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**Microwave Measurements of the
ETA Accelerating Cavity**

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Microwave Measurements of the ETA Accelerating Cavity*

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Abstract

Microwave measurements of the ETA accelerating cavities have shown eleven resonances in the frequency range of 0 - 850 MHz. These modes have been identified according to their similarity with the modes of a cylindrical cavity. Measurements of the "Q"'s of the modes yielded values from 14 to 70 and transverse shunt impedances ($R_{\perp}/Q$) of 9 to 12Ω . In particular, beam breakup mode (TM_{110}) has a Q of 20 to 40 and a transverse impedance of 9Ω . A later report will describe proposed modifications to the cavity for the ATA accelerator that reduce the Q's substantially.

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Introduction

The current which can be accelerated through any structure is ultimately limited by the interaction of the beam with beam induced fields in the accelerator structure.¹ All linear accelerators are subject to beam instabilities resulting from the coherent interaction of the beam with resonant modes in the accelerating cavities.² Resonant modes with transverse fields in the accelerating gap, "Beam-Breakup Modes", may cause amplification of transverse deflections from one cavity to the next with the coupling provided by the beam.³ In this case the maximum current which can be transported before the beam intercepts the transport tube depends on the accelerator length, voltage gradient, focusing field, etc. and the transverse shunt impedance, resonant frequency, and Q of the responsible resonant modes. Because of its longer length, there is concern that an instability such as BSU may limit ATA to an unacceptably low current.

While RF instabilities are not a limiting factor in the ETA Accelerator the detection of low level RF beam current oscillations and the need to finalize the ATA accelerating cavity design prompted an investigation into RF resonances in the ETA accelerating structure. The theoretical treatment of the interaction of these RF resonances with the accelerated electron beam in an ATA scale accelerator has previously been investigated and will not be discussed here.⁴

In this work we report RF measurements on the ETA Accelerating Cavity. The investigation of Q's and resonant frequencies was confined to a frequency range of 0 - 850 MHz and the determination of transverse shunt impedance to the TM_{110} , TM_{112} and TM_{130} modes.

RF Measurement Techniques

Figure 1 presents a sketch of the ETA accelerating cavity tested. The high voltage feed ports were terminated with the characteristic impedance of the drive lines in order to simulate the actual operating conditions. The cavity was driven by a Wavetech sweeper and its response measured by a Tektronix spectrum analyzer. These instruments were coupled to the cavity through a set of four moveable probes. By sampling the fields at different points and determining their magnitude and direction, a partial map of each mode was constructed. This information was used as an aid in both identifying the mode and determining its transverse shunt impedance.

The B and E field probes mounted in the cavity top plate while fixed in radial position could be moved angularly about the cavity axis by rotating the cavity top plate. A second pair of probes was used to measure the fields in the accelerating gap.

Additional information on the field geometry was obtained by measuring the effects of various dielectric and metallic cylinders inserted into the cavity as a function of their position.

Figure 2 shows an example of how one of these mapping techniques can be used to identify resonant modes. Here the cavity is being driven by the $\hat{B}$ probe mounted on the cavity top plate. The normal axis of this magnetic dipole has been aligned to detect only the radial magnetic field. The angular position of the probe has been chosen to correspond with the predicted peak radial magnetic fields of both the TM_{+120} and TM_{+210} modes.

The oscilloscope traces in Figure 2a shows the response of the electric field as a function of drive frequency and position in the accelerating gap. The individual scope traces measure the electric field amplitude as a function of frequency. The center frequency of each trace is 650 MHz and frequency varies by 20 MHz in each horizontal division. The frequency resolution of the spectrum analyzer was set at 300 kHz. Fig. 2 b&c illustrate roughly the predicted field shape for these two modes.

The oscilloscope traces also point out that these modes possess an additional splitting which would not be present if the cavity were a perfect right circular cylinder. This behaviour should be anticipated as the cavity is most certainly not a perfect circular cylinder. The two component modes of this splitting are denoted α and β . This splitting, while varying in degree, appeared on all of the modes examined.

The information from these different mapping techniques was combined to produce Figure 3. Figure 3 gives the center frequency and tentative identification of the detected cavity resonances in the left column. The right column gives the lowest resonant frequencies of the resonant modes in an ideal right circular cylinder. The dimensions of this cylinder are given at the top right hand side of the figure.

Although this model is very crude, it is also very simple and gives surprisingly good agreement on all but the TM_{+2n0} modes. This encourages one to believe that a prediction of the field geometries based on this model should also be fairly accurate. The type of measurements illustrated in Fig. 2 also confirm this idea. The explicit expressions for the fields of the TM_{mn0} modes in a right circular cylinder are

$$E_z = E_0 J_m(x_{mnp}) \sin(n\theta) e^{-i\omega t}$$

$$H_{\theta} = -i E_0 J'_m \left(\frac{x_{mn} \rho}{R} \right) \sin(n\theta) e^{-i\omega t}$$

$$H_r = \frac{i E_0 n R}{\rho x_{mn}} J_m \left(\frac{x_{mn} \rho}{R} \right) \cos(n\theta) e^{-i\omega t}$$

and the resonant frequencies are given by

$$\nu_{mn\theta} = \frac{c}{2\pi R} x_{mn}$$

where x_{mn} is the n^{th} zero of J_m and R is the cavity radius.

By substituting

$$R_{\text{eff}} = \frac{c}{2\pi} \frac{x_{mn}}{\nu_{mn\theta} \text{Exp}} \quad \text{For } R$$

and P_{eff} for ρ

where $\nu_{mn\theta}$ is the measured resonant frequency and P_{eff} includes the phase shifts induced by the presence of the holes for the transport tube, the above expressions for the fields can be fitted into an even closer correlation with the measured fields. It is these modified expressions which were used to calculate the transverse shunt impedances.

Determination of Transverse Shunt Impedance

Although we have found that there are many modes of oscillation in the ETA Cavity, the resonances of primary concern are of the form TM_{1M0} . These modes have been named the "Beam Breakup Modes" because of the common property of producing a transverse magnetic flux in the transport tube.

The longitudinal electric fields of the TM_{0M0} modes may tend to produce some bunching in the first few accelerator sections, but because the beam impedance in the longitudinal direction increases rapidly with beam energy these oscillations should not propagate.

The longitudinal impedance of an electron beam is given by

$$Z_z = \frac{m_0 c}{n e^2} \frac{z}{\gamma^3}$$

where β_z is the longitudinal velocity divided by the speed of light and

$$\gamma = \left(1 - \beta_z^2\right)^{-1/2}$$

The interaction of the beam with the TM_{1M0} modes is best characterized by the Q, resonant frequency, and the transverse shunt impedance.

The transverse shunt impedance is defined as

$$\frac{R_{\perp}}{Q} = \frac{\left| \int B_{\perp} dz \right|^2}{2 \cdot \omega \cdot \text{Energy Stored}}$$

where $B_{\perp}$ is the average transverse magnetic flux threading across the transport tube.

The RF energy stored in the cavity on resonance is given by

$$\omega = 2\omega_e = \epsilon_0 \overline{\langle \vec{E} | \vec{E} \rangle}$$

for the TM_{1M0} modes this reduces to

$$\omega = \frac{\epsilon_0}{z} \int_0^R \int_0^{2\pi} \int_0^d \langle E_z \rangle^2 \rho d\theta dp dz$$

where $\langle E_z \rangle = E_z(\rho, 0) = \langle E_z(\rho, 0, t) \rangle = \frac{1}{\sqrt{2}} E_{z0}$

Expressions for the fields were obtained by combining experimental results with the predicted fields as described above. These expressions were substituted into the definition for the transverse shunt impedance. The results are presented in Table I. Also, included in Table I are the measured resonant frequencies and Q's for all of the modes investigated.

Effect of Drive Port Termination on Beam Breakup Modes

The ETA Accelerating Cavity possesses four high voltage feed ports. During normal operation, one pair of opposing ports is driven by the blumleins through 25 Ω transmission lines and the other pair is connected to compensating resistive terminations. This is done to minimize the effect of beam loading on the accelerating voltage.

All new ETA Cavities will possess only two drive ports. The compensating loads will be mounted on the transmission lines. For this reason only two feed ports were terminated during these RF tests.

The mode mapping measurements revealed that the $TM_{(+,-)1M0}$ modes align themselves with a field (maximum, minimum) at the drive ports. Figure 5 illustrates the effect of drive port termination on the mode Q's. As anticipated, first there is no effect on the TM_{-1M0} modes and second the Q of the TM_{+1M0} modes is minimized by terminating the port with a matched load. It should also be observed that an open termination results in higher Q's for the TM_{+1M0} mode than the TM_{-1M0} modes. This can be understood by examining Figure 1. The extra extensions on the two ports normally terminated in 25 Ω , produce greater shielding of the mode from the lossy ferrite core in the center of the accelerating cavity.

Summary

RF tests on the ETA Accelerating Cavity have revealed a large number of resonances, but as a result of the large volume of ferrite housed in the cavity the Q's are all quite low. The highest Q measure in the 0 - 900 MHz range was only 70. Comparison of the resonant frequencies of these modes with those of a right circular cylinder showed a good agreement, and this model combined with mapping measurements has been used to calculate what is hopefully an accurate value for the transverse shunt impedance.

Table I: Resonant Mode Parameters

Mode	ν_a (MHz)	ν_B (MHz)	Q	$\frac{R_1}{Q}$
TM ₀₁₀	220	185	14	
TM ₊₁₁₀	350	320	20	8.8 Ω
TM ₋₁₁₀	345	315	40	8.8 Ω
TM ₀₂₀	535	490	34	
TM ₊₁₂₀	610	580	44	10.2 Ω
TM ₋₁₂₀	605	575	50	10.2 Ω
TM ₊₂₁₀	665	630	46	
TM ₋₂₁₀	Unable to resolve			
TM ₀₃₀	770	735	70	
TM ₁₃₀	830	800	56	12.2 Ω
TM ₋₁₃₀	850	830	60	12.2 Ω

Appendix I

The Energy stored in the cavity in TM_{110} mode

$$\omega = 2 \omega_c = \frac{\epsilon_0}{2} \int_0^R \int_0^{2\pi} \int_0^d \langle E_z \rangle^2 \rho d\theta dp dz$$

where $\langle E_z \rangle$ is the time averaged E field

$$\langle E_z \rangle = E_0 J_1 \left(\frac{\chi_{mn} \rho}{R} \right) \sin \theta$$

$$\text{where } E_0 = \frac{E_{\text{peak}}}{1.72}$$

$$2 \omega_e = \frac{\epsilon_0}{2} E_{\text{peak}}^2 \cdot 2.96 \cdot d \int_0^R \int_0^{2\pi} J_1^2 \left(\frac{\chi_{mn} \rho}{R} \right) \rho d \rho \sin^2 \theta d\theta$$

$$= \frac{\epsilon_0}{2} E_{\text{peak}}^2 \cdot 2.96 \cdot d \cdot \pi \int_0^R J_1^2 \left(\frac{\chi_{mn} \rho}{R} \right) \rho d\rho$$

$$= \frac{\epsilon_0}{2} E_{\text{peak}}^2 \cdot 2.96 \cdot d \pi \frac{R^2}{2} \left[J_2(\chi_{mn}) \right]^2$$

$$= \frac{\epsilon_0}{2} E_{\text{peak}}^2 \cdot \text{Volume} \cdot .24$$

$$= \frac{1}{2 \mu_0} B_{\text{peak}}^2 \cdot \text{Volume} \cdot .24 = \frac{B_{\text{peak}}^2}{2 \mu_0} \cdot Vol \cdot f$$

where f is the filling factor

Mode	f
TM_{110}	.24
TM_{120}	.135
TM_{130}	.093

Appendix II

B-Field Enhancement at $R < R_{\text{Transport}}$ Tube in TM_{110} Mode

A B-Field enhancement exists at the transport tube hole which can be calculated from experimental measurements.

Measurement Technique --

The fields of the TM_{110} are

$$E_z = E_0 J_1 \left(\frac{X_{nm}}{R_{\text{eff}}} \rho_{\text{eff}} \right) \sin \theta e^{-i\omega t}$$

$$H_\theta = -i E_0 J_1 \left(\frac{X_{nm}}{R_{\text{eff}}} \rho_{\text{eff}} \right) \sin \theta e^{-i\omega t}$$

$$H_r = -i E_0 \frac{R_{\text{eff}}}{\rho X_{nm}} J_1 \left(\frac{X_{nm}}{R_{\text{eff}}} \rho_{\text{eff}} \right) \cos \theta e^{-i\omega t}$$

$$R_{\text{eff}} = \frac{c X_{nm}}{2\pi \nu_{110}} = 58 \text{ cm}$$

$$\text{at any point } \left| \frac{H_r}{H_\theta} = \frac{R}{\rho X_{nm}} \cdot J_1 \frac{\left(\frac{X_{nm}}{R} \rho_{\text{eff}} \right)}{\frac{X_{nm} \rho_{\text{eff}}}{R}} \cot(\theta) \right|$$

Measurements taken at $\theta = 30^\circ$ and $\rho = 15.25$ cm show $P_{\text{eff}} = 17$ cm meaning a B-Field enhancement in the transport tube of enhancement factor

$$\gamma = \frac{P_{\text{eff transport tube}}}{P_{\text{actual}}} = \frac{9.05}{7.3} = 1.24$$

Mode	Reff	$P_{\text{eff}}(\vec{B}_p)$	Exp. Enhancement
TM ₁₁₀	58	17	1.24
TM ₁₂₀	55	17.5	1.31
TM ₁₃₀	57	18.5	1.45

Why can we use modes for TM and not TEM?

The question reduces to the problem — What is the impedance of the beam compared to frequency space?

$$Z = \frac{dV}{dl} \quad l = nev = nec\beta \quad \therefore dl = necd\beta$$

$$V = \frac{(\text{Energy} - m_0c^2)}{e}$$

$$E = m_0c^2\gamma$$

$$\therefore dV = \frac{dE}{e} = \frac{m_0c^2}{e} d\gamma$$

$$\therefore Z = \frac{m_0c}{ne^2} \frac{d\gamma}{d\beta} = \frac{m_0c}{ne^2} \frac{d}{d\beta} (1 - \beta^2)^{-1/2}$$

$$= \frac{m_0c}{ne^2} \beta\gamma^3 \gg 377\Omega$$

Appendix IV

Calculation of Shunt Impedance,

$$\begin{aligned} \text{Shunt Impedance} = \frac{R_L}{Q} &= \frac{\left| \int_{\text{Thru Cavity Inner opening}} (\vec{\nabla} \times \vec{A}) \cdot \hat{z} \, dz \right|^2}{2 \cdot \omega \cdot \text{Energy Stored}} \quad (\text{cgs}) \\ &= c^2 \frac{\left| \int B_z \, dz \right|^2}{2 \cdot \omega \cdot \text{Energy Stored}} \end{aligned}$$

For the TM_{110} , where in the transport tube $B_z = B^{(1)}_{\text{peak}} \cdot \gamma \quad (2)$

$$\begin{aligned} \frac{R_L}{Q} &= \frac{c^2 B_{\text{peak}}^2 \cdot \gamma^2 \cdot d^2}{2 \cdot \omega \cdot \text{Energy Stored}} \\ &= \frac{c^2 B_{\text{peak}}^2 \gamma^2 d^2}{2 \cdot \omega \cdot \frac{1}{2\mu_0} B_{\text{peak}}^2 \cdot \text{Volume}_{\text{Eff}} \cdot f^{(4)}} \quad (= .24 \text{ for } TM_{110}) \\ &= \frac{\mu_0}{.24} \frac{c^2 \gamma^2 d^2}{V_{\text{eff}} \cdot \omega} = \frac{4\pi \cdot 10^{-7}}{.24} \cdot \frac{(3 \cdot 10^8)^2 \cdot (1.24)^2 \cdot (2.54 \cdot 10^{-2})^2}{\pi \cdot (.58)^2 \cdot 3.15 \cdot 10^8 \cdot 2\pi} \\ &= \underline{8.8 \, \Omega} \end{aligned}$$

Mode	Reff	f	$\gamma (= B \text{ enhanc})$	R_L/Q
TM_{110}	58	.24	1.24	8.8 Ω
TM_{120}	55	.135	1.31	10.2 Ω
TM_{130}	57	.093	1.45	12.2 Ω

(4) Filling factor is calculated in A-I

(3) d = Accelerating Gap Width

(1) $B_{\text{peak}} = B_{\text{peak } F_n}$ the TM_{110} in an equivalent perfect RCC

(2) γ = Enhancement factor; the net flux threading across the transport tube is greater than the flux would be without the hole there. See A-II.

Appendix V

Calculation of transverse shunt impedance for the TM_{110} mode, based on metal bead measurements.

The average magnetic field in a region of space can be determined by displacing the fields in that region and measuring the frequency shift. Here we describe the results on inserting an aluminum rod down the center of an ETA Cavity.

For the TM_{110} mode the electric field in the center can be neglected. The displacement of the magnetic fields by the rod causes a frequency shift given by

$$\begin{aligned} \frac{\Delta(h\nu)}{h\nu} &= \frac{\Delta\nu}{\nu} = \frac{1}{2\mu_0} \frac{\int_{\text{Displaced Region}} B^2 d^3x}{\omega_{e \text{ total}} + \omega_{B \text{ total}}} \\ &= \frac{\int_{\text{Region}} B^2 d^3x}{2 \int_{\text{cavity}} B^2 d^3x} \\ &= \frac{B^2_{\text{region}} \cdot n \cdot \Delta \text{ Volume}}{2 \cdot B^2_{\text{ave}} \cdot \text{Volume}_{\text{Total}}} \end{aligned}$$

where n is the demagnetizing ratio;

for a long cylinder $n = .5$

The Aluminum Rod was 4" in diameter and the effective radius of the cavity for the TM_{110} mode is 58 cm.

$$\therefore \frac{\Delta \text{ Volume}}{V_{\text{eff}}} = \frac{R_{\text{cylinder}}^2}{R_{\text{eff}}^2} = \left(\frac{5.08 \text{ cm}}{58 \text{ in}} \right)^2 = .77 \cdot 10^{-2}$$

and it follows that

$$\frac{B^2_{\text{region}}}{B^2_{\text{ave}}} = \frac{2 \cdot \text{Vol eff}}{n \cdot \Delta \text{Volume}} \cdot \frac{\Delta v}{v} = 5.21 \cdot 10^2 \frac{\Delta v}{v}$$

A frequency shift of 4 MHz was measured upon inserting the bar for the TM_{110} mode with a normal center frequency of 305 MHz.

$$\therefore \frac{B_{\text{center}}}{B_{\text{ave}}} = 6.84$$

$$\text{and } \frac{R_s}{Q} = \frac{\left(\int B_z dz \right)^2}{2 \omega \cdot \text{Energy Stored}} = \underline{9.35 \Omega}$$

Fig. 1 Sketch of Configuration Tested

Fig. 2a A Sample E-Field Map of the TM_{+210} and TM_{+120}
 S-A Pictures of $B(r)$ at 650 MHz Center Frequency
 20 MHz/ Horiz. Division

Fig. 2b

Center freq: 540 MHz
Scale: 20 MHz/div

(α, β)
TM+120 $\nu_o = 610 \text{ MHz } Q_o \approx 23$
580^(\beta) MHz

Fig. 2c

Center freq: 650 MHz
Scale: 20 MHz/div

TM+210 $\nu_o = 665^{(\alpha)} \text{ MHz } Q_o \approx 23$
630^(\beta) MHz

Fig. 3 Experimental Measurements Theory
Energy Levels of ETA Cavity

Fig. 4 The Effect of Drive Port Termination on the TM_{140} Modes Asymptotic Behaviour Shown is Derived from $R_{LOAD} = 0.0$ (not shown;)

TM010 TM010

Center freq: 200 MHz
Scale: 20 MHz/div

TM010 $\nu_o = 220^{(\alpha)} \text{ MHz}$ $Q_o \approx 7$
 $185^{(\beta)} \text{ MHz}$

TM020 TM020

Center freq: 540 MHz
Scale: 20 MHz

TM020 $\nu_o = 535^{(\alpha)} \text{ MHz}$ $Q_o \approx 17$
 $490^{(\beta)} \text{ MHz}$

TM030 TM030

Center freq: 750 MHz
Scale: 20 MHz/div

TM030 $\nu_o = 770^{(\alpha)} \text{ MHz}$ $Q_o \approx 35$
 $735^{(\beta)} \text{ MHz}$

TM+110 β α

Center freq: 350 MHz
Scale: 20 MHz/div

(α, β)
TM+110 $\nu_o = 350^{(\alpha)} \text{ MHz } Q_o \approx 10$
 $320^{(\beta)} \text{ MHz}$

TM-110 β α

Center freq: 350 MHz
Scale: 20 MHz/div

(α, β)
TM-110 $\nu_o = 345^{(\alpha)} \text{ MHz } Q_o \approx 20$
 $315^{(\beta)} \text{ MHz}$

Center freq: 543 MHz
Scale: 20 MHz/div

(α, β)
TM+120 $\nu_o = 610^{(\alpha)}$ MHz $Q_o \approx 22$
580 $^{(\beta)}$ MHz

Center freq: 540 MHz
Scale: 20 MHz/div

(α, β)
TM-120 $\nu_o = 605^{(\alpha)}$ MHz $Q_o \approx 25$
575 $^{(\beta)}$ MHz

TM+210 β α

Center frequency: 650 MHz
Scale: 20 MHz/div

$$\text{TM+210 } \nu_o = 665^{(\alpha)} \text{ MHz } Q_o \approx 23$$

$$630^{(\beta)} \text{ MHz}$$

TM-210 As yet unable to resolve
test configuration gives
too much overlap with
TM030

Center freq: 750 MHz
Scale: 50 MHz/div

(α, β)
 T+130 $\nu_o = 830^{(\alpha)}$ MHz $Q_o \approx 28$
 $800^{(\beta)}$ MHz

Center freq: 830 MHz
Scale: 20 MHz/div

(α, β)
 TM-130 $\nu_o = 850^{(\alpha)}$ MHz $Q_o \approx 30$
 $830^{(\beta)}$ MHz

Appendix VII

Experimental Results: ETA Cavity $B(r)$ on Center for TM_{110} Mode

TM-110 $B(r)$ in transport tube

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