DOE/ER/10397-3

DOE/ER/10397--3

DE82 006920

MASTER

THEORY OF ELECTRON-INDUCED REACTIONS

Progress Report for Period

May 1, 1981 - April 30, 1982

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December 1981

Prepared for:

The United States Department of Energy
Grant No. DE-AC02-79ER 10397

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Progress Report December 1981

In the current contract year we have concentrated on the following aspects of the general problem of electron-induced nuclear reactions;

- (a) finite-size and charge corrections to the virtual photon spectrum,
- (b) electron pair production, and (c) electroproduction of pions.

(a) Finite Size and Charge Corrections to Virtual Photon Spectrum (VPS)

Work on the corrections to the VPS is summarized here. The appendix contains the short text of three talks given at meetings of the American Physical Society by D. Onley, P. Durgapal and F. Zamani Noor: these will be referred to as refs (1), (2) and (3) below.

The correction to the VPS due to the finite extent of the nucleus, insofar as it depends on only the gross parameters of the nucleus (true at least up to 200 MeV), depends on the nuclear radius and hence on the mass number A : we refer to it as the A -correction. Its counterpart due to the distorting effect of the nuclear charge is naturally called the Z -correction. The 'uncorrected' spectrum in this context refers to the conventional VPS attributed either to Dalitz and Yennie⁽⁴⁾ or Thie, Mullins and Guth⁽⁵⁾.

The A -correction is invariably negative and the Z -correction positive. The A -correction is dominant at high energy (say ≥ 100 MeV) and the Z -correction at low energy (say ≤ 10 MeV). In between both are operative and since both modify the same low electron partial-wave contributions, they interfere strongly.⁽¹⁾

Our calculations using the distorted-wave technique start with the conventional uncorrected spectrum and calculate the correction in two steps. First the result of substituting Coulomb waves (i.e. solutions of the Dirac equation in the potential of a point charge) for plane waves for the lowest angular momenta (approx. $\kappa \leq 7$): at this point we have essentially the results of our earlier calculation⁽⁶⁾. Then the result of substituting finite-radius solutions for the point-charge solutions constitutes the A-correction. We find that integrals involving the difference between finite-radius and point-charge solutions converge more rapidly as a function of radial variable than does either integral separately.

As long as the Z-correction is linear in Z it is possible to solve the same problem using the second-order Born approximation which has many advantages. We have done this case for M1 transitions⁽²⁾ and checked that the two methods agree. The corresponding calculation for E1 is almost complete. Aside from being a useful check on the distorted wave calculations, the second-order Born calculation is more rapid and has no energy limitations (other than those inherent in the assumption of a virtual photon spectrum). Using the distorted wave method we have attempted to pin down regions where the behavior is not linear in Z. These appear at high Z and seem to be associated with singular behavior of all the cross sections in the limit $\alpha Z \rightarrow 1$. In the case of M1 transitions particularly, there is a rapid increase at the high end of the range of naturally occurring Z's (80-100) which could stand further investigation.

For certain purposes, especially where angular distributions of nuclear decay products are being measured in the final state, it is necessary to have the VPS broken down into magnetic substates for each multipole. Although the method is exactly the same as that described as the first method

above (distorted wave) the calculation is somewhat more tiresome (there are a number of interference terms which otherwise drop out). This calculation is described in ref (3). One result of the corrections is the considerable enhancement of the $m = 0$ component in both electric and magnetic dipole cases, this term distinguishes a virtual spectrum from real radiation (for which the $m = 0$ transition is forbidden, of course), and plays a particularly important role in analyses which stress the difference between the electroproduction and photoproduction cross sections.

(b) Electron Pair Production

The long and tedious computer evaluation of electron-positron pair production from a point nucleus, with exact Coulomb wavefunctions is essentially completed, and the details of the analysis and the results are being prepared for publication. The preliminary analysis of pair production produced by 20 MeV photons on Uranium indicates that the interpolation formula of Maximon and Gibb(6) is better than the interpolation formula of Øverbø(7). The computer code that produces these results will be made available to anyone interested, but a great deal of computation time is required to obtain pair production cross sections good to the needed 1% precision.

(c) Electroproduction of Pions from Nuclei

The main effort has been in theoretical analyses of pion electroproduction in anticipation of future experiments and to help interpret some current experiments--particularly those done at the Bates Linear Accelerator, where pions are produced by a combination of real and virtual photons.

The pion electroproduction operator we reported last year(8) has been applied to the deuteron assuming that the electron, pion and a nucleon are detected in coincidence. A paper(9) reporting these results has been

submitted to Nuclear Physics, and our calculations should assist in designing experiments for the new generation of LINACS with continuous beams or high duty cycle.

In the last couple of months one of us (LEW) has examined the application of the virtual photon spectrum concept to pion electroproduction in collaboration with L. Tiator at the University of Mainz. We find that the virtual photon concept works extremely well for pion production from the proton and ^3He for photon energies within 50 MeV of the endpoint, but that the conventional Dalitz-Yennie⁽⁴⁾ virtual spectrum formula is not accurate. We derived a reasonably simple and quite accurate virtual spectrum formula which takes into account the nuclear recoil: this has been submitted to Nuclear Physics⁽¹⁰⁾. This result should help sort out the discrepancy between the experimental and theoretical ratios of pion production by real and virtual photons observed at MIT⁽¹¹⁾.

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Appendix

Since only the abstracts of references (1), (2) and (3) appear in print, the texts of these talks (in note form) are appended here.

a

Abstract Submitted
for the Asilomar Meeting of the
Nuclear Physics Section of APS

Physics and Astronomy
Classification Scheme
25.20 or 25.30

October 28-30, 1981 APS

Suggested Session:
Photonuclear

Virtual Photon Spectrum Corrected for Nuclear Size and Charge, D.S. ONLEY, Ohio Univ.*--The virtual photon spectrum for an electron incident on a nucleus is subject to two corrections; distortion due to nuclear charge (Z-correction) and the effect of nuclear size (A-correction). In the plane wave (PW) Born approximation and the long-wavelength limit, the spectrum is independent of both Z and A. Hitherto the Z-correction has been obtained by substituting distorted waves (DW) for PW¹ and the A-correction by introducing a form factor in the PW calculation². This calculation uses DW taking both effects into account and since both modify low partial waves they interfere strongly. We calculate directly the differences between finite-size and point-nucleus matrix elements since the latter can be evaluated analytically, and use these to correct the calculation of ref. 1. At energies where the A-correction appeared minimal in PW (e.g. 10-50 MeV), we find interference with the Z-correction renders it important. The largest corrections are for M1 transitions where enhancement by a factor 4-5 is found for ²³⁸U; this is compared with a factor 20 for the Z-correction alone.

*Supported by the U.S. Department of Energy

¹L. Wright & C. Soto Vargas, Comp.Phys.Com.20(1980)337.

²P. Durgapal & D. S. Onley, Bull.Am.Phys.Soc.25(1980)756.

Please schedule to precede
accompanying paper by
Durgapal and Onley

Signed

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It is an old idea that the field of a passing charged particle is equivalent to a burst of radiation as registered for example by a nearby target capable of absorbing radiation.

Field of a passing charge
equivalent to a burst of
radiation on a nearby target

What I will call the conventional expressions for the spectrum of such radiation is calculated in the plane wave Born Approximation and long wavelength limit and results in simple expressions such as those shown here for E1, M1 and E2 radiation. Note they depend only on electron kinematic variables--initial and final energies E_1 E_2 and momenta p_1 p_2 , the mass m and photon energy E_γ .

$$N^{(E1)}(E_1, E_\gamma) = \frac{\alpha}{\pi} \left\{ \frac{E_1^2 + E_2^2}{p_1 p_2} \ln \frac{E_1 E_2 + p_1 p_2 - m^2}{m E_\gamma} - 2 \frac{p_1}{p_2} \right\}$$

$$N^{(E2)}(E_1, E_\gamma) = \frac{\alpha}{\pi} \left\{ \frac{8 p_2 (E_1 E_2 + m^2)}{3 p_1 E_\gamma^2} + \frac{E_1^2 + E_2^2 - 2 m^2}{p_1^2} \ln \frac{E_1 E_2 + p_1 p_2 - m^2}{m E_\gamma} \right\}$$

$$N^{(M1)}(E_1, E_\gamma) = \frac{\alpha}{\pi} \left\{ \frac{E_1^2 + E_2^2 - 2 m^2}{p_1^2} \ln \frac{E_1 E_2 + p_1 p_2 - m^2}{m E_\gamma} \right\}$$

The yield in terms of photo-cross section $\sigma_\gamma(E_\gamma)$ is then

$$\sigma(E_1) = \sum_{\lambda L} \int_0^{E_1} N^{(\lambda L)}(E_1, E_\gamma) \sigma_\gamma^{(\lambda L)}(E_\gamma) dE_\gamma/E_\gamma$$

The idea, of course, is that the electron cross section for any process is then related to the photon cross section by integrating the corresponding σ_γ over the spectrum. Unlike real plane wave radiation however, we have to identify the multipole components involved since they don't share the same spectrum. In this respect it is unlike plane wave radiation in which all multipoles have the same spectrum. Virtual radiation is generally greater for E2 and M1 multipoles than E1 and therefore used to sort out different multipole contributions in the cross section.

That the conventional spectrum does not depend on any of the parameters of the target is a result of the approximations used. In fact one should correct for the influence of the charge of the target on the electron and also for the finite size of the target which will sample the radiation over a volume which will even overlap with the electron wavefunction.

A correction for the target's charge we call the Z-correction and for the finite size--since this depends primarily on the nuclear mass number A--we call the A-correction.

The Z-correction is made by substituting Coulomb waves for plane waves in the Born approximation in which case the spectrum has to be written in terms of a sum over partial waves as shown.

Virtual Spectrum written as a sum over partial waves κ_1, κ_2 :

$$N^{(\lambda L)} = \sum_{\kappa_1 \kappa_2} N_{\kappa_1 \kappa_2}^{(\lambda L)}$$

$$N_{\kappa_1 \kappa_2}^{(\lambda L)} = \frac{\alpha}{\pi} \frac{p_2}{p_1} \frac{E_1 E_2}{2L+1} \omega^4 S(\lambda) (2j_1+1)(2j_2+1) |(j_1, -\frac{1}{2}; j_2, \frac{1}{2} | L0) \cdot R_{\kappa_1 \kappa_2}^{\lambda L}|^2.$$

Radial integrals:

$$R_{\kappa_1 \kappa_2}^{EL} = \int \left\{ h_{L-1}(\omega r) \left[\sqrt{\frac{L}{L+1}} (f_{\kappa_1} g_{\kappa_2} - g_{\kappa_1} f_{\kappa_2}) + \frac{\kappa_1 - \kappa_2}{\sqrt{L(L+1)}} (f_{\kappa_1} g_{\kappa_2} + g_{\kappa_1} f_{\kappa_2}) \right] \right. \\ \left. - \sqrt{\frac{L}{L+1}} h_L(\omega r) (f_{\kappa_1} f_{\kappa_2} + g_{\kappa_1} g_{\kappa_2}) \right\} r^2 dr$$

$$R_{\kappa_1 \kappa_2}^{ML} = \int h_L(\omega r) \frac{\kappa_1 + \kappa_2}{\sqrt{L(L+1)}} (f_{\kappa_1} g_{\kappa_2} + g_{\kappa_1} f_{\kappa_2}) r^2 dr$$

Z-correction:

$$\Delta N = \sum_{\kappa_1 \kappa_2}^{\kappa_{\max}} [N_{\kappa_1 \kappa_2}^{(\lambda L)}(Z) - N_{\kappa_1 \kappa_2}^{(\lambda L)}(0)]$$

Coulomb waves are solutions of the Dirac equation with a point-charge Coulomb potential and as such are known analytic functions. The radial integrals R obtained here are then also analytic functions (if somewhat obscure ones) and hence are precisely calculated. The differences between planewave and finite- Z results are confined to low partial waves and hence when applied as a correction to the conventional result we avoid a sum over an excessive number of partial waves.

The A-correction requires the introduction of a charge distribution and a transition-current distribution for the nucleus. The electron wave

functions are no longer known analytic functions and the radial integrals are more complicated because of interpenetration.

$$R_{\kappa_1 \kappa_2}^{(\lambda L)}(E_1, \omega) = \int \left(\vec{A}^{(\lambda L)} \cdot \vec{J}^{(\lambda L)} - \Phi^{(\lambda L)} \rho^{(\lambda L)} \right) r^2 dr$$

$$J^{(EL)} = - \frac{i}{\sqrt{L(L+1)}} \left\{ L(f_{\kappa_1} g_{\kappa_2} - g_{\kappa_1} f_{\kappa_2}) + (\kappa_1 - \kappa_2)(f_{\kappa_1} g_{\kappa_2} - g_{\kappa_1} f_{\kappa_2}) \right\}$$

$$J^{(ML)} = - \frac{i}{\sqrt{L(L+1)}} (\kappa_1 + \kappa_2)(f_{\kappa_1} g_{\kappa_2} + g_{\kappa_1} f_{\kappa_2})$$

$$\rho^{(EL)} = (f_{\kappa_1} f_{\kappa_2} + g_{\kappa_1} g_{\kappa_2}), \quad \rho^{ML} = 0$$

$$\begin{aligned} A^{(EL)} &= h_{L-1}(\omega r) \int_0^r \left[-j_{L+1}(\omega r') \sqrt{\frac{L}{L+1}} J_{L+1}(r') + j_{L-1}(\omega r') J_{L-1}(r') \right] r'^2 dr' \\ &+ j_{L-1}(\omega r) \int_r^\infty \left[-h_{L+1}(\omega r') \sqrt{\frac{L}{L+1}} J_{L+1}(r') + j_{L-1}(\omega r') J_{L-1}(r') \right] r'^2 dr' \\ &- \sqrt{\frac{L}{L+1}} \frac{2L+1}{\omega^3} r^{L-1} \int_r^\infty J_{L+1}(r') r'^{-L} dr' \end{aligned}$$

$$A^{(ML)} = h_L(\omega r) \int_0^r j_L(\omega r') J_L(r') r'^2 dr' + j_L(\omega r) \int_0^\infty h_L(\omega r') J_L(r') r'^2 dr'$$

$$\begin{aligned} \Phi^{(EL)} &= \sqrt{\frac{L}{L+1}} h_L(\omega r) \int_0^r \left[-j_{L+1}(\omega r') \sqrt{\frac{L}{L+1}} J_{L+1}(r') + j_{L+1}(\omega r') J_{L-1}(r') \right] r'^2 dr' \\ &+ j_L(\omega r) \int_r^\infty \left[h_{L+1}(\omega r') \sqrt{\frac{L}{L+1}} J_{L+1}(r') + j_{L-1}(\omega r') J_{L-1}(r') \right] r'^2 dr' \\ &- \sqrt{\frac{L+1}{2L+1}} \frac{1}{\omega^2} r^L \int_r^\infty J_{L+1}(r') r'^{-L} dr' \end{aligned}$$

$$\vec{J}_{\text{NUCLEAR}}(r) = \sum_{ML} \sum_{L'=L, L+1} J_{L'}(r) \vec{Y}_{LL'}^M(\hat{r})$$

We therefore have to do the integrals numerically--a thorny enterprise because they converge very poorly as a function of the radial variable. (The result of multiplying three wavelike functions together and integrating--there are four beat frequencies one of which is very small indeed).

Convergence problem with radiative radial
integrals.

We get around this by subtracting the corresponding integrand with point-charge wavefunctions which gives us reasonable convergence in the mean.

Thus we apply the A-correction to the Z-correction, the result being a rather clumsy but reliable way of calculating the virtual photon spectrum--reliable as long as everything converges.

We can actually introduce any shapes for the nuclear charge and current distributions but find--as is indeed essential--that details are unimportant, only the radii (RMS radius and transition radius) are important and these we relate to the mass number A .

The Z-correction is dominant for low energies and is an enhancement and the A-correction for high energies and depresses the spectrum.

Since both affect the low partial waves they interfere strongly between 10 and 100 MeV (roughly).

General Behavior

Detailed shapes of charge and transition current distributions unimportant, only the radii

$R_{\text{rms}}, R_{\text{transition}}$

need be specified - both are related to A .

Z-correction is dominant for low energies and is an enhancement of the spectrum.

A-correction is dominant for high energies and is a depression of the spectrum.

The two interfere strongly between electron energies in the range 10 MeV to 100 MeV.

The net correction is roughly linear for low Z but rises sharply for $Z \geq 80$.

Abstract Submitted
for the 1981 Fall Meeting of the
Division of Nuclear Physics

Session
Electron
Scattering

Classification
25.20 or 25.30

Virtual Photon Spectrum in Second Order Born Approximation, P. DURGAPAL & D. ONLEY, Ohio Univ.*--An expression for the virtual photon spectrum (VPS) is evaluated in second order Born Approximation (SOBA). The method adopted is similar to that of Cutler¹ and Bergstrom² but includes finite mass and energy loss for the electron which is vital for this calculation. We expand the cross section in integrals over the nuclear inelastic form factor with a kernel which is evaluated in SOBA and derived from the elastic form factor. The kernel can be evaluated provided the elastic form factor contains only poles (we have used a Yukawa form factor). We have found³ that the VPS is insensitive to the details of the inelastic form factor over a range $E_e \leq 300$ MeV and depends only on the transition radius. Therefore we can choose an inelastic form factor which also has only poles, which enables us to evaluate the matrix element analytically. The remaining integral in the VPS (over momentum transfer) is performed numerically. We have evaluated the VPS for various transitions and compared our results with DWBA.

*Supported by the U.S. Department of Energy

¹L. S. Cutler, Phys.Rev.157,885(1967).

²J. C. Bergstrom, Phys.Rev.C,11,1514(1975).

³P. Durgapal & D. Onley, Bull.Am.Phys.Soc.25,746(1980).

Standard session

Please schedule to follow
accompanying paper by
Onley

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In this paper we present the virtual photon spectrum (VPS) obtained after making charge and finite size corrections using second order Born approximation (SOBA). The method adopted is similar to the one used by Cutler⁽¹⁾ and Bergstrom⁽²⁾. Both these authors have neglected the mass and energy loss of the electron in their calculations which would be out of the question for this calculation. The method consists in expanding the scattering cross section in terms of integrals over the nuclear inelastic form factor with a kernel which is evaluated in SOBA and is derived from the elastic-electron scattering form factor $F(q)$. The equivalent matrix element can then be written as an integral over intermediate momentum transfers (q, q') as

$$|H_{fi}|^2 = \frac{4\alpha}{\pi^2(2\ell+1)} \int dq dq' \frac{q^2 q'^2 F_{\sigma\ell}^{if}(q) F_{\sigma\ell}^{if}(q')}{(q^2 - \omega^2)(q'^2 - \omega^2)} G_{\ell}(q, q', \Delta, \omega)$$

where $F_{\sigma\ell}^{if}(q)$ is the nuclear form factor of multipolarity $\sigma\ell$, ω is the transition energy and G_{ℓ} is the kernel.

In deriving this expression, we use an electron wave function which is an improvement over the plane wave solutions.

$$\Phi_{p_i}^{\pm}(\vec{r}) = e^{i\vec{p}_i \cdot \vec{r}} \left[1 + \frac{\alpha Z}{2\pi^2} \int \frac{d^3k (\vec{\alpha} \cdot \vec{k} + 2E_i) e^{i\vec{k} \cdot \vec{r}} F(k^2)}{(k^2 + \lambda^2)(k^2 + 2\vec{k} \cdot \vec{p}_i + i\epsilon)} \right] u(\vec{p}_i)$$

$\lim_{\substack{\lambda \rightarrow 0 \\ \epsilon \rightarrow 0}}$

$\vec{p}_i$ - Momentum of particle i

E_i - energy of particle i

$\vec{\alpha}$ - Dirac spin operator

$F(k^2)$ - elastic electron scattering form factor

$u(\vec{p}_i)$ - spin function

The kernel G_ℓ can be written as a sum of two terms, the first one of which leads to the usual PWBA term. The second term gives the correction and can be expressed as an integral over the solid angle $d\Omega_q$

$$G_\ell(q, q', \Delta, \omega) = G_\ell^{\text{BA}} + G_\ell^{\text{corr}}$$

$$G_\ell^{\text{corr}}(q, q', \Delta, \omega) = \frac{\alpha Z}{2\pi^2} \int \lim_{\lambda \rightarrow 0} d\Omega_q \frac{F[(\vec{\Delta} - \vec{q})^2]}{(\vec{\Delta} - \vec{q})^2 + \lambda^2} S(\vec{q}, q', \vec{\Delta}, \omega)$$

The expression for S is obtained after carrying out summation over spin and other indices. The integration over solid angle can be performed provided the elastic form factor contains only simple poles. We have used a Yukawa form factor

$$F(q^2) = \frac{\gamma^2}{q^2 + \gamma^2}$$

where

γ - Yukawa parameter

Although this may not seem a good form factor, the shape of the charge distribution turns out to be largely irrelevant, and we can also use a sum of Yukawa-like form factors if necessary.

The solid angle integral is then performed using symmetry arguments and a technique due to Feynman*. The integration over q' is simple since it involves a delta function. We can then write the expression for the correction part of the kernel showing its q -dependence explicitly.

*R. P. Feynman, Phys. Rev. 76, 769 (1949)

$$\begin{aligned}
G_{\ell}^{\text{corr}}(q, \Delta, \omega) = & \frac{4}{\Delta^2} \left[\omega(p_1^2 - p_2^2) - (E_1 + E_2) \Delta^2 \right] \left[2 \ln \left| \frac{q - \Delta}{q + \Delta} \right| - \ln \left(\frac{(q - \Delta)^2 + \gamma^2}{(q + \Delta)^2 + \gamma^2} \right) \right] \\
& + \left[\frac{\gamma^2}{\Delta} + \frac{(q^4 + C_1 q^2 + C_2)}{\Delta(q^2 - \Delta^2)} \right] \left[\left(\frac{E_2}{p_2} - \frac{E_1}{p_1} \right) \ln \left| \frac{q - \Delta_{mn}}{q + \Delta_{mn}} \right| - \left(\frac{E_2}{p_2} + \frac{E_1}{p_1} \right) \ln \left| \frac{q - \Delta_{mx}}{q + \Delta_{mx}} \right| \right] \\
& - \frac{(E_1 q^4 + C_3 q^2 + C_4)}{\Delta \sqrt{p_1^2 q^4 + C_5 q^2 + C_6}} \ln \left| \frac{(q^2 - \Delta^2) V_2 + \gamma^3 V + 2q \sqrt{p_1^2 q^4 + C_5 q^2 + C_6}}{(q^2 - \Delta^2) V_2 + \gamma^2 V_1 - 2q \sqrt{p_1^2 q^4 + C_5 q^2 + C_6}} \right| \\
& - \frac{(E_2 q + C_{3x} q^2 + C_{4x})}{\Delta \sqrt{p_2^2 q^4 + C_{5x} q^2 + C_{6x}}} \ln \left| \frac{(q^2 - \Delta^2) V_1 + \gamma^2 V_2 + 2q \sqrt{p_2^2 q^4 + C_{5x} q^2 + C_{6x}}}{(q^2 - \Delta^2) V_1 + \gamma^2 V_2 - 2q \sqrt{p_2^2 q^4 + C_{5x} q^2 + C_{6x}}} \right|
\end{aligned}$$

$$V_1 = q^2 - (p_1^2 - p_2^2), \quad V_2 = q^2 + p_1^2 - p_2^2$$

C_i are functions of $p_1, p_2, E_1, E_2, \Delta, \gamma$

$$\Delta_{mn} = p_1 - p_2 \quad \text{min. value of momentum transfer}$$

$$\Delta_{mx} = p_1 + p_2 \quad \text{max. value of momentum transfer}$$

Our final expression for the corrected VPS can then be written as the sum of the first order and the correction term which involves two

integrations--one over the physical momentum transfer between kinematic limits and the other over intermediate momentum transfer q .

$$N^{M1}(E_i, \omega) = N_{BA}^{M1}(E_i, \omega) + N_{corr}^{M1}(E_i, \omega)$$

where

$$N_{corr}^{M1}(E_i, \omega) = \frac{\alpha^2 Z}{2\pi^2} \frac{\omega^2}{p_i^2} \int_{\Delta_{mn}}^{\Delta_{mx}} d\Delta \frac{\Delta F_{M1}^{if}(\Delta)}{(\Delta^2 - \omega^2)} \frac{I_q}{|F_{M1}^{if}(\omega)|^2}$$

$$I_q = \int_0^\infty dq \frac{F_{M1}^{if}(q)}{(q^2 - \omega^2)} G_\ell^{corr}(q, \Delta, \omega)$$

All q -integrals are to be evaluated in the complex plane shown. There are simple poles at $q = \pm\Delta$ and there are four branch cuts in the positions shown.

Although in principle it is possible to do the two integrations numerically, things get somewhat clumsy. Therefore we decided to do the

integration over q analytically. This can be achieved only by making an appropriate choice for the inelastic form factor.

Before doing the second order calculations, we had studied the first order term including a form factor to take the finite size effect into account. We had observed that the VPS is insensitive to the inelastic distribution and depends only on the transition radius for a large range of energies⁽³⁾. This observation gives us the freedom of choosing an inelastic distribution for which the form factor has only poles and we have chosen a modified form of the exponential distribution

$$F_{M1}^{if}(q) = \frac{Nq}{(a^2 + q^2)^4} \left[1 + (16\epsilon - 1) \frac{q^2}{a^2} \right]$$

This form factor together with the kernel then has a suitable analytic structure for the q -integrals to be evaluated by contour integration

$$I_q = \int_0^\infty dq \frac{F_{M1}^{if}(q)}{(q^2 - \omega^2)} G_\ell^{corr}(q, \Delta, \omega)$$

We have evaluated the q -integrals both analytically and numerically-- the two modes provide a vital check on each other. Finally the integration over the physical momentum transfer is performed to give the VPS.

Results

Fig. 1 shows the square of the form factor for the plane wave solution and the equivalent quantities for the second-order are distorted-wave calculations with electron energy 100 MeV on ${}^{40}_{20}\text{Ca}$. The SO and DW compare very well. Note that $|F|^2$ is not normalised.

Fig. 2 shows the effective VPS for a wide range of energies for $^{40}_{20}\text{Ca}$. At very low energies around 10 MeV the finite size does not contribute and the correction is mainly due to the charge. Therefore the corrected VPS is above that of the long wavelength limit. Around 40 MeV the charge and the finite size corrections neutralize each other's effect and almost coincide with the VPS evaluated in the long wavelength limit. Around this energy a cross over occurs and at 100 MeV we see that the finite size correction takes over. In all the three cases the agreement between SO and DW calculations is pretty good. However, this agreement is not quite as good at higher Z and low energies as can be seen from Fig. 3, the graph for $Z=48$ ($^{114}_{48}\text{Cd}$). At 10 MeV the DW contribution is much larger than the SO contribution. But for higher energies the two cases compare quite well. Around 100 MeV we get good agreement even for $Z=56$.

From the graphs shown so far we see that for higher energies the Z correction goes to zero and we are left with an increasingly important finite size correction. At these energies if we compare the corrected VPS with the DW term, we find that they coincide (Fig. 4) and neither SOBA nor DW produce any effective corrections.

Fig. 5 compares the VPS obtained using SOBA, where a Yukawa form factor is used for elastic and a modified exponential distribution for inelastic form factor, and the VPS obtained from DW calculation using a Fermi charge distribution and Fermi-like current distribution. The rms and transition radii are the same in the two cases and we get identical results. In all the graphs, the graph for DW starts at higher values of ω , and the reason for this is that at small ω the DW program starts having convergence problems.

Finally we make the following conclusive remarks.

1. Except for high Z and low energies the second order results compare very well with the distorted wave results.
2. The second order expression can evaluate VPS even for very small values of ω , where the distorted wave program runs into convergence problems.
3. In comparison to DW, very little computer time is consumed in the SO calculations.

Fig 1.

Fig 2.

Fig 3.

Fig 4.

Fig 5

Abstract Submitted

for the Ohio Section Meeting of the

American Physical Society

October 23-24, 1981

DATE

Physical Review
Analytic Subject Index
Number 25.20 or 25.30

Bulletin Subject Heading in
which Paper should be placed.
Nuclear

Virtual Photon Method Applied to Angular Distribution of Fission Fragments Following Electroexcitation.

F. ZAMINI-NOOR and D.S. ONLEY, Ohio University*--We want to investigate angular distributions following fission induced by electron scattering. For this we need to formulate the spectrum of virtual photons emitted by the scattering electron and examine its dependence on quantum numbers of angular momentum L , M , transferred between electron and nucleus. For electric and magnetic dipole transitions we have derived virtual photon spectrum for different M -values using Plane Wave Born Approximation and long-wavelength limit. The PWBA results have then been corrected in low partial waves for the finite charge and size of the nucleus, using distorted wave methods.

* Supported by the U.S. Department of Energy.

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We suppose the division follows excitation of symmetric-top-like states typically $D_{MK}^J(\theta, \phi, \psi)$. The purpose is to find relative strength of M-substates in the final state and hence the angular distribution of fission fragments.

For electrofission angular distribution we expect, for a given electron energy (E) and nuclear excitation energy (ω),

$$w_{\kappa}^{\lambda L}(\theta, E, \omega) = \sum_M w_{\kappa M}^L(\theta) N^{\lambda LM}(E, \omega) / N^{\lambda L}(E, \omega),$$

where

$$w_{\kappa M}^L(\theta) = \left(\frac{2L+1}{2} \right) |d_{\kappa M}^L(\theta)|^2$$

is the square of the wave function for a symmetric top.

$N^{\lambda LM}(E, \omega)$ is the virtual photon spectrum broken down into the contributions of different M-substates, and

$$N^{\lambda L}(E, \omega) = \sum_M N^{\lambda LM}(E, \omega)$$

λ is type of transition (electric or magnetic).

In the traditional Virtual Photon Approximation we assume, for the electron, Plane waves + Long Wavelength Limit. Electro-excitation depends on same matrix elements as photo-excitation, but has $M=0$ component in addition. We have broken down E1 and M1 transition into M-substates, for the plane-wave case we get following closed forms:

$$N^{E10} = \frac{\alpha}{2\pi k_1^4} \left\{ k_1 k_2 (E_1 E_2 + 3m_e^2) + m_e^2 E (3E_1 + E_2) \ln \xi \right\}$$

$$N^{E11} = \frac{\alpha}{4\pi k_1^4} \left\{ k_1 k_2 (-4E_1^2 - E_1 E_2 + m_e^2) + [2E^2 (k_1^2 + k_2^2) - m_e^2 E^2] \ln \xi \right\}$$

$$N^{M10} = \frac{\alpha}{2\pi k_1^4} \left\{ k_1 k_2 (E_1 E_2 - m_e^2) - m_e^2 E^2 \ln \xi \right\}$$

$$N^{M11} = \frac{\alpha}{4\pi k_1^4} \left\{ k_1 k_2 (m_e^2 - E_1 E_2) + [2E_1^2 (E_1^2 + E_2^2) - m_e^2 (E_1 + E_2)^2 - 4m_e^2 k_1^2] \ln \xi \right\}$$

$$\xi = \frac{E_1 E_2 + k_1 k_2 - m_e^2}{m_e E}$$

The next step is to correct plane-wave approximation by using distorted wavefunctions and correct the long wavelength limit by considering finite size effects. These have been calculated in partial wave form and correction applied to lower partial waves. We still need plane-wave results to complete the sum with high angular momentum contributions.

Now we compare plane wave long wavelength limit with corrected E1, M1 transitions, and compare (e,f) and (γ ,f) angular distributions for E1 transitions.