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FEEDBACK STABILIZATION OF AXISYMMETRIC MODES IN TOKAMAKS

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Feedback Stabilization of Axisymmetric Modes in Tokamaks

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Noncircular tokamak plasmas can be unstable to ideal MHD axisymmetric instabilities. Passive conductors with finite resistivity will at best slow down these instabilities to the resistive (L/R) time of the conductors. An active feedback system far from the plasma which responds on this resistive time can stabilize the system provided its mutual inductance with the passive coils is small enough.

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Since the effective "external field curvature" of noncircular tokamaks can be negative, the plasma is potentially unstable to axisymmetric magnetohydrodynamic instabilities [1,2]; i.e., a rigid vertical displacement of the entire plasma column. This ideal MHD instability occurs on the poloidal Alfvén time scale.

$$\gamma \sim (B_p^2 / \mu_0 \rho_0)^{1/2} \frac{1}{a}$$

which is on the order of $(\mu\text{sec})^{-1}$ for typical tokamak parameters. Passive conductors such as shaping coils and the vacuum vessel walls will stabilize these modes through the formation of eddy currents [3]. However, these stabilizing eddy currents will decay on the order of the L/r time of the passive conductors, leaving the plasma unstable on this slower time scale (~ 100 msec). In order to stabilize the plasma on this slow time scale, an active feedback system is required. The intent of this paper is to investigate the feasibility of locating the active feedback coils far from the plasma as would be required in a reactor environment.

We first consider the situation without active feedback. The circuit equation for a single coil (i) can be written

$$L_i \frac{dI_i}{dt} + r_i I_i + \sum_{j \neq i} M_{ij} \frac{dI_j}{dt} + M_{ip} I_p \frac{dz}{dt} + \alpha_i Z = 0.$$

Here L_i is the self inductance of the coil which carries current I_i , has resistance r_i , and may have a voltage applied to it proportional to the plasma vertical displacement Z with proportionality constant α_i . If this is a passive coil $\alpha_i = 0$. M_{ij} is the mutual inductance between coils i and j , and $M_{ip}(Z)$ is the inductive coupling between the plasma and the coil. Primes

denote derivatives with respect to Z . The plasma experiences a force due to the external field gradient, and due to the eddy currents or active feedback currents in the coils. Thus, its equation of motion is

$$m \frac{\partial^2 Z}{\partial t^2} = I_p \sum_i M'_{ip} \hat{I}_i + I_p \frac{\partial B_{\text{ext}}}{\partial Z} Z.$$

Here we have represented the plasma as a wire loop of mass m carrying a constant current I_p , and having a fixed radius but free to oscillate vertically. The currents in the coils are $I_i = \bar{I}_i + \hat{I}_i$, where $\bar{I}_i$ is the equilibrium current. The term $(\partial B_{\text{ext}}/\partial Z) \equiv \sum_i M'_{ip} \bar{I}_i$ represents the effects of external field gradients, while the terms $M'_{ip} I_p \hat{I}_i$ represents the forces on the plasma due to perturbed currents in the coils.

If we consider a single passive coil only (see Fig. 1), this system of equations has roots (to the lowest order in $\gamma_r^2/w_0^2 \ll 1$)

$$\gamma_1 = -\gamma_r,$$

$$\gamma_2 = \gamma_r w_0^2 / (w_1^2 - w_0^2),$$

$$\gamma_{3,4} = \pm i [w_1^2 - w_0^2]^{1/2} - \frac{\gamma_r}{2} \frac{w_1^2}{w_1^2 - w_0^2}.$$

Here $\gamma_r = r/L$ is the inductive resistive decay rate, $w_0^2 = I_p^2/m (\partial B_{\text{ext}}/\partial Z)$ is the unstabilized growth rate, and $w_1^2 = I_p^2 M_{1p}'^2/mL$ represents the stabilizing effect of the passive coils.

The first mode, γ_1 , is always stable, and simply decays with the L/r time of the coil. If the passive term is large enough, i.e., $w_1^2 > w_0^2$, then the system is neutrally stable (oscillatory) on the fast time scale. The

introduction of coil resistance is seen to damp the oscillatory modes (3 and 4). However, mode 2, which has zero growth rate in the absence of resistivity, is now unstable. This mode corresponds to a new equilibrium point moving away from $Z = 0$ on the slow time scale.

Now let coil number one be the passive coil, and coil number two be the active (only two coils). The active feedback circuit response is assumed to be limited by a capacitive input impedance so that $\alpha = \beta / (1 + i\omega/w_c)$ where $i\omega = \gamma$ is the growth rate, β is the feedback gain, and w_c is the cutoff frequency, of order $(L/r)^{-1}$. The system is then described by the set of equations

$$L_1 \frac{\partial \hat{I}_1}{\partial t} + M \frac{\partial \hat{I}_2}{\partial t} + r_1 \hat{I}_1 + I_p M'_{1p} \frac{\partial Z}{\partial t} + \frac{\beta}{1 + i(\omega/w_c)} Z = 0 \quad , \quad (1a)$$

$$L_2 \frac{\partial \hat{I}_2}{\partial t} + M \frac{\partial \hat{I}_1}{\partial t} + r_2 \hat{I}_2 + I_p M'_{2p} \frac{\partial Z}{\partial t} = 0 \quad , \quad (1b)$$

$$m \frac{\partial^2 Z}{\partial t^2} - I_p \hat{I}_1 M'_{ip} - I_p \hat{I}_2 M'_{2p} - I_p \frac{\partial B_{\text{rext}}}{\partial Z} Z = 0 \quad . \quad (1c)$$

Examination of the normal modes of Eq. (1) shows that there are two high frequency damped modes (which are purely oscillatory without resistivity or feedback), and there are two other modes given by the dispersion relation

$$\tau_1 \gamma = \frac{1}{2\tau_2 D} \left\{ \tau_2 \left(1 - \frac{\gamma_2^2}{\gamma_0^2} \right) + \tau_1 \left(1 - \frac{\gamma_1^2}{\gamma_0^2} \right) - \frac{(\gamma_f/\gamma_0)}{1 + (\gamma/w_c)} \right. \\ \left. \left[\frac{\gamma_1}{\gamma_0} \tau_2 - \left(\tau_2/\tau_1 \right)^{1/2} \frac{\gamma_2}{\gamma_0} \tau_{12} \right] \pm F^{1/2} \right\} \quad . \quad (2)$$

Here

$$\begin{aligned}
 F = & \left\{ \tau_2 \left(1 - \frac{\gamma_2^2}{\gamma_0^2} \right) - \tau_1 \left(1 - \frac{\gamma_1^2}{\gamma_0^2} \right) - \tau_2 \frac{(\gamma_f/\gamma_0)}{1 + (\gamma/w_c)} \left[\frac{\gamma_1}{\gamma_0} - \frac{\gamma_2}{\gamma_0} \frac{\tau_{12}}{(\tau_1 \tau_2)^{1/2}} \right] \right\}^2 \\
 & + 4 \left[\tau_{12} - (\tau_1 \tau_2)^{1/2} \frac{\gamma_1 \gamma_2}{\gamma_0^2} \right] \left[\tau_{12} - (\tau_1 \tau_2)^{1/2} \frac{\gamma_1 \gamma_2}{\gamma_0^2} \right. \\
 & \left. + \frac{(\gamma_f/\gamma_0)}{1 + (\gamma/w_c)} \left((\tau_1 \tau_2)^{1/2} \frac{\gamma_2}{\gamma_0} - \tau_{12} \frac{\gamma_1}{\gamma_0} \right) \right] ,
 \end{aligned}$$

$$D = \frac{\gamma_1^2 + \gamma_2^2}{\gamma_0^2} - 1 + \frac{\tau_{12}^2}{\tau_1 \tau_2} - 2 \frac{\gamma_1 \gamma_2}{\gamma_0^2} \frac{\tau_{12}}{(\tau_1 \tau_2)^{1/2}} .$$

The dimensionless and time variables are defined as

$$\tau_1 = \frac{L_1}{r_1} , \quad \tau_2 = \frac{L_2}{r_2} , \quad \tau_{12} = \frac{M}{(r_1 r_2)^{1/2}} ,$$

$$\left(\frac{\gamma_1}{\gamma_0} \right)^2 = \frac{(\partial M_{1p}/\partial z)^2 I_p}{L_1 (\partial B_{\text{rext}}/\partial z)} , \quad \left(\frac{\gamma_2}{\gamma_0} \right)^2 = \frac{(\partial M_{2p}/\partial z)^2 I_p}{L_2 (\partial B_{\text{rext}}/\partial z)} ,$$

$$\left(\frac{\gamma_f}{\gamma_0} \right)^2 = \frac{\beta^2}{r_1} \frac{L_1}{I_p (\partial B_{\text{rext}}/\partial z)} .$$

The feedback enters the rapidly oscillating modes only to order $(r/L)^2$. Since to first order in r/L , this mode is damped, we can turn attention to the two slow time scale modes. In Figs. 2 and 3, we have plotted the growth rate $\tau_1 \gamma$ given by Eq. (2) as a function of the feedback proportionality

parameter (γ_f/γ_0) for different values of the active feedback system rolloff frequency $\tau_1 \omega_c$. In both these plots $\tau_2/\tau_1 = 1$, $\gamma_2/\gamma_0 = 1.5$, $\gamma_1/\gamma_0 = 0.5$. In Fig. 2, $\tau_{12}/\tau_1 = 0.4$ while in Fig. 3 $\tau_{12}/\tau_1 = 0.2$.

With no active feedback one mode is purely growing and one mode is purely decaying. As (γ_f/γ_0) is increased the modes start coming together until they coalesce at a value of $2 \lesssim \gamma_f/\gamma_0 \lesssim 3$ (see Fig. 2b and 3b). Increasing γ_f/γ_0 still farther, these roots form a pair of complex conjugate pairs (See Fig. 2a and 3a). In both cases, lowering the rolloff frequency, ω_c , is destabilizing i.e., causes the roots path in the complex plane to shift upward towards the positive real part (unstable). Comparing Figs. 2a and 3a shows that lowering the mutual inductance between the active and passive circuits is stabilizing, i.e., shifts the curves towards negative real part. Another interesting effect is that increasing γ_f/γ_0 can be stabilizing (as in Fig. 3a with $\tau_1 \omega_c = \infty, 10$) or destabilizing. Thus, in general, we expect the system to be stable only in a window. Figure 3a suggests that a stable situation may be obtained with $\tau_1 \omega_c \sim 2$ which is not overly restrictive.

The mutual inductance is destabilizing since it shields the plasma from the feedback currents in the active coil. The mutual inductance between these coils (or between an active coil and the vacuum vessel wall) must be kept to a minimum. for if it is too large the active stabilization may not work regardless of the gain of the feedback system.

The power requirement for the feedback system are given by the complex power

$$P(Z) = \left(\frac{N}{D}\right) \tau_1^{-1} I_p \frac{\partial B_{\text{ext}}}{\partial Z} Z^2 \quad (3)$$

Here

$$N = (\gamma\tau_1)^2 \left[\left(\frac{\gamma_1}{\gamma_0}\right) - \frac{\tau_{12}}{(\tau_1\tau_2)^{1/2}} \left(\frac{\gamma_2}{\gamma_0}\right) \right] + (\gamma\tau_1) \left(\frac{\gamma_1}{\gamma_0}\right) \left(\frac{\tau_1}{\tau_2}\right) \\ + \left(\frac{\gamma_f}{\gamma_0}\right) \left[\left(\frac{\tau_1}{\tau_2}\right) + (\gamma\tau_1) \right] / \left[1 + (\gamma\tau_1) / (w_c \tau_1) \right] ,$$

$$D = \left(1 + \frac{\gamma\tau_1}{w_c \tau_1} \right) \{ (\gamma\tau_1)^2 [1 - \tau_{12}/(\tau_1\tau_2)^{1/2}] \\ + (\gamma\tau_1) [1 + (\frac{\tau_1}{\tau_2})] + (\frac{\tau_1}{\tau_2}) \} .$$

If the plasma column is displaced from the midplane a distance Z due to random fluctuations or field errors, $\text{Re} \{P(Z)\}$ gives the dissipative power required from the feedback power supply to center the plasma. For the parameters of Fig. (3) and with $\tau_1 w_c = 2$ and $\gamma_f/\gamma_0 = 3$, we have $\text{Re} \{N/D\} = 1.28$. For the proposed INTOR tokamak, typical values would be $\tau_1 = .01$ sec., $I_p = 5 \times 10^6$ amps, $\partial B_{\text{ext}}/\partial Z = 0.2$ Tesla/meter, $Z = .05$ meters. Evaluation of Eq. (3) for these parameters yields a power requirement of the order of 10^6 watts.

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FIGURE CAPTIONS

- Fig. 1. System is modeled by a point plasma of constant current free to oscillate vertically, a passive conductor, and an active feedback coil.
- Fig. 2. High mutual inductance between coils, $\tau_{12}/\tau_1 = 0.4$. γ_f/γ_0 measures gain of feedback system. For $0 < \gamma_f/\gamma_0 < 2$, two purely growing (decaying) modes exist (b). Increasing γ_f/γ_0 further causes roots to form complex conjugate pairs (a). Stable regime exists only for narrow range of parameters.
- Fig. 3. Low mutual inductance between coils, $\tau_{12}/\tau_1 = 0.2$. Stable regime exists for $\gamma_f/\gamma_0 \gtrsim 2.5$ and $\tau_1 \omega_c \gtrsim 2$.

Fig. 1

Fig. 2

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Fig. 3