Review of Wind Simulation Methods For Horizontal-Axis Wind Turbine Analysis

D. C. Powell

J. R. Connell

June 1986

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SUMMARY

This report reviews three reports on simulation of winds for use in wind turbine fatigue analysis. The three reports are presumed to represent the state of the art. The authors of the models and the names given the models in the present report are

1. R. M. Sundar and J. P. Sullivan, Purdue University (Purdue method)
2. P. S. Veers, Sandia National Laboratories (Sandia method)
3. J. R. Connell and D. C. Powell, Pacific Northwest Laboratory (PNL method).

The Purdue and Sandia methods simulate correlated wind data at two points rotating as on the rotor of a horizontal-axis wind turbine. The PNL method at present simulates only one point, which rotates either as on a horizontal-axis wind turbine blade or as on a vertical-axis wind turbine blade.

The spectra of simulated data are presented from the Sandia and PNL models under comparable input conditions, and the energy calculated in the rotational spikes in the spectra by the two models is compared. Although agreement between the two methods is not impressive at this time, improvement of the Sandia and PNL methods is recommended as the best way to advance the state of the art. Physical deficiencies of the models are cited in the report and technical recommendations are made for improvement.

The report also reviews two general methods for simulating single-point data, called the harmonic method and the white noise method. The harmonic method, which is the basis of all three specific methods reviewed, is recommended over the white noise method in simulating winds for wind turbine analysis.

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NOMENCLATURE

$A_{jk}(n)$	Cross spectral amplitude of (u_j, u_k) at frequency n , $(m/s)^2/Hz$
$A(m)$	Fourier wave amplitude at harmonic m , (m/s) , Appendix A
$Atan$	Arctangent, an angle whose tangent is?
$c(m)$	Complex Fourier coefficient for harmonic m , Appendix A
$Co_{jk}(n)$	Cospectrum of (u_j, u_k) at frequency n , $(m/s)^2/Hz$
$Cr_{jk}(n)$	Complex cross-spectrum of (u_j, u_k) at frequency n , $(m/s)^2/Hz$
$Coh(n)$	Squared coherence at frequency n
COHDEC	Code name for coherence decay factor
D	Code name for coherence decay factor, same as COHDEC
Ein0	Code name for rotational frequency, Hz
k	Ordinal number of members of a time series, Appendix A
L_x	Integral scale of u-component turbulence in alongwind direction
L_y	Integral scale of u-component turbulence in crosswind direction
m	Harmonic number, Appendix A
n	Cyclic frequency, Hz
PNL	Pacific Northwest Laboratory
$Qu_{jk}(n)$	Quadrature spectrum of (u_j, u_k) at frequency n
r	Radius of rotation, m
$S(n)$	Spectral density at frequency n , $(m/s)^2/Hz$ or m^2/s ; integrates from zero to infinity to yield the variance
$S2(n)$	Two-sided spectral density at frequency n , half of $S(n)$ for positive n , $(m/s)^2/Hz$; integrates from minus infinity to plus infinity to yield the variance
T	Total time span of a time series or wind record, s
U	Mean wind speed, m/s

	The alongwind component of turbulence, m/s; generalized name for a time series, Appendix A.
z_0	Surface roughness length, m
1P,2P,3P,...	Rotational frequency, second, third,. . . harmonics
Δt	Discretization or sampling interval for a time series
	rms of u-component turbulence, m/s
	Frequency in radians/s

1.0 INTRODUCTION

Computational analysis of wind turbine responses to winds using an aerodynamic model often requires input of wind speed fluctuation in the form of time series. If selective variation of wind input characteristics is required in the investigation of wind turbine response to the wind, there is an obvious advantage to using mathematically simulated wind data whose character may be varied readily. The alternative to mathematical simulation is costly measurements from multianemometer arrays under the not-made-to-order conditions provided by nature.

The principal purpose of this report is to present an evaluation of wind simulations now under consideration in the modeling of wind turbine response to the wind. This report also includes basic discussion of spectral simulation.

The Pacific Northwest Laboratory (PNL) has developed a simulation of wind speeds that mimics the fluctuations in speed encountered around the disk of wind turbine blade rotation (Connell 1982). Two simulations other than the PNL simulation of the rotationally sampled wind speed have been considered for use in the U.S. Department of Energy's wind turbine design program: one developed at Purdue University (Sundar and Sullivan 1983) and one developed at Sandia National Laboratories (Veers 1984). The three wind simulations, designated as the Purdue method, the Sandia method, and the PNL method, are described in this report.

In these methods, simulation involves a transformation from frequency spectra to wind speed variations as a function of time. The required starting information for the simulations is a spectrum of horizontal wind speed. Simulation consists of incorporating the spectral information and phase assumptions into a wind-speed time series.

The PNL method explicitly includes a simulation of wind speed in a rotating frame of reference, as would be experienced by a point on the blade of a horizontal-axis wind turbine. The Purdue and Sandia methods both generate Eulerian, or fixed-point, time series at many points in the y-z plane (the plane of horizontal-axis wind turbine blade rotation). The Purdue and Sandia

methods obtain rotational time series by a simple rotational sampling routine applied to the simulated field of Eulerian wind speed. All three methods use some spectral model of the wind speed as the basis for simulation.

The second section of this report consists of some basic remarks about wind speed time series simulation from a spectral basis. This section can be readily bypassed by those more interested in the resulting simulations and their evaluation. The third section is a comparative review of all three methods. The fourth section compares output of the Sandia and PNL models. The fifth section contains concluding remarks and recommendations.

One reservation applies to all of the methods described in this report. Turbulence is not exactly a set of fluctuations whose magnitudes are distributed in a normal, or Gaussian, manner. It is now believed to consist usually of intermittent elements of intense turbulence embedded in longer periods of relatively quiescent flow. This is sometimes called the Gaussian patch hypothesis (Panofsky and Dutton 1984). A satisfactory simulation from this point of view is one that would incorporate the patchiness feature. To incorporate patchiness, one must know from real data analysis the proportions of the time series in each mode and one must be able to model the variance in each mode as a function of the more basic parameters, such as the mean wind speed, atmospheric stability, and surface roughness. Mechanically, this type of simulation appears to be not too difficult, provided that one knows the required proportions for such a model. But this information will have to come in the future. Therefore the simulation methods described in this report are simple Gaussian.

2.0 BASICS OF SPECTRAL SIMULATION OF WIND IN A ROTATIONAL FRAME

The general principles involved in this study may be separated into two problems. One is that of simulation of time series of wind speed fluctuations for a stationary, stochastic process using the continuous wind speed variation frequency spectrum. The other is that of constructing the spectral information such that a time series of horizontal wind speed at a point rotating at constant angular velocity in a circle in a vertical plane perpendicular to the direction of the mean wind may be simulated. The path is that of a point on the blade of a horizontal-axis wind turbine. These problems are discussed briefly in the next two subsections. They are treated separately and in greater detail in Appendix A.

2.1 SIMULATION OF A STOCHASTIC PROCESS GIVEN ITS SPECTRUM

Two quite different methods of simulating a stochastic process may be encountered in the literature. For brevity they are called the harmonic method and the white noise method. The former is the easier one to work with, and, from our point of view, the more advantageous.

2.1.1 The Harmonic Method

Any time series of evenly spaced discrete data may be converted into a Fourier series of discrete harmonics. If the original data set, which we shall assume to be real, i.e., without imaginary parts, has N points (degrees of freedom), the Fourier series will have $N/2$ harmonics, each with an amplitude designation and a phase designation such that when the $N/2$ harmonics and the mean value are summed, the original data are retrieved. In the Fourier series the original N degrees of freedom in the data are converted to $N/2$ amplitude and $N/2$ phase degrees of freedom. (N is assumed to be an even integer.)

The power spectrum of the data is formed by squaring the amplitudes and multiplying by twice the total running time for the data. When this power spectrum is used as a basis for simulating the original data, the $N/2$ degrees of freedom that are in the phase part are not included in the spectral information and must be resupplied by some assumption. The usual assumption is that

the phase is an uncorrelated random variable from harmonic to harmonic. If so, the resulting simulated time series will be of approximately Gaussian distribution. It will not be identical with the original series, but will have the same statistical characteristics if the original series was also Gaussian.

2.1.2 The White Noise Method

The power spectrum of the original data is also the basis of the white noise simulation. The spectrum is converted to a transfer function by taking the square root. (This conversion can be performed if it is assumed that the time length of the data sample is unity, which can be done without loss of generality.) The Fourier transform of the transfer function is then taken, which yields the impulse response function. The final data are then created by convolution integration of the impulse response function with Gaussian white noise. All the degrees of freedom in the simulation are provided by the Gaussian white noise. References are Bendat and Piersol (1971) and Frost and Moulden (1977).

2.2 SIMULATION OF ROTATIONAL DATA

Two very different methods for simulating rotational data, the PNL and Sandia methods, will be reviewed in more detail. Connell (1982) derived an autocorrelation function for wind speed at a rotating point (see Section 3.0 of this report). The autocorrelation function was transformed into the corresponding spectrum of cyclic frequencies of fluctuation. Therefore, the PNL method uses the spectral transformation of that autocorrelation function. The PNL simulation provides the phases of the cyclic components of the simulated time series by exercising a random number generator from which an even, random distribution of phase angles between 0 and 2π results. This assumption does not necessarily produce the best description of real winds. The PNL simulation only computes winds for the single point that is revolving around a circle.

The Sandia and Purdue methods are quite different from the PNL method in that they must compute wind values for many points throughout the plane within the rotor disk at each time interval. Because the blade rotates in the y-z plane, the intent of these methods is first to generate sufficient spectral

information that the wind field in the plane as a function of (t,y,z) is generated. Following this relatively difficult step, the simulated rotationally sampled data are easily selected from the whole field.

The three methods depend on numerical engineering that should produce useful results in spite of objections that may be raised on theoretical grounds. Because the scale of turbulence increases with height in the atmospheric boundary layer, real turbulence is not homogeneous in a vertical plane. Therefore, spectral description of turbulence in such a rotational plane involves difficulties that have not received definitive treatment in any of the models reviewed.

The basic materials that the Sandia and Purdue methods draw upon in different ways are the simulation functions of several variables (and multivariate processes) by Shinozuka (see Shinozuka 1971 and Shinozuka and Jan 1972). The work of Shinozuka is difficult to comprehend because of certain greater generality in his assumptions than is required for present purposes. Further discussion of the Shinozuka approach is given in Appendix B. The direct description and review of the three simulation methods is given in the next section.

3.0 REVIEW OF THE THREE METHODS

In this section, three methods for simulating the rotationally sampled horizontal wind speed time series flowing parallel to the axis of rotation of a wind turbine are examined in some detail. These methods were developed by

1. R. M. Sundar and J. P. Sullivan of Purdue University (Purdue method)
2. P. S. Veers of Sandia National Laboratories (Sandia method)
3. J. R. Connell and D. C. Powell of the Pacific Northwest Laboratory (PNL method).

All three use the harmonic method rather than the Gaussian white noise method for simulating time series.

3.1 THE PURDUE METHOD

The method reviewed here is as given by Sundar and Sullivan (1983). The Purdue method, like the Sandia method, generates many time series of wind each at a specified location on the y-z plane, which is the plane perpendicular to the assumed mean wind direction. That is, each generates a wind field $u(t,y,z)$. The Sandia and the Purdue methods pick out rotational series from the total wind field. Both methods are ultimately derived from previous work of Shinozuka (1971) and Shinozuka and Jan (1972).

The Purdue model differs from the Sandia model in that a three-dimensional spectrum is required. Since none is available in the meteorological literature, certain physical assumptions must be implied in order to arrive at an assumed form for the three-dimensional spectrum. The form given in the Purdue model implies that turbulence in the lateral direction and turbulence in the vertical direction depend on turbulence in the longitudinal direction but are independent of each other. This appears to be a reasonable engineering assumption provided that the scales of turbulence in the two lateral directions are not too different, which holds in the model as parameterized.

There are two other aspects of the paper describing the model that are difficult to understand. The first equation from the paper is

$$\frac{nS(n)}{C_T V_{m.33}^2} = \frac{4X^2}{(1 + X^2)^{4/3}} \quad (3.1)$$

where

$V_{m.33}$ = the mean wind speed in m/s at a height of 10 m (33 ft)

C_T = a ground roughness factor (not the surface roughness length z_0)

$X = 4000 n/V$ in m^{-1} , a wave number.

The three-dimensional spectral equation, Sundar and Sullivan's Equation (4), is:

$$\frac{S(n, k_y, k_z)}{V_{m.33}^2} = \frac{4C_T}{n\pi^2} \left(\frac{X^2}{(1 + X^2)^{4/3}} \right) \left(\frac{\alpha \text{ abs}(\omega)}{(\alpha^2 \omega^2 + k_z^2)} \right) \left(\frac{\beta \text{ abs}(\omega)}{(\beta^2 \omega^2 + k_y^2)} \right) \quad (3.2)$$

where α and β are functions of correlation in the y and z directions. ω is evidently $2\pi n$.

In order to use Equation (3.2) the parameters α and β must be determined. The development involved can be best traced by starting with Equation (2) in the Sundar and Sullivan paper, which defines the correlation function. This equation is written:

$$\text{Cor} = \exp[-C n L/V] \quad (3.3)$$

by which the authors must mean

$$\text{Cor}(L)|(i) = \exp[-C(i) n L/V] \quad , \quad (3.4)$$

where

L = the argument, the spatial length to which the spatial correlation is applied

$i = 1, 2, 3$ for longitudinal, lateral, or vertical correlation directions

n = spectral frequency

V = mean wind speed

$C(i)$ = a dimensionless constant to be determined for each direction.

If the turbulence is assumed to be isotropic, the $C(i)$ for two lateral directions will still differ from that in the longitudinal direction.

The form on the right side of this equation is difficult to understand. There is no spectral frequency parameter or spectral wave number parameter in a correlation function. A reasonable form for this spatial correlation is

$$\text{Cor}(L)|(i) = \exp -L/\text{Length}(i) \quad (3.5)$$

or

$$\text{Cor}(L)|(i) = \exp -C(i) L/\text{Length} \quad (3.6)$$

In Equation (3.6) we assume that the Length is a measured correlation length and that $C(i)$ is a necessary adjustive constant. In Equation (3.5) we assume that the adjustment has been incorporated into 'Length(i)'. Equation (3.4) would be formally acceptable if the n had a particular value, say that at which maximum energy occurs in the spectrum, which, incidentally, is the unknown we are after.

A second source of confusion is the equation given for a , which reads:

$$a = C/(2\pi V) \quad (\text{per m}) \quad (3.7)$$

This equation implies that C must carry a dimension of per second.

Because of these difficulties and the large amount of computer time apparently required to run the model, no further comparison of the simulated results with simulations from other models is made.

3.2 THE SANDIA METHOD

3.2.1 Geometry

The Sandia method is developed in polar coordinates, the natural system for a rotating blade. A time series $u(t, r, \theta)$ is simulated, where r and θ are the usual polar coordinates: r and θ are in the y - z plane, the rotational plane of a horizontal-axis wind turbine. To select a rotational time series from u , it is sufficient to simulate u at many different angles from the origin in one radial circle. Actually the model can simulate $u(t, r, \theta)$ in as many as three concentric radial circles. (The code is easily modified so that the simulation can be arbitrarily made in many radial circles.) The configuration is shown in Figure 1.

The formal difference between the Sandia method and the Purdue method is that Sandia replaces the requirement for a three-dimensional spectrum with a matrix of one-dimensional spectra where the principal diagonal of the matrix consists of the power spectrum at each observation point and the off-diagonal elements are the cross-spectra applicable between the points. From this information $u(t, r, \theta)$ may be simulated by inverse Fourier transformation, and one or more rotational series may be picked out, which the program does. The method derives from the work of Shinozuka (1971), Shinozuka and Jan (1972), and Smallwood (1982).

3.2.2 Eulerian Power Spectra

The code calculates an Eulerian u -component power spectrum to apply at each sampling point on Figure 1. Because this spectrum is a function of height z and mean wind speed $U(z)$, the spectrum is different at different sampling points. Let us assume that we wish to simulate a series of N data at each of M azimuths from the wind turbine hub and at $NRAD$ radial distances from the hub. A rotation rate, ROT , and a sampling interval, DT , are given, where DT is the time it takes the blade to pass from one azimuth to the next. Thus,

FIGURE 1. Layout of Simulation Points on Horizontal-Axis Wind Turbine Rotation Disk

$$DT = 1/(N*ROT)(seconds) \quad . \quad (3.8)$$

The spectral density functions $S(k,j,m)$ are generated, where k varies with azimuth, j varies with radial distance, and m varies with spectral harmonic number:

$$\begin{aligned} k &= 1, N \\ j &= 1, NRAD \\ m &= 1, M, \quad M = NDATA/2. \end{aligned}$$

Each m corresponds to a cyclic frequency:

$$n = m/(NDATA * DT), \quad m = 1, M \quad (3.9)$$

Note that DT is the sampling interval that determines the maximum computable frequency for the spectra.

The Sandia model uses a u-component power spectrum from Frost et al. (1978, page 4.1). We will call this the Frost spectrum. When this spectrum is computed with the Kaimal u-component spectrum for neutral stability (Kaimal et al. 1972), which is considered state of the art by meteorologists, the result is as shown in Figure 2.

FIGURE 2. Comparison of u-Component by Kaimal and by Frost

The Frost spectrum matches the Kaimal spectrum at low frequency. However, the important frequency range for rotational sampling is the higher frequencies^(a) where the Frost spectrum yields values that are about 2.5 times those of the Kaimal spectrum.

(a) It may be added that there is greater accuracy in the higher frequency part of any state-of-the-art meteorological spectrum than there is in the lower frequency part. Therefore, a user-created mathematical model of boundary layer turbulence power spectra should match the recognized state-of-the-art spectrum on the high-frequency slope in preference to the low-frequency slope when both cannot be conveniently be matched.

The spectral density functions for the $N \times N$ points provide the principal diagonal of the spectral matrix. The cross-spectra between the locations provide the other members of the matrix.

3.2.3 Cross-Spectra Between Sampling Points

To derive the final correlated velocities $u(t, r, \theta)$, the relatedness of the winds at the different points in Figure 1 are expressed in a cross-spectral matrix. The number of columns and rows in the matrix is the product of the number of azimuth angles multiplied by the number of radii, $N \times N$. Let us call this product NN . Then the cross-spectral amplitude matrix is

$$\left[A_{jk}(n), j = 1, NN, k = 1, NN \right] . \quad (3.10)$$

The A_{jk} are given by

$$A_{jk}(n) = \sqrt{S_j(n) S_k(n) \text{Coh}_{jk}(n)} \quad (3.11)$$

where

$$\text{Coh}_{jk}(n) = \left[A_{jk}(n) \right]^2 / (S_j(n) S_k(n)) . \quad (3.12)$$

If Δ_{jk} represents the separation distance between location j and location k on the plane of rotation, the coherence is given by

$$\text{Coh}_{jk}(n) = \exp \left[-D n \Delta_{jk} / U \right] \quad (3.13)$$

where Δ_{jk} is separation distance between points j and k , and where D should be in the 10 to 15 range for coherence over a lateral separation distance. (See Kristensen 1979.) On the diagonal of the matrix A , the coherence is 1 and the A 's are the spectral densities S . The amplitudes are then converted to a transfer function matrix, H , that is lower triangular such that

$$H(n) H^T(n) = A(n) \quad (3.14)$$

where $H^T(n)$ is the transpose of $H(n)$. Note that there is a separate H matrix for each frequency n .

The next step is to form random phase designations $\exp(i\theta(n)_k)$ into the diagonal of a matrix X where all of the diagonal elements are zero. Elements of the product matrix, HX , are of the form

$$H_{jk}(n) \exp(i\theta(k)) ,$$

where θ is an evenly distributed random variable between zero and 2π . The final velocity vector for each location (r, θ) is then calculated by

$$V_i(t, r, \theta) = \text{Inverse Fourier Transform} \left(H_{jk}(n) X(n) \right) \{1\} \quad (3.15)$$

where $\{1\}$ refers to a column vector of ones. From the complete field of $V(t, r, \theta)$, the rotational series are easily selected.

The correlation of the separate velocity vectors $V(t)$ with each other reduces with increased separation distance, because the cross-correlations depend on the coherence specified, which varies inversely with separation distance. For the same reason, the cross-correlation is highest at low frequencies. This can be seen in the low phase differences between phase designations at the same frequency n when n is small for two correlated velocity vectors.

The Sandia model is the only one of the three reviewed here to take at least partial account of vertical inhomogeneity of turbulence. This is done by using a height-dependent specification of the power spectrum at each sampling point in the simulation.

3.3 THE PNL METHOD

The PNL method first computes a rotational spectrum and uses the harmonic simulation method to simulate one time series. No extraneous data (for unused

locations in the disk) are either computed in the spectral domain or simulated in the time domain. A single time series is computed for a single selectable radius of rotation.

The PNL model has been extended to apply to different wind components in relation to the vertical-axis turbine. Also, the simulated time series from the PNL model include the mean wind effect.

3.3.1 Rotational Theory

The rotational theory has been previously reported in detail (Connell 1982 and Powell et al. 1985). Only the briefest summary of material from these reports will be given here.

The basic statistical theory for turbulence as experienced by a point moving in a vertical plane perpendicular to the mean wind direction in the wind turbine layer has been given by Connell (1982). The autocorrelation function of u-component turbulence in the path of the rotating point is given by

$$R_{11}(r) = \frac{2\sigma^2}{\Gamma(1/3)} \left(\frac{r}{2L}\right)^{1/3} \left\{ K_{1/3}\left(\frac{r}{L}\right) - \frac{r}{2L} \left[1 - \frac{x^2}{r^2}\right] K_{2/3}\left(\frac{r}{L}\right) \right\} \quad (3.16)$$

where:

- Γ = the gamma function
- σ^2 = the turbulence variance
- r = the separation distance, as described below
- L = a length characteristic of the turbulence
- $K_{1/3}$ = the modified Bessel function of the second kind of fractional order 1/3
- x = the alongwind component of the separation distance.

To physically understand the r , one must visualize the two ends of the segment defining r as follows. The one end is fixed to an arbitrary point on the blade and thus describes a circular path in the disk of rotation, parallel to the mean wind direction, as the blade rotates. The other end is downstream of its original position on the blade at a distance x , or Ut where U is mean wind speed and t is travel time. Mathematically this is expressed by

$$r^2 = \left[2\rho \sin(\Omega t/2) \right]^2 + (Ut)^2 \quad (3.17)$$

where ρ is the radius of rotation of the one end and Ω is the rotation rate (see Connell 1982). The two terms express the rotational and translation effects, respectively, as shown in Figure 3. The second term, $(Ut)^2$, dominates the first, $[2\rho \sin(\Omega t/2)]^2$, as t becomes large.

P_0 is position of P at initial time

$\Omega t = \theta + 2n\pi$ where n is number of revolutions in time t

FIGURE 3. Geometry of r in Equation (3.17)

It is easily seen that the correlation function of Equation (3.16) may be transformed into one with time for the argument if we substitute from Equation (3.17) into Equation (3.16) and further substitute Ut for x in Equation (3.16).

These steps not only yield a time correlation function, but also incorporate the rotational feature **by** specifying r according to the rotational formula in Equation (3.17). The rotational feature is passed onto the spectrum in the transformation:

$$S(n) = 4 \int_0^\infty R_{11}(t) \cos(2\pi n t) dt \quad (3.18)$$

The problem transformation computation is facilitated by defining the following dimensionless variables. The first two relate turbulence and machine properties; the last is time-related to blade rotation rate:

$$\alpha = \rho/L_y \quad (3.19)$$

$$\beta = U/L_x \Omega \quad (3.20)$$

$$\bar{\tau} = \Omega t/2 \quad (3.21)$$

where

a = a radial distance from the center of rotation divided by the lateral scale of u-component turbulence; this a is not related to a in Equation (3.7)

U = the mean wind speed

Ω = blade rotation rate in radians per second

β = the period of rotation made dimensionless by dividing by the period $U(2\pi/L_x)$

$\bar{\tau}$ = the correlation time made dimensionless by multiplying by the rotation frequency in radians.

The parameters L_x and L_y are two length scales, in place of the one that is in Equation (3.16). This is actually a modification of the theory of Equation (3.16), motivated by the fact that the length scale in a is associated with the radius, which extends in all lateral directions, while the length scale in β is associated with the longitudinal mean wind speed. Thus, it is hypothesized that more than one length scale may be physically relevant.

We note that

$$x/L_x = 2\beta\bar{\tau} \quad (3.22)$$

Substitution of Equations (3.19), (3.20), and (3.21) into (3.16) yields

$$\rho(\bar{\tau}) = \left[2/\Gamma(1/3) \right] \left(\alpha^2 \sin^2(\bar{\tau}) + \beta^2 \bar{\tau}^2 \right)^{1/6} (A - B) \quad (3.23)$$

where

$$A = K_{1/3} \left[2 \left(\alpha^2 \sin^2(\bar{\tau}) + \beta^2 \bar{\tau}^2 \right)^{1/2} \right]$$

$$B = \frac{(\alpha^2 \sin^2(\bar{\tau}))}{(\alpha^2 \sin^2(\bar{\tau}) + \bar{\tau}^2 \beta^2)^{1/2}} K_{2/3} \left[2 \left(\alpha^2 \sin^2(\bar{\tau}) + \beta^2 \bar{\tau}^2 \right)^{1/2} \right] .$$

The function $\rho(\bar{\tau})$ is a correlation in the dimensionless time variable $\bar{\tau}$ in terms of the dimensionless parameters α and β .

The correlation function may be transformed into the spectrum $S(n)$ as follows:

$$S(n) = 4\sigma^2 \int_0^\infty \rho(t) \cos(2\pi nt) dt \quad (3.24)$$

where

$$\sigma^2 = \int_0^\infty S(n) dn .$$

3.3.2 Simulation

The harmonic method was used to simulate real-time data from the rotational spectrum. The Gaussian white noise method was first used. On the average only 79% of the variance in the first five harmonic spikes (1P, 2P, 3P, 4P, 5P) of the spectrum in Equation (3.24) was recaptured in the spectrum of the corresponding spikes of the simulated data. Therefore, the method was dropped in favor of the harmonic method.

In the next section we compare the spectra of simulated time series with the spectra used as inputs to the simulation.

4.0 COMPARISON OF SANDIA AND PNL METHODS

In this section we compare rotational spectra of simulated wind speed time series data from the Sandia method and the PNL method. The Sandia model was formed around the Kaimal spectrum; the PNL model about the von Karman spectrum. However, this difference is not expected to contribute significant differences to the results.

4.1 INPUT PARAMETERS

The methods are not strictly comparable because of the different physics in the input parameters. The input parameters required to run either model are

- hub height
- surface roughness length
- mean wind speed at hub height
- blade rotation rate
- radius of rotation of a point on the blade
- sampling interval
- number of data in one time series.

The input parameters peculiar to each model are

- Sandia model - coherence decay factor for u-component turbulence coherence in lateral directions
- PNL model - longitudinal length scale--integral scale of u-component turbulence in longitudinal direction L_x
lateral length scale--integral scale of u-component turbulence in rotation plane L_y .

The requirement for a longitudinal length scale (L_x) comes from the autocorrelation function (see Equation (3.16)), which is the basis of the rotational simulation. A von Karman autocorrelation function (von Karman 1948) results if the rotational feature is shut off. The length scale L_x may be taken to be about 4 times the hub height. If so, this is equivalent to using z for a length scale, which the Kaimal and Frost spectra do directly. The other length scale in the PNL model is for the lateral direction (L_y). Physically, this requirement corresponds to that for a lateral coherence decay

factor in the Sandia model. In principle, the lateral length scale and the lateral coherence decay factor vary inversely. The PNL model may also read in a turbulence variance as part of the input. Turbulence variance is obtained for the Sandia model by integrating the spectrum.

Powell et al. (1985) report that variances in the harmonic spikes calculated by the PNL model matched observed results quite well when the lateral length scale was used in both roles.

4.1.1 Correspondence of Length Scales and Coherence Decay Parameters

The coherence decay factor is used in conjunction with the assumption that coherence is an exponential function of frequency. Such an expression was written by Davenport (1961) for coherence in the x-direction

$$\text{Coh}(n) = \exp\left[-D n \Delta x/U\right] \quad (4.1)$$

where D is the coherence decay factor, n is frequency in Hz, Δx is separation distance, and U is the mean wind speed. D is presumably different for each of u , v , and w . Note that coherence decreases as D increases.

In fact, Equation (4.1) is an empirical expression. There is no theoretical justification for the exponential formulation. Therefore, the relations assumed between D and turbulence parameters may not be well founded.

Longitudinal coherence has been formulated with considerable sophistication (see Kristensen 1979). However, a good approximation holds that the decay factor is proportional to turbulence intensity, σ_u/U . Specifically, for the u -component the decay factor is about 15 times the intensity (see Chan et al. 1983 and Powell 1974). For neutral stability the relation of intensity to height may be given by

$$\sigma_u/U = 0.96/\ln(z/z_0) \quad (4.2)$$

where z_0 is the surface roughness length. Thus D is proportional to the inverse of $\ln(z/z_0)$ for logarithmic profile winds.

There is no satisfactory power relation of integral scales for the horizontal components to height. Panofsky and Dutton (1984) recommend against using the parameter altogether and state that results of different investigators on this subject are widely scattered. Counihan (1975) gives a graphical formulation of L_x , which is impossible to reduce to a power or logarithm of z . If one considers state-of-the-art spectra only, it would appear that turbulence length scales are linear functions of height but best efforts at observation show that the increase of L_x and L_y with height is significantly less than the first power.

Therefore, we know of no way from the literature to compare one turbulence length L_x with the corresponding coherence decay parameter. Panofsky and Dutton (1984, p. 218) give a relation between L_y and coherence. But the integration is weighted so heavily on $S(n)$ at low frequencies as to be useless unless one arbitrarily sets a lower frequency limit of integration other than zero.

The Sandia and PNL models are derived from different modeling bases and are not closely related theoretically. However, it is the output time series and their properties that are most important to compare.

4.2 COMPARISON OF PARTICULAR CASES

The PNL model has already been shown to produce rotational spectra that match quite well those produced by creating rotational time series from a vertical plane array (VPA) of anemometers at Clayton, New Mexico, for the neutral and unstable atmospheres (see Powell et al. 1985). In the comparison below, one set of input parameters is used for the PNL model. These parameters are

Rotation rate	0.6667 Hz	Mean wind speed	8 m/s
Radius of rotation	20 m	Surface roughness	0.01 m
Hub height	30 m	L_x	120 m
Sampling interval	0.125 s	L_y	48 m
Sample length	1024 data		

The spectrum of simulated data generated with these specifications was compared with spectra generated with the Sandia model when the coherence decay

parameter was varied from run to run. The values used were 2, 7.5, 20, and 50. The version of the Sandia code that was transmitted to us by Sandia used 7.5 for coherence decay. Otherwise, the parameters used in the Sandia model were the same as used in the PNL model, except for the length scales that are not part of the Sandia model.

The results from the Sandia model are compared with the four results from the PNL model in Figure 4. The graphs are arranged in order of decreasing coherence in the Sandia model. These graphs clearly show that the harmonic spikes become better defined in the Sandia output as coherence decreases. Likewise, Powell et al. (1985) found that the harmonic spikes become better defined as the lateral length scale, L_y , decreases.

FIGURE 4. Comparison of Spectra of Simulated Data - Sandia Model and PNL Model, Four Values of Coherence

The contrast between peak and valley is considerably greater in the PNL model spectra than in the Sandia model spectra, particularly when the Sandia model uses a lateral coherence decay factor in the range that is believed to be correct, say 10 to 15. Although we cannot be certain which model is more nearly correct, there is material in the report by Powell et al. (1985) that supports the PNL model in this respect, but not without qualification. On the negative side, when model-produced spectra were compared with observed spectra, the definition of the harmonic spikes in the theoretical spectrum is slightly stronger than that in the observed spectrum. On the positive side, Connell and George (1984) state that the spectrum of measured flapwise bending moment corresponds better with that computed using VPA rotationally sampled data when a low pass filter is used such that the minimum between the harmonic spikes is lower. Also on the positive side, the falloff of the peaks of harmonic spikes with frequency more nearly approaches the expected exponential falloff than is the case with the falloff of harmonic peak spikes in the spectra from the Sandia model. These peaks maintain more constancy than would be expected. In other words, there appears to be too much overtone strength at high frequencies in the spectra produced by the Sandia model.

Another perspective is provided by looking at the variances calculated by integrating a spectrum over each harmonic spike. These variances for the one PNL run and for the four Sandia runs are listed in Table 1.

TABLE 1. Variances in the Harmonic Spikes

<u>Run</u>	<u>Variances - Rotational Spectrum (m/s)²</u>						<u>COHDEC</u>
	<u>Total</u>	<u>1P</u>	<u>2P</u>	<u>3P</u>	<u>4P</u>	<u>5P</u>	
PNL	0.8371	0.3592	0.1114	0.0547	0.0343	0.0244	
VEK0004	0.5688	0.0487	0.0223	0.0152	0.0122	0.0099	2
VEK0000	0.5457	0.0718	0.0425	0.0291	0.0253	0.0229	7.5
VEK0002	0.5323	0.0859	0.0648	0.0444	0.0430	0.0396	20
VEK0003	0.5455	0.0942	0.0855	0.0619	0.0681	0.0608	50

For comparison, these ratios were calculated between 'corrected' values of the PNL variances and the variances from the Sandia runs. The correcting was done by using the ratios calculated (see Powell et al. 1985) between the modeled harmonic spike variance and those observed for the quasi-neutral case studied in that report. These ratios were

<u>1P</u>	<u>2P</u>	<u>3P</u>	<u>4P</u>	<u>5P</u>
1.38	1.15	1.02	0.91	0.74

Thus the corrected variances from the PNL run are

<u>1P</u>	<u>2P</u>	<u>3P</u>	<u>4P</u>	<u>5P</u>
0.263	0.097	0.054	0.0377	0.0330

The ratios between the variances produced by the Sandia runs and the above are given in Table 2 and in Figure 5.

TABLE 2. Ratios of Harmonic Spike Variances Calculated Using the Sandia Model to 'Corrected' Variances from the PNL Model

<u>1P</u>	<u>2P</u>	<u>3P</u>	<u>4P</u>	<u>5P</u>	<u>COHDEC</u>
0.19	0.23	0.28	0.32	0.30	2.0
0.28	0.44	0.54	0.67	0.70	7.5
0.33	0.67	0.83	1.14	1.20	20.0
0.36	0.88	1.15	1.81	1.85	50.0

It appears that the Sandia model may seriously underestimate the energy in the 1P and 2P region, and that the variance estimates for the higher frequencies are reasonably good if a coherence decay parameter of about 20 is used.

FIGURE 5. Ratios of Harmonic Spike Variances Calculated Using the Sandia Model to 'Corrected' Variances from the PNL Model

As an afterthought, the Sandia model was run once more with the Frost spectrum and the coherence decay value of 7.5 that was provided by Veers. The result is

Variances $(m/s)^2$ (using Frost spectrum and COHDEC = 7.5)

<u>Total</u>	<u>1P</u>	<u>2P</u>	<u>3P</u>	<u>4P</u>	<u>5P</u>
1.1013	0.1524	0.0888	0.0597	0.0523	0.0468

Ratios of these variances to corrected PNL values are

$\frac{1P}{0.59}$	$\frac{2P}{0.92}$	$\frac{3P}{1.11}$	$\frac{4P}{1.39}$	$\frac{5P}{1.42}$
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The spectral comparison is shown in Figure 6.

FIGURE 6. Comparison of Spectra of Simulated Data - Sandia Model Using Frost Spectrum and PNL Model

This comparison is obviously better than that obtained when the Kaimal spectrum, which is presumed to be more accurate, is used in the Sandia model. This type of dilemma can occur as long as we do not understand all the physics that should go into these models. However, **if** we are to have confidence in such models, **it** seems that the only direction appropriate to their present state of development is to continue to improve them by incorporating as much of the right physics as we can, and hope at least for an eventual convergence of results for the right reasons when the output of different models is compared.

5.0 CONCLUSIONS AND RECOMMENDATIONS

Of the three models reviewed, two simulations of turbulence as seen by a point on a rotating wind turbine blade have been found to be the most promising: the Sandia and the PNL models. The Sandia model has the distinction of simultaneously simulating rotational winds, as seen by two or more points on the blade. The rotational time series in the Sandia model are selected from simulation of turbulence as a function of time, radial distance from the hub, and azimuth at points in the (r, θ) plane. The PNL model simulates one rotational series without simulating unused data, but does not yet include simultaneous time series at two or more points.

The two models are not exactly comparable in terms of basic physics because they use different as well as common input parameters. When comparisons of rotationally sampled output time series are made, the results are significantly different and the Sandia model seems to be the less accurate. In particular, the variance of the 1P and 2P harmonic spikes of the spectra of the simulated rotationally sampled wind speed from the Sandia model seems deficient. Further, the decrease of harmonic spike amplitude with increasing harmonic number seems to be insufficient in the Sandia model.

An ideal model would be one that simulates required portions of a physically realizable flow field without using large amounts of computer time. Such a model would generate realistically correlated rotationally sampled wind fluctuation data at two or more points having characteristic rotational spectra at two or more points without the generation of superfluous wind data.

Further development of simulations of the rotationally sampled wind speed must concentrate on increased accuracy and on realistic phase and correlation relationships between wind speeds at different points in the disk of the turbine rotor.

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APPENDIX A

MATHEMATICAL DETAIL OF SIMULATION METHODS

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A 1 HARMONIC METHOD

The harmonic method may best be illustrated by a general example. Let us begin with the digital representation of $u(t)$, $0 < t \leq T$, which is

$$u(k) = u(k\Delta t), k = 1, 2, 3, \dots, N; N = T/\Delta t . \quad (A.1)$$

This time series has N degrees of freedom. The power spectrum of $u(k\Delta t)$ is related to the complex coefficients of harmonic analysis of $u(k)$. These are given by

$$c(m) = (1/N) \sum_{k=1}^N u(k) \exp(-i 2\pi km/N) , \quad (A.2)$$

$$m = 1-N/2, 2-N/2, \dots, -1, 0, 1, \dots, N/2.$$

These coefficients satisfy the relationships:

$$c^*(-m) = c(m) , \quad (A.3)$$

where the $*$ denotes the complex conjugate,

$$\text{Mean}\{u\} = c(0) , \quad (A.4)$$

and

$$\text{variance}\{u\} = \sum_{m=1}^{N/2} c(m) c^*(m) . \quad (A.5)$$

Without losing essential generality, we may assume

$$\text{Mean}\{u\} = 0 , \quad (A.6)$$

and thereby make the remainder of the writing easier.

The inverse Fourier transformation relation is given by:

$$u(k) = \sum_{m=1-N/2}^{N/2} c(m) \exp(i 2\pi km/N) \quad . \quad (A.7)$$

This reversal of the operation shows that all the information that was in the real time series $u(k)$ (without imaginary parts) is also in the complex series $c(m)$.

The harmonic concept of Fourier transformation may be more easily seen if we define the amplitude and phase of each harmonic from the $c(m)$'s according to

$$A(m) = c(m) + c^*(m) \quad (A.8)^{(a)}$$

$$\theta(m) = \text{Atan} \left[\text{Imaginary } c(m) / \text{Real } c(m) \right] \quad . \quad (A.9)$$

where θ is one symbol for phase angle. Then $u(k)$ is the sum of harmonics in terms of the amplitude and phase of each according to

$$u(k) = \sum_{m=1}^{N/2} A(m) \cos \left[2\pi km/N + \theta(m) \right] \quad . \quad (A.10)$$

(a) Because the m in $c(m)$ may be positive or negative, it is customary in theoretical writing to recognize a two-sided spectrum that is an even function of n and which integrates to yield the variance according to:

$$\text{Variance } \{u\} = A_n \sum_{m=1}^{N/2} S_2(n).$$

where $S_2(n)$ is the two-sided spectrum. The one-sided and two-sided spectra are related according to

$$\begin{aligned} S(n) &= 2 S_2(n), & n > 0 \\ S(n) &= 0, & n < 0. \end{aligned}$$

The one-sided spectrum is the the version generally used in physical applications.

The one-sided spectrum is related to the c's by

$$S(n) = 2T \left[c(m) c^*(m) \right], \quad n = m/T, \quad m = 1, 2, \dots, N/2, \quad (A. 11)$$

$$n = 1/T, 2/T, \dots, 1/2\Delta t, \quad$$

where n is the cyclic frequency, and to the variance by

$$\text{Variance } \{u\} = A_n \sum_{m=1}^{N/2} S(n) \quad . \quad (A. 12)$$

The relation of the amplitudes to the spectrum is

$$S(n) A_n = 0.5 A^2(m), \quad n = m/T,$$

which may be written

$$S(n) = 0.5 T A^2(m), \quad n = m/T, \quad$$

$$m = 1, 2, \dots, N/2, \text{ or}$$

$$n = 1/T, 2/T, \dots, 1/2\Delta t \quad . \quad (A. 13)$$

The variance is given in terms of the amplitudes by

$$\text{Variance } \{u\} = 0.5 \sum_{m=1}^{N/2} \left[m = 1, N/2 \right] \{A^2(m)\} \quad . \quad (A. 14)$$

A.2 THE WHITE NOISE METHOD

The white noise method consists of the following steps. Begin with the spectrum, $S(n)$; take the square root of $S(n)$ to get a transfer function, $H(n)$; do the Fourier transform to get the impulse response function, and convolve the impulse response function with Gaussian white noise.

Now let $S_{2wh}(n)$ be the two-sided spectrum of discrete Gaussian white noise of unit variance. Then we can write:

$$S_2(n) = H(n) H^*(n) S_{2wh}(n) \quad . \quad (A.15)$$

Since the Gaussian white noise is of unit variance,

$$S_{2wh}(n) = \Delta t \quad , \quad (A.16)$$

for all discrete n ,

$$|(n)| = 1/T, 2/T, \dots, 1/2\Delta t$$

so that this spectrum used in Equation (A.16) gives a variance of unity. Then Equation (A.17) becomes

$$S_2(n) = H(n) H^*(n) \Delta t \quad . \quad (A.17)$$

If we remove all of the phase variation from the transfer function, the transfer function is now real.

$$H(n) = H^*(n) \quad . \quad (A.18)$$

Then the transfer function is numerically the square root of the spectrum. That is,

$$H(n) = \sqrt{[S_2(n)/\Delta t]} \quad , \quad (A.19)$$

which may be reduced numerically to

$$H(n) = \sqrt{S_2(n)} \quad (A.20)$$

because for the simulation purpose Δt is a free parameter that may be set at unity.

We notice that $H(n)$ and the amplitudes $A(m)$ of Fourier coefficients are analogous since both are the square roots of a dimensionally adjusted product including $S(n)$. The impulse response function is obtained by

$h(\tau) = \text{Inverse Fourier transform of } H(n) \text{ or}$

$$= \sum_{m=1-N/2}^{N/2} [m=1-N/2, N/2] \{H(m) \exp(i 2\pi n\tau)\} , \quad (A.21)$$

$n = m/T$.

Let us write

$$h(k) = h(\tau), \quad k = \tau/\Delta t \quad .$$

The final operation is the convolution of the impulse response function with Gaussian white noise. If $us(k)$ are the simulated data, we have

$$us(k) = \sum_{p=-P}^P \{h(p) G(k-p)\} \quad . \quad (A.22)$$

where $G()$ is Gaussian white noise of unit variance and P is a time cut-off point that must be selected after inspecting the $h()$ function. The subjective determination of P is a disadvantage of this method that has no counterpart in the harmonic method. We programmed this method and found that the variance of data simulated by the white noise method was invariably less than the variance obtained by integrating the spectrum used for the simulation. We found no advantages in using the white noise method.

APPENDIX B

THE CONCEPT OF SPECTRAL DENSITY AS PROBABILITY DENSITY

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THE CONCEPT OF SPECTRAL DENSITY AS PROBABILITY DENSITY

Shinozuka (1971) has an interesting concept of a time series as the summation of unit amplitude harmonics at randomly spaced frequencies where a probability density $g(n)$ applies to the frequencies. The quotient of spectral density $S(n)$ divided by the variance is this probability density. That is,

$$g(n) = S(n)/\text{variance} \quad .$$

Because either side of the equation integrates to unity, the quantity on the right does indeed have that property of a probability density. This is important because simplifications may be affected in spectral work by justified application of probability theory. For example, in two dimensions we define the spectrum such that

$$\text{Variance} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(n_x, n_y) \, dn_x \, dn_y$$

and $S(n_x, n_y)/\text{Variance}$ has some properties of a joint probability density. More important, a joint probability density $P(x, y)$ is the product of two individual densities $P(x)$ and $P(y)$ if the processes in x and y are mutually independent. Therefore, the two-dimensional spectrum $S(n_x, n_y)$ is the product of two individual spectral densities $S(n_x)$ and $S(n_y)$ if the variation of the two original processes in the x and y directions is mutually independent; otherwise not.

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