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**EFFECTS OF PRESSURE ANISOTROPY
ON PLASMA TRANSPORT**

by

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EFFECTS OF PRESSURE ANISOTROPY ON PLASMA TRANSPORT

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ABSTRACT

In a recent paper¹ a new set of generalized two-fluid equations is derived which describes plasma transport along the field lines of a space and time dependent magnetic field. These equations are valid for collisional to weakly collisional plasmas; they reduce to the conventional fluid equations of Braginskii² for highly collisional plasmas. An important feature of these equations is that the anisotropy in the ion pressure is explicitly included. In this paper, these generalized transport equations are applied to a model problem of plasma flow through a magnetic mirror field. The profiles of the plasma parameters (density, flow speed, and pressures) are numerically calculated for plasmas in different collisionality regimes. These profiles are explained by examining the competing terms in the transport equation. The pressure anisotropy is found to profoundly impact the plasma flow behavior. As a result, the new generalized equations predict flow behavior more accurately than the conventional transport equations. A large density and pressure drop is predicted as the flow passes through a magnetic mirror. Further, the new equations uniquely predict oscillations in the density profile, an effect missing in results from the conventional equations.

I. INTRODUCTION

The problem of calculating the characteristics of plasmas flowing along spatially varying magnetic field lines arises in a number of plasma configurations. An important case is the flow at the edge of fusion systems such as the parallel flow into magnetic divertors in toroidal devices³. Other examples include axial plasma flow in various mirror devices such as a single mirror⁴, multiple mirrors^{5,6} and tandem mirrors⁷. In these various contexts, plasma parameters span a wide range of collisionality and the plasma is rarely in the highly collisional regime. Nevertheless, the conventional fluid equations (e.g. Braginskii's²) are applied to these problems. This type of solution of course is valid only for the highly collisional plasmas with $\lambda/L \ll 1$ where λ is the collisional mean free path and L is the smallest of the scale length of the gradient in the magnetic field, L_B ($L_B \sim B/(dB/ds)$) or the macroscopic plasma parameters (e.g. velocity scale length, $L_u \sim u/(du/ds)$). Application of the conventional fluid equations, valid only for highly collisional regimes, to plasmas which are not highly collisional casts doubt on the accuracy of the results.

In a recent paper¹ (hereafter referred to as paper I) we derived a new set of generalized two-fluid equations which describes plasma transport along the field lines of a space and time dependent magnetic field. These equations are valid for collisional to weakly collisional plasmas. They were shown to reduce to the conventional fluid

equations of Braginskii² for highly collisional plasmas and they resemble the double adiabatic or Chew, Goldberger, and Low (CGL) equations⁸ in the collisionless limit. An important feature of these equations is that the anisotropy in the ion distribution function is explicitly included. It is shown that¹ in the highly collisional limit, these anisotropy terms reproduce the viscosity terms of Braginskii². This examination also shows that the scale length of the flow velocity, L_u , approximately scales as $L_u \sim L_B (1-M^2)$, where M is the Mach number of the plasma flow. Thus the conventional fluid equations can contain large errors when $M \rightarrow 1$ even for $\lambda/L_B \ll 1$.

It is also found that¹ the plasma flow and the profiles of the plasma parameters are strongly affected by the anisotropy in ion pressure which, in turn, is directly related to the ratio of the collisional mean free path to the scale length of the gradient in the magnetic field. Some numerical comparisons of the solutions to the conventional transport equations with those of the new generalized equations are reported in paper I for a model problem of flow through a mirror. It is found that the conventional transport equations considerably underestimate the density and pressure drop across the mirror even for relatively small λ/L_B . Furthermore, some features of the flow which are the direct result of the pressure anisotropy, such as formation of bumps in the density profile, are missing in the results of the conventional fluid equations (see Figs. 3, 4, and 5 of paper I).

The primary motivation of the work in this paper is to use the new generalized transport equations derived in paper I to study

further the effects of ion pressure anisotropy in ion pressure on plasma parallel transport. In section II we review the transport equations derived in paper I. As a model problem of a spatially varying magnetic field, we choose the flow through a single magnetic mirror. The physical model and boundary conditions are discussed in section III. In section IV, we present numerical results with explanations based upon the new transport equations. The paper concludes with a summary in section V.

II. GENERALIZED FLUID EQUATIONS

For completeness we present here a brief description of the generalized fluid equations derived in paper I (for detailed derivation one is referred to paper I). These equations are derived from an underlying kinetic equation in magnetic flux coordinates expanded in the inverse of gyro-frequency (i.e. Ω^{-1}) and the larmor radius, ρ , retaining only lowest order terms. The moment equations, namely, continuity, momentum, and parallel and perpendicular pressure equations, are obtained by multiplying the resultant gyro-averaged kinetic equation by 1 , $m\mathbf{v}_{\parallel}$, $m\mathbf{v}_{\parallel}^2$ and $\frac{1}{2} m\mathbf{v}_{\perp}^2$ respectively, and integrating the result over the velocity space (where $\mathbf{v}_{\parallel}$ and $\mathbf{v}_{\perp}$ are the relative parallel and perpendicular particle velocities respectively). Certain approximation and assumptions are introduced in paper I to close the moment equations. The electron distribution function is assumed to be isotropic, while the ion distribution function can be strongly anisotropic. The ion

moment equations are closed by ignoring the ion conduction which is in general much smaller than ion convection.

The generalized transport equations for a quasineutral plasma with no external sources ($n_i \approx n_e \approx n$, $u_i \approx u_e \approx u$) and time independent magnetic field is summarized below:

Continuity Equation:

$$B \frac{d}{dt} (n B^{-1}) + n \frac{\partial u}{\partial s} = 0, \quad (1a)$$

Momentum Equation:

$$(m_i + m_e) n \frac{du}{dt} = - \frac{\partial}{\partial s} (p_i + p_e) - B^{3/2} \frac{\partial}{\partial s} (\delta p_i B^{-3/2}) \quad (1b)$$

Ion Parallel Pressure Equation:

$$\frac{dp_{\parallel}^i}{dt} = - 3 p_{\parallel}^i \frac{\partial u}{\partial s} + p_{\parallel}^i \frac{u}{B} \frac{\partial B}{\partial s} + \frac{2 (p_{\perp}^i - p_{\parallel}^i)}{\tau_i} + \frac{(p_e - p_{\parallel}^i)}{\tau_{ei}} \quad (1c)$$

Ion Perpendicular Pressure Equation:

$$\frac{dp_{\perp}^i}{dt} = - p_{\perp}^i \frac{\partial u}{\partial s} + 2 p_{\perp}^i \frac{u}{B} \frac{\partial B}{\partial s} - \frac{(p_{\perp}^i - p_{\parallel}^i)}{\tau_i} + \frac{(p_e - p_{\perp}^i)}{\tau_{ei}} \quad (1d)$$

Electron Pressure Equation:

$$\frac{dp_e}{dt} = - \frac{5}{3} p_e \frac{\partial u}{\partial s} + \frac{5}{3} p_e \frac{u}{B} \frac{\partial B}{\partial s} - \frac{\partial}{\partial s} (q_e B^{-1}) - \frac{p_e - p_{\perp}^i}{\tau_{ei}} \quad (1e)$$

with $p_i \equiv (p_{\parallel}^i + 2 p_{\perp}^i)/3$ and $\delta p_i \equiv p_{\parallel}^i - p_i = 2 (p_{\parallel}^i - p_{\perp}^i)/3$ resulting in $p_{\parallel}^i = p_i + \delta p_i$ and $p_{\perp}^i = p_i - \delta p_i/2$. Also, $d/dt \equiv \partial/\partial t + u \partial/\partial s$.

Here n is the number density, assumed to be the same for electrons and ions (the plasma is assumed to consists of electrons and a single ion species of charge number one), B is the magnetic field strength, u is the average flow velocity along the magnetic field lines which is the same for electrons and ions (no external sources), $p_{\parallel}^i$ and $p_{\perp}^i$ are respectively, the ion pressures parallel and perpendicular to the magnetic field lines, p_i is the total ion pressure, δp_i is the anisotropy in ion pressure, p_e is the electron pressure which is assumed isotropic, q_e is the electron conduction term, m_i and m_e are the ion and electron masses, $\ln \Lambda$ is the coulomb logarithm, τ_{ei}^E is the electron-ion energy exchange time, and τ_i is the ion pressure anisotropy relaxation:

$$\tau_{ei}^E = \frac{3}{8 \sqrt{2\pi}} \frac{m_i}{\sqrt{m_e} (e^2/4\pi\epsilon_0)^2 \ln \Lambda} \frac{T_e^{3/2}}{n} \quad (2a)$$

$$\tau_i = \frac{15}{8 \sqrt{\pi}} \frac{\sqrt{m_i}}{(e^2/4\pi\epsilon_0)^2 \ln \Lambda} \frac{T_{eff}^{3/2}}{n} \quad (2b)$$

Here ϵ_0 is the premitivity of free space and

$$T_{eff} \equiv T_{\parallel}^i \left\{ \frac{15}{4} A^{-2} (-3 + (A+3) \frac{g(A)}{\sqrt{|A|}}) \right\}^{-2/3}, \quad (2c)$$

with $A \equiv (T_{\perp}^i - T_{\parallel}^i)/T_{\parallel}^i$ and $g(A) \equiv \tan^{-1}(\sqrt{A})$ for $A > 0$ and $g(A) \equiv \tanh^{-1}(\sqrt{-A})$ for $A < 0$.

Equation (1a) is the ion continuity equation. For steady state with no external sources, it is simply the conservation of particle flux, Γ_i ($\Gamma_i = n_i u_i A$ where A is the area of the plasma column and is inversely proportional to the magnetic field).

Equations (1b) is the force equation for both ions and electrons. The last term on the right hand side (RHS) is the ion parallel viscosity term due to pressure anisotropy (acceleration and deceleration of a particle moving in a nonuniform magnetic field). Note that the viscosity term has been evaluated rigorously with no restriction on the anisotropy of the ion distribution function or the collisionality regime. We have also shown in paper I that in the highly collisional limit the viscosity term in Eq. (1b) reduces to the classical viscosity².

Equation (1c) is the parallel ion pressure equation. The first two terms on the RHS are the consequence of the longitudinal adiabatic invariant $J = \int v_{\parallel} ds$. The remaining terms on the RHS of Eq. (1c) give the relaxation rate between the ion parallel and perpendicular pressures and the energy exchange between electrons and ions. The factor of two in the pressure relaxation term is due to two perpendicular degrees of freedom feeding energy into the one parallel degree of freedom.

Equations (1d) is the ion perpendicular pressure equation. The terms in this equation have similar meaning as the corresponding ones in Eq. (1c). The first two terms on the RHS are due to the second

adiabatic invariant $\mu \equiv mv_{\perp}^2/2B$ and the remaining terms represent the relaxation process.

Equation (1e) is the electron pressure equation. Various models for q_e are identified in paper I that can be used in regimes of interest.

Finally, the ion parallel and perpendicular pressure equations can be combined using $\tau_{ei}^E \gg \tau_i$ and the definitions of p_i and δp_i to produce two equations for the rate of change of total ion pressure and the ion pressure anisotropy:

$$\frac{dp_i}{dt} = -\frac{1}{3} (5 p_i + 2 \delta p_i) \frac{\partial u}{\partial s} + \frac{1}{3} (5 p_i - 2 \delta p_i) \frac{u}{B} \frac{\partial B}{\partial s} + \frac{p_e - p_i}{\tau_{ei}^E} \quad (3a)$$

$$\frac{d\delta p_i}{dt} = -\frac{1}{3} (4 p_i + 7 \delta p_i) \frac{\partial u}{\partial s} - \frac{2}{3} (p_i - 2 \delta p_i) \frac{u}{B} \frac{\partial B}{\partial s} - \frac{3 \delta p_i}{\tau_i} \quad (3b)$$

It is important to point out that the momentum equation (1b) contains a physical singularity. This singularity occurs when the flow speed reaches Mach 1 and the plasma flow behavior changes from subsonic to supersonic flow. It is shown in paper I that the sound speed, c_s , can be written as

$$c_s = [(\gamma_i p_i^1 + \gamma_e p_e) / n(m_i + m_e)]^{1/2}, \quad (4)$$

where $\gamma_e = 1$. The values of γ_i and the sound speed as functions of

the plasma collisionality are numerically calculated in paper I using the presence of the physical singularity in the transport Eqs. (1) (see Figs. 1 and 2 of paper I). It is found that γ_1 is in the range from 5/3 in the highly collisional regime to 3 in the collisionless limit, as expected. Furthermore, as one scales from the collisional to weakly collisional regimes, γ_1 increases while $p_{\parallel}^i$ rapidly decreases, resulting in decreasing values of $\gamma_1 p_{\parallel}^i$ and the sound speed. The sound speed ranges from $c_s = ((5T_i/3 + T_e)/m_i)^{1/2}$ in the collisional regime to $c_s = (T_e/m_i)^{1/2}$ in the collisionless limit.

III. PHYSICAL MODEL AND BOUNDARY CONDITIONS

The physical situation we wish to model is that of a steady state plasma flow along field lines in a spatially varying magnetic field. A model problem which clearly illustrates the effects of plasma pressure anisotropy on plasma transport is the case of plasma flowing through a magnetic mirror field. The new transport equations (1a) through (1d) at steady state ($\partial/\partial t=0$) are solved numerically as an initial value problem in space (along s). The initial conditions are the ion and electron density, the pressures, the anisotropy in ion pressure and the average flow velocity at the inlet of the mirror (upstream). In general, the initial flow velocity, u_0 , and the initial anisotropy in ion pressure, δp_{i0} , are not known. (Hereafter subscripts 0 and 1 respectively denote the 'inlet or upstream and outlet or downstream parameters). The initial flow velocity is determined for different Mach numbers at the mirror throat using a

series of trial calculations (shooting methods) until the required Mach number is achieved.

The anisotropy in the ion pressure upstream (at the inlet of the mirror) depends on the ion distribution function at that point. In general, when the plasma is in a dynamic equilibrium ($\partial f / \partial t = 0$), the pressure anisotropy relaxation length can be determined from the transport equations. The pressure anisotropy relaxation length is found from Eq. (3b) to be $\lambda_f \equiv u r_i / 3$ (see also Eq. (7)). Thus, in general, the pressure anisotropy upstream can be ignored when the pressure anisotropy relaxation length, λ_f , is smaller than the magnetic field gradient scale length, $\lambda_f / L_B \sim (u/c_s)(\lambda/L_B) \ll 1$.

In the weakly collisional limit, $\lambda_f / L_B \sim O(1)$, most of the particles in the loss cone facing the mirror are transmitted. In this limit the initial anisotropy in ion pressure cannot be ignored. As a first approximation the initial anisotropy in ion pressure can be found by integrating the quantity $(2m v_{\parallel}^2 - m v_{\perp}^2) f / 3$ over velocity space. For small electrostatic potential ($e\phi/kT_e \ll 1$), the integration yields $\delta p_{i0}/p_{i0} = \sqrt{1-R} / 3R$. When the ambipolar potential is not negligible, it increases the size of the ion velocity space loss region and the initial pressure anisotropy.

To investigate the effect of neglecting the initial pressure anisotropy on plasma parameters downstream in the limit $\lambda_f / L_B \sim O(1)$, the transport equations are solved numerically both for cases where the initial anisotropy is ignored and for various initial anisotropy models (e.g. $\delta p_{i0}/p_{i0} = \sqrt{1-R} / 3R$). No significant difference between the solutions of the transport equation is found as

long as $\lambda_e/L_B \lesssim 0(1)$. As such, the pressure anisotropy upstream is ignored in our numerical calculations ($\delta p_{i0} \approx 0$). This result can be explained by noting that the pressure anisotropy generated in the mirror due to variations in the magnetic field is much larger than the initial pressure anisotropy. Also the initial pressure anisotropy is damped in the mirror region due to ion-ion collisions.

The magnetic field profile for the model problem (Fig. 1) is assumed Gaussian and is given by

$$B(s) = B_0 \{ 1 + (R - 1) \exp[-(s/\lambda_B)^2] \} . \quad (5)$$

Here, B_0 is the magnitude of the magnetic field away from the mirror, R is the mirror ratio ($R \equiv B_m/B_0$), B_m is the magnetic field at the mirror throat which is located at $s = 0$, and λ_B is the decay length of the magnetic field. For the numerical calculations, we set $\lambda_B = 7.5$ m and $R = 5$ which results in a minimum magnetic field scale length, $L_{B,min} = 6.25$ m. The plasma parameters at the mirror inlet are chosen to resemble the edge of a fusion plasma, i.e. $T_{e0} = T_{i0} = 50$ eV and $n_0 \sim 5 \times 10^{11} - 2 \times 10^{14} \text{ cm}^{-3}$. Variation of the density rather than the temperature is used to change the ion mean free path (i.e., the collisionality regime). Under these conditions, the electron temperature is found to be essentially constant. This has been verified by solving the equations using a variety of electron conduction models.

IV. NUMERICAL RESULTS

To examine the effects of the pressure anisotropy on plasma flow, one can attempt to isolate the anisotropy from other processes which can affect plasma flow. This is better accomplished by studying first the structure and solutions of the generalized transport equations in the highly collisional limit, $\lambda/L_g \rightarrow 0$, setting $\delta p \rightarrow 0$ in the transport equations (1).

In this limit Eqs. (1) can be combined to get:

$$\frac{1}{n} \frac{dn}{ds} = - \frac{M^2}{(1 - M^2)} \frac{1}{B} \frac{dB}{ds}, \quad (6a)$$

$$\frac{1}{u} \frac{du}{ds} = \frac{1}{(1 - M^2)} \frac{1}{B} \frac{dB}{ds}, \quad (6b)$$

$$\frac{1}{p_1} \frac{dp_1}{ds} = - \frac{5}{3} \frac{M^2}{(1 - M^2)} \frac{1}{B} \frac{dB}{ds}. \quad (6c)$$

Here M is the local Mach number, $M = u_1 / \{(5T_i/3 + T_e)/m_i\}^{1/2}$. Note that the Mach number, M , is a function of s since u and T_i are both functions of s .

For plasmas in this regime that are flowing at small Mach number ($M \ll 1$), the flow speed scales linearly with the magnetic field strength while the density and pressure profiles remain rather flat. When the Mach number is less than but close to one, the flow velocity varies more rapidly than the magnetic field due to the factor of

$1/(1 - M^2)$ in Eq. (6b). One also finds that the flow is sonic exactly at the mirror throat and there is no density or temperature drop across the mirror ($n_1 = n_0$ and $p_{11} = p_{10}$). All profiles of plasma parameters are symmetric about the mirror throat.

The plasma density, flow velocity, pressure and temperature change behavior when Mach number, M , passes from subsonic ($M < 1$) to supersonic ($M > 1$). This is due to denominator of Eqs. (6), $(1 - M^2)$, changing sign as the Mach number crosses Mach one. When the Mach number is larger than but close to one ($M^2 - 1 \ll 1$), the plasma parameters change rapidly with the variation in the magnetic field. As M becomes large ($M \gg 1$), the flow velocity variations are much smaller than the magnetic field variation by a factor $1/M^2$ through Eq. (6b). However, the density scales linearly with the magnetic field for M much greater than one while the ion pressure scales as $(B/B_m)^{5/3}$.

In general, λ/L_B is finite and the anisotropy in ion pressure cannot be ignored. The plasma flow behavior changes from the description just given and is governed by competition among the terms in the transport equations. These effects can be seen in the numerical results from the generalized transport equations given in Figs. (2) through (10) for six quantities: the anisotropy in ion pressure, flow speed, plasma density, ion parallel pressure, ion perpendicular pressure, and the total ion pressure.

The nonlinear nature and nonsymmetric profiles of the pressure anisotropy is fundamental in determining the characteristics of the plasma flow in the mirror region. Figure 2 presents the profiles of

the pressure anisotropy for plasmas in different collisionality regimes as a function of the distance along the field line (note that $s = 0$ represents the mirror throat). To explain these profiles, we rewrite Eq. (3b) at steady state as:

$$\frac{d\delta p_i}{ds} = -\frac{p_i}{3} \left(\frac{4}{u} \frac{du}{ds} + \frac{2}{B} \frac{dB}{ds} \right) - \frac{\delta p_i}{3} \left(\frac{7}{u} \frac{du}{ds} - \frac{2}{B} \frac{dB}{ds} \right) - \frac{3 \delta p_i}{u \tau_i} \quad (7)$$

As magnetic field increases (entering the mirror, $dB/ds > 0$) the flow accelerates, $du/ds > 0$. Since $p_i > 0$, Eq. (7) results in $d\delta p_i/ds$ being less than zero: negative pressure anisotropy is generated, i.e. $\delta p_i < 0$. This result is due to the invariance of μ . As a particle moves from a weak field region to strong field region in the course of its motion, it experiences the effect of increasing B . Therefore, its $v_\perp$ and $T_\perp$ ($T_\perp \sim m v_\perp^2/2$) must increase to keep μ constant. However, since the total particle energy remains constant, one finds that $v_\parallel$ and $T_\parallel$ ($T_\parallel \sim m v_\parallel^2$) decrease.

As pressure anisotropy grows (δp_i becomes more negative), the last term in Eq. (7) which is positive starts to grow. Depending on the value of τ_i (regime of collisionality) this term will arrest the growth of δp_i . For highly collisional plasmas (small τ_i), this collision term is large enough that only a small degree of pressure anisotropy is generated (line I in Fig. 2). In less collisional systems, the pressure anisotropy is larger (see lines II, III, and IV in Fig. 2).

As one passes beyond the mirror throat, the magnetic field

strength and the flow speed start to decrease ($dB/ds < 0$ and $du/ds < 0$). This combined with $\delta p_1 < 0$, result in $d\delta p_1/ds > 0$ and large. The anisotropy, δp_1 increases, passes zero and becomes positive. Collisions again cause the profile to relax and the pressure anisotropy downstream and away from the mirror goes back to zero.

We point out in section III that for steady state flows, the pressure anisotropy relaxation length (hereafter referred to as the "fluid" mean free path) is given by $\lambda_f \equiv u r_1 / 3$, which is the denominator of the collision term in Eq. (7). That this is the right parameter can be clearly seen in Fig. 3 where profiles of pressure anisotropy are plotted for flows of different Mach number, each in a fixed collisionality regime. As expected, the magnitude of the pressure anisotropy decreases with any decrease in Mach number.

For supersonic flows, the profile of δp_1 changes on the downstream side of the mirror. This is caused by the decreasing magnetic field ($dB/ds < 0$), while flow is accelerating, ($du/ds > 0$). In this case, the spatial rate of change in pressure anisotropy ($d\delta p_1/ds$) is still positive but not as large as before. Hence, the pressure anisotropy relaxes more gently (compare graphs f and g in Fig. 3). It is important to point out that in the supersonic region the total ion pressure drops rapidly (see Fig. 9c). Thus, the magnitude of the pressure anisotropy is small in the supersonic region. As such, the flow characteristic is similar for the collisional and weakly collisional cases in the supersonic region on the downstream side of the mirror throat. The reason is that in the supersonic region the electrostatic potential is the dominating factor

in determining the plasma flow behavior.

In summary, the ion pressure anisotropy is directly related to the ratio of the fluid mean free path, $\lambda_f \equiv u r_i / 3$, and to the scale length of the magnetic field, L_B . It diminishes as λ_f / L_B tends to zero in highly collisional systems. As collisionality decreases, the maximum anisotropy increases and saturates at a value of $\delta p_i / p_i \sim -1$. The ion pressure anisotropy profile is asymmetric about the mirror throat even though the magnetic field profile is symmetric. Further for subsonic flows, the viscosity related term, $(\delta p_i / B)(dB/ds)$ is less than zero except for a small region on the down stream side of the mirror throat where the magnetic field is decreasing but δp_i is still negative. In the following sub-sections, we study the effects of the pressure anisotropy on other plasma parameters.

A. Velocity Profiles

The profiles of the flow velocity as a function of the distance along the magnetic field lines are plotted in Fig. 4 for four different values of plasma collisionality. To explain these graphs, we rewrite the momentum equation (1b) using the conservation of the particle flux and noting that $dp_e/ds \approx (p_e/n) dn/ds$ to obtain:

$$\left((p_e + 3 p_{\parallel}^i) - (p_e + \gamma_i p_{\parallel}^i) M^2 \right) \frac{1}{u} \frac{du}{ds} = \frac{p_e + p_{\perp}^i}{B} \frac{dB}{ds} - \frac{3 \delta p_i}{u r_i} + \frac{p_e - p_i}{u r_{ei}} \quad (8)$$

where M is the local Mach number, $M = u / c_s$, and c_s is given by Eq. (4). Since $\gamma_i \leq 3$, the coefficient of $(1/u)du/ds$ in Eq. (8) is

always positive for subsonic flow. Also, the ion-electron relaxation term, the last term on the RHS, is usually negligible.

As one moves toward the mirror, the magnetic field increases ($dB/ds > 0$) while negative anisotropy is generated ($\delta p_i < 0$, see Fig. 2). As a result, the RHS of Eq. (8) is positive and the flow accelerates. The effects of pressure anisotropy is clearly seen in Eq. (8). The larger the magnitude of δp_i , the larger is $(1/u)du/ds$ and thus the value of the normalized flow velocity at mirror throat increases. (For a fixed Mach number at the mirror throat, this translates into a reduction in the inlet flow velocity). This effect can be seen by comparing lines I to IV of Fig. 4: the normalized velocity at the mirror throat increases with decreasing plasma collisionality.

Equation (8) also shows that the maximum flow velocity (where $du/ds = 0$) will not appear at the mirror throat ($dB/ds = 0$). The anisotropy terms produce a shift in the location of the maximum flow velocity to a position just beyond the mirror throat where the terms due to the magnetic field variation and anisotropy just cancel each other (see Fig. 4). Since the anisotropy profiles (Fig. 2) are not symmetric about the mirror throat, the velocity profiles are also asymmetric. The difference between the inlet and outlet velocities is therefore directly related to δp_i and grows as the system become less collisional (see Fig. 4).

The flow velocity profiles are plotted in Fig. 5 for a fixed collisionality regime but for flows with different Mach number at the mirror throat. As pointed out earlier, the magnitude of δp_i is

directly related to the "fluid" mean free path, $\lambda_f \equiv u\tau_i/3$. For flows with small Mach number, the flow appears collisional and the velocity profiles are rather symmetric. As the Mach number increases, the flow profile become asymmetric, the ratio of the flow speed at the throat to the inlet velocity increases, and the maximum flow velocity moves to a position beyond the mirror throat. These effects can be seen in lines b to f in Fig. 5. They are the direct result of the pressure anisotropy and are missing in the conventional fluid equations.

B. Density Profiles

The variation of plasma density as a function of the distance along the magnetic field lines is plotted in Fig. 6 for plasmas in different collisionality regimes. To explain these profiles, we use the ion pressure equation (3a) and the conservation of particle flux to rewrite the momentum equation (1b) as

$$\left(p_e + p_i + \frac{2}{3} p_i^{\perp}\right) \frac{1}{n} \frac{dn}{ds} = -m_i n u \frac{du}{ds} + \frac{17}{6} \frac{\delta p_i}{B} \frac{dB}{ds} - \frac{d\delta p_i}{ds} - \frac{p_e - p_i}{u\tau_{ei}} \quad (9)$$

Ignoring the ion-electron relaxation term (the last term on the RHS of Eq. (9) which is usually very small), the behavior of the density profile is the result of the competition between the ion inertial term (first term on the RHS) and the viscosity terms containing δp_i .

As one enters the mirror field, the magnetic field increases ($dB/ds > 0$) the flow accelerates ($du/ds > 0$) and negative anisotropy is produced ($\delta p_i < 0$). As a result, both the inertial and viscosity

terms are negative. The density profile is sharply depressed in the mirror throat (Bernolli flow). Equation (9) shows that the density drop is directly related to δp_i and thus increases as the plasma become less collisional. This is seen in lines I to IV in Fig. 6.

Downstream of the mirror, the inertial term is positive ($du/ds < 0$) while the viscosity term remains negative ($\delta p_i > 0$ and $dB/ds > 0$). The density variation is the result of these competing terms and even the shape of profile itself is critically dependent on the magnitude of δp_i . For small pressure anisotropy, the density profile is basically determined by the inertial term and is symmetric about the mirror throat (line I in Fig. 6). As δp_i increases (for less collisional plasmas), the profile become asymmetric and the density drop across the mirror (the difference between the inlet and outlet densities) grows. The pressure anisotropy can become large enough to make the viscosity terms in Eq. (9) dominant over the inertial term. In this case (lines III and IV in Fig. 6), the density continue to drop even beyond the mirror throat. In addition, the RHS of Eq. (9), the expression for the density gradient, can have more than one zero, depending on the magnitude of δp_i and the Mach number. This is the cause of the "oscillations" in the density profiles to the right of the mirror throat as seen in Fig. 6.

We pointed out that the magnitude of δp_i is directly related to the "fluid" mean free path, $\lambda_f \equiv \nu r_i / 3$. As such both the ion inertial and viscosity terms in Eq. (9) depend on flow velocity in the mirror region (Mach number). Figures 7 and 8 present the density profiles as functions of distance along the magnetic field line for

flows with different Mach number at the mirror throat and for two different degrees of plasma collisionality. As expected, the density in the mirror throat decreases with increasing Mach number since both the Mach number and δp_i are increasing, making larger both the inertial and viscosity terms in Eq. (9). The density profile changes from flat for small Mach number ($M \ll 1$) (line a in Figs. 7 and 8) to a significantly depressed one as the Mach number increases (line f in Figs. 7 and 8).

The density profile downstream also depends significantly on the Mach number and anisotropy in ion pressure. For the more collisional case (Fig. 7), the inertial term is dominant. The density drop across the mirror increases with increasing Mach number but the density bumps are not yet formed and the density profile is rather symmetric about the mirror throat. For the less collisional case (Fig. 8), at low Mach number, the viscosity term dominates everywhere and the density monotonically decreases (line a in Fig. 8). At higher Mach numbers, the inertial term in Eq. (9) is dominant near the mirror throat while the viscosity terms are dominant elsewhere. This causes the density to increase beyond the mirror throat until the viscosity term balances the inertial term. Away from the mirror throat the viscosity term dominates the flow behavior, resulting in a further decrease in the density (graph f of Fig. 8).

In summary, the density variation along the magnetic field line and the spatial shape of the profile itself are critically dependent on the plasma pressure anisotropy. The large density drop across the mirror and the formation of "bumps" in the profile are new features

that are missing in the results of the conventional fluid equations.

C. Pressure Profiles

The profiles of the ion parallel pressure, ion perpendicular pressure, and total ion pressure are given in Figs. 9a, 9b, and 9c respectively, for plasmas in different collisionality regimes. These profiles in Fig. 9 can be explained by rewriting the parallel and perpendicular pressure equations (1) as

$$\frac{dp_{\parallel}^i}{ds} = -2 \frac{p_{\perp}^i}{u} \frac{du}{ds} - \frac{3 \delta p_{\perp}^i}{u r_i} + \frac{p_{\parallel}^i}{n} \frac{dn}{ds}, \quad (10a)$$

$$\frac{dp_{\perp}^i}{ds} = + \frac{p_{\perp}^i}{B} \frac{dB}{ds} + \frac{3 \delta p_{\parallel}^i}{2 u r_i} + \frac{p_{\perp}^i}{n} \frac{dn}{ds}. \quad (10b)$$

where the electron-ion relaxation terms are ignored since they are small. The first terms on the RHS of Eq. (10) are the results of the adiabatic invariants and reflect the change in the parallel and perpendicular energies as the particle moves in a nonuniform field. The second terms are the anisotropy relaxation terms and the last terms give the effect of the density variation ($p \sim n T$).

As the plasma encounters the magnetic mirror field ($dB/ds > 0$) the flow accelerates ($du/ds > 0$) and the plasma density decreases ($dn/ds < 0$). The adiabatic invariants require the parallel energy of the particle to decrease while the perpendicular energy increases. In the parallel pressure equation (10a), all terms on the RHS are negative except for the anisotropy relaxation term. This results in a

sharp decrease in the parallel pressure as seen in Fig. 9a. The anisotropy relaxation term affects the rate of decrease in $p_{\parallel}^i$: the smaller the plasma collisionality (or longer τ_i), the larger the parallel pressure drop as seen in lines I to IV in Fig. 9a.

In the ion perpendicular pressure equation (10b), initially the perpendicular pressure increases corresponding to the increase in the perpendicular energy (the first term on the RHS of Eq. (10b)). However, the other two terms are negative. As the size of $\delta p_{\perp}$ and the density gradient grows, the corresponding terms arrest the growth of the perpendicular pressure and thus $p_{\perp}^i$ starts to decrease toward the mirror throat. This is seen in Fig. 9b. Again, the smaller the plasma collisionality (the longer τ_i), the smaller the relaxation term will be, and thus the perpendicular pressure grows larger initially at the mirror inlet. This is seen in lines I to IV in Fig. 9b.

On the downstream side of the mirror field, the magnetic field is decreasing ($dB/ds < 0$) and the flow is decelerating ($du/ds < 0$). For more collisional cases, the density increases beyond the mirror (see Fig. 6) resulting in increases in both parallel and perpendicular pressures. For less collisional cases, the density profile beyond the mirror is rather flat (see Fig. 6). The parallel pressure increase somewhat beyond the mirror (the first term on the RHS of Eq. (10a) is positive) while the perpendicular pressure decreases further (first term on the RHS of Eq. (10b) is negative). This pattern continues until the relaxation terms take over and the profile moves toward an isotropic pressure downstream. The effects of collisionality on the

profiles can be seen by comparing lines I to IV in Figs. 9a and 9b.

The total ion pressure profile of course follows the profiles of $p_{\parallel}^i$ and $p_{\perp}^i$ since $p_i \equiv (p_{\parallel}^i + 2 p_{\perp}^i)/3$. Graphs of p_i are given in Fig. 9c. In general, the total pressure profile corresponds closely to the density profiles since the total ion energy increases only slightly as it passes through the mirror field (due to electrostatic potential).

The effects of the flow speed on the pressure profiles can be seen in Figs. 10a, 10b, and 10c. Generally, the shape of the profiles are insensitive to the flow speed while the magnitude of the pressure variations increases with the Mach number. This is expected since the denominator of the pressure relaxation term in Eqs. (10) contains u , the flow speed. (This effect is directly related to the "fluid" mean free path, $\lambda_f \equiv u \tau_i/3$).

Another point of interest is the drastic drop in the perpendicular, parallel, and total pressures at the mirror outlet for supersonic flows. This is caused by the fact that $du/ds > 0$ on the downstream side, thereby making $dp_{\parallel}^i/ds$ and $dp_{\perp}^i/ds$ negative and large. The region between lines f and g in Figs. 10 indicates the regime where a shock should form in the mirror outlet in order for the flow pressure to match the pressure reservoir downstream. Note that we have chosen the Mach number in the mirror throat as our boundary condition, rather than trying to match the pressure downstream.

In short, the total pressure profile basically follows the density profile. As a result, the pressure drop across the mirror increases with increasing flow speed and with any decrease in the plasma collisionality.

VII SUMMARY AND CONCLUSIONS

The new generalized transport equations derived in paper I¹ are used to study the effects of plasma pressure anisotropy on the plasma transport along magnetic field lines. Plasma flow through a magnetic mirror field is used as an example. The transport equations are solved numerically as an initial value problem.

At steady state, the ion pressure anisotropy is found to relax on the scale length of the fluid mean free path, $\lambda_f \equiv v_{Ti}/3$, rather than the collisional mean free path. The magnitude of the pressure anisotropy, δp_i , is directly related to the ratio λ_f/L_B . The pressure anisotropy decreases as λ_f/L_B tends to zero, while for less collisional systems, it increases and saturates at about $\delta p_i/p_i \sim 1$. The ion pressure anisotropy profile along the field line is also found to be asymmetric about the mirror throat. The asymmetry in δp_i causes a shift in the location of the maximum flow velocity to a position just beyond the mirror throat. The difference between the inlet and outlet flow speeds also increases with increasing δp_i (or with decreasing plasma collisionality).

The spatial shape of the density profile is found to depend strongly on δp_i . For highly collisional systems (small δp_i), the density profile is rather symmetric. For large δp_i (less collisional plasmas), the density drop across the mirror throat grows and the profile become quite asymmetric. The pressure anisotropy can become large enough that the density profile continue to decrease beyond the mirror throat. In this case, some oscillations in the

density profile is observed. The pressure profile follows the density rather closely. The pressure drop across the mirror increases with increasing $\delta p_{\perp}$ (less collisional plasmas).

The profiles of the plasma parameters are found to depend strongly on the pressure anisotropy. As a result, the new generalized equations are more accurate in describing the plasma flow behavior as compared to the conventional fluid equations. They predict large pressure and density drops across the mirror and oscillations in the density and pressure profiles downstream. These effects are missing in the results from the conventional equations. The new generalized equations will thus be an important new tool in the investigation of the plasma flow parallel to magnetic field lines.

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FIGURE CAPTIONS

Fig. 1. Normalized magnetic fields, B/B_0 , as a function of the normalized distance along the field line, s/L , for the model problem given by Eq. (5). Note that $s = 0$ is located at the mirror throat, $L = 60$ m, and the minimum magnetic field scale length, $L_{B,min} = 6.25$ m.

Fig. 2. Normalized ion pressure anisotropy, $3\delta p_i/2p_i$, as a function of the distance along the field lines for sonic flow at the mirror throat. Ratio of the initial mean free path to the minimum magnetic field scale length, $\lambda_0/L_{B,min}$, is: I) 0.044; II) 0.13; III) 0.4; and IV) 0.8.

Fig. 3. Normalized ion pressure anisotropy, $3\delta p_i/2p_i$, as a function of the distance along the field lines for $\lambda_0/L_{B,min} = 0.4$. The value of the Mach number, $M = u/c_s$, at the mirror throat is: a) 0.0; b) 0.2; c) 0.4; d) 0.6; e) 0.8; f) $1 - \epsilon$; and g) $1 + \epsilon$ where $\epsilon < 10^{-3}$.

Fig. 4. Normalized flow velocity as a function of the distance along the field lines. Ratio of the initial mean free path, to the minimum magnetic field scale length, $\lambda_0/L_{B,min}$, is: I) 0.44; II) 0.13; III) 0.4; and IV) 0.8.

Fig. 5. Normalized flow velocity as a function of the distance along the field lines for $\lambda_0/L_{B,\min} = 0.4$. The value of the Mach number, $M = u/c_s$, at the mirror throat is b) 0.2; c) 0.4; d) 0.6; e) 0.8; f) $1 - \epsilon$; and g) $1 + \epsilon$ where $\epsilon < 10^{-3}$.

Fig. 6. Normalized density as a function of the distance along the field lines for sonic flow at the mirror throat. Ratio of the initial mean free path to the minimum magnetic field scale length, $\lambda_0/L_{B,\min}$, is: I) 0.644; II) 0.13; III) 0.4; and IV) 0.8.

Fig. 7. Normalized density as a function of the distance along the field lines for $\lambda_0/L_{B,\min} = 0.2$. The value of the Mach number at the mirror throat is: a) 0.0; b) 0.2; c) 0.4; d) 0.6; e) 0.8; f) $1 - \epsilon$; and g) $1 + \epsilon$; where $\epsilon < 10^{-3}$.

Fig. 8. Normalized density as a function of the distance along the field lines for $\lambda_0/L_{B,\min} = 0.4$. The value of the Mach number at the mirror throat is: a) 0.0; b) 0.2; c) 0.4; d) 0.6; e) 0.8; f) $1 - \epsilon$; g) $1 + \epsilon$; where $\epsilon < 10^{-3}$.

Fig. 9. Normalized ion pressures (a: parallel pressure, b: perpendicular pressure, and c: total pressure) as a function of the distance along the field lines for sonic flow at the mirror throat. Ratio of the initial mean free path to the minimum magnetic field scale length, $\lambda_0/L_{B,\min}$, is I) 0.044; II) 0.13; III) 0.4; and IV) 0.8.

Fig. 10. Normalized ion pressures (a: parallel pressure, b: perpendicular pressure, and c: total pressure) as a function of the distance along the field lines for $\lambda_0/L_{B,\min} = 0.4$. The value of the Mach number at the mirror throat is: a) 0.0; b) 0.2; c) 0.4; d) 0.6; e) 0.8; f) $1 - \epsilon$; and g) $1 + \epsilon$; where $\epsilon < 10^{-3}$.

Fig. 1

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8

Fig. 9a

Fig. 9b

Fig. 9c

Fig. 10a

Fig. 10b

Fig. 10c