Polarization Characteristics of Synchrotron Radiation Sources
And a New Two Undulator System*

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Polarization Properties of Synchrotron Radiation Sources

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Abstract

The polarization properties of the usual synchrotron radiation sources are briefly reviewed. The recently proposed two undulator system can provides synchrotron radiation with arbitrarily adjustable polarization. The characteristics of this device both as a source of spontaneous radiation and also as a free electron laser system are discussed.

I. Introduction

With the advent of insertion devices [1] based on the permanent magnet technology, the available synchrotron radiation sources have become increasingly intense. However, the polarization capability of the usual synchrotron radiation sources, bending magnets, wigglers and undulators, has been rather limited; although these sources provide a highly polarized radiation, the polarization state can not be changed easily.

The recently proposed two undulator system [2,3] opens a new possibility because the polarization can be adjusted arbitrarily and rapidly. The magnetic structure of this system is similar to that of the optical klystron [4].

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except that the two undulators are placed perpendicular rather than parallel
to each other. If the structure is placed in an optical resonator, the sys-
tem operates as a free electron laser [5].

In Section II the polarization characteristics of the usual synchrotron
radiation sources are briefly reviewed. Section III discusses the spontane­
ous radiation from the new system. Finally Section IV discusses free elec­
tron laser operation.

II. Polarization Characteristics of the Usual Radiation Sources

a) Bends

It is well known that electrons passing through bending magnets
generate linearly polarized radiation in the plane of the electron orbit.
If the radiation is observed at an angle $\psi$ with respect to the orbit plane,
it is elliptically polarized with the polarization vector [6]

$$\mathbf{E}_{\text{BEND}} \propto K_{2/3}(\xi) \hat{x} + i \frac{\gamma \psi}{\sqrt{1+(\gamma \psi)^2}} K_{1/3}(\xi) \hat{y}.$$  \hspace{1cm} (1)

In the above, $\hat{x}$ is a unit vector in the orbit plane, $\hat{y}$ is the unit vector
along the vertical direction, $\gamma = \text{electron energy/electron rest energy}$, $K$'s
are the modified Bessel's function and

$$\xi = \frac{1}{2} \frac{\omega}{\omega_c} \left(1 + \gamma^2 \psi^2\right)^{3/2}, \quad \omega_c = \frac{3}{2} \frac{\gamma^3 c}{\rho}.$$  \hspace{1cm}

Here $\omega$ is the angular frequency of the observed radiation, $\omega_c$ is the
critical frequency, $c$ is the velocity of light and $\rho$ is the radius of
curvature of the electron trajectory. The sign in Eq. (1) depends on the magnet polarity.

When the magnitudes of the vertical and horizontal components become equal, the polarization becomes circular. A group in BESSY [7] utilizes this property to obtain circularly polarized radiation from the bending magnet source. However, the radiation intensity will decrease as $\psi$ increases.

b) Wigglers

An electron passing through a wiggler experiences a sequence of bending forces in the alternating directions. The radiation emitted in the bending plane is linear as in the bending magnet case. Away from the bending plane, however, the radiation is linearly rather than elliptically polarized. This fact is often not appreciated and is due to the fact that the radiation from one bending section interferes with the radiation from the next bending section with the opposite polarity. The polarization vector is given by

$$\xi_{\text{WIG}} \propto K_{2/3}(\xi) \cos \phi \hat{x} \pm \frac{\gamma \psi}{\sqrt{1+\gamma^2 \psi^2}} K_{1/3}(\xi) \sin \phi \hat{y}. \quad (2)$$

Here

$$\phi = k \int (1 - \vec{v} \cdot \vec{n}) \, dz, \quad (3)$$

where $k = \omega/c$, $\vec{v} = \text{electron velocity}$ and $\vec{n}$ is the direction of the observation. The integral extends from one bending section to the next.
c) Undulators

The polarization from a planar undulator is similar to the case of the planar wigglers discussed in the above, and has been studied in detail in the literature [8].

Helical undulators produce circularly polarized radiation. K. Halbach has proposed a design of helical undulator [9] using permanent magnet blocks.

III. A New Two Undulator System

A new type of insertion device proposed recently [2,3] has the capability of modulating the polarization rapidly. A schematic of the system is shown in Fig. 1. It consists of two identical parts, one of which is rotated 90° relative to the other. Each part consists of a planar $N$ period undulator with period length $\lambda_u$ and an additional magnet shown as the shaded block. The latter magnet serves the function of modulating the polarization and also of introducing a strong dispersion for free electron laser operation. This magnet will be called the dispersion-modulation magnet. It can be considered as a variable field single period undulator of length $\lambda$.

Radiation from electrons travelling through this device has the following polarization vector:

$$\xi = \frac{1}{\sqrt{2}} (\hat{x} + e^{i\alpha} \hat{y}),$$  (4)

where

$$\alpha = 2\pi \frac{\gamma_r^2}{\gamma} (N + N_d)$$
\[ N_d = \lambda_M (1 + K_M^2/2) + D, \quad \gamma_r^2 = (1 + K^2/2) \frac{\lambda_u}{2\lambda}. \] (5)

Here \( D = D_1 + D_2 \), and \( K \) and \( K_M \) are the deflection parameters of the undulator and the dispersion-modulation magnet. The deflection parameter is 0.934 times the peak magnetic field in tesla times the period length in cm. One is interested in frequencies near the first harmonic of the undulator spectrum at which \( \gamma = \gamma_r \).

Equation (4) corresponds to the general case of elliptically polarized light. As \( \alpha \) varies between 0 and \( 2\pi \), the polarization ellipse changes as shown in Fig. 2. Therefore one can obtain any desired polarization by adjusting \( \alpha \).

The magnet arrangement shown in Fig. 1 is similar to the magnetic structure of the optical klystron [4] except that the two undulators are perpendicular rather than parallel to each other.

The phase \( \alpha \) can be changed by either of the following two methods: First, the distance \( D \) can be changed mechanically. This method has the advantage that the radiation intensity remains constant during the modulation. However, the modulation rate may not be greater than \( \sim 1 \) Hz. Alternatively, the magnetic field of the dispersion-modulation magnet can be changed. This can be done very rapidly, \( > 1 \) kHz, but the intensity will also modulate. By choosing the distance \( D_1 \) properly, the intensity modulation can be minimized [3].

The angular spread \( \sigma_\theta \) of the electron beam and the finite resolution \( \Delta \omega \) limits the attainable degree of polarization. This effect is studied in ref. (3).

Table I gives parameters of two example systems, one based on the VUV Ring at NSLS and another on a future storage ring such as that proposed by
LBL [10] or SSRL [11]. These examples shows that the system is a very promising polarized source.

IV. Free Electron Laser with the Two Undulator System

If the magnetic structure discussed in the previous section is placed in an optical resonator, the system will operate as a free electron laser. The system is studied in the small signal regime [12], and it is found that the gain of the system is very similar to that of the optical klystron.

It was also found, perhaps surprisingly, that the polarization of the laser radiation at maximum gain differs from that of the spontaneous radiation. The polarization vector of the laser radiation $\mathbf{e}_0$ is given by

$$
\mathbf{e}_0 = \frac{1}{\sqrt{2}} \left( \hat{x} + e^{i\alpha_0} \hat{y} \right). \tag{6}
$$

The small signal gain is proportional to

$$
- \frac{d}{dx} \left( \frac{\sin x/2}{x} \right)^2 \left( 1 + \cos(\alpha - \alpha_0) \right), \tag{7}
$$

where $x = 4\pi N\gamma / \gamma_r$. This result is very similar to the case of the optical klystron. In fact the gain of the latter can be obtained by replacing the factor $1 + \cos(\alpha - \alpha_0)$ in (7) by $2(1 + \cos \alpha)$. Therefore the gain of the present system can be made large by making the ratio $N_d/N$ large.

The maximum gain occurs at $x = 0$ and $\alpha = \alpha_0 = -\pi/2$ in the strong dispersion limit $N_d/N \rightarrow \infty$. In the other limit of weak dispersion, the maximum occurs at $x = 1.02$ and $\alpha = \alpha_0 = -1.4$. Thus the polarization
phase $\alpha$ of the spontaneous radiation and $\alpha$ of the laser radiation differ by $-\pi/2$ at maximum gain. This means, for example, that the polarization of the laser radiation at maximum gain is circular when the polarization of the spontaneous radiation is linear.

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References


Figure Captions

Fig. 1. Schematic of the system.

Fig. 2. Evolution of the polarization ellipse as $\alpha$ increases from $-\pi/2$ to $5\pi/4$. The $\alpha$ values are shown below the ellipses.
Figure 2
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