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DYNAMICS OF A SPACE-CHARGE-DOMINATED BEAM

C.M. Celata, I. Haber, L.J. Laslett, L. Smith,
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Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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**Naval Research Laboratory, Washington, D.C. 20375.

THE EFFECT OF INDUCED CHARGE AT BOUNDARIES ON TRANSVERSE DYNAMICS OF A SPACE-CHARGE-DOMINATED BEAM*

C.M. Celata, I. Haber², L.J. Laslett, L. Smith, and M.G. Tiefenback
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

A particle simulation code has been used to study the effect on transverse beam dynamics of charge induced on focusing electrodes. A linear transport system was assumed. The initial particle distribution was taken to be that of a uniform elliptical beam with a Gaussian velocity distribution. For misaligned, highly space-charge-dominated beams (betatron phase advance per lattice period $\leq 10^\circ$), a large oscillation of the rms emittance occurred in a beat pattern. Linearized Vlasov analysis shows the oscillation to be a sextupole oscillation, driven by the beam coherent betatron motion. Emittance growth accompanied the oscillation. Preliminary experimental results from the Single Beam Transport Experiment (SBTE) are consistent with the code results. Addition of a dodecapole nonlinearity to the computational focusing field greatly reduces the oscillation amplitude.

Introduction and Description of the Model

We report here the results of a theoretical study of the transverse dynamics of a space-charge-dominated charged particle beam subject to forces due to charge induced by the beam on conducting boundaries. We find that these nonlinear "image" forces can cause a collective oscillation of the transverse rms emittance. We have investigated this phenomenon using computer simulation, and explained the results analytically.

We assume a single species charged particle beam in a linear alternating-gradient transport system. The beam is assumed to enter the transport channel with uniform density and elliptical cross-section, with major and minor radii matched to the focusing system. The initial velocity distribution is locally Maxwellian with a unique temperature throughout the beam. Four conducting cylinders, whose axes are parallel to the transport channel, are used to simulate the conducting boundary provided by electrostatic focusing electrodes (see Fig. 1). The induced charge on these electrodes causes the image forces which are the subject of study. We assume here values of $R = 24$ mm, $d = 21.00$ mm (see Fig. 1), which is consistent with the absence of any dodecapole nonlinearity in the focusing field.¹ No longitudinal acceleration is applied, and the calculation is non-relativistic.

The computer simulation used is a two dimensional (transverse plane) particle-in-cell simulation. Periodic boundary conditions are applied along the boundary shown as a dashed line in Fig. 1, but the beam is well shielded from its periodic replicas by the electrodes. The quadrupole focusing is applied using a thin lens approximation.

Simulation Results

The results of the PIC code show that with image forces present the shape of the beam is changed slightly, with the sections of the beam edge which are near the electrodes being pulled out by the attractive force. For beams centered in the transport channel there is no other effect of the induced charge, unless the beam fills $> 85\%$ of the aperture. For these large beams, particle loss occurs.²

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Fig. 1 The geometry of the model, showing the transverse plane, with conducting electrodes (present at all z) and the boundary of the simulation grid (dashed).

If the beam is misaligned, however, and the tune is sufficiently depressed, ($a/\sigma_0 \leq 0.1$), where σ and σ_0 are the phase advance per period, with and without the space charge force), an oscillation is seen in the rms emittance, accompanied by a net growth of emittance. These two effects are shown in Fig. 2. The parameters for this figure are: $\sigma_0 = 60^\circ$, $\sigma = 6^\circ$, $a = 11$ mm, where "a" is the beam major radius, and the beam is offset by 2.75 mm at $\theta = 45^\circ$. (Note: The frequency of the oscillations in Fig. 2 appears to be half its true value because, due to computer time restrictions on this long run, the emittance was measured with a low sampling rate.) For the same parameters as these runs, but with $\sigma = 12^\circ$, the effect was negligible.

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Fig. 2 y emittance (normalized to its initial value) vs. z for the parameters given in the text. x emittance is similar.

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²Naval Research Laboratory, Washington, D.C. 20375.

We have studied the oscillation of emittance, and an explanation which is quantitatively consistent with the simulation results will be discussed in the next section. The net growth of emittance which accompanies the oscillation does not occur in the linearized theory shown below, and is assumed to be nonlinear evolution of the collective mode. The net growth increases as the amplitude of the emittance oscillation increases. Study of the scaling of the oscillation amplitude will therefore provide some indication of the amount of emittance growth to be expected. We will now describe the oscillation in more detail, and then proceed to the theoretical explanation.

The oscillation occurs in a beat pattern, with the higher frequency approximately equal to twice the coherent betatron frequency. The amplitudes of the x and y emittance oscillations are approximately equal. The amplitude of the oscillation increases as Δr , a , or σ_0 is increased, or σ or d are decreased. Thus this effect could present a limit to the current density which can be transported without emittance growth. The rms beam radii remain constant during the oscillation, while the moments oscillate at the same frequency as the emittance. The beam is therefore heating, while its shape, except for the minor distortion mentioned above, is unchanged. Phase plots indicate that the multipole mode of the oscillation is an odd mode, i.e., the mode potential is proportional to $\cos m\theta$ with "m" odd. Results are similar for a KV distribution.

Results of Linearized Vlasov Analysis

The above emittance oscillation can be examined analytically for the case of a round uniform beam with a KV velocity distribution in a cylindrical pipe. Assume for simplicity an initial offset in the x direction of magnitude h . Using the smooth approximation we obtain the particle equations of motion:

$$\begin{aligned} \frac{dv_x}{dt} &\approx -\omega_0^2 x + (\omega_0^2 - \omega^2) a^2 \left[\left(\frac{x - \bar{x}}{a^2} \right) + \frac{\frac{b^2}{x} - x}{\left(\frac{b^2}{x} - x \right)^2 + y^2} \right] \\ \frac{dv_y}{dt} &\approx -\omega_0^2 y + (\omega_0^2 - \omega^2) a^2 \left[\frac{y}{a^2} - \frac{y}{\left(\frac{b^2}{x} - x \right)^2 + y^2} \right] \end{aligned} \quad (1)$$

where $\omega_0 = \sigma_0 v_z / (2L)$, b = pipe radius, $2L$ = lattice period length, $\omega = \sigma v_z / (2L)$, the bar signifies an average over particles, and $(\omega_0^2 - \omega^2) a = \omega_p^2 a^2 / 2 = 2Ne^2 / m$, with ω_p = plasma frequency and N = charge per unit longitudinal distance. Averaging over x , substituting the result in Eq. (1), and going into the beam frame so that $x_B = x - \bar{x}$, we have

$$\begin{aligned} \ddot{x}_B &= -\omega^2 x_B + (\omega_0^2 - \omega^2) \frac{a^2}{b^2} \left[\frac{x_B}{b^2} + \frac{3}{b^3} \frac{x_B^2 - y^2}{b} \right] \\ \ddot{y} &= -\omega^2 y - (\omega_0^2 - \omega^2) \frac{a^2}{b^2} \left[\frac{x_B}{b^2} y + 2 \frac{3}{b^4} x_B y \right] \end{aligned} \quad (2)$$

In obtaining Eq. (2) it has been assumed that $\bar{x}^2 / b^2 \ll 1$. This has the effect of dropping higher order multipole terms. We now assume that $\bar{x} = h \cos \omega_c t$, where ω_c is the coherent betatron frequency, $\omega_c^2 = \omega_0^2 - (\omega_0^2 - \omega^2) a^2 / b^2$. We also neglect for simplicity the

correction to the quadrupole term given by the first term in brackets in Eq. (2), though this term is not small. Dropping higher frequency terms proportional to $\cos \omega_c t$, we have

$$\begin{aligned} \ddot{x} &= -\omega^2 x + \frac{3}{4} (\omega_0^2 - \omega^2) \frac{a^2}{b^2} \frac{h^3}{b^3} \frac{x^2 - y^2}{b} \cos \omega_c t \\ \ddot{y} &= -\omega^2 y - \frac{3}{2} (\omega_0^2 - \omega^2) \frac{a^2}{b^2} \frac{h^3}{b^3} \frac{xy}{b} \cos \omega_c t \end{aligned} \quad (3)$$

The subscript "B" has been dropped. We see that the lowest order nonlinear term introduced by the image forces appears in the beam frame as a sextupole potential, $V = [(\omega_0^2 - \omega^2) a^2 h^3 / (4b^6)] r^3 \cos 3\theta$, modulated at the coherent betatron frequency.

We now put the forces of Eq. (3) into the Vlasov equation, using a KV distribution $f_0 = \frac{N_0}{\pi} \delta \left[v_x^2 + v_y^2 - \omega^2 (a^2 - r^2) \right]$. We assume a perturbation of the form $V_1 = V_0(t) \frac{r^3}{a^3} \cos 3\theta$, and linearize, assuming the image force to be small. Integrating over unperturbed orbits gives the perturbed part of the distribution function. The density perturbation is then

$$\begin{aligned} n_1 &= -\frac{3}{2} \frac{N_0}{\omega} \frac{r^3 \cos 3\theta}{a^3} \delta(r^2 - a^2) \int_0^t dt' \left[\frac{eV_0(t')}{m} \right. \\ &\quad \left. - \frac{1}{4} (\omega_0^2 - \omega^2) \frac{a^5 h^3}{b^6} \cos \omega_c t' \right] [\sin \omega(t-t') + \sin 3\omega(t-t')] \end{aligned} \quad (4)$$

We integrate using the method of Laplace transform and use the Poisson equation to obtain the perturbed potential as a function of time:

$$\begin{aligned} \frac{eV_0}{m} &\approx \frac{\omega_p^2}{8} \omega_0^2 \frac{a^5 h^3}{b^6} \frac{\cos \omega_c t - \cos \Omega t}{\Omega^2 - \omega_c^2} \\ &\approx \frac{\omega_p^2}{2} \frac{a^5 h^3}{b^6} \sin \left[\frac{\Omega_+ + \omega_c}{2} t \right] \sin \left[\frac{\Omega_+ - \omega_c}{2} t \right], \end{aligned} \quad (5)$$

where the tune depression has been assumed large, so that $\omega_c / \omega_0^2 \ll 1$, and the resonant frequency, $\Omega_+^2 = 5\omega_c^2 + \omega_0^2 / 4 + \sqrt{\omega_0^4 / 16 + \omega_c^2 \omega_0^2 + 16\omega_c^4} \approx \omega_0^2 + 6\omega_c^2$ is the third order (sextupole mode) frequency.³ Equation (5) shows the beat pattern seen in the simulation results, caused by the driving of the sextupole mode, whose frequency approaches ω_0 from above as $\omega_c^2 / \omega_0^2 \rightarrow 0$ by the image force modulated by the coherent betatron oscillation, whose frequency is slightly lower than ω_0 . Note that the oscillation is stable. The frequencies obtained, $(\Omega_{\pm} \pm \omega_c)/2$, are in good agreement with the simulation results, if it is recalled that the frequency should be doubled for comparison with the emittance. The scaling of the computational results with other parameters agrees fairly well with the theoretical predictions, though the simulation results scale slower with aperture width and σ_0 than this model would predict. This is not surprising, perhaps, since the boundary geometry is different in the analytical model, and quadrupole correction terms were left out.

Addition of a Dodecapole Force

Equation (5) and the simulation results both suggest, as expected, that increasing the aperture size would decrease the emittance growth due to induced charge at the boundaries. If this is done for the electrode geometry shown in Fig. 1, a nonlinear (dodecapole) potential of the form $V \sim r^6 \cos 6\theta$ is added to the focusing field potential. This component is only absent when the electrode radius, R , is such that $R/d = 1.1457$.¹ This potential has been shown to cause emittance growth for misaligned beams.⁴ We find that the addition of the dodecapole force with the image force suppresses emittance growth dramatically, if the dodecapole strength is chosen correctly. Setting $R/d = (0.75)(1.1457)$ works well, decreasing the emittance growth in Fig. 2, for instance, to ~5%, in 200 periods. This appears to be due to the fact that in the beam frame the dodecapole and image forces present sextupole components of opposite sign. Reversing the sign of the dodecapole field will enhance, rather than suppress, emittance growth, thus supporting this hypothesis. This is demonstrated in Fig. 3.

Fig. 3 y emittance (normalized to its initial value) vs. z for the same parameters as Fig. 2, except for the addition of a dodecapole force. Solid line has dodecapole for $R/d = 0.744$; dashed has sign of dodecapole reversed.

Experimental Results

Preliminary experimental results from the Single Beam Transport Experiment (SBTE) at Lawrence Berkeley Laboratory seem to be qualitatively consistent with the simulation results. SBTE has the geometry of Fig. 1, with 60% lens occupancy. The dodecapole field component is negligible. Figure 4 shows experimental measurements of emittance using contours including either 100% or 95% of the beam current, for beams displaced by 1.2 mm and 1.8 mm. Though the calibration of the measurement from one z location to another is less reliable, the emittance difference between the two beams is accurately measured, to a few percent. This difference grows with z . The magnitude of the growth is approximately what the simulation would give, using the 95% contour, but much more data are required to make an adequate comparison.

Summary

We have observed the effect of image forces on space-charge-dominated non-relativistic beams, using a particle simulation code. For misaligned beams with very

low betatron phase advance per lattice period ($\sigma/\sigma_0 \leq 0.1$), an oscillation of the rms emittance occurs in a beat pattern in both transverse planes. Associated with the oscillation is a net growth of transverse rms emittance. The beam size is constant during this oscillation, while its transverse momentum increases. Analytical solution of the Vlasov equation for a KV distribution, including the image force of a round pipe, indicates that the third order $j = 0$ $m = 3$ mode is being driven in a stable oscillation by the image force. This occurs due to the near-equivalence of the third order mode frequency and the coherent betatron frequency, which is the frequency of oscillation of the image force amplitude for a misaligned beam. Preliminary experimental evidence from the Single Beam Transport Experiment at Lawrence Berkeley Laboratory is consistent with the simulation results.

Since the emittance changes caused by this driven mode may limit the transportable charge density in a beam, it is important to decrease its effect as much as possible. We have found that the addition of a dodecapole force of the proper strength suppresses the mode. This is probably due to a partial cancellation of the sextupole component of the image force in the beam frame by the same component of the dodecapole.

Fig. 4 Experimental measurements of rms x emittance (circled points) for two off-center beams. The upper two curves are computed using contours enclosing 100% of the beam current; the lower two, 95%. $\sigma_0 = 60^\circ$; $\sigma = 8^\circ$.

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