A Nearly Ideal Lens Optimization Procedure

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Abstract

The Brixner lens optimization procedure, in which a lens design moves steadily toward
diffraction-limited performance, samples lens performance with bundles of precisely traced
skew rays, analyzes performance by calculating the image-spot sizes and positions, and
optimizes performance in a least squares system that minimizes the lateral ray deviations
from their ideal image points. Minimizing the rms image-spot size minimizes the rms optical
path differences (OPDs). Minimizing the rms OPDs also optimizes the diffraction modulation
transfer function (DMTF). Minimizing the image-spot size and position errors also minimizes
and balances the Seidel and higher-order aberrations.

Introduction

The Brixner lens optimization procedure is nearly ideal because it is versatile, fast,
and precise, and the designer can be sure when the optimum minimum region has been reached.
The procedure uses the parameters specified by the designer to make the images as nearly
diffraction limited as possible, an effort that may be frustrating by simultaneously trying
to meet other difficult imaging requirements. As optimization moves lens performance into
the optimum minimum region, the main identifying characteristics develop, parameter
gradients that are all very small and of similar size. When the parameter gradients are
very small, the effect of manufacturing errors on image quality is at a minimum. As a re-
result of this image optimization, the Seidel and higher-order aberrations are minimized and
balanced for the specified image field. To obtain the results described above, the optimi-
sation procedure samples lens performance with bundles of precisely traced skew rays,
analyzes performance by calculating the sizes and positions of the image spots, and
optimizes performance by minimizing the lateral ray deviations from their ideal image
points. Minimizing the spot sizes also minimizes the optical path differences and therefore
optimizes the modulation transfer function.

This paper gives a summary of the further developments of the Los Alamos general-purpose
lens optimization program since it was introduced twenty-two years ago in the Applied Optics
issue that featured Lens Design and Computers. At that time, the Los Alamos code was the
only known user of my procedure, which optimizes lens performance by means of a statistical
analysis of many-ray image spot size and position data in a least squares system. By 1970,
T. C. Doyle had speeded calculations so much that a crude 10-surface lens design could be
optimized enough to reach the optimum-performance region in 20-60 seconds of CDC-7600
machine time. Although the programs break with tradition because they make no use of
classical aberration theory, they do not replace the use of Seidel analysis as a guide when
designing lenses. Simply stated, our programs drive the error function of a starting
prescription to the optimum-minimum region, where maximum optical correction is obtained
from each variable parameter. The Los Alamos programs optimize a lens by making
prescription changes that lead directly toward diffraction-limited performance. The optimi-
sation is achieved by minimizing the OPDs in each image-forming beam. The OPDs are
minimized by minimizing the lateral ray deviations in each image spot at representative
image positions on the focal surface of interest. Minimizing the lateral ray deviations
minimizes the OPDs because the rms image-spot size and the rms OPDs in the ray bundle are
proportional quantities. Minimizing the rms OPDs also optimizes the DMTF because both mea-
sure the approach to diffraction-limited performance. Although my experimental demonstra-
tions of these relationships have been published, analytic derivations of these relation-
ships are too difficult for us to attest at present. Minimizing the image-spot size and
position errors minimizes and balances all seven Seidel and higher-order aberrations because
all cause lateral deviations of the rays from their ideal image points. However, the Seidel
aberrations are approximations of the actual image errors and their optimized values are
usually not zero. The lens is optimized in three steps: first, bundles of precisely traced skew rays
are used to form image spots that sample the performance of the lens; second, lens performance is
analyzed statistically by calculating the errors in the sizes and positions of the image spots; and,
third, a least squares system tries to reduce all the image errors to zero by suitable changes of the lens parameters. During the past
twenty-five years this procedure has proved effective and versatile for discovering more
than 140 complex lens designs that met a variety of imaging requirements.
The development of unconventional lens optimization codes at Los Alamos was started in 1958 after I decided that minimizing the lateral ray deviations in the ray bundle should lead toward diffraction-limited performance. There was little evidence for the validity of that conclusion when I got C. A. Lehman to start making the first statistical code to my specifications, a unique problem that was soon taken over and completed by J. C. Holladay. The eight-significant-figure calculation then available was sufficient for most lateral deviation calculations, but calculation of the OPDs for statistical analysis was tedious and not precise enough to be generally useful without double precision, which was even more tedious. By the time fifteen-significant-figure calculation became routine, my intuitive conclusion had already been confirmed by the observed performance of lenses designed at Los Alamos. The next confirmation came when I compared a series of OPD and DMTL lens performance evaluations with their corresponding image-spot sizes. Our additional evaluation have confirmed the rms-OPD/rms-spot-size constant ratio at constant relative aperture for several simple and complex lenses operating over a large range of focal lengths and relative apertures. Although use of the rms image-spot-size criterion for image evaluation in lens optimization programs was not generally accepted when the Los Alamos codes were developed, it is now. Recently reviewed are methods of accurately calculating the effective size of the raytraced image spot, which has been questioned for many years.

The Los Alamos lens optimization programs

In the Los Alamos lens optimization programs, optical image errors are analyzed in much the same way that they are measured visually on the optical bench. Analysis is accomplished by providing a relatively simple and quick method of tracing many bundles of specified skew rays, of determining where the rays are going, of specifying where they should go, and, finally, of getting them to go where they should go. Lens-performance optimization is directed by the Los Alamos single-number least squares error function, which includes only those performance errors that continually improve imaging as they approach zero. Some types of errors, such as distortion in a very wide angle lens, may impair imaging if they are included in the error function. The performance errors are discovered, first, by tracing numerous bundles of skew rays to secure a comprehensive sampling of the image spots, and second, by making a statistical evaluation of the image-spot sizes and positions. Root-mean-square averaging of the many-ray data is used for the statistical evaluation of lens performance errors. The rms radius of the rays in an image spot generated by a bundle of rays is an example of a performance error. By weighting each type of image error, the designer can emphasize, balance, or neglect selected performance characteristics to suit the needs of each imaging problem. This image analysis by statistical methods is the foundation of the Los Alamos lens optimization programs.

In the program a large number of rays from several object points are traced through the optical system, and the system has a number of adjustable parameters, \(a_1, a_2, \ldots, a_M\). The parameters are varied to reduce the error function. If the rays come from a selected group of object points with heights \(H_1, \ldots, H_m\), and are traced through entrance pupil points \(P_1, \ldots, P_n\), then their intersections with the image plane give a matrix of values \(g_{ij}\) and \(h_{ij}\) for the \(x\) and \(y\) intersections that measure the departure from perfect focusing. The error function is

\[
M_j = \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \left( g_{ij}^2 + h_{ij}^2 \right) \right)^{1/2}.
\]

In the lens optimization calculation the lens parameters are changed, with the \(M_j\)'s and \(P_j\)'s held fixed, so the \(M_j\)'s are functions of the \(a_i\)'s. An iteration in the program is defined to be a change of the \(a_i\)'s that reduced \(M_j\), so at each iteration a new value of \(M_j\) is produced and these \(M_j\)'s make a monotonically decreasing sequence, \(M_1, M_2, \ldots, M_k\), where the subscripts just number the program iterations.

At each program iteration it is also possible to calculate the optical path difference, \(W\), for each ray, and to sum the squares of the differences to get another error function. There is a path difference for each ray from each object point, so they also sum as an error function

\[
W_j = \left( \sum_{i=1}^{m} \sum_{j=1}^{n} W_{ij}^2 \right)^{1/2}.
\]

It seems likely that these two error functions will both decrease as the lens improves, and one is urged to look for a relationship between them. An analytic derivation of a relationship is too difficult and an experimental study using the lens optimization computer code was well worthwhile. The main topic of this paper is that relationship.
It has been shown, experimentally, for several lenses, that there is a linear relationship between these two error functions so that

$$m_k = A u_k$$

(3)

where $A$ is a constant. Figure 1 shows how well this equation holds for a well-corrected lens.

It follows then, that either error function will serve to optimize a lens, and that optimizing using one will optimize the other. However, the image spots are easier to optimize because they are much larger than the OPDs and easier to calculate.

**Discussion**

Although the function of a lens is to produce the optical counterpart of an object, the images produced by both simple and compound lenses generally lack a fine focus throughout the image field. To obtain a fine focus, the optical paths from object point to image point should all be equal within a small fraction of a wavelength of light.\(^\text{10}\) Expressed another way, every ray of light entering the lens from an object point should go to its ideal image point. The Los Alamos programs optimize directly toward that target.\(^\text{2}\) Even though most lens designs cannot be improved enough to give diffraction-limited performance, the Los Alamos programs achieve the nearest approach to that condition and therefore yield the best lens for most purposes.

The Los Alamos program's image-quality optimization is obtained by evaluating the individual ray deviations (image errors) from their ideal image points on the focal surface and then minimizing those deviations by varying the lens parameters in a least-squares system. The most useful variable parameters are the surface curvatures, the distances between the surfaces, and the size and position of the entrance pupil. Infrequently used variable parameters are the conic eccentricity of the surface, the polynomial coefficients for an aspheric surface, the translations and tilts for an off-axis lens surface, the size and position of the exit pupil, and the refractive index. At each iteration of the least-squares system, the optical path differences of the rays in the image spots become proportionately smaller as the lateral deviations in the image spots become smaller.\(^\text{3}\)

Because a linear least squares system is used to solve the nonlinear lens optimization problem, the optimization iterations are repeated until the lens performance gains are insignificant. These are the main features of the Los Alamos lens optimization programs, which are fast, precise and versatile in their calculation capabilities. An extensive bibliography and many details of the calculation procedures are given in ref. 11.

Although the Seidel and higher-order aberration coefficients are not evaluated by the Los Alamos programs, there are two evaluations of Los Alamos-designed lenses by other programs that show the aberrations minimized and balanced in unexpected but surprisingly satisfactory ways.\(^\text{12,13}\) All seven types of aberrations cause lateral deviations of the rays from their ideal image points, hence when the ray deviations are minimized all of the aberrations are simultaneously minimized and balanced. Aberrations, such as spherical aberration, coma, astigmatism, field curvature, and longitudinal chromatic aberration, which enlarge the image spots, are minimized when the image-spot sizes are minimized. Distortion and lateral chromatic aberration, which displace the images laterally, are minimized when the lateral deviations of the image-spot positions are minimized. All of the aberrations are balanced as far as possible because those aberrations causing the largest lateral deviations indirectly become main targets for the least-squares minimizing system. The aberration balance that produces the smallest image spots has shown few near-zero aberrations in the complex lenses studied so far.

The latest and most versatile version of the Los Alamos lens optimization program is a code that uses a Monte Carlo selection of object points and rays to achieve random statistics.\(^\text{14}\) Although this code performs well, more experience will be needed before the particular advantages of random statistics can be evaluated.

Before concluding this discussion, it should be noted that the widespread belief that local minima exist in the least-squares lens-optimization error function is not confirmed by the Los Alamos optimization programs.\(^\text{15}\) The Los Alamos programs find what we believe to be the optimum-minimum region, which is characterized by small parameter gradients of similar size, small performance improvement per iteration, and many designs that give similar performance. Local minima and unique prescriptions have not been found in many-parameter problems. The reason is that the error function is not confirmed in that a small change in one parameter can be compensated by changes in the remaining parameters. Parameter compensation also accounts for the absence of local minima in the error function is minimized. However, false local minima are frequently found and five causes have been identified: loss of rays...
that are being traced to the image surface, stopping parameter movement at a boundary, encountering a local minimum on the damping-number search series, a singularity in the program's matrix calculation, and a variable parameter that does not change during optimization.\textsuperscript{15}

The conditions that cause a variable parameter to move slowly or not at all are the subject of my paper on stagnation in lens optimization.\textsuperscript{16} An important cause of stagnation, previously unrecognized, was found to be a wide range of parameter-gradient sizes during optimization by least squares. When the parameter gradients were approximately equalized, most of the stagnation was avoided. The result was much faster convergence. It was also found that the parameter gradients often can be approximately equalized by adjusting the size of the lens being optimized. A sample problem was run with three different gradient ranges to demonstrate the importance of parameter-gradient equalization and to show how the gradients control optimization progress.

**Lens-optimization example**

To illustrate the results obtained by the Los Alamos program, prescriptions obtained during the optimization of a copy lens for microfilm photography were analyzed. A new design was needed because no commercial lens could be found that met the precise imaging requirements. The most difficult specification was diffraction-limited resolution (400 lines/mm) throughout a 30-mm-diameter flat image at f/4.3 effective relative aperture. The design that was found and built satisfies the resolution and four other specifications needed for good performance in the desired application.\textsuperscript{17} The successful five-glass prescription was found by a 34-second 40-iteration run on the CDC-7600 computer. That run used eight sets of six-ray bundles, two colors from each of four object points. Images at a representative image position were evaluated to obtain the data presented here, the a-light images from the 2\textsuperscript{nd} off-axis object point. Prescriptions selected from the 40-iteration run were evaluated by another code to obtain the plot data. Details of the evaluation procedure are given in ref. 3. Reference 17 describes the search procedure that was used to find the lens and gives the performance evaluations that the Los Alamos code calculates.

Figure 1 shows a plot of the rms image-spot sizes and the corresponding rms OPDs calculated for the prescriptions selected from the 40-iteration optimization run. Note that the points move directly toward zero image error. The graph indicates that the rms spot size is about 50 times larger than the rms OPD throughout the run that optimized the lens at f/4.3 effective relative aperture. The small deviations from the linear decrease are possibly caused by variations in the competitive relation that this image has with the other seven images as the least squares reduction progresses.

Figure 2 shows the DMTFs calculated for lens prescriptions selected from the 40-iteration optimization run. Steady progress of the curves toward the diffraction-limited modulation transfer function is clearly indicated. At iteration 40 the curve is slightly below the theoretical limit, possibly because the prescription that was generated by six-ray bundles is evaluated by a many-ray analysis on the focal plane selected by the least squares system for all eight image spots. Examination of a wave aberration map shows the wavefront deviations to be well below 1/4\textsuperscript{rd} except for a narrow marginal zone occupying about 1/ of the entrance pupil area. The rms OPD is 0.034\textsuperscript{rd}, much smaller than the 0.28\textsuperscript{rd} extreme range.

Optimization with six-ray image spots has generated a lens design that gives performance close to the diffraction limit, as indicated by the DMTF. The effectiveness of optimization with six-ray bundles was discovered soon after the first code was developed, and its use in optimizing a great variety of lenses has confirmed that finding. However, to obtain a still better statistical approximation to the actual imaging problem, bundles of twenty-six rays and three colors were used to generate the refined lens prescription that was built.\textsuperscript{17} That prescription gave diffraction limited images throughout the field.

**Some unusually difficult lens-optimization examples**

The Los Alamos program has discovered successful lens designs for several unusually difficult problems, after the previous designers were unable to satisfy the requirements. The situations occurred when those designers failed to obtain satisfactory lens designs after prolonged study and many design runs with optimization programs based on classical aberration theory. The frustrated designers then appealed to users of the Los Alamos lens optimization program for solutions to their problems. Because documentation is readily available, the solutions to four of these frustrating design problems are noted below.

The first frustrating design problem was the redesign of a seven-power wide-field eyepiece design that needed two paraboloidal surfaces to achieve the specified resolution. Because the manufacturer could not mass produce the paraboloidal surfaces for a military application, we reoptimized the system to eliminate the need for aspherical surfaces. The
redesign was obtained quickly with no change in the number or arrangement of the lens elements, and with performance exceeding that of the original design. The substitution capability of the Los Alamos program was used to simulate the eye's scanning of the large eyepiece pupil.

The second frustrating design problem was a 10.5-inch-efl, f/1.8, three-mirror ultraviolet objective for use in stellar photographic photometry experiments on the U. S. National Aeronautics and Space Administration (NASA) Apollo orbiting spacecraft. An aberration coefficient approach had produced a good design for a 4° field, but comatic flare degraded the images at larger angles. Optimization with the Los Alamos program eliminated comatic flare in an 8° field and made what the previously frustrated designer called "a superb design".

The third frustrating design problem was a 10-inch-efl, f/10 collimator for use with the 600-inch-efl, f/10 U. S. Naval Observatory Telescope (at Flagstaff, Arizona) to make photographs at f/2.0 aperture with a 2-inch-efl lens. An aberration coefficient approach, with about 200 design runs made during eighteen months, produced a best design that still had a serious problem with astigmatism. The Los Alamos program corrected most of this astigmatism with about 15 optimization runs made during one month. The small image spots at the field edge of the Los Alamos design indicated that the energy concentration was at least ten times better than was obtained by the best aberration design. Grain-limited resolution in the galaxy arms was obtained when an f/2.0 camera was used with the telescope-collimator combination.

The fourth frustrating design problem was a 508-mm-efl, f/2.5, light-weight, high resolution telescope for NASA's Mariner 1969 and 1971 spacecraft that returned television pictures from Mars. To study various possible telescope designs and optimize the most promising ones, the Los Alamos program was used for all performance evaluation and optimization at the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California. The best of the many designs that were optimized was the long Equi-Radii Baker design, which met all construction and resolution requirements and made the excellent television pictures that were returned from Mars. Tests of the prototype telescope showed a minimum visual resolution of 180 lines/mm at all parts of the image field.

Conclusion

The four Los Alamos lens optimization codes optimize lens performance by a statistical analysis that minimizes the lateral ray deviations in the image spots from their ideal image points, a procedure that simultaneously minimizes and balances the diffraction errors and the Seidel and higher-order aberrations. A nearly ideal lens optimization procedure has been invented and demonstrated.

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References

22. Reference 13, p. 8, 22 and 41.

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**Figure 1.** Lens optimization progress in minimizing rms image-spot sizes and rms OPDs.

**Figure 2.** Lens optimization progress toward a diffraction-limited MTF.