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*Mixing of
Final-State Spin Components
in the High-Resolution Spectrometer*

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Wayne Cornelius

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MIXING OF FINAL STATE SPIN COMPONENTS IN THE HIGH RESOLUTION SPECTROMETER

by

Wayne Cornelius

ABSTRACT

The mixing of scattered proton spin components by precession in the High Resolution Spectrometer (HRS) dipole magnets is discussed in terms of focal plane polarimeter (FPP) experiments. Explicit expressions are derived for calculation polarization transfer parameters from focal plane polarizations. Also discussed are the effects of out-of-plane scattering at small laboratory angles on FPP measurements as well as analyzing power measurements.

I. INTRODUCTION

This report concerns the mixing of final-state spin components through spin precession in the High Resolution Spectrometer (HRS) magnets and through out-of-plane scattering (namely, nonzero azimuthal angles). To begin with we have four basic coordinate systems associated with the scattering--the coordinate system of the incident beam (x_0, y_0, z_0) ; the coordinate system defined by the spectrometer (x_1, y_1, z_1) ; and the coordinate systems of the scattering plane defined by the momentum vectors $\vec{k}_{in}$ and $\vec{k}_{out}$ -- (x, y, z) for the incident particle and (x', y', z') for the scattered particle. These reference frames are defined as follows. (x_0, y_0, z_0) are defined in the laboratory to be with $\hat{z}_0$ along $\vec{k}_{in}$, $\hat{y}_0$ is the up-down laboratory direction, and $\hat{x}_0$ is defined consistent with a right-handed coordinate system. The spectrometer reference frame is most simply described as a rotation of the beam coordinates about the $\hat{y}_0$ -axis by the scattering angle Θ . However, in keeping with the usual spectrometer convention, I will define the $\hat{y}_1$ -direction to be parallel to the

spectrometer field $\vec{B}$ and $\hat{x}_1$ perpendicular to that in a right-handed system. This is the normal spectrometer reference frame where x provides excitation energy information and y provides information about the scattering angle.

The scattering reaction is described in the plane defined by the incident momentum vector $\vec{k}_{in}$ and the scattered momentum vector $\vec{k}_{out}$. For the incident particle, $\hat{z}$ is defined to be along $\vec{k}_{in}$ with $\hat{y}$ being along $\vec{k}_{in} \times \vec{k}_{out}$ and $\hat{x}$ defined in a right-handed coordinate system. The scattered particle coordinate system has $\hat{z}'$ along $\vec{k}_{out}$ and $\hat{y}'$ along $\vec{k}_{in} \times \vec{k}_{out}$ (e.g., $\hat{y}$ and $\hat{y}'$ are parallel). $\hat{x}'$ is again defined by a right-handed coordinate system. The relationship between these four coordinate systems is shown in Fig. 1.

In what follows I will be using Θ_{scatt} as the true scattering angle, θ as the horizontal plane scattering angle, ϕ as the azimuthal scattering angle, and ζ as the angle between the horizontal plane and the scattering plane (Fig. 2). With these definitions, a scattering reaction from the proton's point of view will look like Fig. 2 (the incident beam is into the page).

The description of particle polarizations will be in the reference system $(\hat{s}, \hat{n}, \hat{\ell})$, where $\hat{\ell}$ (longitudinal) is along the particle momentum ($\vec{k}$), $\hat{n}$ (normal) is perpendicular to the scattering plane (parallel to $\hat{y}$ and $\hat{y}'$), and $\hat{s}$ (sideways) is perpendicular to these in a right-handed coordinate system (Fig. 2). This description follows the "Argonne convention" of notation. In this notation we have in general five triple-scattering observables corresponding to the five Wolfenstein parameters. These are $D_{ss'} (=R)$, $D_{\ell s'} (=A)$, $D_{nn'} (=D)$, $D_{s\ell'} (=R')$, and $D_{\ell\ell'} (=A')$. These five parameters, together with the polarization $P(\theta)$ and analyzing power $A(\theta)$, completely describe the scattered proton spin in a general parity-conserving and time-reversal-invariant form.

$$I = I_0 (1 + P_n A(\theta)) , \quad (1a)$$

$$I P_{s'} = I_0 (P_s D_{ss'} + P_{\ell} D_{\ell s'}) , \quad (1b)$$

$$I P_{n'} = I_0 (P(\theta) + P_n D_{nn'}) , \quad \text{and} \quad (1c)$$

$$I P_{\ell'} = I_0 (P_s D_{s\ell'} + P_{\ell} D_{\ell\ell'}) . \quad (1d)$$

Fig. 1. Diagram of the relationship between the various coordinate systems describing polarized proton scattering at HRS. (x_0, y_0, z_0) is the coordinate system of the incoming beam in the laboratory and (x_1, y_1, z_1) is the coordinate system of the HRS. (x, y, z) is the microscopic coordinate system of a single incident proton referenced to the microscopic scattering plane defined by $\vec{k}_{in}$ and $\vec{k}_{out}$. (x', y', z') denotes the microscopic coordinate system of the scattered proton referenced to the scattering plane. The scattering angles θ (HRS ϕ_{tgt}) and ϕ (HRS θ_{tgt}) are explicitly shown as is their relationship to the true scattering angle θ_{scatt} .

Fig. 2. View of Fig. 1 as seen from the incident beam direction. Shown is the relationship between the laboratory coordinate system (x_0, y_0, z_0) and the microscopic coordinate systems (x, y, z) and (x', y', z') . Also shown are the microscopic coordinate systems defining the three orthogonal components of proton spin in the incident proton frame (s, n, l) and in the scattered proton frame (s', n', l') . The figure shows the scattering plane tilted with respect to the laboratory frame by an angle ζ .

II. METHODOLOGY

Now I will take each of the possible incident beam polarization directions and derive a general form for the spin-transfer observables in the scattered particle coordinate system. I will then relate this reference frame to the coordinate system describing the HRS focal plane and determine what the up-down and left-right asymmetries really measure in terms of the scattered particle polarizations.

If we consider the incident beam to be polarized in the $\hat{n}$ -direction (up-down in the lab), then the incident beam polarization and incident particle polarization are related by a rotation of ζ about $\hat{z}_0$ (Fig. 3). Thus if the incident beam polarization is P_i , then $P_n = P_i \cos \zeta$ and $P_s = P_i \sin \zeta$. Using these in Eqs. (1) leads to the following.

$$I = I_0 (1 + P_i \cos \zeta A(\theta)) , \quad (2a)$$

$$I P_{s'} = I_0 (P_i \sin \zeta D_{ss'}) , \quad (2b)$$

$$I P_{n'} = I_0 (P(\theta) + P_i \cos \zeta D_{nn'}) , \text{ and } \quad (2c)$$

$$I P_{\ell'} = I_0 (P_i \sin \zeta D_{s\ell'}) . \quad (2d)$$

I will first examine the $\hat{n}'$ -polarization, namely,

$$P_{n'} = \frac{P(\theta) + P_i D_{nn'} \cos \zeta}{1 + P_i A(\theta) \cos \zeta} . \quad (3)$$

By using spin up (+) and spin down (-) incident beam, we get two expressions:

$$P_{n'}^{\pm} = \frac{P(\theta) + P_i^{\pm} D_{nn'} \cos \zeta}{1 + P_i^{\pm} A(\theta) \cos \zeta} \text{ and } \quad (4a)$$

$$P_n^- = \frac{P(\theta) + P_1^- D_{nn}' \cos \zeta}{1 + P_1^- A(\theta) \cos \zeta} \quad (4b)$$

These two expressions can be combined by eliminating $P(\theta)$ to give

$$D_{nn}' \cos \zeta = \frac{(P_n^+ - P_n^-) + A \cos \zeta (P_1^+ P_1^+ - P_1^- P_1^-)}{(P_1^+ - P_1^-)} \quad (5)$$

Fig. 3. End-on view of the microscopic polarizations in the incident proton reference frame for incident n-type beam polarization.

I note at this point that the sign of the polarization is implicitly contained in the symbol ($P^+ = +|\vec{P}|$ and $P^- = -|\vec{P}|$), thus $(P_1^+ - P_1^-)$ is equal to $2|\vec{P}|$ and not zero. I note also that + and - need not refer specifically to positive and negative polarizations but, in general, refer to two oppositely oriented spin directions (normal and reverse).

We obtain similar expressions for the $\hat{s}'$ and $\hat{\ell}'$ polarizations from the same elimination process.

$$D_{ss}' \sin \zeta = \frac{(P_s^+ - P_s^-) + A \cos \zeta (P_s^+ P_1^+ - P_s^- P_1^-)}{(P_1^+ - P_1^-)} \quad \text{and} \quad (6)$$

$$D_{s\ell'} \sin \zeta = \frac{(P_{\ell'}^+ - P_{\ell'}^-) + A \cos \zeta (P_{\ell'}^+ P_1^+ - P_{\ell'}^- P_1^-)}{(P_1^+ - P_1^-)} \quad (7)$$

Now if the incident beam is polarized in the $\hat{s}$ -direction (sideways in the lab system), we again can relate the beam and incident particle reference frames by a rotation about $\hat{z}_0$ (Fig. 4). Under this rotation $P_s = P_i \cos \zeta$ and $P_n = -P_i \sin \zeta$, leading to the expressions

$$I = I_0 (1 - P_i \sin \zeta A(\theta)) \quad , \quad (8a)$$

$$I P_{s'} = I_0 (P_i \cos \zeta D_{ss'}) \quad , \quad (8b)$$

$$I P_{n'} = I_0 (P(\theta) - P_i \sin \zeta D_{nn'}) \quad , \text{ and} \quad (8c)$$

$$I P_{\ell'} = I_0 (P_i \cos \zeta D_{s\ell'}) \quad . \quad (8d)$$

Fig. 4. End-on view of the microscopic polarizations in the incident proton reference frame for incident s-type beam polarization.

These expressions can be reduced to

$$D_{ss'} \cos \zeta = \frac{(P_{s'}^+ - P_{s'}^-) - A \sin \zeta (P_{s'}^+ P_1^+ - P_{s'}^- P_1^-)}{(P_1^+ - P_1^-)} \quad , \quad (9)$$

$$D_{nn'} \sin \zeta = \frac{-(P_n^+ - P_n^-) + A \sin \zeta (P_n^+ P_1^+ - P_n^- P_1^-)}{(P_1^+ - P_1^-)} , \text{ and} \quad (10)$$

$$D_{s\ell'} \cos \zeta = \frac{(P_{\ell'}^+ - P_{\ell'}^-) - A \sin \zeta (P_{\ell'}^+ P_1^+ - P_{\ell'}^- P_1^-)}{(P_1^+ - P_1^-)} . \quad (11)$$

For incident beam polarized in the $\hat{\ell}$ -direction (longitudinal in the lab system), no rotation of reference frames is necessary because of the rotational symmetry about the $\hat{z}$ or $\hat{z}_0$ axes. Equations (1) then become

$$I = I_0 , \quad (12a)$$

$$I P_{s'} = I_0 (P_1 D_{\ell s'}) , \quad (12b)$$

$$I P_n = I_0 (P(\Theta)) , \text{ and} \quad (12c)$$

$$I P_{\ell'} = I_0 (P_1 D_{\ell \ell'}) . \quad (12d)$$

These can be reduced immediately, but I have chosen to maintain a symmetry with the previous derivations. Thus we get similar expressions for the spin transfer quantities

$$D_{\ell s'} = \frac{(P_{s'}^+ - P_{s'}^-)}{(P_1^+ - P_1^-)} \quad \text{and} \quad (13)$$

$$D_{\ell \ell'} = \frac{(P_{\ell'}^+ - P_{\ell'}^-)}{(P_1^+ - P_1^-)} . \quad (14)$$

So there is only one general form for the determination of the transferred parameters, namely

$$D_{ab} \left(\frac{\sin \zeta}{\cos \zeta} \right) = \frac{(P_b^+ - P_b^-) + A \left(\frac{\sin \zeta}{\cos \zeta} \right) (P_b^+ P_a^+ - P_b^- P_a^-)}{(P_a^+ - P_a^-)} \quad (15)$$

This form is very useful because the false asymmetries of the polarimeter are very nearly cancelled in first order. This is easily demonstrated. If the measurement of the final state polarization has a built-in false asymmetry leading to a false polarization P_f , then this false polarization enters as an additive constant, namely,

$$D_{ab} = \frac{[(P_b^+ + P_f) - (P_b^- + P_f)] + A[(P_b^+ + P_f)P_a^+ - (P_b^- + P_f)P_a^-]}{(P_a^+ - P_a^-)} \quad (16)$$

$$= D_{ab} + AP_f \left(\frac{\sin \zeta}{\cos \zeta} \right) \quad (17)$$

The false polarization merely adds to the actual value of the measured transfer coefficient in a predictable way, hence we have a measure of the error induced by any intrinsic polarimeter geometry factor.

III. RELATIONSHIP TO FOCAL PLANE MEASUREMENTS

The spectrometer coordinate system (x_1, y_1, z_1) is related to the scattered particle system first by a rotation about the $\hat{z}'$ -axis by $-(\zeta + \pi/2)$ and then a rotation about the $\hat{y}'$ -axis by $-\phi$. The second rotation can be visualized by examining the z' -direction in the (x', y', z') -system (Fig. 1). By rotating the viewer's frame of reference through an angle $-\phi$, the z' -vector points up at an angle $+\phi$. The polarization then precesses about the $\hat{y}_1$ -axis in the field of the spectrometer by an angle $-\chi$ (Fig. 5).

χ is defined as the net rotation with respect to the momentum vector $\vec{k}$ and is given by

$$\chi = \gamma \left(\frac{g}{2} - 1 \right) \alpha \approx 269 \gamma, \quad (18)$$

where g is the proton gyromagnetic ratio, α is the HRS bend angle (150°) and γ is the usual Lorentz contraction factor. We are concerned with precession of the polarization vector about an axis perpendicular to the momentum vector. The net rotation of the bend-plane polarizations with respect to the momentum vector is given by χ and can be equivalently described as a rotation of the reference frame through an angle of $-\chi$.

Therefore the transformation of the polarizations in the scattered particle reference frame to the spectrometer focal plane is given by the matrix

$$M = \begin{pmatrix} -\sin\zeta\cos(\phi+\chi) & -\cos\zeta\cos(\phi+\chi) & -\sin(\phi+\chi) \\ +\cos\zeta & -\sin\zeta & 0 \\ -\sin\zeta\sin(\phi+\chi) & -\cos\zeta\sin(\phi+\chi) & +\cos(\phi+\chi) \end{pmatrix}. \quad (19)$$

But we really need the inverse transformation that tells us what the scattered frame polarizations are in terms of the measured quantities at the focal plane [in terms of $P_n(fp)$, $P_g(fp)$, and $P_\theta(fp)$]. This inverse transformation is given by

$$M^{-1} = \begin{pmatrix} -\sin\zeta\cos(\phi+\chi) & +\cos\zeta & -\sin\zeta\sin(\phi+\chi) \\ -\cos\zeta\cos(\phi+\chi) & -\sin\zeta & -\cos\zeta\sin(\phi+\chi) \\ -\sin(\phi+\chi) & 0 & +\cos(\phi+\chi) \end{pmatrix}. \quad (20)$$

Fig. 5. Relationship between the HRS coordinate system at the target (x_1, y_1, z_1) and the coordinate system of the focal plane (x_{fp}, y_{fp}, z_{fp}) . The components of proton spin orthogonal to the magnetic field are precessed through an angle of 269γ , where γ is the Lorentz contraction factor.

So we can derive the scattered frame - polarizations from the polarizations measured at the focal plane, hence

$$P_{s'} = -\sin\zeta\cos(\phi+\chi)P_n(fp) + \cos\zeta P_s(fp) - \sin\zeta\sin(\phi+\chi)P_\ell(fp) , \quad (21)$$

$$P_{n'} = -\cos\zeta\cos(\phi+\chi)P_n(fp) - \sin\zeta P_s(fp) - \cos\zeta\sin(\phi+\chi)P_\ell(fp) , \text{ and} \quad (22)$$

$$P_{\ell'} = -\sin(\phi+\chi)P_n(fp) + \cos(\phi+\chi)P_\ell(fp) . \quad (23)$$

From Eqs. (21)-(23) it is evident that we can get substantial mixing of final state spin-components at the HRS focal plane depending on ζ and ϕ .

Now we need to examine the reaction more closely to see what determines the allowed values of ζ and ϕ . Figure 6 shows how the four angles describing the scattering reaction are related. The two angles measured by the HRS are $\theta(\phi_{tgt})$ and $\phi(\theta_{tgt})$. The true scattering angle θ_{scatt} is then computed from these two quantities

$$\tan\theta_{scatt} = \tan\theta \sqrt{1 + \frac{\tan^2\phi}{\sin^2\theta}} . \quad (24)$$

ζ is defined as the angle between the horizontal plane (defined by the beam line and the spectrometer) and the scattering plane and is given by

$$\tan\zeta = \frac{\tan\phi}{\sin\theta} . \quad (25)$$

Fig. 6. Relationship between the four angles describing a single proton scattering event in both the microscopic and laboratory reference frames.

Because the spectrometer has a vertical acceptance of $\pm 2^\circ$ ($-2^\circ < \phi < +2^\circ$), the allowed ζ values vary from $\pm 90^\circ$, $\theta = 0^\circ$ to $\pm 2^\circ$ at $\theta = 90^\circ$; see Fig. 7. From Fig. 7 it is also evident that not only do we have a substantial mixing of final state spin-components at small angles, but it is possible to get important mixing at larger angles as well (for example at $\theta = 8^\circ$, ζ can be as large as 14° , mixing the spin-components by as much as $\sin \zeta \approx 0.24$).

This derivation is valid for spin-mixing in any single proton scattering event. In practice, however, we analyze a relatively large number of events, more or less evenly distributed in ζ . Because of this we must average over the allowed values of ζ , hence

$$\overline{\sin \zeta} = \frac{\int_{-\zeta_{\max}}^{+\zeta_{\max}} \sin \zeta \, d\zeta}{\int_{-\zeta_{\max}}^{+\zeta_{\max}} d\zeta} = 0 \quad \text{and} \quad (26)$$

$$\overline{\cos \zeta} = \frac{\int_{-\zeta_{\max}}^{+\zeta_{\max}} \cos \zeta \, d\zeta}{\int_{-\zeta_{\max}}^{+\zeta_{\max}} d\zeta} = \frac{\sin \zeta_{\max}}{\zeta_{\max}} \quad (27)$$

For the average values of $\sin \phi$ and $\cos \phi$ we get similar expressions. Thus $\overline{\sin \phi} = 0$ and $\overline{\cos \phi} = 0.9998$. And from Eqs. (26) and (27)

$$\sin(\overline{\phi} + \chi) = +\sin \chi \overline{\cos \phi} = +\sin \chi \quad , \quad (28)$$

$$\cos(\overline{\phi} + \chi) = +\cos \chi \overline{\cos \phi} = +\cos \chi \quad , \quad (29)$$

and therefore we obtain the following.

$$P_{S'} = 0 \cdot P_n(fp) - \overline{\cos \zeta} P_S(fp) + 0 \cdot P_L(fp) \quad , \quad (30)$$

$$P_{n'} = +\overline{\cos \zeta} \cos \chi P_n(fp) + 0 \cdot P_S(fp) - \overline{\cos \zeta} \sin \chi P_L(fp) \quad , \text{ and} \quad (31)$$

$$P_{L'} = \sin \chi P_n(fp) + 0 \cdot P_S(fp) + \cos \chi P_L(fp) \quad . \quad (32)$$

Figure 8 shows a plot of the "effective spin purity" ($\overline{\cos \zeta}$) as a function of scattering angle θ_{scat} . Note that, as expected, we get important mixing only at small angles.

These expressions are not entirely correct for incident s-type beam polarizations. The analyzing power $A(\theta)$ will produce an asymmetry in the up-down distribution of events. This will alter the average values of $\sin \zeta$ and $\cos \zeta$. The effect of these terms is most pronounced only at small angles where the analyzing power is small. Hence the above representations of $\overline{\sin \zeta}$ and $\overline{\cos \zeta}$ should be adequate.

Fig. 7. Maximum out-of-plane scattering angle ζ as a function of true scattering angle θ_{scatt} . The dashed line is an expanded scale of θ_{scatt} (0-90°).

Fig. 8. Proton "spin purity" as a function of true scattering angle θ_{scatt} . The effective proton polarization for s- and n-type beam polarization is the product of the measured beam polarization and the spin purity factor. The dashed line is an expanded scale of θ_{scatt} (0-90°).

IV. EFFECTS ON FOCAL PLANE MEASUREMENTS

We must now examine more closely just what we can hope to measure with the HRS. It is clear from Eqs. (30)-(32) that we get substantial mixing of final state $\hat{n}'$ - and $\hat{\ell}'$ - components through the precession in the HRS. At first the situation appears hopeless because we cannot measure $P_{\ell}(fp)$. However, if we examine the reaction observables more closely, we see that we cannot mix unknown quantities.

The measurements of D_{ss}' and $D_{\ell s}'$ are simple because no precession occurs in the HRS. This polarization is easily determined from the up-down asymmetry in the focal plane polarimeter. When measuring the other parameters we get mixing of spin components, but we never get mixing of two unknown polarizations. If we are measuring D_{nn}' , the incident polarization is transverse to the reaction plane and does indeed precess in the HRS. However, because of parity conservation, the scattering reaction cannot produce any polarization in the $\hat{\ell}'$ -direction to mix with the $\hat{n}'$ -polarization. Hence, we have only P_n' transported to the focal plane which, from Eq. (19) becomes

$$P_n(fp) = -\cos\bar{\zeta} \cos\chi P_n' \quad . \quad (33)$$

So measuring the right-left asymmetry at the focal plane is sufficient to determine P_n' and, therefore, D_{nn}' .

Similarly, when we measure $D_{s\ell}'$ or $D_{\ell\ell}'$, we must determine the outgoing $\hat{\ell}'$ -polarization of the scattered protons. Because the beam is never polarized in a direction normal to the scattering plane, we can only get polarization transferred to the $\hat{\ell}'$ -direction from the $\hat{\ell}$ -direction or from the $\hat{s}$ -direction. Parity conservation forbids production of $\hat{n}'$ -polarization by any mechanism except the usual polarization function $P(\theta)$. This quantity is well determined either by $P=A$ for elastic scattering or as a by-product of the D_{nn}' measurement. In addition, $P(\theta)$ is a simple function of the scattering angle and is not dependent on the beam polarization. Thus it will cancel out of the expressions for $D_{s\ell}'$ and $D_{\ell\ell}'$ as shown in Eq. (16). We can therefore only mix a "known" polarization into the $\hat{\ell}'$ -direction and can unambiguously determine P_{ℓ}' from measurement of the left-right asymmetry at the focal plane.

Thus, from Eq. (19)

$$P_n(fp) = +\cos\bar{\zeta} \cos\chi P(\Theta) - \sin\chi P_\ell' \quad (34)$$

V. CONCLUSION

Out-of-plane scattering ($\theta_{tgt} \neq 0$) is an important consideration in the analysis of any single proton event. The out-of-plane scattering coupled with precession in the HRS substantially mixes final-state spin components, making interpretation of focal plane polarizations difficult if not impossible. However, because we will be analyzing a large number of scattered protons more or less evenly distributed in θ_{tgt} , these effects are dependent only on the average values of $\sin\zeta$ and $\cos\zeta$; essentially eliminating mixing of components and at the same time reducing the sensitivity of the measurements by the average value of $\cos\zeta$. Hence we can interpret focal plane asymmetries in terms of scattered proton spin directions and therefore determine the polarization transfer coefficients. Another important result of this derivation is that the effective beam polarization in an analyzing power measurement is not P_1 but $P_1\cos\bar{\zeta}$. This effect is negligible at laboratory angles greater than 25° but becomes increasingly important as one goes to smaller angles (Fig. 8). It is also important to note that $\sin\zeta$ and $\cos\zeta$ in Eqs. (2)-(17) must also be replaced with their average values, which will alter the form of some of those equations.

When all of the above effects are considered, we obtain Eqs. (35)-(42), which describe the measurement of polarizations and polarization transfer using the HRS focal-plane polarimeter.

VI. DETERMINATION OF SCATTERED PARTICLE POLARIZATIONS

$$\text{outgoing } s' \quad P_s(fp) = +\cos\bar{\zeta} P_s' \quad (35)$$

$$\text{outgoing } n' \quad P_n(fp) = -\cos\bar{\zeta} \cos\chi P_n' \quad (36)$$

$$\text{outgoing } \ell' \quad P_n(fp) + \cos\bar{\zeta} \cos\chi P(\Theta) = -\sin\chi P_\ell' \quad (37)$$

VII. POLARIZATION TRANSFER OBSERVABLES

$$D_{ss'} = \frac{(P_S^+ - P_S^-)}{(P_S^+ - P_S^-)} \quad (38)$$

$$D_{sl'} = \frac{(P_l^+ - P_l^-)}{(P_S^+ - P_S^-)} \quad (39)$$

$$D_{nn'} = \frac{(P_n^+ - P_n^-) + A \cos \bar{\zeta} (P_n^+, P_n^+ - P_n^-, P_n^-)}{(P_n^+ - P_n^-)} \quad (40)$$

$$D_{ls'} = \frac{(P_S^+ - P_S^-)}{(P_S^+ - P_S^-)} \quad (41)$$

$$D_{ll'} = \frac{(P_l^+ - P_l^-)}{(P_l^+ - P_l^-)} \quad (42)$$

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