NOTICE

MAGNETIC-FIELD MEASUREMENTS OF SUPERCONDUCTING MAGNETS FOR
THE COLLIDING-BEAM ACCELERATOR*

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I. Introduction

An important aspect of the development and production of superconducting magnets for the Colliding Beam Accelerator is the measurement of the magnetic field in the aperture of these magnets. The measurements have the three-fold purpose of determining the field quality as compared to the lattice requirements of the CBA, of obtaining the survey data necessary to position the magnets in the CBA tunnel, and lastly, of characterizing the magnetic fields for use in initial and future orbit studies of the CBA proton beams. Since for a superconducting storage accelerator it is necessary to carry out these detailed measurements on many (~1000) magnets and at many current values (~1000), we have chosen, in agreement with previous experience,¹⁻⁵ to develop a system which Fourier analyses the voltages induced in a number of rotating windings and thereby obtains the multipole field components. The important point is that such a measuring system can be fast and precise. It has been used for horizontal measurements of the CBA ring dipoles.

II. Field Representation

Within the free aperture of a superconducting magnet of the CBA, the magnetic field is essentially two dimensional and its value at any point (r, θ) is equal to a sum of harmonic multipoles. Thus, for cylindrical coordinates (Fig. 1),

$$\vec{B}(r, \theta) = \sum_{n=1}^{\infty} C(n) \left(\frac{r}{R}\right)^{n-1} \left[\vec{r}_n \sin n(\theta - \alpha_n) + \vec{t}_n \cos n(\theta - \alpha_n) \right], \quad (1)$$

where $C(n)$ is the amplitude (tesla) of the n^{th} magnetic multipole at a reference radius R , while α_n is the angular orientation of the multipole with a value in the range: $0 < \alpha_n < 2\pi/n$. We note that such a harmonic expansion can be developed, consistent with Maxwell's equations, about any chosen center ($r=0$) inside the free aperture and the associated region of validity of the expansion extends to the nearest current carrying conductor. If we now make the expansion about a center that is offset from the initial one by a displacement (r_0, ω) , expressed in cylindrical coordinates, we can write

$$\vec{B}(\rho, \phi) = \sum_{\ell=1}^{\infty} \vec{C}(\ell) \left(\frac{\rho}{R}\right)^{\ell-1} \left[\vec{r}_\ell \sin \ell(\phi - \bar{\alpha}_\ell) + \vec{t}_\ell \cos \ell(\phi - \bar{\alpha}_\ell) \right]. \quad (2)$$

Here the multipole amplitude and phase $(\vec{C}(\ell), \bar{\alpha}_\ell)$ defined at the new center (r_0, ω) are related to the previous ones $(C(n), \alpha_n)$ according to

$$\vec{C}(\ell) \cos \ell \bar{\alpha}_\ell = \sum_{n=\ell}^{\infty} C(n) C_{n-\ell}^{n-1} \left(\frac{r_0}{R}\right)^{n-\ell} \cos[(n-\ell)\omega - n\alpha_n], \quad (3)$$

and

$$\vec{C}(\ell) \sin \ell \bar{\alpha}_\ell = - \sum_{n=\ell}^{\infty} C(n) C_{n-\ell}^{n-1} \left(\frac{r_0}{R}\right)^{n-\ell} \sin[(n-\ell)\omega - n\alpha_n], \quad (4)$$

where the binomial coefficient is

$$C_{n-\ell}^{n-1} = \frac{(n-1)!}{(n-\ell)!(\ell-1)!}. \quad (5)$$

Though, as we shall see, the cylindrical multipoles $(C(n), \alpha_n)$ which appear in Eq. (1) are particu-

larly easy to work with when discussing the voltage induced in a rotating coil, another equivalent field representation is more convenient to use in the study of accelerator beams. In this case, the magnetic field components perpendicular to and lying in the median plane are expressed in terms of the median plane coefficients (b_n, a_n) . For a dipole magnet the median plane is defined by the dipole field $(C(1) = B_0, \alpha_1 = 0)$, and Eq. (1) with $(r=x, \theta=0)$ results in the series

$$B_y = B_0(b_0 + b_1 x + b_2 x^2 + \dots + b_n x^n) \quad (6)$$

and

$$B_x = B_0(a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n), \quad (7)$$

where the normal (b_n) and skew (a_n) coefficients are related to the cylindrical coefficients by

$$b_0 = \cos \alpha_1 = 1, \quad b_n = \frac{C(n+1)}{C(1)} \frac{\cos[(n+1)\alpha_{n+1}]}{R^n}, \quad (8)$$

and

$$a_0 = \sin(-\alpha_1) = 0, \quad a_1 = \frac{C(2)}{C(1)} \frac{\sin(-2\alpha_2)}{R}, \quad a_n = \frac{C(n+1)}{C(1)} \frac{\sin[-(n+1)\alpha_{n+1}]}{R^n}. \quad (9)$$

Although the representation of the magnetic field by cylindrical harmonics [Eq. (1)] is appropriate for any magnet, the median plane representation as given by Eqs. (6) and (7) applies only to dipole magnets. (In this summary paper, we shall confine our discussion to dipole magnets.) The harmonic coefficients (b_n, a_n) tend to be less sensitive to the operating field level, since the dominant dependencies on iron saturation and hysteresis are carried by the dipole field (B_0) .

An alternative to the median plane multipole coefficients is the closely related pair of fractional field coefficients (or primed coefficients) specified at a particular radius $(R_0 = 4.4 \text{ cm for CBA magnets})$. They are defined as follows:

$$b'_n = R_0^n b_n, \quad a'_n = R_0^n a_n. \quad (10)$$

From the viewpoint of the magnet designer, the fractional field coefficients have a certain advantage. If the reference radius R_0 is stipulated to be equal to a given fraction ($\sim 2/3$) of the radius at which the current conductors are located, then these coefficients provide a good numerical comparison of the relative field quality of different magnet designs. In addition, for a given magnet the fractional field coefficients are conveniently all of the same order of magnitude. We can readily express the rectangular components of the magnet field in the aperture using the fractional field coefficients (b'_n, a'_n) . For a dipole,

$$B_y(r, \theta) = \sum_{n=0}^{\infty} B_0 \left(\frac{r}{R_0}\right)^n \left[b'_n \cos n\theta - a'_n \sin n\theta \right], \quad (11)$$

and

$$B_x(r, \theta) = \sum_{n=0}^{\infty} B_0 \left(\frac{r}{R_0}\right)^n \left[a'_n \cos n\theta + b'_n \sin n\theta \right]. \quad (12)$$

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In the median plane ($r=x$, $\theta=0$) of the dipole, the conventional series are

$$B_y = B_0 \left(b'_0 + b'_1 \frac{x}{R_0} + b'_2 \frac{x^2}{R_0^2} + \dots + b'_n \frac{x^n}{R_0^n} \right), \quad (13)$$

and

$$B_x = B_0 \left(a'_0 + a'_1 \frac{x}{R_0} + a'_2 \frac{x^2}{R_0^2} + \dots + a'_n \frac{x^n}{R_0^n} \right). \quad (14)$$

III. Tangential Coil for Dipoles

The tangential coil assembly designed for measuring the magnetic field in the dipoles of the CBA consists of six independent windings wound on the outer surface of a fiberglass tube about 17 ft in length. This tube rotates on a set of non-magnetic bearings fixed to a stationary, centrally located, stainless spline. To minimize the final overall sag, the central spline has been prebent. Figure 2 is a sketch showing the longitudinal and circumferential positions of the various windings. In Table I we have listed the principal properties of the six windings as well as their basic functions. Winding number 1 is the dipole field reference winding and provides the most sensitive measure of the dipole field integral. In addition, it serves, in conjunction with winding 2, as a source of voltage for bucking the large dipole voltage appearing in the harmonic measuring windings 3 through 6. We have placed all the windings at the periphery of the sleeve primarily for two reasons. First, the harmonic measuring windings which subtend the smaller angles are then more sensitive to the higher harmonics. Second, the central region of the coil becomes available for mechanical support over a greater part of the aperture and over the full length of the coil.

IV. Measurement System

In Fig. 3 we present a block diagram of the overall system. The measurement cycle starts with a trigger pulse from the shaft encoder. The voltage from each rotating tangential winding is digitized at 128 points spaced equally over one shaft rotation by the HP 3456A DVM's. This basic data is processed by the HP 9826 computer, and the calculated magnetic field multipoles values ($C(n)$, α_n) are transmitted to the PDP 11/60 where the central axis of the magnetic field as well as the angular orientation of the median plane are determined. The system has been designed to measure all the required magnet parameters in a completely automatic way.

V. Basic Theory

When a tangential winding rotates in a multipole field, represented by Eq. (1), the voltage induced in it is

$$V(t) = - \sum_{n=1}^{\infty} C(n) \left(\frac{R_c}{R} \right)^{n-1} 2\ell R_c N \omega \sin \frac{n\Delta}{2} \cos n(\omega t - \delta + \alpha_n), \quad (15)$$

where (See Fig. 4)

- Δ - the angle subtended by the winding,
 - δ - the initial angular offset of the middle of the winding,
 - R_c - the radial location of the winding,
 - ω - the angular (clockwise) speed of rotation,
 - N - the number of turns,
 - ℓ - the axial length of the winding,
- and $C(n)$ and α_n characterize the harmonics.

To illustrate the procedure followed to determine the harmonics measured by a given coil, we shall consider the specific case of the long harmonic winding number 3 when windings 1 and 2 are employed for digi-

tal bucking of the dipole voltage.* Referring to the flow chart (Fig. 5), one sees that a Fourier analysis is first performed on the digitized voltage signals ($x(m)$, $\bar{x}(m)$, $\bar{x}(m)$; $m = 1 \dots N_p$). The required bucking factors (f_1, f_2) are then calculated by satisfying the bucking condition which balances the fundamental component (F_1) in winding 3 against a fraction of the fundamental as observed by windings 1 and 2. With f_1 and f_2 determined, the net digitized bucked voltage, $y(m)$, is now constructed and a Fourier analysis performed on it. Since the n th harmonic of this bucked voltage is given by

$$\begin{aligned} v(n) = & -C(n) \left(\frac{R_c}{R} \right)^{n-1} 2\ell R_c N \omega \sin \frac{n\Delta}{2} \cos n(\omega t - \delta_3 + \alpha_n) \\ & - f_1 C(n) \left(\frac{R_1}{R} \right)^{n-1} 2\ell_1 R_1 N_1 \omega \sin \frac{n\Delta}{2} \cos n(\omega t - \delta_1 + \alpha_n) \\ & - f_2 C(n) \left(\frac{R_2}{R} \right)^{n-1} 2\ell_2 R_2 N_2 \omega \sin \frac{n\Delta}{2} \cos n(\omega t - \delta_2 + \alpha_n), \end{aligned} \quad (16)$$

one can now calculate the n th cylindrical multipole field ($C(n)$, α_n).

VI. Centering in a Dipole Magnet

The multipole coefficients measured by the tangential coil are referred to the axis of rotation of the coil. Since a CBA dipole has a built-in, allowed, sextupole component ($C(3)$, α_3) whose value goes through zero for a magnet current in the vicinity of 2000 A, it is difficult to specify a unique central axis over the full range of currents for this sextupole field, particularly in the presence of a residual, non-allowed, quadrupole component. We have therefore taken advantage of the fact that the sextupole field varies non-linearly over the magnet current excursion and defined the central axis as that axis which yields a quadrupole component that is most linear with respect to the dipole field. Thus, on the assumption that the built-in residual quadrupole field is proportional to the dipole field, this procedure chooses an axis that minimizes the feed-down to the quadrupole component from the non-linear sextupole component.

To facilitate the calculation of the necessary offset from the axis of the tangential coil, we combine Eqs. (3) and (4) into a simpler equation by introducing complex quantities. For the k th multipole at the offset position we consequently write

$$\bar{C}(\ell) e^{i\ell\bar{\alpha}_\ell} = \sum_{n=\ell}^{\infty} C(n) C_{n-\ell}^{-1} \left(\frac{Z_0^*}{R} \right)^{n-\ell} e^{i n \alpha_n}. \quad (17)$$

Here $Z_0^* (= r_0 e^{-i\omega})$ is the complex conjugate of a displacement from the coil axis. To first order in this offset then, the quadrupole is

$$\bar{C}(2) e^{i2\bar{\alpha}_2} = C(2) e^{i2\alpha_2} + 2 \left(\frac{Z_0^*}{R} \right) C(3) e^{i3\alpha_3}. \quad (18)$$

Constraining this quadrupole field to be linearly related to the dipole field, we obtain the relation

$$mC(1) = C(2) e^{i2\alpha_2} + 2 \left(\frac{Z_0^*}{R} \right) C(3) e^{i3\alpha_3}. \quad (19)$$

It is this equation to which we make a least square fit in order to obtain the location of the central axis (r_0, ω) over the range of operating fields.

VIII. Results

As an example of typical data, we present some measurements made on a recent CBA dipole, LM20. In Fig. 6, we display the normal sextupole, decapole, and 26-pole fractional field coefficients as a function of current. The measurements were made in 200 A steps from injection current (~ 250 A) to approximately full

current for 400 GeV/c operation (~ 3750 A). The b'_2 and b'_4 components show the behavior characteristic of CBA dipoles: superconductor magnetization at low current and a variation due to iron saturation beginning at ~ 2000 A. The circled point at 1000 A on the b'_2 plot is the result of a calculation of this component, in good agreement with the measured value. In Table II, we list the observed values of all the harmonic coefficients through the 26-pole.

Acknowledgments

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References

1. M. Wake, D.A. Gross, and R. Yamada, Cryogenic Magazine, May 1981.
2. L. Walckiers, CERN Report ISR-BOM/18-15, March 1981.
3. C. Wyss, Proc. 5th Int. Conf. on Magnet Technology, Frascati, 1975.
4. B.C. Brown, et al., Proc. 1983 Particle Accelerator Conference (to be published).
5. J. Allinger, et al., Proc. 4th Int. Conf. on Magnet Technology, Brookhaven, 1972.
6. The idea of digitized bucking was first suggested by J.G. Cottingham.

TABLE I. Properties of the tangential coil for measuring dipole magnets.

Winding No.	Length	Radial Position	Turns	Winding Starting Angle	Winding Subtended Angle	Function
1	205.25"	1.510	2	0°	180°	Dipole reference. Bucking of dipole field
2	205.25"	1.510"	2	12°	180°	Bucking of dipole field.
3	205.25"	1.510"	20	90°	12°	Measurement of multipoles for full length magnet
4	30"	1.468"	40	-78°	12°	Measurement of multipoles in center
5	30"	1.455"	4	-54°	60°	Measurement of end multipoles. Location of axis.
6	30"	1.455"	4	-54°	60°	Measurement of end multipoles. Location of axis.

TABLE II. Fractional field coefficients ($\times 10^{-4}$) at 1200 A for CBA dipole LM20.

n	b'_n	a'_n
1	1.2	1.5
2	-28.5	-0.6
3	0.3	0.7
4	1.7	-0.2
5	0.2	-0.1
6	0.6	-0.2
7	0	0.5
8	0	0
9	0	-0.3
10	0.2	0.1
11	0.1	-0.1
12	-0.6	0

Figure 1 Coordinate system for the free aperture of the CBA magnets.

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Figure 2 Schematic layout for windings on dipole tangential coil.

Figure 4 Notation for a tangential coil.

Figure 3 Block diagram of overall magnetic field measurement system.

Figure 5 Flow chart for digital harmonic analysis.

Figure 6 Normal fractional field coefficients vs. magnet current for LM20 dipole.

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