Technical Information Memorandum

TASC
THE ANALYTICAL SYSTEMS CORPORATION

To: Dr. A.M. Kaufman Lawrence Livermore Laboratory Post Office Box 808 Livermore, California 94550	TIM No. 1797-5
	Date: 1 August 1979
	Contract No. 4250209
	From: S.G. Oston
Program: Nuclear Waste Systems Studies	Approval: T.J. Kabele
Subject: A PRELIMINARY ANALYTICAL REPOSITORY FLOW MODEL	

ABSTRACT

MASTER

This memo describes a preliminary model that predicts the flow of dissolved nuclear waste in an underground repository. The model is based on the flow of water that enters regions of the repository in which the waste is actually contained; it is expected to be an improvement over the current approach that assumes the waste flow is proportional to total flow of water along various pathways. The repository flow model is intended for implementation in the TASC WASTE computer code.

The model treats the repository as a homogeneous two-dimensional slab contained in a geologic formation and penetrated by a shaft leading to an aquifer. Flow of water both perpendicular and along the central corridor is determined in order to compute the heads of the repository as a function of position. The heads are then used to estimate the fraction of waste that leaks to the formation and the fraction that travels up the shaft. The waste-splitting fractions are presented as closed form analytical expressions to facilitate rapid calculations. Numerical results for a typical geometry are also given.

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, advice, product, or process disclosed, or represents its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

1.

SUMMARY

TASC has modeled a repository as a continuous, conducting, two-dimensional medium containing a central corridor which connects to a shaft. Flow perpendicular to the central corridor along branch corridors and flow along the central corridor are accounted for. Leakage to the formation occurs at all points of the repository. The hydrology calculation determines the heads in the repository as a function of position. The predicted heads are used in conjunction with the Golder/NMA storage room flow model to determine flows into and out of backfill regions. The important final result is an estimate of the fractions of waste leaving the repository through the shaft and by leakage to the formation. Calculation of the waste splitting fractions is readily accomplished since the model supplies fairly simple, closed-form analytical expressions. The waste-splitting fraction can be used in conjunction with a single node repository model in a flow network. The model requires further study and refinement, but we feel it has great promise for repository design and waste migration calculations.

2.

INTRODUCTION

Early versions of the WASTE computer code (Ref. 1) consider a repository as a single node providing a source of nuclear waste to a network of conducting pipes leading to aquifers. The pipes represent shafts and various types of pathways through the surrounding geologic formation. This approach permits rapid calculation of waste transport. However, it over-simplifies the description of the repository which, in reality, is a complex of tunnels and storage areas covering a sizeable area.

During the past year the WASTE code has been improved (Ref. 2) by the addition of the capability to include a multi-node repository and also to consider water flow paths in and around storage rooms. These improvements were suggested originally by Golder Associates and were substantially taken from their REPFLO computer program (Ref. 3). In essence, this approach determines the flow rates of ground water that either by-pass storage rooms, enter the rooms and continue down tunnels, or enter the rooms and return to the formation. These branching ratios are determined by the various storage room heads, which in turn are computed by means of numerical solution of a repository flow resistance network.

The REPFLO method has been criticized, in particular by New Millenium Associates (Ref. 4), on the following grounds:

- Storage room flow paths must be determined by an external means, e.g., a finite-element code

- Leakage of waste to the formation is not properly accounted for by use of the receiving unit concept, i.e., lumping together ensembles of storage rooms and tunnels into receiving units and perfectly conducting pipes.

Furthermore, the use of the multi-node repository in the WASTE code is not efficient for rapidly performing large numbers of sensitivity studies or Monte Carlo runs.

The new repository flow model discussed here attempts to remove the potential weaknesses in the single node and multi-node methods. It computes in terms of closed-form analytical expressions both the leakage flow of waste to the formation and the flow of waste through the repository to a shaft. The repository itself is modeled in the two horizontal dimensions, with tunnel flow and leakage flow calculated continuously as a function of position. For a given set of formation, shaft, and repository flow resistances, the model determines the waste-splitting fraction, i.e., the fraction of waste leaving the repository by way of the shaft (as opposed to leakage to the formation). The model is, therefore, able to consider some of the detail of the repository/formation geometry to arrive at the waste-splitting fraction, a number that can readily be associated with a single node repository. Both rapidity of calculation plus a realistic treatment of leakage flow are thereby achieved.

This memo first describes the geometry assumed for the repository flow model. Next, the hydrology or flow of water in the repository is examined, principally to characterize the repository heads. Then, the flows of water in the vicinity of storage areas are computed; these flows determine the movement of dissolved waste material and enable the calculation of the desired waste-splitting fraction. Finally, some

THE ANALYTIC SCIENCES CORPORATION

numerical results for a typical unflawed salt repository configuration are presented.

3.

MODEL GEOMETRY

In Fig. 3-1 the repository model is formulated as a two-dimensional slab with a main corridor running parallel to the x-axis. The repository slab has a length L , a half-width ℓ , and a height h . Lower and upper aquifers at fixed heads H_0 and 0 , respectively, each lie a distance z from the slab. The formation is characterized by a permeability k_F . The central corridor, whose cross-sectional area is A_c , is connected to the upper aquifer by a shaft with flow resistance R_s . Flow in the repository is also determined by the backfill flow permeability k_B and extraction ratio e .

Figure 3-1 Repository Geometry

4.

REPOSITORY HYDROLOGY CALCULATION

The hydrology calculation starts by considering flow of water in the storage rooms and branch corridors whose flow properties are homogenized to determined flow in the y-direction. In the volume element shown in Fig. 4-1 we allow leakage from the lower aquifer proportional to $H_0 - H$ and leakage to the upper aquifer proportional to H . Flow in the y-direction is governed by Darcy's law through an effective fraction of the material equal to e , the extraction ratio. The resulting differential equation, based on a steady-state mass balance, is:

$$ek_B h \frac{d^2 H}{dy^2} = \frac{k_F}{z} [2H - H_0] \quad (4-1)$$

We invoke the boundary condition

Figure 4-1 Volume Element for Branch Corridor Flow

$$\frac{dH}{dy} = 0 \text{ at } y = 0 \quad (4-2)$$

which imposes zero flow at the repository edge.* At the junction with the main corridor ($y=l$) the head, H , and the Darcy velocity, q , are related by

$$\begin{aligned} q(l) &= -k_B \frac{dH}{dy}(l) \\ &= -k_B \mu \left[H(l) - \frac{H_0}{2} \right] \tanh \mu l \end{aligned} \quad (4-3)$$

where

$$\mu = \sqrt{\frac{2k_F}{ek_B h^2}} \quad (4-4)$$

Now flow along the main corridor can be examined. In the volume element shown in Fig. 4-2 we include a contribution from the branch corridor flow given by Eq. 4-3 in addition to the leakage term, giving

R-46986

Figure 4-2 Volume Element for Central Corridor Flow

*This assumption remains to be verified.

$$k_B A_c \frac{d^2 H}{dx^2} + 2heq(\ell) = \frac{k_F A_c}{zh} [2H - H_o] \quad (4-5)$$

or

$$\frac{d^2 H}{dx^2} = \lambda^2 \left[H - \frac{H_o}{2} \right] \quad (4-6)$$

with

$$\lambda^2 = 2 \left[\frac{k_F}{zhk_B} + \frac{he\mu}{A_c} \tanh \mu \ell \right] \quad (4-7)$$

Equation 4-6 was solved subject to the two boundary conditions:

$$\frac{dH}{dx} = 0 \text{ at } x = L \quad (4-8)$$

(again, no flow at the edge of the repository) and

$$R_s k_B A_c \frac{dH}{dx} + H = 0 \text{ at } x = 0 \quad (4-9)$$

which relates flow and head across the shaft resistance.

Combining the solutions to Eqs. 4-1 and 4-6 yields

$$H(x,y) = \frac{H_o}{2} \left[1 - \frac{\cosh \mu y \cosh \lambda(L-x)}{\cosh \mu \ell \left(\cosh \lambda L + \frac{\lambda L R_s}{R_c} \sinh \lambda L \right)} \right] \quad (4-10)$$

R_c is the flow resistance of the main corridor:

$$R_c = \frac{L}{k_B A_c} \quad (4-11)$$

Total leakage flows of water from the lower aquifer and to the upper aquifer can be obtained by integrating $H_0 - H$ and H , respectively, over the surface areas of the repository and the main corridor. The difference represents the flow up the shaft.

5.

WASTE FLOW CALCULATION

The water flow calculations given in Section 3 describe the repository hydrology and are needed to evaluate the spatial variation of the heads. Of greater interest, however, is the flow of waste through backfilled regions, such as storage rooms, and its leakage to the formation. An assumption made at this point is critical: the flow of waste is proportional to the flow of water that contacts the waste.

Golder (Ref. 3) defines Q_B as the flow of water from the lower aquifer that passes through a storage area and defines $-Q_A$ as the flow of water that exits a storage area and flows to the upper aquifer. Thus, the quantity $Q_A + Q_B$ is the flow of water that moves into the storage area and enters the system of corridors. The various water flow paths near a storage room are depicted in Fig. 5-1. Although the detailed structure of the repository does not appear explicitly in our model, we can infer that the flow of waste branches at any point in the repository according to Q_A and $Q_A + Q_B$.

In Ref. 5 Golder has published polynomial fits for Q_A and Q_B in terms of drawdown, based on finite-element code results. Here, drawdown is defined as the difference between the actual head of a storage area and the background head. The Golder fitted expressions have been analytically verified by NMA in Ref. 4. In terms of our nomenclature the Q_A and Q_B fits can approximately be expressed as:

Figure 5-1 Water Flow Paths Near a Storage Room

$$-Q_A = \frac{k_F H_O}{z} \left(\frac{H}{H_O} \right)^2 \quad (5-1)$$

and

$$Q_B = \frac{k_F H_O}{z} \left(1 - \frac{H}{H_O} \right)^2 \quad (5-2)$$

Define F_{waste} as the fraction of waste in the repository that exists via the shaft. We can obtain this fraction by integrating the leakage currents over the entire surface area of the repository, i.e.,

$$F_{\text{waste}} = \frac{\iint dx dx (Q_B + Q_A)}{\iint dx dy Q_B} \quad (5-3)$$

The denominator of Eq. 5-3 contains a contribution from water inflow to the main corridor, but in the present model that flow has been found to be very small compared to the overall inflow.

To compute F_{waste} substitute Eq. 4-10 for $H(x,y)$ into Eqs. 5-1 and 5-2 and perform the integrals indicated in Eq. 5-3. The result is:

$$F_{\text{waste}} = \frac{4\phi \frac{\tanh \mu \ell}{\mu \ell}}{1 + 2 \frac{\tanh \mu \ell}{\mu \ell} \phi + \left[\frac{1 + \frac{\sinh \mu x \cosh \mu \ell}{\mu \ell}}{\cosh^2 \mu \ell} \right] \left[\frac{1 + \frac{\sinh \lambda L \cosh L}{\lambda L}}{\sinh^2 \lambda L} \right] \frac{(\lambda L)^2 \phi^2}{4}} \quad (5-4)$$

where

$$\phi = \frac{\sinh \lambda L}{\lambda L \left[\cosh \lambda L + \lambda L \frac{R_s}{P_c} \sinh \lambda L \right]} \quad (5-5)$$

A similar expression for the fraction of total water flowing to the shaft, based on the hydrology calculation, is:

$$F_{\text{water}} = \frac{2\phi \frac{\tanh \mu \ell}{\mu \ell}}{1 + \phi \frac{\tanh \mu \ell}{\mu \ell}} \quad (5-6)$$

6.

NUMERICAL RESULTS

Waste and water splitting fractions were computed for four ratios of k_F/k_B and are presented in Table 6-1. Column one is a key parameter, the ratio of the formation to backfill permeability. The second column is the waste splitting fraction given by Eq. 5-4. In column three is the water splitting fraction determined from Eq. 5-6. A comparable water splitting fraction taken from REPFLO calculations (Ref. 6) is given in the fourth column.

TABLE 6-1
WASTE AND WATER SPLITTING FRACTIONS

k_F/k_B	F_{waste} (TASC)	F_{water} (TASC)	F_{water} (REPFLO)
10^{-3}	0.016	0.008	0.045
10^{-4}	0.13	0.066	0.15
10^{-5}	0.59	0.36	0.34
10^{-6}	0.97	0.84	0.89

For largest k_F/k_B ratios, the repository becomes "leaky." The TASC hydrology model predicts greater leakage than does the REPFLO approach and therefore gives a lower shaft flow in this range. For all permeability ratios the fraction of waste to the shaft is greater than the fraction of total water. This is attributable to the significant amount of water that by-passes the backfill entirely (Q_p in Fig. 5-1).

Estimates were also made of the flows Q_A , Q_B , and Q_A+Q_B through various branch corridors as a function of corridor position along the x-axis. The corridor at $x=0$ is nearest the shaft; the corridor at $x=L$ is farthest from the shaft. Results are shown in Table 6-2 for the case $k_F/k_B = 10^{-5}$.

TABLE 6-2
STORAGE ROOM FLOW RATES

x	$-Q_A$	Q_B	Q_A+Q_B	Units of $\frac{k_F H_o \ell}{2z}$
0	.23	2.36	2.13	
L/4	.49	1.69	1.20	
L/2	.67	1.40	.73	
3L/4	.77	1.26	.49	
L	.81	1.21	.40	

Note the leakage to the upper aquifer $-Q_A$ increases rapidly as we move away from the shaft (increasing x). Flow in the backfill toward the main corridor, Q_A+Q_B , decreases rapidly away from the shaft, as does the leakage Q_B from the lower aquifer. These results are expected from the increase in head H as x increases.

REFERENCES

1. Berman, L.E., Ensminger, D.A., Guiffre, M.A., and Koplik, C.M., Oston, S.G., Pollak, G.D., and Ross, B.I., "Analytic of Nuclear Waste Management Options," The Analytic Sciences Corporation, Report No. TR-1103-1-1, U.S.D.O.E. Report No. UCRL-13917, October 1978.
2. Guiffre, M.S., "Improvements in the WASTE Nuclide Transport Model," The Analytic Sciences Corporation, Report No. TIM-1797-1, 6 April 1979.
3. Golder Associates, Inc., "REPFLO Computer Program," Memorandum No. E/78/464, 12 April 1978.
4. Holland, D.H., and Wilson, R.N., "Analysis of the REPFLO Hydraulic Model," New Millenium Associates, Report No. NMA L12, November 1978.
5. Miller, I., "Receiving Unit Flow," Golder Associates, Inc., Memorandum No. E/78/546.
6. "Third Report - Development of Site Suitability Criteria for the High Level Waste Repository for Lawrence Livermore Laboratory," Golder Associates, Inc., U.S.D.O.E. Report No. UCRL-13755-3, February 1977.