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by substituting the granular temperature from Equation (9-284) into the shear given by Equation (9-283). The result is

$$\frac{P_{xy}}{\rho_p d_p^2 \left(\frac{dU_x}{dy} \right)^2} = \frac{4\sqrt{15}}{75\sqrt{\pi}} \epsilon_s^2 g_o \frac{1+e}{(1-e)^{\frac{1}{2}}} \quad (9-304)$$

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APPLICATIONS OF KINETIC THEORY

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Predictive Models of Circulating Fluidized Bed Combustors

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**D. Gidaspow, Principal Investigator
Department of Chemical Engineering
Illinois Institute of Technology
Chicago, IL 60616**

ABSTRACT

The overall objective of this investigation is to develop experimentally verified models for circulating fluidized bed (CFB) combustors.

This report presents the author's derivation of analytical solutions useful in understanding the operation of a CFB. The report is in a form of a chapter that reviews the kinetic theory applications.

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CHAPTER 9 - PART II

APPLICATIONS OF KINETIC THEORY

by

Dimitri Gidaspow

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APPLICATIONS OF KINETIC THEORY

19. GRANULAR SHEAR FLOW

The simplest solution to the Navier-Stokes' equation for a fluid is that of shear or Couette flow (Schlichting, 1966). It is assumed that the flow is steady, incompressible and fully developed. The latter condition insures that the velocity is nonzero in one direction, say in the x direction only. Then the continuity equation with constant porosity shows that the gradient of velocity is zero. Hence all accelerations vanish and the momentum balance for granular flow is simply

$$\nabla \cdot \bar{\bar{P}} = 0 \quad (9 - 273)$$

where $\bar{\bar{P}}$ is given by Equation (9-250). Furthermore, analogous to the fluid pressure one assumes the solids pressure in the x direction to be constant. The momentum balance then assumes the simple form

$$\mu_s \frac{du_s}{dy} = \text{constant} \quad (9 - 274)$$

In the simple shear flow one assumes a no slip boundary condition at one wall. The second wall is assumed to move with velocity U . With the fluid adhering to the wall the boundary conditions are

$$\text{at } y = 0, u_s = 0 \text{ and at } y = h, u_s = U \quad (9 - 275)$$

The velocity profile is then

$$u_s = Uy/h \quad (9 - 275)$$

Although Equation (9-274) does not permit a direct determination of viscosity as is done in the Poiseuille viscometer in which the pressure in Equation (9-273) varies with length of flow, it turns out that the granular temperature balance, Equation (9-211) permits one to estimate the viscosity, as developed below.

19a. DISSIPATION-PRODUCTION BALANCE

With the same assumptions made for the simple shear flow discussed in the previous paragraph, the granular temperature Equation (9-211) with no production due to fluid turbulence or molecular fluid collisions is as follows.

$$\bar{P} : \nabla \vec{v}_s = \mu_s \left(\frac{\partial u_s}{\partial y} \right)^2 = \gamma + 3\beta_a \Theta + \gamma_w \quad (9 - 276)$$

$$\begin{array}{l} \text{PRODUCTION OF} \\ \text{OSCILLATIONS BY} \\ \text{SURFACE FORCES} \end{array} = \begin{array}{l} \text{DISSIPATION} \\ \text{DUE TO} \\ \text{INELASTIC} \\ \text{COLLISIONS} \end{array} + \begin{array}{l} \text{DISSIPATION} \\ \text{IN THE} \\ \text{FLUID} \end{array} + \begin{array}{l} \text{DISSIPATION} \\ \text{BY THE WALL} \end{array} \quad (9 - 276a)$$

In obtaining the dissipation by the wall in Equation (2-210) it was assumed that the granular heat flux in Equation (9-211) is a constant and that the granular heat flow was developed, as is done in the thermal forced convection problems. For an elastic particle collision with a non-vibrating wall the dissipation by the wall vanishes. In Equation (9-276) γ is the symbol used for the collisional source contribution defined below

$$\gamma = N_c \left(\frac{m}{2} C^2 \right) \quad (9 - 277)$$

where N_c was given by Equation (9-192). Although the value of this integral was given previously by Equation (9-213) it is instructive to present an approximate derivation of this integral. Physically the integral given by Equation (9-192) can be regarded as the product of the loss of the kinetic energy of the particles per collision

$$\Delta\psi = \Delta\left(\frac{1}{2}mC^2\right) \quad (9-278)$$

and a mean binary collision frequency given by Equation (9-51). As a very rough approximation we can assume that this collision frequency is independent of C^2 in Equation (9-278). Then it is given by Equation (9-63), while the change in kinetic energy is given by Equation (9-45). In terms of particle diameter d_p and the granular temperature Θ , the latter becomes as follows.

$$\Delta E = \frac{\pi}{4}(e^2 - 1)\rho_p d_p^3 \Theta \quad (9-279)$$

Multiplication of ΔE above by N_{12} from Equation (9-63) gives an approximate average value of γ that is three times the value given by Equation (9-213).

The use of the exact value obtained from Equation (9-213) gives

$$\gamma = \Delta E_{AV} \cdot N_{12} = 12\epsilon_s^2 \rho_p g_o (e^2 - 1) \left(\frac{\Theta}{d_p}\right) \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} \quad (9-280)$$

Equation (9-280) shows that the dissipation is proportional to the cube of the random velocity and to the square of the particle concentration. The latter dependence arose due to the assumptions that the binary probability of collision is the product of the single frequency distributions, the so called "chaos" assumption. The dissipation also strongly depends upon the empirically based value of g_o which depends upon ϵ_s . Furthermore, the restitution coefficient is not a constant but depends upon the collision velocity and the fluid medium.

Shen and Ackerman (1982) in analyzing the energy dissipation due to the collisions, ΔE given by Equation (9-279), also include a dissipation due to a change of

velocity in a direction perpendicular to the normal direction. This gives rise to a coefficient of kinetic friction. Using such a concept, Equation (9-280) for the dissipation γ would include the restitution coefficient and the kinetic friction. For rough spherical particles, Shen and Ackerman's (1982) kinetic friction should be related to the tangential restitution coefficient of Araki and Tremaine (1986). The latter authors consider particle translation and rotation. They have extended the analysis presented in Chapman and Cowling's chapter eleven (1961) for rough spherical molecules to include kinetic energy losses due to normal and tangential restitution coefficients.

The dissipation due to the presence of a fluid is given by the product of the friction coefficient and the random velocity which is expressed in terms of the granular temperature in Equation (9-276). For low particle Reynolds numbers and dense suspensions the friction coefficient is obtained from the first term in the Ergun equation. It becomes as follows.

$$\beta_A = \frac{18\mu_f \epsilon_s}{d_p^2} \quad (9 - 281)$$

It involves the fluid viscosity, μ_f and a reciprocal of a permeability.

19 b. GRANULAR TEMPERATURE FOR DENSE FLOW

For a dense suspension in a fluid of a small viscosity, the production- dissipation balance, Equation (9-276) reduces itself to

$$P_{xy} \left(\frac{\partial u_s}{\partial y} \right) = \gamma \quad (9 - 282)$$

The shear P_{xy} is a component of the general stress given by Equation (9- 250). For the dense suspension or for a Maxwellian velocity distribution it is

$$P_{xy} = \frac{4}{5} \epsilon_s^2 \rho_p d_p g_o (1 + e) \left(\frac{\Theta}{\pi} \right)^{\frac{1}{2}} \left(\frac{\partial U_s}{\partial y} \right) \quad (9-283)$$

The use of γ given by Equation (9-280) gives an equation for the granular temperature, Θ ,

$$(1 - e)\Theta = \frac{1}{15} \left(\frac{\partial U_s}{\partial y} \right)^2 d_p^2 \quad (9-284)$$

or for the random velocity, $\langle C^2 \rangle^{\frac{1}{2}}$,

$$\sqrt{1 - e} \langle C^2 \rangle^{\frac{1}{2}} = \frac{1}{\sqrt{5}} \left| \frac{dU_s}{dy} \right| d_p \quad (9-285)$$

Equation (9-285) gives rise to the dimensionless group R introduced by Savage, (1983)

$$R = \frac{d_p \left| \frac{dU_s}{dy} \right|}{\langle C^2 \rangle^{\frac{1}{2}}} = \frac{\text{particle diameter} \cdot \text{shear rate}}{\text{random velocity}} \quad (9-286)$$

Equation (9-285) shows that the value of R depends only upon the restitution coefficient, as shown below.

$$R = \sqrt{5(1 - e)} \quad (9-287)$$

This value of R differs from Savage's (1983) value given by his Equation (4.4) because he used an earlier version of his kinetic theory.

An equation similar to Equation (9-285) was obtained by Shen and Ackerman (1982). It involves their kinetic friction coefficient, as well as the drag term from Equation (9-276).

To obtain kinetic coefficients, such as viscosity and conductivity, the granular temperature must be known. However, Equation (9-285) makes it clear that the random velocity cannot be estimated without a knowledge of the restitution coefficient. The example below shows the problem.

For mm size particle diameters used by Savage and shear rates as high as 10^3 sec^{-1} with a restitution coefficient of 0.9 used by Savage, we have using Equation (9-285),

$$\langle C^2 \rangle^{\frac{1}{2}} = \frac{1}{\sqrt{0.5}} \cdot 10^3 \text{ sec}^{-1} \times 10^{-3} \text{ m} \approx 1 \text{ m/sec} \quad (9 - 288)$$

Such oscillations are quite reasonable in view of hopper, critical flow rates and Carlos and Richardson's (1968) direct measurements of oscillations in a fluidized bed, as simulated by Gidaspow, et al (1991). However, for flow of 100 micron particles in a vertical pipe where shear rates are of the order of 10^2 sec^{-1} , we have

$$\sqrt{1 - e} \langle C^2 \rangle^{\frac{1}{2}} = \frac{1}{\sqrt{5}} 10^2 \text{ sec}^{-1} \times 10^{-4} \text{ m} \quad (9 - 289)$$

For an oscillation velocity of one meter per second, we have roughly

$$1 - e \approx 10^{-4} \quad (9 - 290)$$

In this theory the restitution coefficient must be much closer to one to obtain reasonable values of viscosities. Computations of a restitution coefficient from an analysis of a collision of two slightly non-elastic spheres (Kurabara and Kono, 1987) shows that the restitution coefficient approaches unity as the velocity of collision is reduced.

19C. GRANULAR TEMPERATURE FOR DILUTE FLOW

The use of the dilute limit viscosity given by Equation (9-183) in the balance given by Equation (9-276), and neglecting the dissipation due to the walls and the fluid, with γ given by Equation (9-280) produces an expression for the granular temperature shown below.

$$12\epsilon_s^2(1 - e^2)\Theta = \left(\frac{5\pi}{96}\right)\left(\frac{\partial U_s}{\partial y}\right)^2 d_p^2 \quad (9 - 291)$$

This expression is similar to Equation (9-284). The essential difference is the presence of the square of the particle concentration that multiplies the granular temperature. Since the right hand side of Equation (9-291) is finite as the particle concentration approaches zero the granular temperature approaches infinity. This apparently strange behavior is present throughout the theory of Lun, et al. (1984). It also occurs in the molecular dynamics computer simulations of Campbell and Brennan (1983) and Walton and Braun (1986).

To clarify the behavior exhibited by Equation (9-291), this equation can be rewritten in terms of the mean free path given by Equation (9-72). The mean free path, except for the constant in Equation (9-72), is a ratio of the particle diameter to the volume fraction of the solid. Hence (9-291) becomes

$$(1 - e^2)^{\frac{1}{2}}\Theta^{\frac{1}{2}} = \left(\frac{5\pi}{18}\right)^{\frac{1}{2}}\ell \cdot \left(\frac{\partial U_s}{\partial y}\right) \quad (9 - 292)$$

$$\frac{\text{RANDOM}}{\text{VELOCITY}} = \frac{\text{MEAN FREE}}{\text{PATH}} \cdot \frac{\text{SHEAR}}{\text{RATE}} \quad (9 - 292a)$$

The highest mean free path that a particle can achieve equals the diameter of its containing vessel. This restricts the highest value that can be achieved by the

granular temperature. Hence the mathematical singularity in the dilute limit in Equation (9-291) does not occur. Equation (9-291) should be restricted by the inequality

$$\text{For } \ell = \frac{1}{6\sqrt{2}} \left(\frac{d_p}{\epsilon_s} \right) \geq D_{Pipe}, \quad \ell = D_{Pipe} \quad (9-293)$$

Sinclair and Jackson (1989) use such a concept for modeling developed riser flow. Equation (9-285) for the random velocity for dense flow is then essentially the same equation as the dilute flow Equation (9-292). In (9-285) the mean free path is the particle diameter divided by a constant, since the porosity is about one half.

Then the constant shear viscosity for this dilute limit, using Equation (9-183), is as follows

$$\mu_s = \frac{5\pi\sqrt{5}}{288\sqrt{2}} \rho_p d_p \ell \left(\frac{\partial U_s}{\partial y} \right) \frac{1}{\sqrt{1-e^2}} \quad (9-294)$$

For a constant shear the viscosity increases as the particle concentration decreases or as the mean free path between the particles increases. The viscosity is also non-Newtonian.

In the presence of a fluid, there is also dissipation of oscillations by the fluid, as given by Equation (9-276). Consider the limiting case of no dissipation between the particles, that is restitution coefficient of unity, then using Equation (9-281) the expression for granular temperature becomes as follows.

$$\Theta^{\frac{1}{2}} = \frac{5}{864\sqrt{\pi}} \left(\frac{\rho_p}{\mu_f n} \right) \left(\frac{\partial U_s}{\partial y} \right)^2 \quad (9-295)$$

Again the granular temperature is high for dilute flow, that is for small number density of particles, n . As expected, the particle oscillations become small in a fluid

of a high viscosity, μ_f . The unexpected result is that the particle viscosity will vary with the square of the shear gradient. Hence the shear in such a situation will vary with the cube of the shear rate. Again, in Eq. (9-295) the number density n cannot be too low. Below some low value of n , the dissipation by the wall, as given by Equation (9-276) becomes dominant. In such a situation the granular temperature is finite for a given shear due to the finite γ_w in (9-276). Hence in a complete analysis of dilute transport dissipation by the wall must be considered.

19d. TRANSITION FROM VISCOUS TO GRAIN INERTIA REGIME

For dilute viscous flow of elastic particles Savage's dimensionless R given by Equation (9-286) can be evaluated by substituting Equation (9-295) for the oscillation velocity. The result is

$$R = \frac{5184\epsilon_s}{5\sqrt{3}\pi} \frac{\mu_f}{\rho_p \left| \frac{dU_x}{dy} \right| d_p^2} \quad (9 - 296)$$

It is a ratio of the fluid viscosity to a particular phase viscosity due to collisions. Except for the concentration dependence its inverse is the dimensionless shear strain group found by Bagnold (1954).

$$N_{Bagnold} = \frac{\rho_p d_p^2 \left| \frac{dU_x}{dy} \right|}{\mu} = \frac{\text{inertial stress}}{\text{viscous stress}} \quad (9 - 297)$$

Equation (9-297) suggests that for a sufficiently small $N_{Bagnold}$, fluid shear will dominate the stresses, while for a large, $N_{Bagnold}$ grain inertia will determine the flow processes. $N_{Bagnold}$ can also be looked at as a type of a Reynolds number. In his review Savage (1984) has also included a concentration effect into the definition of this dimensionless group.

Bagnold (1954) constructed an annular shear cell in which he determined the shear by measuring the force exerted on a stationary wall and determined the normal force of neutrally buoyant millimeter size particles suspended in the annular compartment by measuring the fluid pressure. The relations he found were as follows.

$$\text{shear} = \text{constant } (\lambda d_p)^2 \left(\frac{dU_s}{dy} \right)^2 \quad (9 - 298)$$

for large $\frac{dU_s}{dy}$

$$\text{shear} = \text{constant } \lambda^{\frac{3}{2}} \mu_f \frac{dU_s}{dy} \quad (9 - 299)$$

for small $\frac{dU_s}{dy}$

The grain concentration λ is defined as the ratio of grain diameter to the distance between the grains, s , as shown below

$$\lambda = \frac{d_p}{s} \quad (9 - 300)$$

It is related to the radial distribution function g_o by means of the relation

$$g_o = 1 + \lambda \quad (9 - 301)$$

Hence the function g_o can be interpreted as the dimensionless distance between spheres, as shown below

$$g_o = \left[1 - \left(\frac{\epsilon_s}{\epsilon_{s, \max}} \right)^{\frac{1}{3}} \right]^{-1} = \frac{s + d_p}{s} \quad (9 - 302)$$

For uniform spheres, $\epsilon_{s,max} = \pi/3\sqrt{2} = 0.7405$. Bagnold also found that the ratio of the shear to the normal force per unit area, the solids pressure, was constant in the grain inertia regime and equal to about 0.3.

19e. GRAIN INERTIA REGIME

The Bagnold number, Equation (9-297) shows that we will always be in the grain inertia regime for suspensions of particles in air, since the gas viscosity is very low. Hence Savage (1983) and students had constructed an annular shear cell for studying shear and solids pressure for particles suspended in air. Their cell was similar to that of Bagnold (1953), except that the height of the annular compartment could be varied; while the particles were rotated, by putting various weights on top of the cell. The weights gave them a direct measure of the normal pressure. The volume fraction of particles was determined from the initial weight of the particles and the height of the compartment during rotation. Only the average value of the volume fraction of solids could be determined. The shear was measured by means of a spring balance. The shear rate was determined as in Bagnold's experiment by means of the formula

$$\frac{dU_s}{dy} = \frac{R_m w}{\delta R} \quad (9-303)$$

where R_m is the mean radius of the assembly, w is the rotational speed and δR is the gap width.

Figures 9.5 and 9.6 show the measured dimensionless shear and pressure, respectively, and a comparison to various versions of the kinetic theory. Ding and Gidaspow (1990) theory only considered the dense regime, similarly to that discussed by Savage (1985). For this situation the dimensionless shear can be obtained

by substituting the granular temperature from Equation (9-284) into the shear given by Equation (9-283). The result is

$$\frac{P_{xy}}{\rho_p d_p^2 \left(\frac{dU_s}{dy}\right)^2} = \frac{4\sqrt{15}}{75\sqrt{\pi}} \epsilon_s^2 g_o \frac{1+e}{(1-e)^{\frac{1}{2}}} \quad (9-304)$$

Similarly, the pressure is obtained by substituting the granular temperature from Equation (9-284) into Equation (9-230).

$$\frac{P_s}{\rho_p d_p^2 \left(\frac{dU_s}{dy}\right)^2} = \frac{\epsilon_s}{15(1-e)} [1 + 2\epsilon_s g_o (1+e) \rho_p] \quad (9-305)$$

The data of Savage and Sayed (1984) obtained from the shear cell for millimeter size particles are close to the values computed from Equations (9-304) and (9-305) using restitution coefficients of 0.8 or 0.9. Shen and Ackerman's (1982) expressions give much lower shear and pressures because they did not include g_o into their theory which is of the order of ten. The shear and the pressures in the more complete theory and that of Lun, et al. (1984) shown in Figures (9.5) and (9.6) have a minimum due to the increase of the mean free path and hence granular temperature at low volume fraction of solids. For high g_o the ratio of shear to the solids pressure is given by

$$\frac{P_{xy}}{P_s} = \frac{2}{5} [15/\pi(1-e)]^{\frac{1}{2}} = \frac{2}{5} \left(\frac{3}{\pi}\right)^{\frac{1}{2}} R \quad (9-306)$$

$$\text{for } \rho_p \epsilon_s g_o \gg 1 \quad (9-306a)$$

The ratio of the shear to the normal stress can be interpreted to be the tangent of the dynamic angle of repose ϕ_d .

$$\frac{P_{xy}}{P_s} = \tan \phi_d \quad (9 - 307)$$

Hence in this dense regime this angle depends only upon the restitution coefficient.

20. FLOW DOWN A CHUTE

As a second simple example, consider fully developed two dimensional, free surface flow of granular material down a rough plane inclined at an angle ξ to the horizontal as shown in Figure 9.7. The fully developed condition implies that flow properties vary only in the g direction, normal to the surface. The momentum equations similarly to single phase flow are then as follows.

Momentum Balance Normal to the Surface

$$\frac{dP_s}{dy} = g \cos \xi \rho_b \quad (9 - 308)$$

Momentum Balance Parallel to the Surface

$$\frac{dP_{xy}}{dy} = g \sin \xi \rho_b \quad (9 - 309)$$

For an elastic, nonvibrating wall there is no flux of granular temperature into the surface. There is also a zero gradient of granular temperature at the free surface. Hence for granular thermally developed flow the granular flux is zero. For steady developed flow the convective derivative of the granular temperature also vanishes. Then the granular temperature Equation (9-211) reduces itself to Equation (9-282). Therefore the granular temperature Θ is a constant and is given by Equation (9-284). In view of the constancy of Θ , the shear is also a constant. The velocity gradient is then given by

$$\frac{dU_s}{dy} = \frac{[15(1-e)\Theta]^{\frac{1}{2}}}{d_p} \quad (9-310)$$

For no slip boundary condition at $y = h$ integration of Equation (9-310) gives the linear velocity distribution obtained by Savage (1983).

$$U_s = [15(1-e)\Theta]^{\frac{1}{2}}(h-y)/d_p \quad (9-311)$$

Similarly, with a zero pressure at the free interphase

$$P_s(0) = 0 \quad (9-312)$$

integration of Equation (9-308) gives

$$P_s = g\rho_p \cos\xi \cdot \int_0^y \epsilon_s dy \quad (9-312)$$

But for constant shear the solids pressure was found to be given by Equation (9-305).

Substitution of Equation (9-310) into (9-305) and use of (9-312) gives the expression for the granular temperature found by Savage (1983).

$$\theta = \frac{g \cos\xi \int_0^y \epsilon_s dy}{2\epsilon_s^2 g_o (1+e)} = \text{constant} \quad (9-313)$$

Then assuming the mean values below are approximately equal

$$\frac{1}{y} \int_0^y \epsilon_s dy \cong \frac{1}{h} \int_0^h \epsilon_s dy \quad (9-314)$$

Savage (1983) finds that the particle concentration distribution across the width of the chute is given by the relation shown below

$$\frac{\epsilon_s^2 g_o(\epsilon_s)}{\epsilon_{sh}^2 g_o(\epsilon_{sh})} = \frac{y}{h} \quad (9-315)$$

To evaluate the concentration profile, a value of the volume fraction at the bottom of the chute, ϵ_{sh} must be specified. Figure 9.8 shows the velocity, granular temperature and concentration profiles corresponding to flow down an inclined chute with a 30° inclination angle studied experimentally by Ishida and Shirai (1979). Their particles were glass beads in the diameter range of 350 to 500 μm . Using a fiber optic probe they obtained velocity distributions for several inclination angles. In constructing the plots in Figure 9.8, Savage assumed the restitution coefficient to be 0.8, a steady height of 6 mm and the volume fraction of the solids at the surface to be 0.5. Numerical simulations performed by Ding roughly agree with Savage's analysis. However, flow never develops fully. Hence the computed behavior is more complex.

A relation between the normal pressure and the shear follows directly from Equations (9-308) and (9-309).

$$\frac{dp_s}{dp_{xy}} = \tan\xi \quad (9-316)$$

Then Equation (9-306) shows that

$$\tan\xi = \frac{2}{5} [15/\pi(1-e)]^{\frac{1}{2}} = \frac{P_s}{P_{xy}} \quad (9-317)$$

This implies that for a given value of a restitution coefficient there is only one angle of inclination for which there exists steady-fully developed flow. Equation (9-317) also shows that for nearly elastic particles, the solids pressure is small compared with

the shear. In view of the incompleteness of the theory caution must be exercised in interpreting relations such as Equation (9- 317).

BUBBLING BED: FLOW PATTERNS

Inviscid two phase flow models were able to predict the formation, the growth and the bursting of bubbles in gas fluidized beds with large jets. In such situations the jet establishes the flow pattern. However, many industrial fluidized beds are built with complex gas distributors that can be approximated by assuming the gas enters the bed uniformly. The gas actually enters the bed through a number of multi-jet tuyers which may have the shape of bubble caps. To prevent these tuyers from being inoperative by blockage of solids a distributor having a pressure drop of at least twenty percent of the bed weight is frequently used (Whitehead, 1971; 1985). Such a two dimensional bed was constructed at Illinois Institute of Technology to determine the flow patterns as a function of gas velocity and bed height (Gamwo, 1992). The purpose of this study was to determine the predictive capability of the hydrodynamic models. Obstacles were also put into the bed to simulate horizontal heat exchange tubes. The computations were made using the dense phase kinetic theory with the Maxwellian distribution (Ding and Gidaspow, 1990). Table 9-6 summarizes the equations used. For the Maxwellian distribution $\mu_{s,dilute}$ and κ_{dilute} are zero. The restitution coefficient was taken to be 0.8, based on Savage's (1983) annular shear cell studies.

For uniform inlet flow, at low gas velocities or for shallow beds, bubbles form between the center of the bed and the wall. Figure 9-9 shows the observed and the computed bubbles and the flow pattern for a two-dimensional bed with a uniform inlet velocity of air of 100 cm/sec. This flow pattern and the bubble formation

can be explained qualitatively based on the inviscid analysis presented in Chapter 7, where an equation for the solids vorticity had been derived. In the absence of viscosity the rate of change of solids vorticity, ζ , can be related to the gradient of porosity as follows,

$$\frac{d\zeta}{dt^s} = -\frac{g}{(1 - \epsilon_{mf})} \frac{\partial \epsilon}{\partial y} \quad (9 - 318)$$

where ϵ_{mf} is the porosity at minimum fluidization. Downward particle motion at the wall sets up the circulation or vorticity. The solids vorticity then gives rise to the vertical porosity gradient, as shown by the above equation. A nonuniform porosity gradient at the bottom of the bed causes nonuniform void propagation. For Geldart type B particles the void moves faster in the dilute region. There is a catch-up effect, an intersection of void paths. Hence bubbles form in the region between the walls and the center of the bed, as shown in the flow patterns in Figure (9-9). The angle that the bubbles make with the horizontal depends on the fluid velocity, the height of the bed, and the geometry. In the case of fluidization with a jet, the bubble always forms at the jet. A strength of the present model is that it can correctly predict bubbling fluidization with a uniform injection velocity.

The downward flow at the center of the bed is due to conservation of mass of the particles and symmetry. Lin, et al (1984) measured such time averaged flow patterns using radio-active tracer particles. As the bubbles form at the bottom, rise and break on top, there is, of course, some variation in the flow patterns, as shown in Figure 9-9.

Figure 9-10 shows the computed time averaged granular temperature. The computed values seem reasonable. The random velocity is higher in the top half of

the bed due to presence of larger bubbles. The peaks are caused by the bubbles breaking at the surface, as seen in Figure 9-9. The time averaged solids viscosity shown in Figure 9-11 is not unreasonable, although it appears to be somewhat low. This may be due to the use of a restitution coefficient that is too low.

The second basic flow pattern for uniform inlet flow occurs for higher flow rates and taller beds. As the bed is filled with particles to a higher and higher level, the bubbles coalesce to a single central bubble. Such a situation is illustrated in Figure 9-12 for a cylindrical fluidized bed. This simulation corresponds to a radio-active particle tracking experiment of Lin, et al (1984). Fig. 9-13 is a comparison of the computed and measured time averaged velocities of the particles. Both the simulation and the experiment show the centrally moving upward region caused by the bubble shown in Fig. 9-12. Conservation of mass then requires that there exist a downward flow near the wall. Lin, et al's (1984) experiments show that as the velocity is reduced to 45 cm/sec. and less, the flow pattern changes to that described for the two dimensional bed simulation at a velocity of 100 cm/sec. The differences in the velocities at which the flow pattern changes for the two dimensional and the cylindrical simulations is probably due to the wall effect.

The two basic flow patterns, central downflow at low velocities and low bed levels and central upflow and bubble coalescence at high velocities and high bed heights are also observed for bubbling gas-liquid flow. These patterns can be explained by two concepts: (1) vorticity generation due to wall effects and (2) large bubble formation due to shock formation caused by the intersection of characteristic paths.

An effort to determine the circulation patterns in bubble columns using a ra-

radioactive particle tracking facility is under way (Moslemian, et al 1990). Their preliminary observations are consistent with the general description given above.

22. LIQUID-SOLID FLUIDIZATION

Carlos and Richardson (1968) measured particle velocities in a liquid-solid fluidized bed with a uniform distribution by photographing the movement of tracer particles in a completely transparent bed of glass particles fluidized with dimethyl phtalate. The latter liquid has the same refractive index as the particles. Hence only a blackened tracer particle is visible. Photographs of the tracer particles were taken at equally spaced periods. From the coordinates of the tracer particles, the velocity was determined. In constructing histograms of radial, tangential and axial velocities, Carlos and Richardson neglected the position dependence of velocities. In view of this to illustrate the technique, an artificial example is shown in Figure 9-14, corresponding to the data in Carlos and Richardson's paper (1968) for a ratio of inlet to fluidization velocities of two. At a given position such data can be obtained by use of a laser-doppler velocimeter. In Fig. 9-14, the hypothetical velocity data has been subdivided into intervals of 0.5 cm/sec.

The ordinate is the frequency, f expressed as the fraction of the total number of velocities falling in the interval, divided by the size of the interval. The area under the histogram is unity. A normal distribution with a standard deviation of unity fits the data. Since the standard deviation is the square root of the granular temperature, these measurements give a granular temperature of one cm^2/sec^2 at the location indicated in Figs. 9-14. Unfortunately Carlos and Richardson lumped all their position dependent data into overall histograms where mean velocities are zero.

Numerical simulations were performed for 122 cm long, 10 cm diameter column filled with 0.889 cm diameter glass beads of density of 2.49 g/cm^3 . Table 9-7

summarizes the flow conditions used in the simulation. The density of the liquid is 1.19 g/cm^3 .

Table 9-7: FLOW CONDITIONS:

$U_{mf} \text{ 4.8 cm/s; height of fluidized bed = 26 cm}$

Superficial liquid velocity U (cm/s)	U/U_{mf}	voidage
7.6	1.6	0.53
9.4	2.0	0.58
11.7	2.4	0.63
15.1	3.1	0.70

The computations were made using the dense phase kinetic theory with the Maxwellian or the normal velocity distribution (Gidaspow et al. 1991). Table 9-6 summarizes the equations used, where for the Maxwellian distribution $\mu_{s,dilute}$ and κ_{dilute} are zero, as in the previous gas-fluidized bed example. Angular symmetry was assumed to exist.

Both the experiment and the computations show that the flow pattern consists of an upward movement of the particles in the center and downward movement at the walls. Figure 9-15 shows a comparison of the computed and the measured mean axial velocities. Figure 9-16 shows marked inward radial movement near the bottom of the bed and outward radial movement higher in the bed. Fig. 9-17 shows the computed time averaged granular temperature. There is a great deal of stirring in the lower section of the bed. In view of this both the computed viscosity shown in

Figure 9-18 and the granular pressure shown in Figure 9-19 are high in the lower section of the bed. The high value of viscosity, one to twenty poises, is expected in such systems due to the large particle diameter.

23. CIRCULATING FLUIDIZED BED LOOP SIMULATION

The complete set of Equations shown in Table 9-6 were solved for the circulating fluidized bed loop shown in Figure (9-20). (Gidaspow, et al. 1990; Gidaspow et al. 1992; Bezbaruah, 1991.) This loop is a simplified version of an actual PYROFLOW circulating fluidized bed system built for Goodrich Co. in Henry, Illinois (Johnk and Wietske, 1989). The main difference is the replacement of the hot gas cyclone by a screen type gas-solid separator that allows the gas to leave but not the solids. To prevent the separators from plugging up it was extended beyond the dimensions shown in PYROFLOW system. An ideal circulating fluidized bed (CFB) normally operates at velocities of 2 to 10 m/sec., with loadings of 10 to 15% solids by volume and with a flux of 400 kg/m²-sec or larger. Since the pressure drop in a CFB is approximately equal to the weight of the solids, such operation requires the use of high pressure blowers. Thus some CFB boilers using inexpensive blowers operate at loadings of only a few percent solids in the riser. In the simulation we assumed that the CFB operates in an ideal mode. To obtain the desirable high loadings in the riser, we assumed that at time zero the CFB was filled with 150 μ m particle solids to a height of 4.5 m, as shown in Fig. 9-20. The solids were assumed to be at minimum fluidization. This height of solids gives a pressure drop of 0.455 atm. To allow for some losses due to friction, the inlet pressure was increased by another 5%. The exit pressure was assumed to be one atmosphere. These requirements give an inlet pressure of 0.153 MPa. Simulations using dense phase kinetic theory

and turbulence in the riser were done at three velocities ranging from 2 to 5 m/sec. (Gidaspow, et al. 1990). To permit operation at high fluxes, the solids in the standpipe were fluidized. To obtain dense flow in the standpipe, the fluidization velocity was kept as low as possible. It was at its minimum fluidization. In the simulation presented by Gidaspow et al. (1990) using dense phase kinetic theory and turbulence in the dilute phase, the results were not sensitive to the restitution coefficient. Thus a restitution coefficient of 0.8 gave satisfactory results. However, with no imposed turbulence and the use of the complete kinetic theory model, the results become sensitive to the restitution coefficient. To obtain values of solids viscosities in agreement with Miller's measurements (1991) a restitution coefficient of 0.999 was used.

Figure (9-21) shows the particle concentrations, velocities, the granular temperature and the solids viscosities 5 seconds after start-up. There is the expected dense flow in the lower section of the downcomer, strong downflow at both walls of the riser, reasonable values of the granular temperature and solids viscosity. A color representation of the particle concentrations shown in Figure 9-21 clearly shows the film like dense annular region in the riser. It also shows the density distributions in the downcomer which cannot be seen in the dot plot representation in Figure 9-21. A color video for an inlet velocity of 2 m/sec shows the turbulent structure of the particles as a function of time. Cluster like dense regions are discharged from the downcomer. They move up in the riser. Velocity plots show a vortex formation at the bottom of the riser. The granular temperature and hence the viscosity was adjusted to have reasonable values by varying the restitution coefficient. Both the granular temperature and the viscosity are very high, however, in the dilute

regions. Probably the restitution coefficient must be made velocity dependent to prevent the occurrence of such high oscillations. Clearly, the restitution coefficient must decrease, as the oscillations rise. Figure (9-22) shows how the solids viscosity varies with the restitution coefficient for an inlet velocity of 2 m/sec. It is interesting to observe that the viscosity is constant in the core of the riser. Experimental measurements by Miller support such an observation. The experimental and the computed solids velocity, at the wall undergoes cyclic oscillations.

Figure (9-23) shows this behavior at the right wall of the riser. Figure (9-24) displays the discharge from the standpipe to the riser. The discharge velocity oscillates about the one meter per second velocity typical of discharge velocities from hoppers. This computed velocity is independent of the gas velocity into the riser (Gidaspow, et al. 1991). It represents the critical discharge velocity. Thus the theory suggests that a restriction in the dense phase of the loop will limit the circulation velocity in the CFB. The next section gives a derivation for the maximum circulation rate which is not limited by this critical velocity.

Figure (9-25) shows a comparison of the kinetic theory model to Ding's model (Gidaspow, et al. 1991) with turbulence for the mass flux as a function of radial distance. The viscosity does not affect the system behavior greatly once its value is reasonable. The strong asymmetry disappears at a higher gas velocity. An average value of the solids flux at velocities of 2 to 3 m/sec is about 350 kg/m²-sec, a value used in some commercial CFB units. Figure (9-26) displays the solids velocities for the same operating conditions and at the same location as in Figure (9-25). The particle velocities are much higher than the inlet velocities because of area blockage by solids. Such behavior is found to be true experimentally (Miller 1991).

Figure (9-27) shows a comparison of the area averaged solids volume fractions for the two models and for two inlet gas velocities. In each case the average volume fraction is higher near the solids inlet, as expected. At the two meter inlet velocity there is no fully developed region. At a velocity of five meter per second the riser particle concentration is smaller and there exists an approximate developed region.

The simulation of the CFB has shown the power of the method to make predictions. The method should be useful for making design improvements.

24. MAXIMUM SOLIDS CIRCULATION IN A CFB

Squires, et al. (1985) have called a circulating fluidized bed a fast bed and have pointed out the advantages of operating in this flow regime. Hence it is of great interest to determine the maximum possible circulation rate. It was already pointed out that if there is an area restriction in the flow, the velocity will reach a critical flow condition. Hence the total rate will be restricted by this maximum possible velocity. It is, nevertheless, useful to derive another maximum velocity in the absence of a critical flow restriction.

To understand the behavior of solids circulation it is useful to obtain a simplified analytical solution to the problem. To do this a number of drastic assumptions must be made in the multidimensional two phase flow equations. Based on order of magnitude estimates it is possible to:

- (1) Neglect gas momentum with respect to solids momentum due to the much larger density of the solids compared with that of the gas
- (2) Neglect the momentum head in the downcomer region due to small solids velocities
- (3) Neglect wall friction and solids pressure

With these assumptions the mixture momentum equations become:

Mixture momentum in the riser region in conservative form

$$\frac{d}{dx}(\rho_s \epsilon_{sr} U_{sr}^2) = -\frac{dP_r}{dx} - g\rho_s \epsilon_{sj} \quad (9-319)$$

Mixture momentum in the downcomer

$$\frac{dP_d}{dx} = -g\rho_s \epsilon_{sd} \quad (9-320)$$

Away from the entrance of the downcomer and the riser region, where the solids are accelerated to be carried pneumatically in the riser region, the momentum equations can be expressed in integrated form. With constant solids volume fraction in each region, we then have the balances

$$\rho_s \epsilon_{sj} U_{sr}^2 + \Delta P_r + g\rho_s \epsilon_{sj} x = 0 \quad (9-321)$$

$$\Delta P_d + g\rho_s \epsilon_{sd} x = 0 \quad (9-322)$$

Equation (9-321) is somewhat similar to Yang and Keairn's (1974) equation (16) for the draft tube. In Equation (9-321) the wall friction has been neglected. Also the kinetic head does not have a factor of 1/2 in Equation (321). This is because the mixture momentum equation has been integrated in the conservative form given by Equation (9-319). Integrating it in the form $\rho_s \epsilon_{sr} U_{sr} \frac{dU_{sr}}{dx}$ leads to the 1/2 in the Yang and Keairn's formula. However, this involves an additional assumption of constancy of ϵ_{sr} throughout the riser. Integration of the mixture momentum equation in the conservative form only assumes that the solid entered the region with a zero velocity. Thus in equation (9-321) only the value of ϵ_{sr} in

the gravity term has been assumed to be a constant. This constant may further be interpreted as an average value over the distance x .

Equation (9-322) states that the pressure drop in the downcomer region is all due to the weight of the solids, as is well known to be approximately true near minimum fluidization. When the downcomer is not fluidized, part of the weight is supported by the solids pressure. Furthermore, the boundary conditions suggest that

$$\Delta P_r = \Delta P_d \quad (9 - 323)$$

Elimination of the pressure drop in Equation (9-321) gives

$$\rho_s \epsilon_{sj} U_{sr}^2 = g \rho_s x (\epsilon_{sd} - \epsilon_{sr}) \quad (9 - 324)$$

From continuity equation for the solid we know that

$$\rho_s \epsilon_s U_s = \text{constant} = W_s \quad (9 - 325)$$

where W_s is the recirculation rate of solids per unit area. Then multiplication of equation (9-324) by $\rho_s \epsilon_{sr}$ and solution for W_s gives the desired formula.

$$\text{Solids circulation} = W_s = \rho_s \sqrt{\epsilon_{sr} g x (\epsilon_{sd} - \epsilon_{sr})} \quad (9 - 326)$$

The above formula for solids circulation flux, W_s , states that circulation is caused by the differences in densities or solids volume fractions, $\epsilon_{sd} - \epsilon_{sr}$, between the two regions. Qualitatively, this has been known to be the cause of circulation in fluidized beds for many years. Davidson and Harrison's group has taken advantage of this

cause to suggest various designs involving blowing gases at different flow rates at various bed positions. The solids circulation formula (9-325) is also related to the orifice formula, Equation (39) of Yang and Keairns (1974) and to the empirical correlations, equations (2) and (3) of Yang and Keairns. All the formulae involve a driving head due to a column of a solids mixture.

In the solids circulation formula the void fractions in the two regions can be estimated similarly to that discussed by Yang and Keairns (1974). ϵ_{sd} can be obtained from the pressure drop which can be related to the friction given by the Ergun type equations. ϵ_{sr} can be approximately obtained from a drag or friction between the phases type expression used by Yang and Keairns.

Approximate Solids Circulation Formula

Away from the entrance region, gradients of relative velocity vanish and we can simply balance gravity with drag between the phases. Then for constant slip,

$$\beta(U_g - U_s) = g \quad (9 - 327)$$

where the friction coefficient β is related to the standard drag coefficient, C_d by the formula

$$\beta = C_d \frac{(1 - \epsilon)\mu}{\epsilon^2 d^2 \rho_s} \quad (9 - 328)$$

Then substituting $W_s = \epsilon_{sr} \rho_s U_{sr}$ into Equation (9-327) the volume fraction of solids in the jet region is given by

$$\epsilon_{sr} = \frac{W_s}{\rho_s} \left(\frac{1}{U_{gr} - g/\beta} \right) \quad (9 - 329)$$

and the solids circulation rate is

$$W_s = \frac{\rho_s \epsilon_{sd} g x (U_{gr} - g/\beta)}{g x + (U_{gr} - g/\beta)^2} \quad (9-330)$$

Although in Equation (9-330) β is implicitly a function of W_s through equation (9-328) and dependence of β on C_d , as in Arastoopour and Gidaspow (1979), the formula shows the dependence of circulation on riser velocity, U_{gr} , bed height, x and particle diameter, d .

For small slip velocity or large bed height,

$$W_s = \rho_s \epsilon_{sd} (U_{gr} - g/\beta) \quad (9-331)$$

Thus for constant β , the circulation rate increases linearly with riser velocity under these conditions. Indeed Gidaspow, et al. (1990) find such an approximate rise in the CFB loop simulation. For the other extreme there is a decrease of circulation with jet velocity. For constant β , there is a maximum circulation rate at the velocity

$$U_{gj, optimum} = \frac{g}{\beta} + \sqrt{g x} \quad (9-332)$$

given by

$$W_{s, max} = \frac{\rho_s \epsilon_{sd}}{2} \sqrt{g x} \quad (9-333)$$

For a downcomer height of 4.5 meters with particles at minimum fluidization,

$$\sqrt{g x} = 6.64 \quad m/sec \quad (9-334)$$

Since this value is much higher than the expected one meter per second critical velocity, the huge discharge rate of $3320 \text{ kg/m}^2\text{-sec}$ given by formula (9-333) will not materialize. However, with sufficient aeration the critical velocity will jump to the order of magnitude higher homogeneous two phase flow velocity. Using such a strategy the circulation rates will be very large and limited by the driving head of the solids, as approximately derived here.

TABLE 9-6: KINETIC THEORY MODEL FOR MULTIPHASE FLOW

1. CONTINUITY EQUATION FOR PHASE $k(= g,s)$

$$\frac{\partial}{\partial t}(\epsilon_k \rho_k) + \nabla \cdot (\epsilon_k \rho_k v_k) = \dot{m}_k, \quad \sum_k \epsilon_k = 1 \quad (9-6.1)$$

2. MOMENTUM EQUATION FOR PHASE $k(= g,s; l= g,s)$

$$\frac{\partial}{\partial t}(\epsilon_k \rho_k v_k) + \nabla \cdot (\epsilon_k \rho_k v_k v_k) = \epsilon_k \rho_k g + \nabla \cdot \bar{\bar{\tau}}_k + \beta(v_l - v_k) + \dot{m}_k v_k \quad (9-6.2)$$

acceleration
of phase k = gravity + stress + drag force + phase change
momentum

3. CONSTITUTIVE EQUATION FOR STRESS

$$\bar{\bar{\tau}}_k = [-P_k + \xi_k \nabla \cdot v_k] \bar{\bar{I}} + 2\mu_s \bar{\bar{S}}_k \quad (9-6.3)$$

where

$$\bar{\bar{S}}_k = \frac{1}{2} [\nabla v_k + (\nabla v_k)^T] - \frac{1}{3} \nabla \cdot v_k \bar{\bar{I}} \quad (9-6.3a)$$

4. Gas Phase Stress

$$\bar{\bar{\tau}}_g = 2\epsilon_g \mu_{ge} \bar{\bar{S}}_g \quad (9-6.4)$$

Turbulent Viscosity (Used for comparison only)

$$\mu_{ge} = \mu_t + \mu_g = \rho_g (c_t \Delta)^2 (2\bar{\bar{S}}_g \cdot \bar{\bar{S}}_g)^{\frac{1}{2}} + \mu_g \quad (9-6.4a)$$

with $c_t = 0.1$ and $\Delta = \sqrt{(\Delta x \Delta y)}$

5. Solid Phase Stress

Kinetic Theory Model (Gidaspow's 1991 extension of Savage, 1983 and Ding and Gidaspow's, 1990 expressions to dilute and dense flow)

Solids Phase Pressure

$$P_s = \rho_p \epsilon_s \Theta [1 + 2(1 + e)g_o \epsilon_s] \quad (9-6.5a)$$

Solids Phase Bulk Viscosity

$$\xi_s = \frac{4}{3}\epsilon_s^2 \rho_p d_p g_o (1+e) \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} \quad (9-6.5b)$$

Solids Phase Shear Viscosity

$$\mu_s = \frac{2\mu_{s,dil}}{(1+e)g_o} \left[1 + \frac{4}{5}(1+e)g_o\epsilon_s \right]^2 + \frac{4}{5}\epsilon_s^2 \rho_p d_p g_o (1+e) \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} \quad (9-6.5c)$$

Solids Phase Dilute Viscosity

$$\mu_{s,dil} = \frac{5\sqrt{\pi}}{96} \rho_p d_p \Theta^{\frac{1}{2}} \quad (T5d)$$

Radial Distribution Function

$$g_o = \frac{3}{5} \left[1 - \left(\frac{\epsilon_s}{\epsilon_{s,max}} \right)^{\frac{1}{2}} \right]^{-1} \quad (9-6.5e)$$

6. FLUCTUATING ENERGY ($\Theta = \frac{1}{3} \langle C^2 \rangle$) EQUATION

$$\frac{3}{2} \left[\frac{\partial}{\partial t} (\epsilon_s \rho_s \Theta) + \nabla \cdot (\epsilon_s \rho_s v_s \Theta) \right] = \bar{\bar{\tau}}_s : \nabla v_s - \nabla \cdot q - \gamma \quad (9-6.6)$$

Collisional Energy Dissipation γ

$$\gamma = 3(1-e^2)\epsilon_s^2 \rho_s g_o \Theta \left(\frac{4}{d_p} \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} - \nabla \cdot v_s \right) \quad (9-6.6a)$$

Flux of Fluctuating Energy q

$$q = -\kappa \nabla \Theta \quad (9-6.6b)$$

Conductivity of Fluctuating Energy

$$\kappa = \frac{2}{(1+e)g_o} \left[1 + \frac{6}{5}(1+e)g_o\epsilon_s \right]^2 \kappa_{dil} + 2\epsilon_s^2 \rho_p d_p g_o (1+e) \left(\frac{\Theta}{\pi}\right)^{\frac{1}{2}} \quad (9-6.6c)$$

Dilute Phase ("Eddy Type") Granular Conductivity

$$\kappa_{dil} = \frac{75}{384} \sqrt{\pi} \rho_p d_p \Theta^{\frac{1}{2}} \quad (9-6.6d)$$

7. GAS - SOLID DRAG COEFFICIENTS

For $\epsilon_s < 0.8$ (based on Ergun equation)

$$\beta = 150 \frac{\epsilon_s^2 \mu_g}{\epsilon d_p^2} + 1.75 \frac{\rho_g \epsilon_s |v_g - v_s|}{d_p} \quad (9-6.7a)$$

For $\epsilon_s > 0.8$ (based on empirical correlation)

$$\beta = \frac{3}{4} C_d \frac{\epsilon \epsilon_s \rho_g |v_g - v_s|}{d_p} \epsilon^{-2.65} \quad (9-6.7b)$$

where

$$C_d = \frac{24}{Re_p} [1 + 0.15 Re_p^{0.687}], \text{ for } Re_p < 1000 \quad (9-6.7c)$$

$$C_d = 0.44, \text{ for } Re_p > 1000 \quad (9-6.7d)$$

$$Re_p = \frac{\epsilon \rho_g |v_g - v_s| d_p}{\mu_g} \quad (9-6.7e)$$

Figure 9-5. Non-Dimensional Shear Stress versus Solid Volume Fraction for the Case of Simple Shear Flow

Figure 9-6. Non-Dimensional Normal Stress versus Solid Volume Fraction for the Case of Simple Shear Flow

Figure 9-7. Gravity flow of granular material down an inclined plane

Figure 9-8. Predicted profiles for velocity, granular temperature and solids fraction during flow down an inclined chute

Figure 9-9. Computed Solids Velocities and Porosity Distributions

(— $\epsilon = 0.8$, - - - $\epsilon = 0.6$) (Ding, Gidaspow and Gamwo,
1990 Annual AIChE Meeting)

Figure 9-10 Time-Averaged Granular Temperature (cm^2/sec^2) in the Bed
(Ding, Gidaspow and Gamwo, 1990 Annual AIChE Meeting)

Figure 9-11 Time-Averaged Solids Viscosity ($\text{Pa} \cdot \text{s}$) in the Bed

Figure 9-12. Computed Bubbles in Cylindrical Bed at Several Times, Inlet Air Velocity 64.1 cm/s. Bed Diameter: 13.8 cm; Static Bed Height: 11.3 cm. Particle Diameter: 500 μm ; Particle Density: 2.5 g/cm^3 . (Ding and Gidaspow, AIChE Journal 36, 523, 1990)

Figure 9-13. Experimental and Computed Solids Velocities in Cylindrical Bed, Inlet Air Velocity 64.1 cm/s Exp.: Lin et al. (1985)

Fig 9-14. An idealized fit of a histogram of mean axial velocities of particles in a circular liquid fluidized bed at the center of the bed, 8 cm above a uniform distributor to a Maxwellian or normal distribution, f_N .

$$f_N(c_Y) = (1/\sqrt{(2\pi)\sigma}) \exp[-(c_Y - v)^2/2\sigma^2]$$

where $\sigma = \text{standard deviation} = \sqrt{(\Theta)}$

$\Theta = \text{granular temperature and the hydrodynamic velocity,}$

$$v = \int_{-\infty}^{\infty} c_Y f_N dc_Y; \quad \Theta = \int_{-\infty}^{\infty} (c_Y - v)^2 f_N dc_Y$$

$$\text{at } c_Y = v, \quad f_N = 0.4 = \frac{1}{\sqrt{2\pi\sigma}}$$

Figure 9-15. Mean Axial Velocities ($U/U_{mr} = 2.0$)
(1 - Carlos & Richardson, 1968).

(Gidaspow, Jayaswal and Ding in "Liquid-Solid Flows," FED-Vol. 118, p. 165, 1991).

Figure 9-16. Mean Radial Velocities ($U/U_{mf} = 2.0$)
 (1 - Carlos & Richardson, 1968).

(Gidaspow, Jayaswal and Ding in "Liquid-Solid Flows," FED-Vol. 118, p. 165, 1991).

Figure 9-17. Computed Granular Temperature in the Bed ($U/U_{mf} = 2$).

(Gidaspow, Jayaswal and Ding in "Liquid-Solid Flows," FED-Vol. 118, p. 165, 1991).

Figure 9-18. Computed Solids Shear Viscosity as a Function of Radial Distance at Various Bed Heights ($U/U_{mf} = 2$).

(Gidaspow, Jayaswal and Ding in "Liquid-Solid Flows," FED-Vol. 118, p. 165, 1991).

Figure 9-19. Computed Solids Pressure as a Function of Radial Distance at Various Bed Heights ($U/U_{mf} = 2$).

(Gidaspow, Jayaswal and Ding in "Liquid-Solid Flows," FED-Vol. 118, p. 165, 1991).

Figure 9-20. Schematic of CFB Loop with Initial and Boundary Conditions.

Figure 9-21. Particle Concentration, velocities, Granular Temperatures (cm/s^2) and Solid Viscosities (Poise) for $V_{gn} = 5 \text{ m/s}$.

Cross Sectional Profile at 7.5 m

Figure 9-22. Effect of Restitution Coefficient on Solids Viscosity.

Figure 9-23. Particle Velocity at the Right Wall of the Riser at 10 m.

**Figure 9-24. Solids Discharge Velocity from Standpipe to Riser
for $V_{gln} = 5$ m/s.**

Figure 9-25. Time Averaged Radial Mass Flux Profile.

Riser Height = 10 m

Figure 9-26. Time Averaged Radial Solids Velocity Profile.

Figure 9-27. Area Averaged Axial Solids Volume Fraction Profile.

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