PHYSICS OF $e^+e^-$ COLLIDERS: PRESENT, FUTURE, AND FAR FUTURE

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1. Introduction

Electron-positron colliders have emerged in the last ten years as tools of major significance in the study of fundamental physics. They have allowed the detailed study of the $\psi$ and $T$ resonances which has given striking confirmation to the quark model of strong interactions, and they have provided the setting for the discovery of the $D$ and $B$ mesons and the $\tau$ lepton. One could easily fill the time I have been allotted in reviewing this glorious history. The history of $e^+e^-$ colliders, however, has been recounted on many occasions and one may read of it in a number of excellent review articles. I have little to add to this story. Indeed, I doubt that this is what one should present to a summer school devoted to the physics of accelerators. You are, I am sure, much more interested in the future of $e^+e^-$ colliders. You want to know what is the importance of the continued development of $e^+e^-$ colliders and, especially, what physics can be done with the machines you will design. I have therefore taken as my topic the glorious future of $e^+e^-$ colliders, reserving just enough time for review to find some principles to use in extrapolating to higher energies.

Before beginning this discussion, I should offer two warnings to the reader. The first is that I am, by trade, a theorist, and worse, a rather speculative one. In some sense, I stand at the opposite end of the profession from designers of accelerators. But I feel that this is an example of the kinship of extremes: Both wide-eyed theorists and creative accelerator designers find themselves drawn to thinking about the highest achievable energies and link their aspirations to the exploration of these elevated realms. The second warning is that I have been asked by the organizers of this school to begin at the very beginning and proceed in one lecture to a high level of sophistication. I have tried to err in the direction of clarity, with the result that most of the material to be presented here will be familiar to most workers in the field of $e^+e^-$ collisions. Still, though the physics of $e^+e^-$ colliders in the regime of present and nearby future energies has been rather thoroughly studied, the physics of the realm of very high energies has been given serious attention only relatively recently. Whatever here is new is the result of a collaboration with John Ellis, whom I thank for many discussions.

The presentation of this lecture will proceed as follows: In Section 2, I will review the features of $e^+e^-$ collisions according to the standard gauge theory of
strong, weak, and electromagnetic interactions. This discussion will review a few of the most important features of $e^+e^-$ collisions at currently accessible energies and the expectations for $e^+e^-$ reactions which produce the intermediate vector bosons $Z^0$ and $W^\pm$. In Section 3, I will review some of the experimental work done at the current generation of $e^+e^-$ colliders; this discussion will emphasize the search for new types of elementary particles. Section 4 will be a theoretical digression, introducing a number of ideas about physics at the energy scale of 1 TeV. Section 5 will discuss (rather superficially) a number of technical aspects of electron-positron colliders designed to reach the TeV energies. Finally, in Section 6, I will discuss various possible effects which could appear in $e^+e^-$ collisions as the result of new physics appearing at 1 TeV or above.

As an introduction to this discussion, however, I should quickly review the present status of $e^+e^-$ colliders, if only in the compact form of Fig. 1. The arrow on the horizontal axis separates operating machines from those under construction. The stars on the vertical axis denote the places where spectacular new physics has been found or is to be expected; I will explain the interest of these energies as we proceed.

2. Features of $e^+e^-$ Collisions, according to the Standard Model

I will begin this discussion by reviewing, along the most basic lines, the physics which we observe in $e^+e^-$ collisions at currently accessible energies. From this base, it will be possible to extrapolate forward a certain distance in energy by studying the predictions of the gauge theories which we now believe describe the structure of the strong, weak, and electromagnetic interactions. Earlier in this school, Chris Llewellyn-Smith has described the successes of the theory which models the strong interactions as a gauge theory based on the group SU(3) and the weak and electromagnetic interactions as a gauge theory based on the group SU(2) x U(1). In this section, I will treat this conglomerate gauge theory as established and refer to it as the standard model. Please allow me to postpone discussion of how far in energy this model remains an adequate description of Nature. I will discuss that question at some length in § 4.

We might start by considering the very most basic process which occurs at $e^+e^-$ colliders, the QED process $e^+e^- \rightarrow \mu^+\mu^-$. The cross-section for this reaction, computable from the Feynman diagram shown in Fig. 2, is given by

$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{3E_{\text{cm}}^2} = \frac{86.8\text{nb}}{(E_{\text{cm}}^{\text{GeV}})^2}$$

This equation defines the quantity I will refer to as 1 unit of $R$. The quantity (2.1) is the basic pointlike annihilation cross-section, and so the $R$ unit sets the
Fig. 1. Past, present, and proposed $e^+e^-$ colliding-beam accelerators, after Ref. 7.

scale for the rates of all processes, leptonic or hadronic, which take place at $e^+e^-$ colliders.

Throughout this lecture, whenever I quote a cross-section, I will quote it in units of $R$. Let me warn you from the outset that this is cheating. One more conventionally imagines building accelerators to obtain a fixed luminosity, and, indeed, the $e^+e^-$ colliders constructed since 1970 have shown a constant or slowly decreasing luminosity as the collision energy has increased. However, this is not an appropriate course for the future. The $R$ unit falls rapidly with energy, as one can see by converting this unit to events/year for a fixed luminosity of $10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$ characteristic of the current generation of colliders. Defining a
practical year to be $10^7$ sec, one finds:

<table>
<thead>
<tr>
<th>$E_{cm}$ (GeV)</th>
<th>events/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>500,000</td>
</tr>
<tr>
<td>10</td>
<td>90,000</td>
</tr>
<tr>
<td>30</td>
<td>10,000</td>
</tr>
<tr>
<td>100</td>
<td>900</td>
</tr>
<tr>
<td>250</td>
<td>140</td>
</tr>
<tr>
<td>1000</td>
<td>9</td>
</tr>
</tbody>
</table>

Thus one must somehow devise a way to keep the number of R units per year fixed as the collider energy is increased. For the most part, I will simply assume that this can be done. I will, however, comment briefly on the problem of working at constant R/yr in §5.

The first reason that the R unit is of interest is that the cross-section for the production of hadrons from $e^+e^-$ is observed to be a number of R units of order 1. The standard model predicts, via the Feynman diagrams of Fig. 3, the relation

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_f Q_f^2 \cdot 3 \cdot \left(1 + \frac{\alpha_s}{\pi} + \ldots\right), \quad (2.2)$$

where $\alpha_s$ is the strong interaction gauge coupling constant, 3 is the number of colors, $f$ runs over quark flavors, and $Q_f$ is the quark electric charge. This relation is quite well satisfied experimentally.\[^9,10^\] Eq. (2.2) predicts a relatively small rate for the production of hadrons, but one which is democratic between familiar and exotic flavors:

$$\sigma(\to \text{charm}) = \sigma(\to \text{up}) = 4\sigma(\to \text{bottom}) = 4\sigma(\to \text{down}) \approx \frac{2}{3} \sigma(\to \tau^+\tau^-) \quad (2.3)$$
At present energies, the production of charmed quark pairs makes up almost half of the total hadronic cross-section, a situation rather different from that found in hadron-hadron collisions. The \( \tau \) lepton is produced as copiously as the muon. This democracy of production provides a great advantage in studying the properties of new and exotic particles.

![Feynman diagram](image)

**Fig. 3.** Feynman diagram contributing to the process \( e^+e^- \rightarrow \text{hadrons} \).

I might illustrate the experimental correctness of Eq. (2.2) by presenting in Fig. 4 the experimental data on the total cross-section for \( e^+e^- \rightarrow \text{hadrons} \).

![Graph](image)

**Fig. 4.** Experimentally determined ratio of cross-sections for production of hadrons and of \( \mu \) pairs in \( e^+e^- \) annihilation, from Ref. 11.

The ratio indicated in (2.3) is exceedingly constant over most of the energy range; however, at certain points this ratio increases markedly to show prominent resonances. The resonances indicated in Fig. 4 have properties which agree in
detail with those predicted for quark-antiquark bound states of spin 1 and odd parity. In general, any resonance with the right quantum numbers to couple to 1 photon (or directly to $e^+e^-$) should be directly visible as an increment of $R$. The size of this increment is given by

$$\int dE_{cm} \Delta \sigma_T(E_{cm}) = \frac{6\pi^2}{m_R^2} \Gamma(\Phi \rightarrow e^+e^-).$$  \hspace{1cm} (2.4)$$

where $m_R$ is the resonance mass and $\Gamma$ is the partial decay width of the resonance into $e^+e^-$. As an illustration of this result, let me display the size of the resonance associated with the lowest spin 1 ($J^P=1^-$) $q\bar{q}$ bound state built, respectively, with charmed quarks, bottom quarks, top quarks. For the purpose of making estimates in the course of this lecture, I will assume that the top quark mass is 40 GeV. The last column of the table below gives the observed (or to-be-observed) peak value of the increment of the hadronic cross-section, in $R$ units. Since $q\bar{q}$ resonances are characteristically very narrow, with total widths of order 100 keV, it is the energy spread of the synchrotron which determines the observed width and height of these resonances.

<table>
<thead>
<tr>
<th>state</th>
<th>mass (GeV)</th>
<th>$\int \Delta \sigma_T(Run/1s) \cdot 10\text{MeV}$</th>
<th>$\Delta R_{\text{obs}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c\bar{c}$</td>
<td>$\psi$</td>
<td>3.10</td>
<td>124</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>$\tau$</td>
<td>9.46</td>
<td>29</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
<td>$\zeta$</td>
<td>90</td>
<td>400</td>
</tr>
</tbody>
</table>

The resonances associated with the $b\bar{b}$ system are shown in more detail in Fig. 5.

Fig. 5. The spectrum of $b\bar{b}$ resonances, as observed by the CLEO experiment, from Ref. 16.
In the energy region above these resonances, the process $e^+e^- \rightarrow \text{hadrons}$ resembles the process shown in Fig. 3 not only in the value of the total cross-section but also in the form of the hadronic states produced. These hadrons characteristically lie in narrow cones, roughly $18^\circ$ in radius at PEP and PETRA energies, oriented about some axis. The orientation of this axis varies from event to event; it must be located anew in each event. A convenient systematic algorithm for finding this axis is that of maximizing a quantity called the thrust, defined, for a given axis, by:

$$T = \sum |p^{(i)}_T|$$

where the sum runs over all particles and $p^{(i)}_T$ is the component of the momentum of the $i$th particle relative to the chosen axis. The distribution of particle momenta relative to the thrust axis obtained by the JADE experiment at PETRA is shown in Fig. 6. Another way to display this collimation of final-state particles, is to label the hadrons clustering about a particular axis as a jet and then to display the distribution of the invariant masses of these jets. The MAC experiment at PEP has done such an analysis, somewhat crudely defining a jet to be all the particles in a given hemisphere with respect to the thrust axis. The resulting distribution of jet masses is shown in Fig. 7. This distribution is sufficiently narrow that bottom quarks, of mass 5 GeV, can be recognized, at least on a statistical basis, by the fact that they produce jets of high invariant mass.

![Fig. 6. Distribution of momentum in $e^+e^-$ annihilations to hadrons, as a function of angle from the thrust axis, from Ref. 18.](image)

Having now given a very brief review of the character of $e^+e^-$ annihilation events at currently available energies, let me turn to a review of the features of
Fig. 7. Jet invariant mass distribution at $E_{cm} = 29 \text{ GeV}$, from Ref. 19. In this analysis, a jet was defined to be the collection of particles within $90^\circ$ of the thrust axis.

e^+e^- annihilation at energies soon to be accessed, as these features are predicted by the standard model. The most striking features involve the production in $e^+e^-$ reactions of the weak-interaction intermediate vector bosons $W^\pm$ and $Z^0$. These bosons couple as strongly to $e^+e^-$ as the photon; thus, their influence becomes comparable to that of the photon when the center-of-mass reaction energy reaches the level of the masses of these bosons, respectively 83 and 91 GeV.\textsuperscript{[20,21]}

The most important effect of the weak bosons arises from the fact that the $Z^0$ has the right quantum numbers to be produced singly as a resonance in the $e^+e^-$ annihilation cross-section; this process is indicated in Fig. 8. The resonance is an enormous one. If the width of the $Z^0$ is dominated by its decay into the known quarks and leptons, the height of the resonance will be about 3000 units of $R$. Such a resonance would produce $3 \times 10^6 Z^0$ events per year at an $e^+e^-$ collider with a luminosity of $10^{21} \text{ cm}^{-2} \text{ sec}^{-1}$. The prediction of the standard model for the total hadronic cross-section as a function of energy is shown in Fig. 9; the $Z^0$ is clearly the dominant feature in this cross-section over the energy range shown.

Like the photon, the $Z^0$ couples to quarks and leptons democratically between heavy and light flavors. Because the $Z^0$ is a component of the weak interactions, however, its coupling is chiral and depends on the fermion helicity. The $Z^0$ charges of the known fermions, in terms of their charges, weak isospins, and helicities, are indicated in Fig. 10. This helicity dependence of the $e^+e^-$ annihilation cross-section produces a number of remarkable effects. The first of these is an energy-dependent forward-backward asymmetry, different for differ-
Fig. 8. Appearance of the $Z^0$ as a resonance in $e^+e^-$ annihilation.

Fig. 9. The total cross-section for $e^+e^- \rightarrow \text{hadrons}$ as predicted by the standard model, showing the effect of the $Z^0$ resonance.

\[ \text{Left-handed:} \]
\[
\begin{array}{c}
\text{right-handed:}
\end{array}
\]

Fig. 10. Charges defining the coupling of the $Z^0$ to fermions. $Q$ and $J^3$ denote, respectively, electric charge (in units of the electron charge) and weak isospin. $e$ is the electron charge, and $\theta_W$ is the Weinberg angle which parametrizes neutral-current weak interactions.

different types of fermions. This effect arises because the processes which produce left- and right-handed fermions from the electron-positron annihilation depend differently on the direction of the new fermion:
for \( e^-(L) + e^+ \rightarrow f(L) + f, \quad e^-(R) + e^+ \rightarrow f(R) + f \):

\[
\frac{d\sigma}{d\Omega} \sim (1 + \cos \theta)^2;
\]

for \( e^+(L) + e^- \rightarrow f(R) + f, \quad e^+(R) + e^- \rightarrow f(L) + f \):

\[
\frac{d\sigma}{d\Omega} \sim (1 - \cos \theta)^2.
\]

where \( \theta \) is the angle between the original electron and the \( f \). The various helicity cross-sections are all equal at low energies, where annihilation through a single photon dominates, but they receive very different weights as one passes through the \( Z^0 \) resonance, producing the forward-backward asymmetry shown in Fig. 11. The small asymmetry in lepton pair production at the relatively low energies now accessible has already been observed at PEP and PETRA. A second effect of the helicity-dependent cross-sections is that the fermions produced in \( Z^0 \) decays will be longitudinally polarized. Figure 12 shows the standard model prediction for the polarization of leptons pair-produced in \( e^+ e^- \) annihilation, as a function of energy. To show the correlation between helicity and angle, I have plotted also the polarization of those leptons emitted into the forward hemisphere. The same dramatic energy-dependence appears in the cross-section for producing lepton pairs from polarized electrons.

In addition to the striking qualitative features of the weak interactions near the \( Z^0 \) resonance, \( e^+ e^- \) annihilation in the vicinity of this resonance offers the
Fig. 12. Longitudinal polarization of leptons pair-produced in $e^+e^-$ annihilation, according to the standard model. The two curves give the polarization integrated over all leptons and also integrated only over those in the forward hemisphere.

possibility of a dramatic improvement in the quantitative comparison of the standard model of the weak interactions with experiment. Two measurements, in particular, can be performed with high accuracy: It should be possible to use the well-defined beam energy available in $e^+e^-$ reactions to measure the position of the peak of the $Z^0$ resonance to better than 100 MeV. In addition, the dramatic effect of the lepton polarization shown in Fig. 12 depends on the quantity $(1 - \sin^2 \theta_W)$, so that a precise measurement of this effect—which might be obtained from the polarization of $\tau$ leptons defined by their decay distributions, or from the measurement of the dependence of the cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ near the $Z^0$ on the initial electron polarization—determines $\sin^2 \theta_W$ to an accuracy an order of magnitude better. These two quantities should provide two independent measurements of $\sin^2 \theta_W$ good to an accuracy of 0.04%. The rapport between these measurements will provide a stringent test of the complete structure of the standard weak-interaction theory, including its radiative corrections.\[^{[23]}\]

The $Z^0$ resonance decays dominantly into quarks and leptons, in proportions determined by their quantum numbers through the charges shown in Fig. 10. The rate of the $Z^0$ decay to the fermions of the standard model, ignoring their masses for the moment, is given by:

$$\Gamma_Z = 90 \text{ MeV} \cdot [1.0 N_t + 3.5 N_u + 4.5 N_d + 2 N_\nu],$$ \hspace{1cm} (2.7)

where $N_t$ is the total number of charged leptons light enough to be pair-produced at the $Z^0$, and $N_u$, $N_d$, and $N_\nu$ are the corresponding numbers of up quarks,
down quarks, and neutrinos. Assuming that one finds only the three generations already observed, and a top quark of mass 40 GeV, and taking into account the suppression due to the top quark mass, one would expect to see 4% of the observed $Z^0$'s decay to each charged lepton, 14% to each up quark (9% for top), and 17% to each down quark. Note that a measurement of the width of the $Z^0$ resonance to 50 MeV allows one to count directly the number of light neutrinos. Since neutrinos account for such a large fraction of the $Z^0$ decays, this counting might also be done by studying the radiative process:

$$e^+e^- \rightarrow \gamma + \text{unobserved neutrals}$$

in the energy region just above the $Z^0$.

In addition to the known quarks and leptons, one can well expect that new, heavier particles will be pair-produced in $Z^0$ decays. In principle, the $Z^0$ can produce any particle that couples to the weak interactions which has a mass less than half the $Z^0$ mass, that is, any mass up to about 45 GeV. The phase space suppression for heavy states is offset by the large number of $Z^0$ events expected. Let us label the velocity of such a new particle by

$$\beta = \left(1 - \frac{4m^2}{E_{\text{cm}}^2}\right)^{1/2}.$$  \hfill (2.8)

Then any new fermion is produced in $Z^0$ decays with the partial width

$$\Delta \Gamma_Z = 90 \text{MeV} \cdot N_c \cdot 8 \cdot \left[(I_3^L - Q\sin^2\theta_w)^2 + (I_3^R - Q\sin^2\theta_w)^2\right] \cdot \beta \left(\frac{3 - \beta^2}{2}\right),$$  \hfill (2.9)

where $I_3^L$, $I_3^R$ are the weak isospin quantum numbers of the left- and right-handed components, respectively, and $N_c$ is the number of color states. The corresponding formula for a new boson is:

$$\Delta \Gamma_Z = 90 \text{MeV} \cdot N_c \cdot \left[(I_3 - 2Q\sin^2\theta_w)^2\right] \cdot \beta^3.$$  \hfill (2.10)

A charged Higgs boson, for example would appear in $1\% \cdot \beta^3$ of all $Z^0$ decays.

One should keep in mind also that, because the number of expected $Z^0$ events is so large, particles which can only be produced by more indirect processes might still be visible experimentally down to branching ratios as small as $10^{-5}$.

The evident interest in $Z^0$ physics has spurred the construction of two new accelerators designed to study $e^+e^-$ collisions at a center-of-mass energy of 100 GeV. This is not the place for a detailed discussion of these machines (such discussions may be found in Refs. 24 and 25), but I would like to briefly review their features. These two machines are of completely different design; they
signal, in a certain sense, the end of one era and the beginning of another. The first of these machines to be proposed is LEP, a large synchrotron of 30 km circumference being built around the CERN site and under the Jura, for a cost of roughly 1 billion Swiss francs. The luminosity is projected to be a $10^{31}$ cm$^{-2}$ sec$^{-1}$; the energy spread of the beam should be roughly 100 MeV. It is scheduled to be completed in 1988. It is not unlikely that LEP will be the last and largest electron synchrotron to be built. Its competitor is the first of a new type of colliding-beam accelerator: the linear accelerator collider. This device, the SLC, is being constructed at SLAC and is designed to collide directly beams extracted from the SLAC linac. A diagram of the apparatus is shown in Fig. 13. The SLC is designed to achieve a luminosity of roughly $6 \times 10^{30}$ cm$^{-2}$ sec$^{-1}$, not by the usual method of establishing a circulating beams which interact repeatedly, but rather by extracting high-density bunches of electrons and positrons from the linac at a sufficiently high rate. The desired luminosity results from extracting $5 \times 10^{10}$ particles/bunch at the SLAC linac rep rate of 180 Hz and focusing these particles to a bunch size of $1.2\mu \times 1.2\mu$ at the collision point. The estimate includes an extra factor of about 6 from the mutual attraction (and disruption) of the electron and positron bunches in the collision process. Since the electrons are extracted directly from the SLAC linac, they can be prepared as a polarized beam.

The design of LEP allows it, in principle, to reach even higher energies by the replacement of conventional with superconducting accelerating cavities. Center-of-mass energies up to 250 GeV are achievable in principle. At these elevated energies, one finds another set of new reactions predicted by the standard model—the pair production of weak-interaction bosons. The largest of these processes is the reaction

$$e^+e^- \rightarrow WW^{-}.$$ (2.11)

This process has its threshold at about 160 GeV but, as Fig. 14 shows, it quickly rises to become the major component of the $e^+e^-$ annihilation cross-section. The dependence on energy and angle of this cross-section, and those of the related reactions

$$e^+e^- \rightarrow Z^0\gamma \text{ and } e^+e^- \rightarrow Z^0Z^0,$$ (2.12)

are predicted precisely by the standard model; these cross-sections were first computed by Sushkov, Flambaum, and Kriplovich and Alles, Boyer, and Buras for $W$ pair production, and by Brown and Mikaelian for the reactions (2.12). The angular distributions of the three reactions at the energy of 250 GeV are shown in Fig. 15. Alles, Boyer, and Buras stress in their paper that the $W$ pair production cross-section is a sensitive test of the detailed structure of the standard model. The reason is indicated in Fig. 16. The
various diagrams which contribute to $W$ pair production do not simply add; rather, they have an intricate cancellation of which the cross-section shown in Fig. 14 is the small residue.

Far above the $Z^0$ resonance, the cross-sections for production of quarks and leptons in $e^+e^-$ annihilation returns to the simple form of Eq. (2.2), in which these cross-sections are essentially constant in units of $R$. One must, of course,
include the modification that the electron and positron can annihilate through either of the light vector mesons $\gamma$ or $Z^0$. Let me conclude my discussion of the predictions of the standard model, then, by tabulating the various contributions to the total cross-section for $e^+e^-$ annihilation, at present energies, far below the $Z^0$, and at energies asymptotically far above:
Components of the $e^+e^-$ annihilation cross-section
(in units of $R$)

<table>
<thead>
<tr>
<th>Present Energies</th>
<th>$E_{cm} &gt;&gt; m_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu, \tau$</td>
<td>$W^+W^-$</td>
</tr>
<tr>
<td>$d, s, b$</td>
<td>$Z^0\gamma$</td>
</tr>
<tr>
<td>$u, c$</td>
<td>$Z^0Z^0$</td>
</tr>
</tbody>
</table>

New fermions $Q^2 \cdot N_c \cdot \beta(\frac{3-\beta^2}{1})

New bosons $Q^2 \cdot N_c \cdot \frac{1}{2} \beta^3$

In this table, $Q$ represents the electric charge, $N_c$ the number of color states.

Fig. 16. Components of the cross-section for $e^+e^- \rightarrow W^+W^-$: (a) The three Feynman diagrams which (in unitarity gauge) must be summed and squared to produce the cross-section; (b) Contributions to the differential cross-section at $\cos \theta = 0$ from products of these diagrams. $\nu Z$, for example, denotes the interference term between the diagram with $\nu$ exchange and that with $Z^0$ exchange.
and $\beta$ the velocity (2.8). The production of new heavy particles at high energies can usually be estimated from the 1-photon contribution, the $Z^0$ making a nontrivial but small correction. The reader should be aware that the cross-sections for (2.11) and (2.12) (expressed in $R$ units) rise with energy. The other pair production cross-sections become constant in units of $R$ at a level set only by the electromagnetic and weak charges of the produced particles, reflecting a perfect democracy between heavy and light species.

3. Particle Search Experiments

Now that we have reviewed the features of $e^+e^-$ annihilation according to the standard model, let us begin to look beyond this model, toward the discovery of new elements of fundamental physics. In this section, I will discuss the search for new physics as it has been carried on in the current experiments at PEP and PETRA. The participants in these experiments have put a great deal of effort into the search for new types of elementary particles. This search for new particles is an important pursuit for deep reasons associated with the deficiencies of the standard model and the possibility that this model will be superseded at very high energies. For my discussion in this section, however, I will discuss this search on its own terms and for its own intrinsic interest. The main question which I would like to explore is the following: We have seen in the previous section that new particles with electromagnetic or weak interactions are produced in $e^+e^-$ annihilation with cross-sections of order 1 unit of $R$. But if such particles are actually being produced, can their presence be observed?

You are well aware that no new particles have yet been discovered at PEP or PETRA. What I will be discussing in this section, then, is a record of the failure to observe new states. This record is, of course, disappointing for the present, but I feel that it is very encouraging for the future. In the course of the search for new particles at present energies, the experimenters at PEP and PETRA have been exploring the backgrounds which might hide the new particles accessible at future machines. They have shown that these backgrounds are, in fact, exceedingly small. In many cases, one can find cuts which would retain a substantial fraction of the signal for the production of a new particle while eliminating virtually all possible background events. I would like to discuss several particle searches in some detail, to show the remarkable extent to which new particles should make themselves visible. Please note that this will not be a systematic survey of all particle searches at PEP and PETRA; however, you may such a systematic review in papers of Wu, $^{14}$ Lau, $^{16}$ and Yamada. $^{16}$

I will focus my discussion on three particular unusual states which have been sought at PEP and PETRA: a scalar particle with muon number $\mu$, a new heavy lepton $L^+$, and a charged Higgs particle $H^+$. The first of these is predicted by
the notion of supersymmetry; the last appears in almost any extension of the standard weak interaction theory. All three are remarkable states which would be, in any event, of great intrinsic interest. Let us ask, then, how carefully experimenters at the present $e^+e^-$ colliders have been able to search for them.

Let us begin with the $\mu$. This is a charged boson, and so it can be pair-produced in $e^+e^-$ annihilation with a cross-section corresponding to $\frac{1}{4}\beta^3$ R units. In the supersymmetric models which require its existence, it is expected to decay via:

$$\mu \rightarrow \mu + \gamma,$$  \hfill (3.1)

in which the $\gamma$ is a fermion with the quantum numbers of the photon, called a photino. In practice, the $\gamma$ should be light and very weakly interacting. If both $\mu$'s of the pair decay by (3.1), $\mu$ production results in the observed signature:

$$e^+e^- \rightarrow \mu^+\mu^- + \text{(missing } E, p_t). \hfill (3.2)$$

There is a background from $\mu$ pair production via the 2-photon process, in which each electron bremsstrahlungs a photon, and these photons interact to produce a $\mu$ pair, but this process only rarely gives substantial transverse momentum to the 2-$\mu$ system.

The CELLO collaboration\cite{Cellosupp} has reported a search for the signature (3.2); this search made use of 11 pb$^{-1}$ of data at $E_{\text{cm}}$ between 33 and 36.7 GeV. This corresponds to about 800 (R units)$^{-1}$, that is, a quantity of data in which a reaction with a cross-section of 1 R unit produces 800 events. They selected for events of the type (3.2) by placing the following relatively loose cuts on the data:

1. less than 8 charged tracks, of which 2 had $p_t > 800$ MeV, $35^\circ < \theta < 145^\circ$;
2. two tracks are identified as muons and have $p > 0.2|E_{\text{beam}}|$;
3. these two tracks are acoplanar by more than 30$^\circ$; and
4. no additional neutral shower is observed. No event in the CELLO data sample passed these cuts. Figure 17(a) shows the number of events to be expected if a $\mu$ indeed existed in the PETRA energy region, and the 95% confidence limits placed by this experiment. Along the bottom of the graph I have recorded the total number of $\mu$-pair events which would be present in the whole data sample for a $\mu$ of that mass. Over a wide range of masses, more than 10% of these events pass the cuts which exclude all background. Figure 17(b) shows the analogous bounds which this experiment places on the existence of a scalar electroweak.

Let me digress to note that, with larger data samples, it is possible to search for supersymmetry even if the $\mu$ and $\tilde{\mu}$ are too heavy to be pair-produced. One way to do this is through the reaction shown in Fig. 18(a), a process introduced by Gaillard, Hinchliffe, and Hall.\cite{Gaillard} The Mark II, MAC, and JADE collaborations\cite{MarkII,JADE} have used this process to set a lower limit of 25 GeV on the...
mass of the 3 (assuming a very light 3). To probe for 3's of higher mass, the MAC collaboration has searched for the process shown in Fig. 18(b), corresponding to the signature

\[ e^+e^- \rightarrow \text{wide angle } \gamma + \text{nothing} \]  

(3.3)

and has reported a (corresponding) lower limit of 37 GeV on the 3 mass. This process is now being sought at PEP with a new specialized detector, AP.\textsuperscript{37} The ASP collaborators expect, with 100 pb\(^{-1}\) of data, to be sensitive to the presence of 3's with mass up to 50 GeV.

Let us now turn to a search for new heavy leptons. The JADE collaboration\textsuperscript{38} has reported a search for heavy leptons produced in association with their own
Fig. 18. Two processes sensitive to the presence of scalar electrons too heavy to the pair-produced in $e^+e^-$ annihilation. The second process has no threshold in the mass of the $\tilde{e}$, though it is suppressed as the $\tilde{e}$ becomes heavy.

neutrinos, leading to the process:

$$e^+e^- \rightarrow L^+ (\rightarrow \text{hadrons} + \nu) + L^- (\rightarrow \text{anything} + \nu), \quad (3.4)$$

They have also searched for neutral heavy leptons with electron number, which might be produced from the electron by $W^\pm$ exchange:

$$e^+e^- \rightarrow E^0 (\rightarrow \text{hadrons}) + \nu. \quad (3.5)$$

In either case, the emission of unobserved neutrinos leads to acoplanarity and missing energy. The JADE analysis (actually, I report only on the 'high-mass' analysis of Ref. 38) made use of 37 pb$^1$ of data at $E_{cm}$ between 27.2 and 36.7 GeV, corresponding to 2800 (R units)$^{-1}$. Events were selected according to the following criteria: (1) a total energy $\geq 3.0$ GeV is deposited in the shower counters, or an energy $\geq 0.4$ GeV is deposited in each endcap shower counter; (2) the event contains at least 4 tracks, excluding the case of 1 isolated track and 3 opposite it; (3) $0.33 < E_{\text{visible}}/E_{\text{tot}} < 1.0$; and (4) the thrust axis points
at an angle such that $|\cos \theta_{\text{thrust}}| < 0.7$. For the sample of events which pass these loose cuts, one can divide the event into hemispheres, compute the thrust axis appropriate to each hemisphere, and plot the distribution of events as a function of the acoplanarity angle between these two axes. Figure 19 shows this distribution, together with the distributions which one would have expected from the processes (3.4) and (3.5). Placing a cut at an acoplanarity angle of 50° yields only 2 candidate events; this excludes the presence of new heavy leptons at the 95% confidence level for $6 \text{ GeV} < m_L < 18 \text{ GeV}$ and for $6 \text{ GeV} < m_E < 24.5 \text{ GeV}$. A similar analysis carried out by the MARK J collaboration [39] excluded new heavy leptons for $m_L < 16 \text{ GeV}$ using a data sample of only 520 $R^{-1}$.

Fig. 19. Distribution of heavy lepton candidate events in acoplanarity angle, comparing data to Monte Carlo distributions for the processes (3.4) (dashed histogram) and (3.5) (solid histogram), from Ref. 38.

Finally, let us consider the search for a charged Higgs scalar $H^+$. The dominant decay modes of the Higgs should be those to heavy leptons ($r^+ \nu_r$) and to heavy quarks ($c\bar{s}$ or $c\bar{b}$). The relative branching ratio into leptons versus hadrons is highly model-dependent; however, and it is necessary to search for the $H^+$ by different techniques depending on the assumed value of this ratio. If the branching ratios into hadrons and leptons are both substantial, one can
search for the reaction

$$e^+e^- \rightarrow H^- (\rightarrow \text{hadrons})$$

$$+ H^+ (\rightarrow \nu + \tau^+ (\rightarrow \mu^+ \nu \bar{\nu}))$$

(3.6)

The final signature is a hadronic jet opposite an isolated acollinear muon. The MARK J collaboration[40] has reported a search for events of this class, based on 40 pb$^{-1}$, or about 3000 R$^{-1}$, of data. They required (1) a muon opposite a hadronic shower; (2) more than 2 tracks in the inner vertex chamber; (3) thrust $< 0.95$; (4) $0.3 < E_{\text{visible}}/E_{\text{cm}} < 0.75$; (5) $|\sum p_{\perp}| > 0.4 E_{\text{visible}}$; and (6) $|\sum p(\perp \text{to plane of } \mu \text{ and beam})| > 0.15 E_{\text{cm}}$. No candidate event passed these cuts. An $H^+$ with a mass of 13 GeV and a branching ratio of 25% to $\tau^+\tau^-$ would have produced 3 events in this sample.

The case in which the Higgs decays dominantly into leptons is also quite straightforward to explore. The case in which the Higgs decays dominantly into hadrons, however, is rather tricky by the standards of $e^+e^-$ annihilation, since the decay products of each Higgs superficially resemble hadronic jets. One can, however, make use of the narrowness and low invariant mass of typical $e^+e^-$ jets to distinguish the Higgs. Let me review an analysis along these lines performed by the TASSO collaboration.[41] This study used 71.5 pb$^{-1}$ of data at $E_{\text{cm}}$ between 33 and 37 GeV, corresponding to about 5400 R$^{-1}$. Since the full analysis is rather complex, I will give only a sketch of it here. The method of the study was to select hadronic events, fit these events to the hypothesis that they contain 4 jets, and then place cuts on the parameters determined by the fit. The authors insisted that each jet should contain at least 2.6 GeV of observed energy and should reconstruct to an energy greater than 3.6 GeV. They then combined jets in pairs and computed the energy $E_{\text{pair}}$, invariant mass $m$, and opening angle $\theta$ of each pair. They selected events in which $\theta$ differed from one side to the other by less than 9°. Events with charged Higgs production, as generated by a Monte Carlo program, generally meet this criterion; these fictitious events also cluster into an ellipse in the three variables $(E_{\text{pair}} - E_{\text{beam}})$, the average $m$, and the average $\theta$. Change variables so that the ellipse becomes a sphere; then Fig. 20 shows the expected number of events in the data sample which should fall at given distances from the center of the sphere and compares this distribution to the data. Clearly, no real events appear in the preferred region. The analysis gives a stringent constraint for charged Higgs masses up to 13 GeV.

Figure 21 summarizes the results of the various experiments which have searched for charged Higgs mesons at PEP and PETRA. It is remarkable that, despite the elusive nature of this particle and the vagueness of theoretical predictions for its decays, $e^+e^-$ annihilation experiments are sensitive to the presence of the charged Higgs throughout almost the whole of the mass range which is accessible kinematically.
Fig. 20. Search for events containing charged Higgs pairs, Ref. 41: The figure shows the number of expected vs. actual events at various distances from the center of the sphere in parameter space defined by the analysis, for $m_{H^\pm} = 10$ GeV.

Fig. 21. Summary of the results of PEP and PETRA experiments which have searched for charged Higgs mesons, from Ref. 30.

I should properly note that the neutral Higgs meson $H^0$ is not easily produced in $e^+e^-$ annihilation; this unfortunate property is, however, shared by all other types of high-energy accelerators. If the mass of the Higgs lives in specific regions, however, it does stand out if it is sought in certain elegant reactions. If
the mass of the neutral Higgs is less than the mass of the lowest \( t\bar{t} \) resonance, the Higgs should appear in the Wilczek process:

\[
\zeta(t\bar{t}) \rightarrow \gamma + H^0. \tag{3.7}
\]

If the mass of the \( \zeta \) is about 80 GeV, the branching ratio for this reaction in the simplest form of the standard model is 3% of all \( \zeta \) decays. The monoenergetic \( \gamma \) is easily distinguished from background. An \( H^0 \) in this general mass range might also be found in the Bjorken process, \(^{[43]} \) the \( Z^0 \) decay

\[
Z^0 \rightarrow \ell^+\ell^- + H^0. \tag{3.8}
\]

The mechanism for this decay is shown in Fig. 22(a), and the rate as a function of Higgs mass is shown in Fig. 22(b); the \( Z^0 \) branching ratio into this mode is greater than \( 10^{-5} \) for \( H^0 \) masses below 30 GeV. At energies above the \( Z^0 \) resonance, one may search for a still heavier Higgs, by looking for the process shown in Fig. 22(c): \(^{[44]} \)

\[
e^+e^- \rightarrow H^0 + Z^0. \tag{3.9}
\]

The rate of this process at \( E_{\text{cm}} = 200 \) GeV as a function of the mass of the \( H^0 \) is shown in Fig. 22(d).

A discussion of the status of the search for new particles would not be complete without a comment on a closely related endeavor, the search for substructure within the quarks and leptons. Like the search for new particles which would force us beyond the standard model, this study has so far been unsuccessful. No form factor effects have been seen in the hard scattering of quarks or leptons. For those two fermions for which the magnetic moment has been accurately measured—\( e \) and \( \mu \)—the value of \( (g - 2) \) agrees quite precisely with the predictions of QED. Thus, if quarks and leptons are composite, their intrinsic size is much smaller than the distances we have currently probed; equivalently, the mass scale \( \Lambda \) which characterizes the internal momentum of quarks and leptons is much larger than the momentum transfers of 30-40 GeV which we now consider well-explored. It is, then, interesting to ask, first, what experiments now provide the best lower bounds on \( \Lambda \), and, secondly, how sensitively \( e^+e^- \) annihilation experiments can probe the existence of substructure.

Actually, two experiments on substructure stand out from the others in placing a lower bound on \( \Lambda \) which is close to 1 TeV. The first of these is the measurement of the muon \( (g - 2) \) value: The fact that this measurement agrees with the QED prediction to an accuracy

\[
\frac{1}{2}|g - 2| - \text{QED} < 1.5 \times 10^{-8} \tag{3.10}
\]

requires, roughly, that \( \Lambda > 800 \) GeV. \(^{[45]} \) The second measurement is one conducted in \( e^+e^- \) annihilation, the accurate test of the standard model prediction
for Bhabha scattering at high energies. If electrons are composite objects, one would expect that electrons and positrons could scatter through a process not included in the standard model in which these particles overlap and exchange their constituents. This process would lead to an additional contribution to the amplitude for Bhabha scattering, of the form of a 4-fermion contact interaction, as shown in Fig. 23. Several experiments have searched for deviations from the standard model prediction. Figure 24 shows a comparison of the angular distribution in Bhabha scattering, as determined by the MAC collaboration, to the standard model and to models with additional contact interactions. The bound which this experiment places on $A$ is sensitive to the Lorentz structure assumed for the contact interactions, but in any case it is quite strong; the MAC group quotes, as 95% confidence limits, $A > 1.2$ TeV for purely left- or right-handed contact interactions and $A > 2.5$ TeV for vector-like contact interactions.
Other analyses, giving bounds of similar quality, have been reported by HRS and JADE.\textsuperscript{[30,47]}

Fig. 23. An additional contact contribution to Bhabha scattering which appears if the electron is a composite object.

Fig. 24. Comparison of the angular distribution for Bhabha scattering obtained by the MAC collaboration (Ref. 46) with the prediction of the standard model and of a model with purely right-handed contact interactions with $\Lambda = 600$ GeV. The overall normalization is arbitrary, within the limits indicated by the arrow. The two predictions of the contact interactions correspond to the two possible signs of the interference.
We have now reviewed several different aspects of the search for physics beyond the standard model as it has been carried out at PEP and PETRA, the highest-energy $e^+e^-$ 'colliders now in operation. The $e^+e^-$ annihilation experiments of the current generation have proved their ability to probe for the presence of new particles definitively and unambiguously. That they have found none presumably reflects only the relatively low energies at which these experiments have been done. Let us now, therefore, turn to the question of how far in energy one must go in order to find new physics, the question of why one might expect to see new particles and new interactions and where, given this expectation, these new states must occur.

### 4. A Digression: 3 Kinds of 1 TeV Physics

To what extent does our present knowledge of physics call for the extension or improvement of the standard model? I know of no experiments which definitively contradict the standard model or require its extension; indeed, I have shown you in the previous section that the standard model has passed many stringent tests. Yet I do believe that a close examination of the details of this model reveals inadequacies whose correction would alter the model profoundly. In this section, I would like to discuss this issue. The question is essentially one of theoretical physics, in one of its more speculative manifestations, yet the answer to the question is obviously of great importance in choosing and planning for future accelerators. Allow me, then, this rather theoretical digression.

The standard model is essentially a theory of gauge symmetries and the interactions of gauge bosons. Its prediction for the couplings of these gauge bosons to fermions is reflected directly in the structure of the weak interactions, and, as such, is brilliantly confirmed by experiment. However, there is another aspect to the fundamental interactions which the standard model touches upon but does not at all illuminate. This is the production of masses for the quarks and leptons and for the weak bosons. It is a slight oversimplification, but, I think, a fair one, to say that the standard model offers as an explanation of the masses of the $W^\pm$ and $Z^0$ bosons and the quarks and leptons only a set of formulae of the form:

\[
m_W = \frac{1}{2}g(\phi) \quad m_Z = \frac{1}{2}(g^2 + g'^2)^{1/2}(\phi) \quad m_t = \lambda_t(\phi) \quad \ldots \quad m_e = \lambda_e(\phi)
\]

(4.1)

Here $g$ and $g'$ are gauge couplings and therefore connected to other aspects of the theory; the $\lambda_f$, however, are new dimensionless numbers whose values range ...
from $10^{-1}$ to $10^{-5}$, and $\langle \phi \rangle$ is a new dimensionful parameter whose value

$$\langle \phi \rangle = 250 \text{ GeV} \quad (4.2)$$

may be determined from the masses of the weak bosons. In the Weinberg-Salam theory, this is the vacuum expectation value of the fundamental Higgs scalar field which that model invokes. In the standard model all of these new parameters must be adjusted to their phenomenologically correct values by hand; more technically, they are all renormalized parameters, like the value of the fine structure constant in QED, whose values must be specified in order to define the theory. Grand Unified Theories, which introduce new structure at extremely large mass scales of order $10^{15}$ GeV, can explain the size of $g$ and $g'$ but say little about the other parameters. The values of the $\lambda_f$ are a mystery from almost every perspective. If we wish to understand more deeply the sizes of the various quantities in Eq. (4.1), it seems that we find as our main clue the value of the dimensionful parameter $\langle \phi \rangle$. It is tempting to suggest that the value of $\langle \phi \rangle$ is caused by something, that $\langle \phi \rangle$ appears as the calculable result of some physical process. If this is true, that process must add physics to the standard model. This new physics must somehow be associated with the mass scale set by $\langle \phi \rangle$, or, to speak roughly, the scale of 1 TeV. Let me sort the possibilities for what might appear into three broad classes, by saying that new physics at 1 TeV might be absent, weak, or strong.

**Absent?** I have already noted that the standard model, with the inclusion of a Higgs scalar meson, is an internally consistent theory which has so far been verified by experiments. It could be exactly correct. In that case, the parameters of Eq. (4.1) will remain a mystery, and, worse, we will see no new physics, except perhaps the Higgs boson itself, at any foreseeable new accelerator beyond the current generation. I do not consider this scenario likely, but one must still remember that it is possible.

**Weak?** I will use this term to refer to models in which the new scale at 250 GeV arise from new interactions which are essentially weak, in the sense that they can be described by Feynman diagram perturbation theory. To understand the structure of such a model in a general way, let us examine the Feynman diagrams which contribute to $\langle \phi \rangle$. Figure 25(a) shows the leading contributions to $\langle \phi \rangle$ in the standard model. The loop diagram is quadratically ultraviolet divergent; this is the same divergence which appears in the renormalization of the Higgs boson mass. If $\langle \phi \rangle$ and the Higgs mass are to be predicted by the theory, rather than being free parameters, we must add some other diagram which contributes to $\langle \phi \rangle$ and can cancel the divergent part of this loop. This new diagram would necessarily involve either new particles with masses in the
range of 100 GeV - 1 TeV or row interactions which become active for particle momenta of this range.

Examples of models of this class are provided by unifying theories based on the idea of supersymmetry. Supersymmetry is a symmetry which transforms bosons into fermions and vice versa. Thus, if Nature is assumed to be supersymmetric, even if this symmetry is broken spontaneously, there must be, for every boson that we see, a corresponding fermion with the same quantum numbers. In particular, the $W^\pm$ and $Z^0$ must have fermionic partners, and these can contribute to $\langle \phi \rangle$ through the diagram shown in Fig. 25(b). In still higher orders of perturbation theory, the quarks and leptons enter the calculation of $\langle \phi \rangle$, as shown in the graph of Fig. 25(c). The cancellation of the quadratic ultraviolet divergences of this diagram requires that the quark and leptons have bosonic partners which contribute, for example, the diagram of Fig. 25(d). If the theory is precisely supersymmetric, the sum of the contributions to $\langle \phi \rangle$ contains no quadratic ultraviolet divergences but only the more controllable divergences associated with coupling constant renormalizations and the wavefunction renormalizations of the various fields.

Strong? I will use this term to refer to models in which the new scale at 250 GeV is produced by new interactions by means of bound state formation or other effects characteristic of strong interactions. What is needed is that these effects break spontaneously the weak interaction gauge symmetry $SU(2) \times U(1)$. Studies of the phenomenology of QCD and of model field theories have offered us several mechanisms by which this symmetry-breaking might occur. Any one of these possibilities requires, again, the presence of new particles of mass roughly 1 TeV. At the most basic level, none of the interactions in the standard model can be strongly coupled at this new scale; thus, we must invoke either some change in the particle content of the standard model or, more likely, some completely new interaction. We might hope for new physics of considerable complexity and interest: There is no reason why these new forces should not build a spectrum of particles as one finds in the conventional strong interactions.

Two distinct realizations of this idea have been discussed prominently in the literature. This first of these introduces so-called technicolor interactions and makes use of the analogue in these new interactions of the strong-interaction symmetry breaking effects which are found in QCD. Some time ago, Weinberg [40] and Susskind[50] noted that the physics which generates large effective masses for the light quarks, and, at the same time, breaks the chiral $SU(3)$ symmetry of the strong interactions, breaks, at the same time, the weak-interaction symmetry $SU(2) \times U(1)$. This effect is rather small compared to the symmetry-breaking effects induced by $\langle \phi \rangle$, but the weak gauge bosons do receive contributions to
their masses in the correct pattern:

\[ m_W = \frac{1}{2} g r_f, \quad m_Z = \frac{1}{2} (g^2 + g'^2)^{\frac{1}{2}} f, \quad m_\tau = 0. \]  

(4.3)

In (4.3), \( f \) is the pion decay constant, equal to 93 MeV. To produce a reasonable theory of \( W \) and \( Z \) masses, we need only postulate a higher-mass copy of the strong interactions, with new fermions and new bound states, such that the analogue of the pion decay constant in this new theory is equal to the value of \( \langle \phi \rangle \).

I should note that no completely satisfactory method is known for coupling this symmetry-breaking to the ordinary quarks and leptons to produce the parameters \( \lambda_f \) which appear in (4.1). The most straightforward such method\(^{[51,52]} \) leads to a complex system of fermion-fermion couplings which produces, in particular, too large an amplitude for \( K^0 - \bar{K}^0 \) mixing.\(^{[53,54]} \) A second possibility for this new dynamics is that it might be based on the formation of the quarks and leptons as bound states of more elementary constituents, with the weak-interaction
symmetry-breaking being driven by the fermion bound-state formation.

Whether the new physics at 1 TeV is weak or strong, new particles and other novel phenomena should be visible to experiments which can provide such large momentum transfers. For the case of 'weak' 1 TeV physics, these new particles will be the primary manifestation of the physics which produces the new mass scale. For the case of 'strong' 1 TeV physics, these new particles provide at least one indication of the existence of new phenomena. Let me emphasize this point by showing you two conjectured mass spectra taken from the theoretical literature: Fig. 26 shows the result of a compilation done by John Ellis\(^7\) of the masses of novel particles predicted in a number of supersymmetric unified models. Figure 27 shows the spectrum of the lowest-mass particles bound by the new forces in two classes of technicolor models.\(^27\) In each case, there is a rich variety of new states to be expected in the mass region up to roughly 1 TeV.

In \(e^+e^-\) annihilation, all of these states which possess electromagnetic or weak interactions are produced with cross-sections of units of \(R\):

\[
\Delta \sigma \approx Q_{E M}^2 \cdot (\text{no. of colors}) \cdot \left(\frac{1}{4} \text{ for bosons}\right) \cdot (\text{phase space}), \quad (4.4)
\]
as long as the center of mass energy is sufficiently high that the state in question can be pair-produced. Since the production of ordinary particles is also at the level of units of \(R\), the new states should clearly manifest themselves above the background. All we need, then, to search for these particles, is a machine which can produce a sufficient total rate: \(10 \text{ R}^{-1}/\text{day}, \) at \(E_{cm} = 1 \text{ TeV}\).

5. Technics for 1 TeV \(e^+e^-\) Collisions

I should, however, express the optimistic theoretical conclusion of the previous section in units which put it in better perspective. At 1 TeV, a luminosity of \(10 \text{ R}^{-1}/\text{day}\) corresponds to

\[
\mathcal{L} \sim 10^{33} \text{cm}^{-2}\text{sec}^{-1}, \quad (5.1)
\]
a value higher by two orders of magnitude than the luminosities which can now be achieved at high-energy \(e^+e^-\) colliders. The design of \(e^+e^-\) colliders for physics in the TeV energy region must confront these two coupled problems of reaching high energy and high luminosity. In this section, I would like to discuss, on a very basic level, some of the technical challenges which must be met to achieve these goals.
To see the basic problems, consider the following estimates, based on extrapolating the technology of the linear collider to TeV energies. The luminosity of a linear collider is given by

\[ \mathcal{L} = \frac{N^2 f}{4\pi\sigma_f^2} \cdot H, \]

(5.2)
where $N$ is the number of particles per bunch, $f$ is the repetition rate, $\sigma_r$ is the transverse size of the bunch at the interaction point, and $H$ is the enhancement resulting from the pinch effect. A straightforward extrapolation of the SLC technology would give, for a typical set of parameters yielding a luminosity of $10^{33}$ cm$^{-2}$sec$^{-1}$, the following: $N = 1.4 \times 10^{10}$, $f = 2000$ Hz (180 Hz cavities operated at 12 bunches/cycle), $H = 6$, and $\sigma_r = 0.14\mu$ (0.1 - the SLC design value). These parameters require a each beam to carry a power

$$P = N \cdot f \cdot (1 \text{ TeV})$$

(5.3)

equal to 4.7 MWatt/beam. This does not seem completely unreasonable until one recalls that the SLAC linac converts electrical line power into beam power with an efficiency of about 3%. A more general relation, in terms of the parameter

$$D = \sigma_r/(\text{focal length}),$$

(5.4)

is:

$$L = \frac{\gamma N^2 f H}{4\pi \sin^2 \beta} = \frac{3.5 \times 10^{31} \cdot P(\text{MW}) \cdot D \cdot H}{\sigma_r(\text{mm}^2)}.$$  

(5.5)

To the above constraints we must add one more. The particle densities in each bunch which are required to achieve high luminosity are sufficiently large that each bunch, as it crosses through the other, emits so much radiation that its overall energy is affected. This effect, called "beamsstrahlung", gives rise to an intrinsic energy spread of each beam which grows according to:

$$\sigma_E/E \propto EL.$$  

(5.6)

In terms of our basic set of parameters,

$$\frac{\sigma_E}{E} = \frac{414 E(\text{TeV}) L(10^{33})}{\sigma_r(\text{mm}^2) f(\text{Hz})}.$$  

(5.7)

A reasonable design should at least maintain $\sigma_E/E < 0.1$. We will examine in the next section what values of $\sigma_E/E$ are required by the physics.

A sample design for a 1 TeV on 1 TeV e$^+$-e$^-$ collider consistent with these constraints is given in Table 1. I should emphasize that this design represents an existence proof for such a collider; it is not an optimized design. The design is based closely on a scaling up of current SLAC technology: The left-hand column assumes a linear accelerator with a gradient equal to that currently available at SLAC; the right-hand column is based on the use of klystrons of basically the same type operating at double the frequency currently used. The
TABLE 1
An incompletely optimized example of a 2 TeV linear collider at $10^{33}$ luminosity and 10% $\sigma E^*/E^*$ for two RF wavelength, from Ref. 56.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_y^*$ (cm)</td>
<td>1</td>
</tr>
<tr>
<td>$D$</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma_x$ (mm)</td>
<td>2</td>
</tr>
<tr>
<td>$\epsilon_n$ (mrad)</td>
<td>$4 \times 10^{-6}$</td>
</tr>
<tr>
<td>$N$</td>
<td>$1.4 \times 10^{10}$</td>
</tr>
<tr>
<td>$f$ (Hz)</td>
<td>2000</td>
</tr>
<tr>
<td>$b$</td>
<td>12</td>
</tr>
<tr>
<td>$P_b$ (MW per beam)</td>
<td>4.7</td>
</tr>
<tr>
<td>$\lambda$ (cm)</td>
<td>10</td>
</tr>
<tr>
<td>$G$ (MV/m)</td>
<td>20</td>
</tr>
<tr>
<td>$L$ (km - each linac)</td>
<td>50</td>
</tr>
<tr>
<td>Number of Klystrons</td>
<td>3500</td>
</tr>
<tr>
<td>Total Input AC Power (MW)</td>
<td>390</td>
</tr>
</tbody>
</table>

cost of a facility based on this technology would be roughly that envisioned for the Superconducting SuperCollider.

One must, however, remember that the art of designing linear $e^+e^-$ colliders has hardly begun to develop. In the past few years, a number of new acceleration methods have been proposed—among them, the laser near-field, wake-field, and plasma beat-wave concepts—which are based on accelerating structures of high power density and correspondingly small transverse dimension and which might achieve gradients of several hundred MeV/meter. It seems likely that one of these methods could form the basis of a design which would appear radical compared to that of Table 1 but which would be relatively economical in terms of construction cost and power bill. I hope that those of you who are attending this summer school (and those of you who are reading this lecture) will consider this prospect as a challenge.
6. Physics of 1 TeV $e^+e^-$ Collisions

What, then, can one hope to discover at very high energy $e^+e^-$ colliders? The most important effects, as I have emphasized repeatedly earlier in this lecture, involve the production of new types of particles. I have already explained that $e^+e^-$ colliders are ideal facilities for the production of new particles, if the necessary luminosity can be provided. In this section, I would like to amplify this viewpoint in several ways. First, I will discuss a few technical points relevant to the physics of 1 TeV $e^+e^-$ reactions. Next, I will explain how to compare the energies of $e^+e^-$ and $p-p$ or $p-p$ colliders. Finally, I will discuss a number of phenomena associated with the case of ‘strong’ 1 TeV physics which will most probably only be visible in the special environment of $e^+e^-$ annihilation reactions.

Let us first dispose of three technical questions. The first concerns a new background not present in current $e^+e^-$ annihilation experiments. I explained in §3 that the process $e^+e^- \rightarrow W^+W^-$ should be a major component of the $e^+e^-$ annihilation cross-section for center-of-mass energies above a few hundred GeV. At first sight, this seemed a good feature, but at very high energies one might worry that $W$ pair production might be a substantial background to other types of new physics. One might worry a bit more on seeing the magnitude of this cross-section, which I show in Fig. 28 for energies up to 1 TeV. Fortunately,
differential cross-section $\frac{d\sigma}{d\cos\theta}$, at angles away from the forward direction. In Fig. 29, I have replotted the three gauge boson production cross-sections shown in Fig. 15 on a linear scale and compared these estimates, made at $E_{cm} = 250$ GeV, to corresponding results at 4 TeV. It is remarkable that the differential cross-section for $W$ pair production, and also, for that matter, the differential cross-sections for $Z^0$ production, how very little energy-dependence except just in the forward direction. Since new-particle production has, as I have argued above, a broad angular distribution, the background from $W$ and $Z$ production turns out to be rather small. [5]

![Angular distributions for the reactions $e^+e^- \rightarrow W^+W^-$, $e^+e^- \rightarrow Z^0\gamma$, and $e^+e^- \rightarrow Z^0Z^0$, for (a) $E_{cm} = 250$ GeV, (b) $E_{cm} = 4$ TeV.](image)

The second technical issue is that of the energy spread of the beams in high-energy $e^+e^-$ collisions. I noted in the previous section that linear colliders have the property that increasing the luminosity also increase the spread of energies in the beam, as a result of “beamstrahlung” (Eq. (5.7)). Thus, one might well have to live with $e^+$ and $e^-$ beams with a 10% spread in energy. This
decreases the cleanliness of the $e^+e^-$ annihilation environment, but not, I think, significantly. Such a beam spread would have almost no effect on new particle searches away from resonances; it would not, for example, have jeopardized any of the experiments discussed in § 3. I will argue below that, if there are new strong interactions at TeV energies, the new resonances associated with these interactions should be rather broad, with widths of order 10% of their masses. Such resonances could be explored without extremely fine energy resolution. The one case in which a fine energy resolution would be valuable is if there existed a new $Z^0$, with mass roughly 1 TeV. For such a particle, one would expect a width of order $\alpha$ times the mass, or $\Gamma/M \approx 0.01$. However, one would also expect the cross-section at the peak to be very large, roughly $(M/\Gamma)^2 \approx 10^4$ units of $R$. Thus, in this case, one could easily sacrifice luminosity for gain in energy resolution. The "beamstrahlung" effect seems, then, not to be a serious problem for physics.

Finally, I would like to note that it may be possible to polarize at least the electron beam at a TeV $e^+e^-$ facility. This situation contrasts markedly with high-energy $p\bar{p}$ collisions, since in that case even polarizing the protons completely gives very little net polarization to the quarks and gluons which are the elementary objects involved in hard reactions. Is polarization useful? Certainly in would be a help in disentangling $e^+e^-$ annihilation through the photon from annihilation through the $Z^0$. It is also possible that new phenomena will be strongly polarization-dependent; I will give an example of one such process in my discussion of models with composite electrons at the end of this section. Is it enough to polarize only the electrons if one cannot polarize the positrons? This should be no problem at all. All of the gauge interactions of the electron preserve the electron's helicity; the only helicity-flip interactions of the electron in the standard model are those which result from the electron mass or other direct couplings of the electron to the Higgs sector. Conversely, if new interactions which one might discover at the TeV energy scale could flip an electron's helicity, they would also be expected to contribute a large term to the electron's mass; thus, the strength of such interactions is strongly restricted. Helicity conservation would imply that left-handed electrons annihilate only right-handed positrons, up to minute corrections of order $(m_e^2/E_{cm}^2)$.

Let us now turn from technicalia to more central issues of physics. At the end of § 4, I stressed that the search for new particles is our most promising route to the discovery of new physics which replaces the standard model at energy scales of order 1 TeV. I have already emphasized that, on their own merits, $e^+e^-$ colliders are ideal instruments for the search for new particles, since they offer low backgrounds and democratic, if not large, production cross-sections. I would like now to assess their capabilities in a somewhat more pointed way—by comparing their capabilities to those of high-energy $p\bar{p}$ or $p\bar{p}$ colliders.
How should one compare the power of \( e^+e^- \) and \( p\bar{p} \) or \( p-p \) colliders to produce novel particles with masses in the range of 1 TeV? On the one hand, conventional technology allows much higher center-of-mass energies for proton colliders than for electron colliders. (The design studies for the Superconducting SuperCollider have argued the technical feasibility of a \( p-p \) collider with \( E_{cm} = 40 \text{ TeV} \).) On the other hand, new particles are produced from proton-proton collisions only in reactions of elementary constituents—quarks and gluons—which in general carry only a small fraction of the proton's momentum. This means that an \( e^+e^- \) collider operating at a fixed energy will have a power to produce new particles equivalent to that of a \( p-p \) collider of a substantially higher energy. We must consider this comparison of energies with some care.

Let us begin with a very simplistic picture. Roughly half the momentum of the proton is carried by gluons, and the rest is divided among the three valence quarks and the sea. Very crudely, then, one might expect that the effective center-of-mass energy of a \( p-p \) collider available for producing new particles will be about \( 1/6 \) of the total center of mass energy, so that colliders with

\[
E_{cm}(e^+e^-) \approx \frac{1}{6} \cdot E_{cm}(pp),
\]

will have comparable power to produce new particles. It is, in principle, quite straightforward to quantify this estimate, by considering comparable production processes, integrating over realistic parton distributions, and including the (generally rather small) effects of QCD scaling violations.

There is, however, one additional consideration which plays a crucial role. Cross-sections for producing new particles necessarily involve exchanges of virtual states which are off-shell by an amount of order the new particle mass. Thus, by dimensional analysis, such cross-sections decrease with the mass \( M \) of the new particle, according to \( \sigma(M) \sim M^{-2} \). This behavior is precisely the same as that seen in the precipitous decrease of the \( R \) unit in \( e^+e^- \) annihilation as \( E_{cm} \) increases. In \( e^+e^- \) colliders, however, the total cross-section is also decreasing as \( E_{cm}^{-2} \), so that one can, at least in principle, compensate these small cross-sections by increasing the luminosity. In \( p-p \) collisions, however, the total cross-section increases with \( E_{cm} \), and the production of jets with transverse momentum greater than some given value represents an increasing fraction of the total cross-section. Since general purpose detectors can tolerate only a certain total event rate, there is a limit beyond which one cannot usefully raise the luminosity. For the conditions of the SSC, it is generally agreed that this limit corresponds to a luminosity of order \( 10^{33}\text{cm}^{-2}\text{sec}^{-1} \) for detectors without precision vertex chambers but may be as low as \( 10^{27}\text{cm}^{-2}\text{sec}^{-1} \) if precision vertex chambers (which might, for example, tag jets with \( b \) quarks) are necessary for specific important experiments. If the luminosity is fixed but the cross-sections
for interesting events decreases with $E_{cm}$, the ability of a $p-p$ collider to produce new particles of high mass will be limited: It is not simply that the proportionality constant in Eq. (6.1) will be smaller but, rather, that the equivalent $E_{cm}$ for $e^+ e^-$ annihilation will grow as a smaller power of $E_{cm}(pp)$.

Let me now discuss this constraint more quantitatively. To do this, I will make use of the recent landmark study of Eichten, Hinchliffe, Lane, and Quigg (EHLQ) on the physics capabilities of high-energy $p-p$ colliders. These authors have computed the magnitudes of signal and background, as a function of the center-of-mass energy of the collider, for a variety of new phenomena predicted by theories of 1 TeV physics; this allowed them to formulate criteria for estimating the largest value of the mass for which a new particle should be detectable in experiments done at a given $E_{cm}$. For clarity, I have selected five representative processes which cover the range of new physics they consider: production of a new $W$ boson, pair production of a new heavy quark $Q$, production of the supersymmetric partner of the gluon (called the gluino or $\tilde{g}$), production of a new heavy lepton $L$ in association with its neutrino, and deviations from the QCD predictions for high-$p_T$ jet production as the result of the presence of a $q\bar{q}$ contact interaction (parametrized by a mass $\Lambda$) of the type discussed for electrons at the end of § 3. For each of these processes, I have converted the mass limit found by EHLQ into an equivalent center-of-mass energy for discovery of the analogous effect in $e^+ e^-$ annihilation in the following way: For $W$ production, I have taken the equivalent $E_{cm}$ to be the mass of the new $W$; though the $W$ can be produced only in pairs, the corresponding $Z$ boson, normally quite close in mass, can be produced directly. For $Q$ and $L$ production, I have taken the equivalent values of $E_{cm}$ to be $2m_Q$ and $2m_L$. For gluino production, I have taken the equivalent $E_{cm}$ to be $3m_g$; the gluino is difficult to produce directly in $e^+ e^-$ annihilation, but in models where the mass of the gluino is as large as 1 TeV, the masses of the partners of the quarks and lepton are comparable or perhaps smaller. For the contact interaction, I have taken the equivalent $E_{cm}$ equal to $\Lambda/30$; this reflects the sensitivity of current probes of $\Lambda$ in Bhabha scattering reported at the end of § 3.

The results of this comparison are shown in Figs. 30 and 31; these figures assume $p-p$ luminosities of $10^{32}$ and $10^{33}$, respectively, integrated over a practical year of $10^7$ seconds. The $W$, $Q$, and $\tilde{g}$ processes have relatively large cross-sections in $p-p$ collisions; for these, the one-sixth rule (6.1) is not a bad approximation for $E_{cm} = 10$ TeV, though it rapidly becomes an overestimate as the $p-p$ center-of-mass energy is increased. Over the range of energies shown, the actual relation between the values of $E_{cm}$ for $e^+ e^-$ and $p-p$ is close to:

$$E_{cm}(e^+ e^-) \sim \sqrt{E_{cm}(pp)}, \quad (6.2)$$

if both energies are expressed in TeV. Except for this last stipulation, this is
Fig. 30. Equivalent energies of $e^+e^-$ and $p-p$ facilities in searches for a variety of novel particles which might result from new physics at 1 TeV. The five reactions are explained in the text; the capabilities of $p-p$ colliders are taken from the calculations of Ref. 58. This figure assumes a maximum $p-p$ luminosity of $10^{32} \text{cm}^{-2} \text{sec}^{-1}$. The dashed line is a suggested fit to the data:

$$E_{\text{cm}}(e^+e^-) = \sqrt{E_{\text{cm}}(pp)}/3,$$

with all energies in TeV.

Fig. 31. Equivalent energies of $e^+e^-$ and $p-p$ facilities, assuming a maximum $p-p$ luminosity of $10^{33} \text{cm}^{-2} \text{sec}^{-1}$. The notation is as in Fig. 30. The dashed line is a suggested fit to the data:

$$E_{\text{cm}}(e^+e^-) = \sqrt{E_{\text{cm}}(pp)}/3.$$

precisely the relation giving the comparison of energies for colliding-beam and fixed-target experiments.

I have now argued that the advantages of $e^+e^-$ colliders in producing novel particles do not appear dimmed even by comparison to $p-p$ colliders of much higher energy. If a multi-TeV $e^+e^-$ collider could be constructed, it would allow searches for new particles as powerful as any which might be conducted
at conceivable \( p+p \) facilities. In my discussion of § 4, I argued that, for the class of models with 'weak' 1 TeV physics, this question of the reach in mass for new particle production is the only relevant question. However, for the case of 'strong' 1 TeV physics, it is only half of the story. Theories in this latter class will contain distinctive effects of bound state and resonance formation; such effects will be dramatically visualized in the controlled environment which \( e^+e^- \) annihilation makes available. At the end of § 4, I described the ideas of technicolor and of compositeness of quarks and leptons as two realizations of strong 1 TeV physics. Let us now consider these implications of these two scenarios for very high energy \( e^+e^- \) annihilation experiments.

The idea of technicolor postulates a new strong interaction sector, quite similar in structure to the familiar one, at a mass scale of 1 TeV. Below 100 GeV, the predictions of this scheme are quite close to those of the standard model, except that some light charged and neutral particles resembling Higgs bosons should be found in this region. Above 300 GeV in the center of mass, a spectrum of new bosons such as that shown in Fig. 27 should appear. All of these new particles, however, are the analogues of the \( \pi \) and \( K \) mesons of this new set of strong interactions. There should also exist analogues of the \( \rho \)–vector bosons bound by the new strong interactions which can be produced as resonances in \( e^+e^- \) annihilation. These vector resonances decay to the pions, that is, to the exotic bosons just mentioned and to \( W \) and \( Z \) pairs. If the new strong interactions are exactly like the familiar ones (except for a change in the number of "light" quarks), one can compute the properties of these resonances with some confidence. A calculation of the shape of the techni-\( \rho \) resonance in one particular the technicolor scheme\(^{[59]}\) is shown in Fig. 32. The peak is a dramatic one, rising to 15 to 20 units of \( R \); the bulk of this cross-section results in the production of pairs of exotic particles.

At energies three or four times the mass of the techni-\( \rho \) (\( E_{cm} \sim 4 \) TeV), one should expect to see the asymptotic behavior of this new strong interaction theory. If these new interactions are just analogous to the familiar ones, built up as an asymptotically free gauge theory, one should see jet-like final states in the new sector—with jets built from multiple \( W \) and \( Z \) production and having characteristic transverse momenta of 500 GeV relative to the jet axis. But this is not the only alternative which the theory allows. Holdom\(^{[68]}\) has argued that technicolor interactions should not exhibit asymptotic freedom but rather should show a more intricate asymptotic behavior; his model predicts that the multi-boson final states are broader in the transverse momentum spectrum:

\[
E \left( \frac{d\sigma}{d^3p} \right) \sim \frac{1}{|p_\perp|^\alpha}, \quad (\alpha < 4).
\]

Alternatively (or in addition), one might find new hard effects, associated with the forces which couple technifermions to ordinary fermions to generate the
Fig. 32. The behavior predicted in a particular technicolor model (model (a) of Fig. 27) for the cross-section for production of exotic boson pairs in the vicinity of the techni-$\rho$ resonance.

The simplest model of these forces would insist that they are the result of exchanges of new vector bosons, often called ETC bosons. You should recall my comment at the end of § 4 that this model has some serious phenomenological difficulties. However, if the scheme makes any sense, it is noteworthy that the scale of the ETC boson masses is not unreasonably high. The mass of the ETC boson which couples to the top quark can be estimated from the relation:

$$\frac{m_t}{250\text{GeV}} \approx \left(\frac{m_T}{m_{\text{ETC}}}\right)^2. \quad (6.4)$$

$m_T$ is the dynamical mass of a technifermion, about 400 GeV; then this ETC boson mass should be roughly 1 TeV. ETC bosons can be produced singly in $e^+e^-$ annihilation, in association with a top quark and a technifermion, producing a final state with a $t$, and $\bar{t}$, and a multi-$W$ jet. If the ETC bosons carry both color and $(SU(4))$ technicolor, the cross-section for this process is reasonably large; this cross-section is shown in Fig. 33 for two possible values of the ETC boson mass.

Let us now turn to the effects of composite structure within the leptons. $e^+e^-$ annihilation experiments are particularly sensitive to the presence of such composite structure because the total cross-section is dominated by the relatively weak processes of 1-photon and 1-$Z^0$ exchange. Strong interactions involving the
composite structure of the electron would lead to an annihilation cross-section which we would estimate geometrically to be

\[ \sigma \sim \frac{1}{\Lambda^2}, \]

(6.5)

where, as in the discussion below Eq. (3.10), \( \Lambda \) is a mass scale whose inverse gives the size of the composite state. The cross-section can become comparable to the electromagnetic point cross-section (2.1) when \( \Lambda \) is still an order of magnitude less than \( E_{\text{cm}} \). We have noted this sensitivity already at the end of section 3 when we described current experiments which probe for the existence of a composite size \( \Lambda^{-1} \). TeV-energy \( e^+ e^- \) annihilation experiments should be similarly sensitive. It is worth noting that at energies of order 1 TeV, well above the scale of weak-interaction symmetry breaking, the left- and right-handed components of the electrons are distinct species with different \( SU(2) \times U(1) \) quantum numbers and should be expected to have different constituents. This can lead to some remarkable effects. As an example, we might consider a theory in which the left-handed electron and the left-handed muon (the components of these particles which couple to the \( W \) ) share some constituents and so are coupled by a contact interaction. The helicity-dependence of the interaction leads to structure in the forward-backward asymmetry and in the polarization asymmetry as a function of \( E_{\text{cm}} \). The first of these is displayed in Fig. 34; this figure indi-
cates that TeV-energy $e^+e^-$ colliders will be sensitive to composite structures of extremely small size. The availability of polarised electron beams would be very useful in untangling the precise space-time structure of the interaction and, from it, the quantum numbers of the underlying constituents.

![Graph showing forward-backward asymmetry and polarization asymmetry](image)

Fig. 34. Effect on the process $e^+e^- \rightarrow \mu^+\mu^-$ of a contact interaction linking left-handed electrons and left-handed muons. The graphs show the behavior of (a) the forward-backward asymmetry and (b) the polarization asymmetry for various values of the compositeness scale $\Lambda$.

On the other hand, it is possible that the composite structure of electrons and muons will actually appear at a more accessible energy. If $\Lambda = 3$ TeV, a value well above the current experimental bounds, the constituent interactions predict huge effects on the $\mu$ pair cross-section for $E_{\text{cm}}$ above 300 GeV. In the multi-TeV region, the couplings due to composite structure would dominate all other contributions to $e^+e^-$ annihilation. To display the cross-sections which would result, it is necessary to change our standard of cross-section from the $\sigma_R$ unit to some absolute level such as the nanobarn. The geometrical cross-section (6.5) associated with a $\Lambda$ value of 3 TeV is 0.1 nb, a value of the same order as the current PEP and PETRA annihilation cross-sections. The behavior of the $\mu$ pair cross-section as a function of $E_{\text{cm}}$ is sketched in Fig. 35. The cross-section should exhibit the typical strong interactions effects of resonant structure; it may even show the Regge asymptotic behavior of strong interaction processes.

We have now explored the physics possibilities of very high energy $e^+e^-$
colliders from several different points of view. We have seen, first, that $e^+e^- \rightarrow \mu^+\mu^-$ colliders are ideal devices for conducting searches for new particles, and I have emphasized that this search will play a central role in our exploration of the TeV energy region. But we have seen also that many pictures of TeV physics predict more remarkable effects, which characterise directly the underlying theory if they can be observed with sufficient clarity. Here, again, the low background levels and the control of the collision energy available in $e^+e^-$ collisions could be of great value. Can such machines be built? Perhaps, with your thought and effort. I hope some of you can find a way to construct these most elegant devices and to realize the beautiful experiments which they promise.

Fig. 35. The cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ for a model with left-handed contact interactions and $\Lambda = 3$ TeV, displayed for $E_{cm}$ of the order of $\Lambda$. 
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