
5	126.	14.
6	144.	14.
7	162.	32.
8	198.	32.
9	234.	14.
10	252.	14.
11	270.	14.
12	288.	32.
13	324.	32.

TABLE II
TFTR COIL SYSTEM

<u>R</u>	<u>Z</u>	<u>Turns</u>	
EF	0.67	± 0.769	40
	0.78	± 0.802	7
	0.816	± 0.835	2
	2.84	± 2.169	- 9
	3.879	± 1.947	-14
	4.881	± 0.809	-26
<hr/>			
VC	0.717	± 1.465	26
	1.605	± 2.020	-15
	2.333	± 2.296	- 4
	5.0	± 0.851	3

FIGURE CAPTIONS

- Fig. 1 Features of the TFTR vacuum vessel pertinent to the eddy current problem.
- Fig. 2 High resistance bellowed Inconel connection and low resistance rib sections.
- Fig. 3 Coordinates and vacuum vessel dimensions.
- Fig. 4 Cylindrically shaped bellows are replaced by wedges for easier parameterization of eddy current paths. Toroidal angle subtended by various sections are indicated for TFTR.
- Fig. 5 Single (a) and triple (b) Maxwellian-loop models of vacuum vessel eddy current paths.
- Fig. 6 A triple-loop section in (θ, ϕ) space. Shaded area is magnetically transparent.
- Fig. 7 Line integral paths used in Neumann formula for calculating vacuum vessel inductance matrix. Internal contribution to self-inductance is ignored.
- Fig. 8 Equilibrium Field (EF) and Variable Curvature (VC) coil positions in TFTR.

Fig. 9 Characteristic L/R decay times for optimized vacuum vessel model.

Fig. 10 Equilibrium Field applied current waveform.

Fig. 11 Toroidally averaged vertical field profiles (a) and time history at $R = R_v$ (b). Dashed curves correspond to fields from external applied currents. Solid dots are experimentally measured values. No plasma is present.

Fig. 12 Corresponding Maxwellian-loop currents in three different sized sections. No plasma is present.

Fig. 13 Toroidally averaged vertical field profiles (a) and time history at $R = R_v$ (b). Plasma motion included self-consistently.

Fig. 14 Corresponding Maxwellian-loop currents in three different sized sections. Plasma motion included self-consistently.

Fig. 15 Plasma position vs. time, i.e., $R_p(t)$.

Fig. 16 Fourier harmonic of perturbed magnetic field across a rational flux surface.

Fig. 1

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Fig. 2

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Fig. 3

Fig. 4

#85T0052

Fig. 5

#85T0073

Fig. 6

#85T0076

Fig. 7

#85T0071

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

#85T0072

Fig. 15

Fig. 16

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