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MASTER

A Simple Solution to a Problem Arising from the Processing of Finite Accuracy Digital Data Using Integer Arithmetic

School ~~Department~~ of Physics
University of Minnesota
Minneapolis, MN 55455

Abstract

The reconstruction of physical events by digital electronic processing of multiparameter analog data introduces artifacts caused by the digitization process. Several methods of minimizing these artifacts are described.

Benjamin F. Bayman and John H. Broadhurst
Physics Department, University of Minnesota
Minneapolis, Minnesota 55455

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1. Introduction

The technique of representing a continuous analog parameter, for example the kinetic energy of a fast ion, by the electronic conversion of an analog of its energy into an integer number was first introduced by Wilkinson[1] and has become the standard technique of data aquisition in both nuclear and elementary particle physics. In the earlier experimental work this converted data directly represented the physical parameter of interest, and a display of the number of events accumulated for each integer number was the normal way of presenting the data for manual analysis. As experimental requirements became more stringent, analog processing of incoming signals was added to this basic arrangement. However no attempt was made to manipulate the digital data electronically.

Once electronic manipulation of this data is attempted, the following shortcoming immediately appears. To a data point representing a number of events is associated a span of the incoming analog signal lying between the numbers N and N + 1. This is normally represented as a histogram of events per unit interval. Electronic processing on the other hand describes all these events as having the unique analog value associated with the number N, thus changing the characteristics of the incoming signal from continuous in its analog parameter to a series of singularities at N, N+1, N+2, ...

A palliative solution to this problem is to digitize the data to a greater resolution than will eventually be needed, thus approximating the continuous signal by several singularities within the basic interval $N \rightarrow N + 1$. This method is wasteful in digitization time and storage space on the medium which is being used to record the events. In addition it is often not technically feasible to increase the resolution of the digitization.

The solution proposed in this paper is to distribute the events ascribed to the digitized value of N over the interval $N \rightarrow N + 1$ by adding an arbitrary fraction in the range $0 \rightarrow 1$ to each event before processing. This can be done in real time, i.e. while the data are being recorded, or it can be done at some later time using the stored digitized data. Since the processing time requirements are somewhat different in these two cases, they are discussed separately in sections 2 and 3.

2. Real-Time Calculation

In this situation it is imperative that the calculations be done as quickly as possible. In order to avoid the longer processing time associated with the computer manipulation of real numbers, it is simpler first to multiply the integers N, N+1, etc. by 2^L and then add a smaller arbitrary integer in the range 0-2^L to it (where L is chosen to fill the computer word, e.g. for 10 bit data and a 16 bit word length, L = 5). This process produces a comb of singularities within the interval $N \rightarrow N + 1$ to represent the continuous function at a spacing determined only by the limitations of the processing computer.

The generation of the small integers can follow one of several guidelines. The simplest procedure is to make their frequency of occurrence constant throughout the range. How to approximate this distribution for a finite set of numbers (corresponding to the finite number of events within the interval) depends on the sensitivity of the data to different types of processing artifact. If the number of events in an interval is small then a series of integers at intervals equal to the reciprocal of the number of events ensure that the events of that interval are equally distributed. It is necessary though to assign these integers in random order if correlation artifacts between two sets of integer data are to be minimized. Once the number of events in the interval becomes large compared with the number of possible values of the added integer, random values of the added integer provides adequate distribution of the events. If neither of these critera are satisfied a pseudo-random sequence reduces the probability of misinterpretation by assuring that the processing artifacts reproduce between different data sets.

Fig. 1 shows the effects of this distribution technique in a simple example of digital processing. In the focal plane of a momentum spectrometer was installed a silicon detector which provided two electrical signals, E and (X.E), where E is the kinetic energy of the incident particle and X is its position relative to one of its ends. These signals were digitized to 1:1024 resolution and a positional spectrum obtained using the integer algorithim

$$S = ((1024.(X.E))/E) \tag{1}$$

where S represents the distance along the focal plane. (The parentheses represent integer truncation of the enclosed quantity.) Plots of number of events against S are shown, in the lower graph without and in the upper graph with the addition of pseudo random distribution of the events over each interval. Note that the smoothing of the processed data increases the information which can be resolved, in this case the double peaking of the events from elastic scattering by the calcium nuclei indicating that the detector was not located correctly in the focal plane.

3. Post-acquisition Reconstruction

The previous section described a procedure that can be used in real time to reduce artifacts caused by digitization. After the original digitized data have been accumulated and stored, somewhat more sophisticated smoothing procedures can be used. This is illustrated below, using the same example as was discussed in section 2, namely the extraction of an X-spectrum when the data consists of digitized E and X.E spectra.

Figure 2 shows a portion of an E - X.E/1024 plane. Each slanting line is the locus of points of a fixed integer value of X. Suppose that there are 1000 events for which the true values of E and X.E/1024 satisfy $501 \le E < 502$ and $101 \le X.E/1024 < 102$. Every such event is recorded as E = 501 and X.E/1024 = 101, as if it corresponded to the lower left-hand corner of the square labelled (0,0). It is seen from Figure 2 that if X-values were calculated from this point alone and digitized, it would be concluded that all 1000 events corresponded to X = 206. This is, in fact, the "standard" procedure using Equation 1 with the recorded integer E and X.E/1024 values. However inspection of Figure 2 shows that the digitized X-values corresponding to points within the square (0,0) can actually have the values 206, 207, 208. Conversely, the "standard" procedure applied to the entire region shown in Figure 2 would yield no digitized X-values corresponding to X = 205 or X = 207, since the slanting bands above the X = 205 and X = 207 loci contain no lattice corners. Thus analyzing digitized data using integer arithmetic has the effect of artificially removing events from some X channels and putting them into others. These effects will be more serious if the slanting bands in Figure 2 are close together, that is if $E/1024 \ll 1$.

The smoothing method presented in Section 1 is based upon the assumption that all the 1000 points are distributed uniformly and at random over the entire (0,0) square. Then the probability of obtaining any of the allowed digitized values is proportional to the area of the corresponding slanted band. This would imply that there were 377.4 events with X = 206, 489.8 events with X = 207 and 132.8 events with X = 208. Although the assumption of a uniform distribution of events over each square is somewhat arbitrary, it is clear that this procedure yields a better representation of the distribution of X-values than is obtained with the untreated digitized data, i.e. by assuming that all 1000 points in the (0,0) square correspond to X = 206.

It is possible to get a closer approximation to the true distribution by using the information contained in the variation of the number of counts from one square to another. Let n(i,j) be the number of events that fall in the (i,j) square. Then construct the function

$$f(x,y) = ax^2 + 2bxy + cy^2 + 2dx + 2ey + g, \qquad (2)$$

where for the case illustrated in Figure 2,

$$x = E - 501, \quad y = X.E/1024 - 101$$

$$a = \frac{n(1,0) + n(-1,0) - 2n(0,0)}{2}$$

$$b = \frac{n(1,1) + n(-1,-1) - n(1,-1) - n(-1,1)}{8} \qquad (3)$$

$$c = \frac{n(0,1) + n(0,-1) - 2n(0,0)}{2}$$

$$d = \frac{n(1,0) - n(-1,0)}{4}$$

$$e = \frac{n(0,1) - n(-1,0)}{4}$$

$$g = \frac{28\, n(0,0) - n(0,-1) - n(0,1) - n(-1,0) - n(1,0)}{24}$$

The integral of the function f(x,y) over each of the five squares (0,0), (0,1), (1,0), (0,-1) and (-1,0) equals the number of events actually observed to occur in that square. Moreover the sum of the integrals of f(x,y) over the squares (1,1) and (-1,-1), minus the corresponding sum for the squares (1,-1) and (-1,1), equals n(1,1) + n(-1,-1) - n(-1,1) - n(1,-1). These six conditions determine the six constants in f(x,y) in a way that should allow f(x,y) to give a reasonably accurate representation of the relative probability of events within the (0,0) square.

To reconstruct the X-spectrum, sets of three random numbers are chosen uniformly on the interval (0,1). The first two determine the (x,y) coordinates within the (0,0) square. The quantity

$$z \equiv \frac{f(x,y)}{f_{max}}$$

is then calculated, with f_{max} the maximum value that f(x,y) achieves within the (0,0) square. If the third random number is greater than z, this (x,y) point is ignored and another set of three random numbers is chosen[2]. If the third random number is less than z, then an X value is calculated from the E and X.E/1024 values corresponding to this point. This process is continued until as many X values have been calculated as there were events whose digitized E and X.E/1024 values placed them in the (0,0) square. In this way a population of (x,y) points is generated within the (0,0) square whose density is proportional to f(x,y). This procedure is repeated for every pair of digitized E and X.E/1024 intervals (every square).

Figure 3 shows a comparison of the methods described here. A population of X-values very similar to the Ca peak in Figure 1 has been artificially generated. It consists of the sum of two Gaussian distributions, each with FWHM of 20, but with means of 210 and 230 and relative heights of 22 and 26, respectively. Similarly, a Gaussian E distribution was generated with a mean of 150 and FWHM of 5. These E and X distributions were then used to generate populations of digitized E and X.E/1024 values, from which attempts have been made to recover the original X-distribution.

The smooth line in Figures 3a through 3d shows the histogram of digitized X-values that would theoretically result if 204,800 values were chosen at random from the X-distribution. The points in Figure 3a show the histogram that actually resulted when 204,800 X-values were selected. These points show some statistical fluctuations around the expected values, but the fact that the population has peaks centered at X = 210 and 230 is clearly evident. The points in Figure 3b show the histogram of X-values obtained by division of digitized X.E/1024 and E values (Equation 1). It is clear that this simplest procedure introduces artifacts that result in a significant loss of resolution. It can also be seen that this procedure shifts so many events from one channel to another that some X-channels end up practically empty, whereas others end up with much greater occupation than they should have. This effect is more pronounced in Figure 3b than in the Ca peak of Figure 1b, because Figure 3b corresponds to a mean E-value of 150, compared to 850 for Figure 1b. Thus the constant X-lines on a plot like Figure 2 would be almost 6 times closer, and statistical fluctuation effects are more prominent.

Figure 3c shows histogram of X-values obtained by assuming that the events are uniformly distributed within each of the squares shown in Figure 2. Here the relative probabilities of the different digitized X-values within each square are taken to be proportional to the areas of the corresponding bands, using real-number arithmetic without truncation. Thus this histogram does not show sampling or truncation irregularities. The histogram shown in Figure 3d was obtained by assuming that the events within each square were governed by the probability distribution given by Equation 2. Since this takes some account of the continuous variation of probability in the E - (X.E)/1024 plane, it can be expected that this produces the best approximation to the original spectrum. However, it is clear from Figure 3 that very little improvement has been achieved compared to the X-histogram calculated using a uniform distribution within each square, especially when compared with the enormous improvement the uniform distribution yields compared to the simple division of digitized data (Equation 1).

These observations are confirmed by a comparison of the r.m.s. deviations between the expected histogram and the histograms derived by the above-mentioned processes. The quadratic probability distribution of Equation 2 is seen to provide a slightly lower r.m.s. deviation than the uniform distribution. Moreover, comparison of Figures 3c and 3d shows that the additional curvature of Equation 2 yields a better approximation to the expected histogram in the regions of maxima and minima.

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References

1. Wilkinson D., Cambridge Phil. Soc. 46 3 508 (1950).
2. Abramowitz, M., and Stegun, I.A., Handbook of Mathematical Functions, National Bureau of Standards, Applied Mathematics Series, No. 55, 1964, pp 949-953.

Figure Captions

Fig. 1. Elastic scattering of 8.5 MeV protons at 20° lab angle on Ca F_2 target. Silicon position sensitive detector in focal plane of magnetic spectrometer.

Fig. 2. A portion of the E - X.E/1024 plane. The sloping lines are the loci of constant X-values. The pairs of numbers in parentheses are used to labcl the squares.

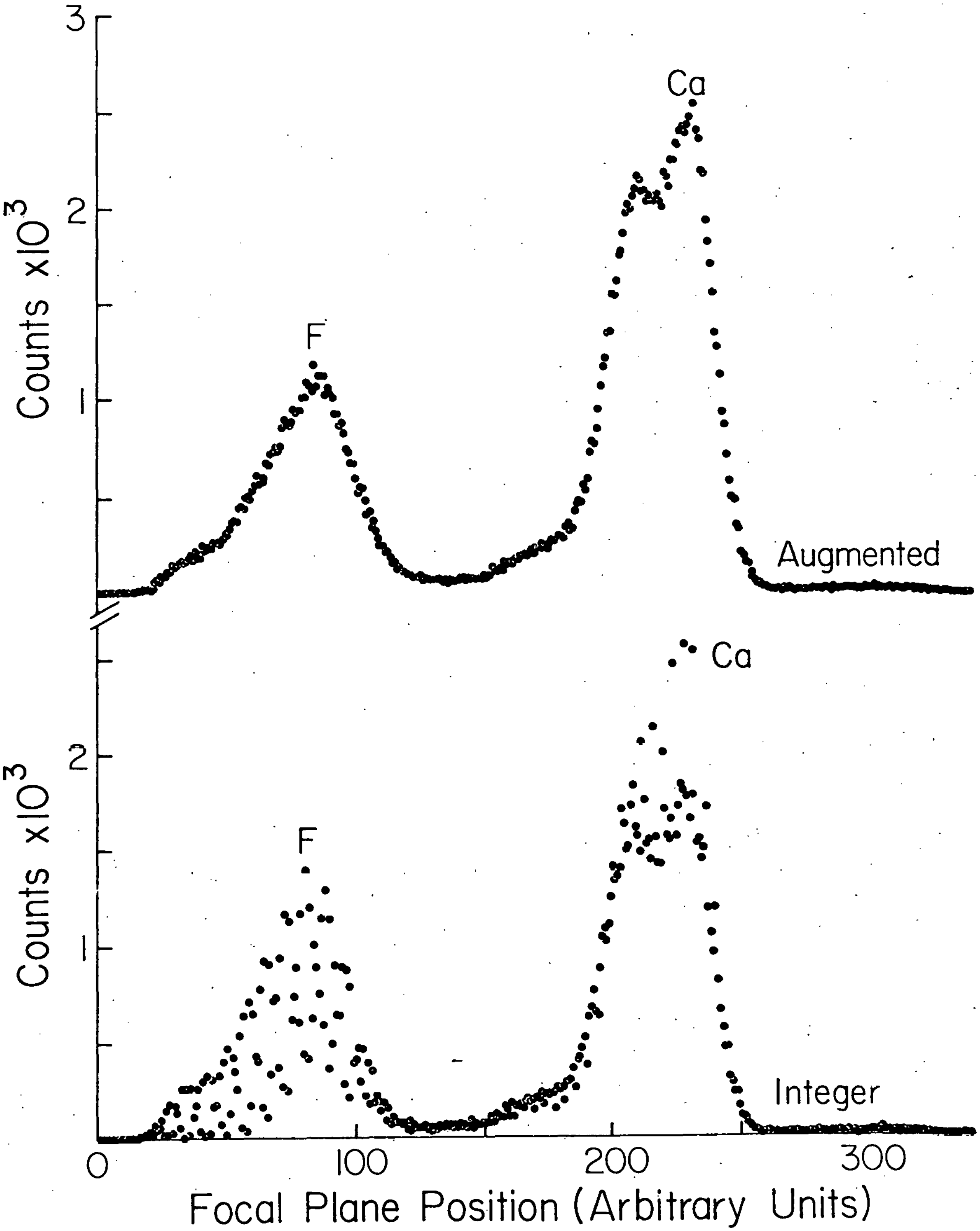
Fig. 3. The continuous lines represents the expected histogram of X-values.

3a) Histogram of 204,800 randomly chosen X-values.

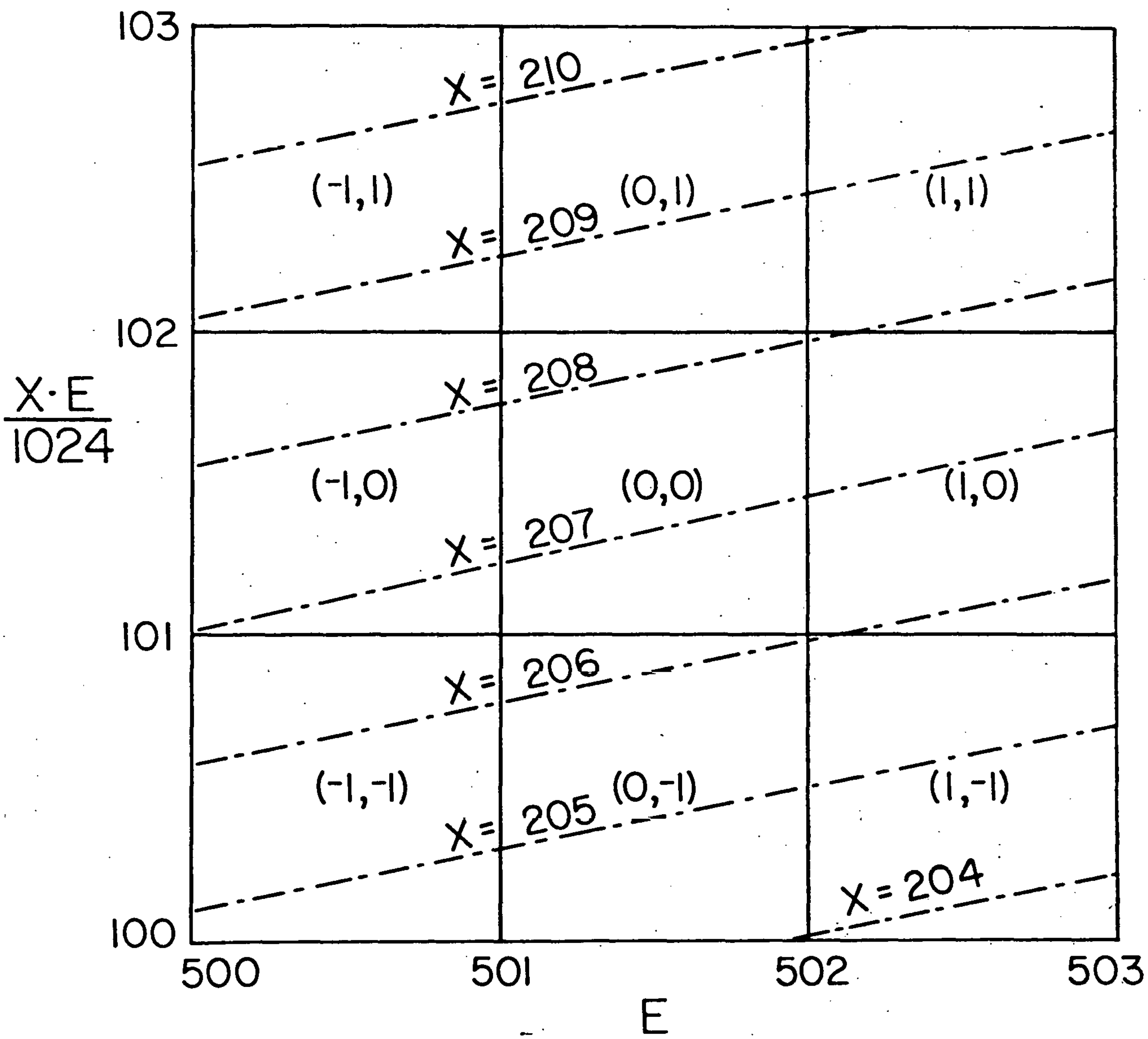
3b) Histogram of X-values calculated from the digitized data using Equation 1.

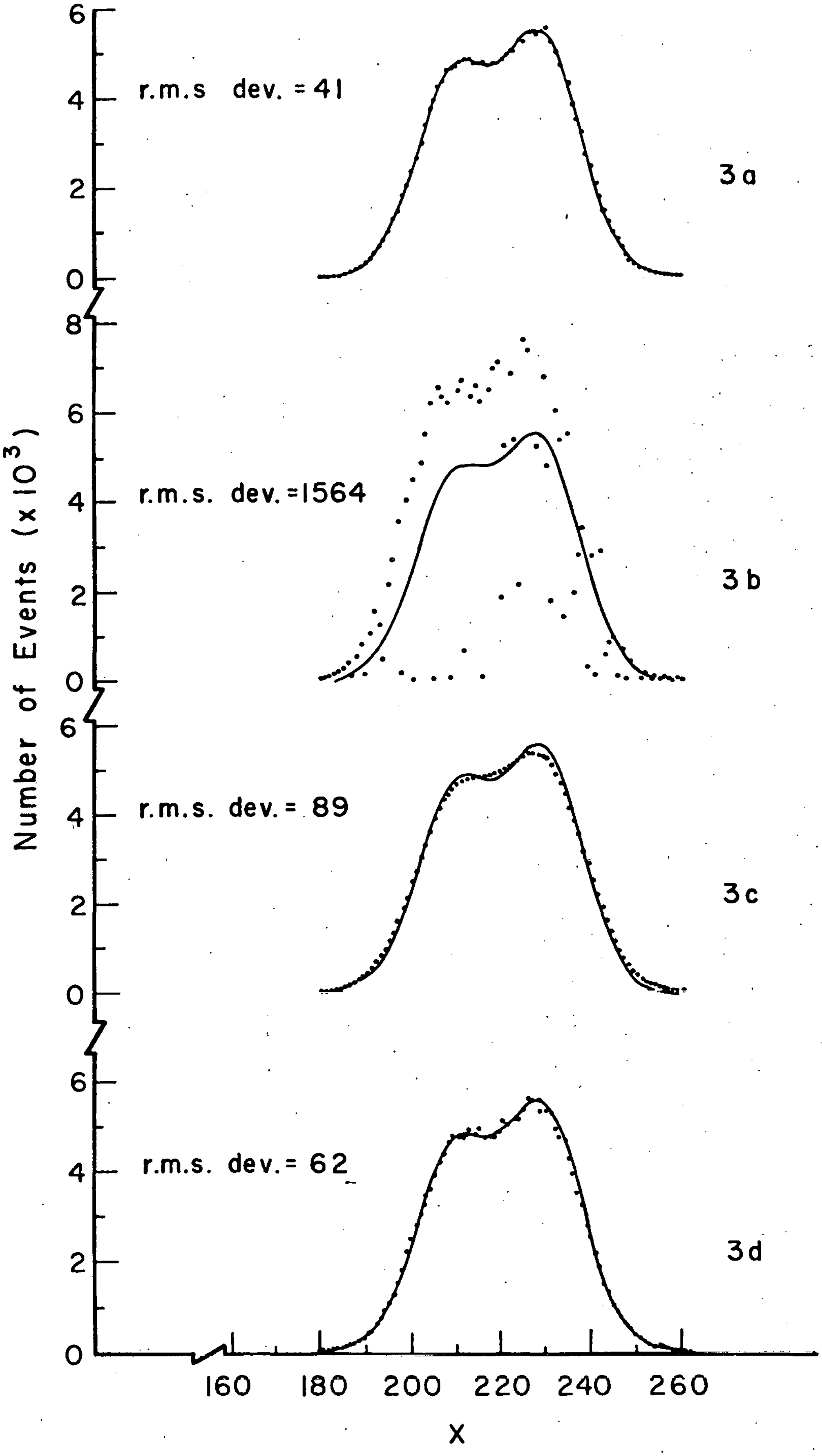
3c) Histogram of X-values calculated assuming a uniform distribution.

3d) Histogram of X-values calculated assuming the quadratic distribution of Equation 2.



3
2
1
Counts x10^3
F
Ca
Augmented
2
1
Counts x10^3
F
Ca
Integer
0
100
200
300
Focal Plane Position (Arbitrary Units)


103
102
101
100
X·E / 1024
500
501
502
503
E
X = 210
X = 209
X = 208
X = 207
X = 206
X = 205
X = 204
(-1,1)
(0,1)
(1,1)
(-1,0)
(0,0)
(1,0)
(-1,-1)
(0,-1)
(1,-1)



r.m.s dev. = 41
3a
r.m.s. dev. =1564
3b
r.m.s. dev. = 89
3c
r.m.s. dev. = 62
3d
Number of Events (x 10³)
X
160
180
200
220
240
260