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Using Perpendicular Electron-Cyclotron
Emission to Diagnose the Thermal Barrier
Electrons in TMX - Upgrade

A Progress Report

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Abstract

Possibilities are explored for diagnosing the hot-electron distribution function for the thermal barriers of tandem mirrors using perpendicular electron-cyclotron emission. Emission from a relativistic bimaxwellian with a loss cone is calculated in the single-particle limit. Formulae are derived for finding $T_{\perp}$ for a bimaxwellian by measuring frequencies of the harmonic intensity maxima at optically thin frequencies, or the intensity at optically thick frequencies. A positive $\partial f / \partial p_{\perp}$ in the distribution function is shown to lead to decreased absorption, and instability for low $T_{\perp}$. Methods are discussed for diagnosing the loss-cone distribution by comparing numerical calculation with measured intensity ratios and frequencies at the harmonic peaks. Finally, the perpendicular source function in the single-particle limit for this distribution function is calculated.

I. Introduction

Electron cyclotron emission can be used to determine parameters of the electron distribution function. In tandem mirrors this is of interest for diagnosing the electrons of the end cell plasmas. Of particular interest are the "hot electrons" of the thermal barrier. The electron distribution function at the thermal barrier will not be Maxwellian, and its form is of interest. The purpose of this work is to devise methods by which the parameters of the thermal barrier electron distribution function can be computed from a measured electron cyclotron spectrum.

This work has been done assuming the experimental parameters and profiles expected in TMX-Upgrade. The results, however, depend on the range of local plasma parameters, rather than on details of the experimental profiles, and therefore, apply more generally to the case of any plasma with average energy ~ 10 -70 keV and electron distribution function similar to those used below. At present, electron cyclotron emission from the TMX-Upgrade end cell is measured at two locations - by a microwave horn which looks down the central field line from the end of the machine ("WECE" or "Whistler Electron Cyclotron Emission" diagnostic)¹, and another looking perpendicular to the axis of the machine and in the midplane, at the axial position of the thermal barrier ("PECE" or "Perpendicular Electron Cyclotron Emission" diagnostic.) The work reported here concerns the perpendicular emission only.

II. The Optical Constants

In order to calculate the radiation intensity emitted by the plasma at a given frequency, one must know both the power radiated per unit volume of plasma, and the attenuation of the emitted radiation per unit length due to reabsorption as the ray traverses the plasma. These are given by the emission coefficient, $j(\omega)$, and absorption coefficient, $\alpha(\omega)$, respectively. The net intensity radiated is then obtained by integrating the radiative transfer equation along the ray path, using the local values of these optical constants. The optical depth of the plasma, defined as $\tau(\omega) = \int \alpha(\omega) dx$, (the integral is over the ray path), is a measure of the importance of absorption at frequency ω .

When the density is low enough that dielectric effects on the emission can be neglected (the "single particle limit"), the emission coefficient can be calculated simply by integrating the emissivity of a single electron over the distribution function. The emission coefficient is then

$$j(\omega) = \int \eta_{ST}(\omega) f(\vec{p}) d^3p \quad (1)$$

where $f(\vec{p})$ is the electron momentum distribution, and η_{ST} , the single particle emissivity, is given by the Schott-Trubnikov formula²:

$$\eta_{ST} = \frac{e^2 \omega^2}{8\pi^2 \epsilon_0 c} \sum_{\ell=1}^{\infty} \left[\left(\frac{\cos \theta - \beta_{\parallel}}{\sin \theta} \right)^2 J_{\ell}^2 + \beta_{\perp}^2 J_{\ell}'^2 \right] \delta \left(\omega - \frac{\ell \omega_c}{\gamma} - \omega \beta_{\parallel} \cos \theta \right). \quad (2)$$

Here $\beta = v/c$, θ is the angle between the propagation vector, $\vec{k}$, and $\vec{B}$, $\beta_{\parallel}$ (or $\beta_{\perp}$) is the component of β parallel (or perpendicular) to $\vec{B}$, and the argument of the Bessel functions is just $\omega \gamma \beta_{\perp} \sin \theta / \omega_c$.

The absorption coefficient in this single particle limit can be calculated from³

$$\alpha(\omega) = - \frac{8\pi^3 c^2}{\omega^2} \int d^3p \frac{n}{p_{\perp}} [m_0 \gamma \frac{\partial f}{\partial p_{\perp}} - \cos \theta (\frac{p_{\parallel}}{c} \frac{\partial f}{\partial p_{\perp}} - \frac{p_{\perp}}{c} \frac{\partial f}{\partial p_{\parallel}})] \quad (3)$$

The single particle approach is valid when⁴

$$\omega_p \ll \omega \quad (4)$$

for frequencies around the cyclotron harmonics, and⁵

$$\frac{\omega_p^2}{\omega_c^2} \frac{c^2}{v_t^2} \ll 1 \quad (5)$$

($v_t = \sqrt{2k_B T_e / m_0}$, $\omega_c = q B / m_0$, and m_0 = electron rest mass) for frequencies near the fundamental cyclotron frequency for near-perpendicular emission. (For the Doppler regime, where $N_{\parallel} > v_t / c$, Eq. (5) becomes⁵: $(\omega_p^2 / \omega_c^2) (c / v_t) / \sqrt{2} \ll 1$.) Equation (5) implies that the single particle approach does not hold for most experimental regimes of interest. In what follows, this single particle model will be assumed. Thus, results do not hold for the fundamental for most experiments and can be questionable for the second harmonic ($\ell = 2$) for high density thermal barrier plasmas. (Note that for this reason none of the figures in this paper include calculations at the fundamental.) Refraction and

dielectric effects on the emission coefficient can be neglected under the same conditions, and these assumptions are made in what follows.

III. Calculation of the Intensity

For a homogeneous slab of plasma of optical depth τ which has incident on it intensity I_i , the radiative transfer equation gives:

$$I = I_i e^{-\tau} + S(\omega) (1 - e^{-\tau}) \quad (6)$$

as the emitted intensity, where S , the source function, is given by the Kirchoff relation:

$$S(\omega) = j(\omega) / (N_r^2 \alpha(\omega)) \quad (7)$$

and N_r^2 , the ray refractive index, is approximately unity in the single particle limit. For plasmas in thermal equilibrium, $S(\omega)$ in the single particle limit is well known to be the Rayleigh-Jeans specific intensity, $\omega^2 k_B T / 8\pi^3 c^2$. For other distribution functions, the source function is in general not known, and must be calculated from $j(\omega)$ and $\alpha(\omega)$ using Eq. (7). From Eq. (6) we can distinguish two important frequency regimes. For frequencies for which the plasma is "optically thick" ($e^{-\tau} \ll 1$), the intensity radiated by a slab homogeneous plasma is just equal to the source function. At "optically thin" frequencies ($\tau \ll 1$), absorption is negligible, and the radiated intensity is given by the integral of the emission coefficient over the ray path. For

these two frequency ranges the transfer equation has a relatively simple solution, whose dependence on local plasma parameters may also be relatively simple. We, therefore, look to these regions of the spectrum for diagnostic possibilities. At "optically gray" frequencies, however, ($\tau \sim 1$), the transfer equation cannot be simplified, in general, and the dependence of the spectrum on the plasma parameters is much more complicated. We therefore concentrate below on optically thick and thin regions of the spectrum.

IV. Optically Thick and Thin Frequency Ranges for a Maxwellian Plasma

Figures (1) and (2) show the quantities $\int j \, dx$ and $I_{BB} = S = \omega^2 k_B T_e / 8\pi^3 c^2$ for a Maxwellian velocity distribution with $T = 30 \text{ keV}$ for two densities: $n_{e0} = 3 \times 10^{11} \text{ cm}^{-3}$, and $3 \times 10^{12} \text{ cm}^{-3}$. These were calculated by numerically integrating the radiative transfer equation over the observation line described in Section I as the observation ray path for the PECE diagnostic on TMX-Upgrade. The hot electrons were assumed to have a density given by

$$n = n_{e0} \exp \left(-\frac{r^2}{a^2} - \frac{z^2}{b^2} \right), \quad (8)$$

where $a = 17.5 \text{ cm}$, $b = 15 \text{ cm}$, $r = \text{radial distance}$ and $z = \text{axial distance}$. T_e was assumed to be spatially uniform. The vacuum magnetic field followed an analytical fit to the vacuum field of

TMX-Upgrade calculated by B. Stallard⁶, with the diamagnetic correction to $\vec{B}$ given by pressure balance. $j(\omega)$ was computed in the single particle limit described above, by numerically integrating Eq. (1). Since T_e was assumed uniform, $\int j \, dx / I_{BB} = \int (j / I_{BB}) \, dx = \int \alpha \, dx = \tau$, so that the ratio $\int j \, dx / I_{BB}$ gives the optical depth of the plasma. Note that due to the decrease in the cyclotron frequency due to the relativistic mass increase of the electrons, all harmonics in Figs. (1) and (2) are down-shifted from the non-relativistic cyclotron harmonic frequencies so that the second harmonic ($\ell = 2$), for instance, has $\omega < 2\omega_c$ (5 kG). With this in mind, one can see from Figs. (1) and (2) that the optically thick frequency range includes the second harmonic and, for the higher density case, the third. The optically thin region of the spectrum consists of harmonics above the third, or the fourth or fifth in the high density case. This then gives a rough idea of the frequency ranges discussed below as optically thick and optically thin, for the temperature range of interest.

V. Non-Maxwellian Emission

A. The Distribution Function

The distribution function chosen for study was the following:

$$f(\vec{p}) = A p_{\perp}^{2\xi} e^{-\sqrt{1 + np_{\parallel}^2 + p_{\perp}^2}} \quad (9)$$

Here A is the normalization constant, and ξ , n , and $\epsilon_{\perp}$ are the distribution function parameters. The quantity ξ will be referred to as the "loss cone index", since the factor $p_{\perp}^{2\xi}$ forces the distribution function to zero at $p_{\perp} = 0$, and therefore models a loss cone⁷. The exponential is a relativistic model for a bimaxwellian, which reduces in the non-relativistic limit to the familiar form: $[\exp(-\frac{1}{2}m_0 v_{\parallel}^2/k_B T_{\parallel})] [\exp(-\frac{1}{2}m_0 v_{\perp}^2/k_B T_{\perp})]$, where $\epsilon_{\perp} \rightarrow k_B T_{\perp}/m_0 c^2$ and $n \rightarrow T_{\perp}/T_{\parallel}$ in this limit. For $\xi = 0$, then, the distribution function will be referred to as a "relativistic bimaxwellian" or simply a "bimaxwellian". Further, we will define " $T_{\perp}$ " for this distribution function as

$$k_B T_{\perp} \equiv m_0 c^2 \epsilon_{\perp}. \quad (10)$$

This distribution function was chosen for study because it has general features which may be found in experimental distribution functions - i.e., $T_{\perp} \neq T_{\parallel}$, and $\partial f / \partial p_{\perp} > 0$ for some range of $p_{\perp}$. Sums of distribution functions of this type have also been used by Y.-J. Chen and G.R. Smith⁸ to model results of the Fokker-Planck code of Y. Matsuda⁹ for electron cyclotron resonance heating. Understanding the spectrum from this distribution function may therefore be useful later in understanding the emission from more complicated and realistic distribution functions.

B. Calculation of the Emission Coefficient

Calculations of the emission coefficient, $j_{\perp}$, in the single particle limit for $\theta = \pi/2$ and the distribution function of Eq. (9) are shown in Fig. 3 for $\xi = 1$, $T_{\perp} = 30$ keV, $n = 1, 3, 10$. From this figure one can see that the dependence of $j_{\perp}$ on n for perpendicular emission is weak. The features of the spectrum usually used to diagnose the distribution function¹⁰ - the frequencies of the harmonic peaks, the ratio of intensities at sequential harmonic peaks, and the absolute magnitude of the intensity, all are only weakly dependent on n . This holds true for other values of ξ ($\xi = 0, 2$) and $T_{\perp}$ ($10 \text{ keV} \leq T_{\perp} \leq 70 \text{ keV}$). Since $T_{\perp}$ was kept constant in these calculations, one can say that the perpendicular spectrum is insensitive to the mean parallel energy, and the perpendicular cyclotron radiation intensity can not be used to measure this parameter. This is to be expected, since the perpendicular energy is the source of the radiation and there is no Doppler shift for $\theta = \pi/2$. Therefore it is $T_{\perp}$ which gives the shape of the spectrum.

Figure 4 shows the dependence of $j_{\perp}$ on $T_{\perp}$ for fixed n , ξ , and density, n_e . The absolute intensity, frequencies of the harmonic intensity maxima, and ratio of $j_{\perp}$ at sequential harmonic peaks, $j_{\perp}(\ell+1)/j_{\perp}(\ell)$, are all seen to be strongly dependent on $T_{\perp}$. As $T_{\perp}$ increases, the average perpendicular energy of the distribution $((\xi + 1)kT_{\perp})$ increases, increasing the intensity radiated. The spectrum shifts down in frequency for increasing

$T_{\perp}$, due to the increase of the mean relativistic mass, and the spectrum flattens, i.e., $j_{\perp}(\ell+1)/j_{\perp}(\ell)$ increases, as is well known for Maxwellian plasmas. Thus any of these features might be used to measure $T_{\perp}$. The results shown in Fig. 4 are generally true for other ξ and n .

Next we consider using these characteristics of the spectrum to obtain $T_{\perp}$ and ξ from a measured spectrum. We consider first the simplest case - $\xi = 0$, the bimaxwellian.

VI. Diagnosing the Bimaxwellian

A. Optically Thin Frequencies

As explained above, the perpendicular cyclotron radiation intensity is not sensitive to the parallel energy of the electrons. The polarization of the radiation is likely to be a measure of the parallel energy, but that has not been investigated since depolarizing reflections from the vacuum vessel make polarization measurements difficult. We will not attempt here, therefore, to discover methods of measuring $T_{\parallel}$. Measuring the electron density, n_e , using the cyclotron spectrum is difficult to do accurately and will be discussed only briefly here.

To measure electron density, one must look at the electron spectrum at optically thin frequencies. At low frequencies where the plasma is optically thick it will be shown below that, as in

the case of a Maxwellian plasma, the intensity radiated is independent of density. At optically gray frequencies, as discussed above, the dependence of the spectrum on n_e is fairly complicated. At optically thin frequencies the intensity radiated by the plasma is simply the sum of the intensities radiated by individual electrons, and therefore is linearly proportional to density. A measurement of absolute intensity at optically thin frequencies can therefore be compared to calculations of the spectrum to infer density. This involves having an absolute calibration of the detection system, including antenna-plasma coupling, which is difficult to do very accurately¹¹. For a Maxwellian plasma, calibration can be done by comparing radiation temperature measured with that given by Thompson scattering, but for a non-Maxwellian plasma Thompson scattering may not be useful. To the accuracy that calibration can be done, the density of radiating electrons can be determined.

We will discuss now methods of measuring $T_\perp$. Consider first the optically thin region of spectrum. If the spectrum has little dependence on n , then the radiation spectrum of a bimaxwellian will be essentially the same as that of a Maxwellian with $T_{\text{Max}} = T_\perp$. Trubnikov has derived an expression for $j_\perp$ for a mildly relativistic Maxwellian which is²:

$$j_\perp \approx \frac{\omega_p^2 \omega}{c} \frac{x k T}{8\pi^3 c^2} \sqrt{2\pi} \frac{\mu^{5/2}}{x^4} \ell^2 \sqrt{\ell^2 - x^2} e^{-\mu(\frac{\ell}{x} - 1)} \frac{e^{2\beta/\gamma}}{\sqrt{16\pi \ell^3 \gamma}} \left(\frac{\gamma - 1}{\gamma + 1}\right)^\ell, \quad (11)$$

where $x = \omega / \omega_c$, ℓ = harmonic number, $\mu = m_0 c^2 / k_B T$ (or $m_0 c^2 / k_B T_\perp$ for the bimaxwellian), and γ is the γ of the resonant electrons, i.e., $\gamma = \ell \omega_c / \omega$. Equation (11) is valid for the extraordinary mode when $\gamma^3 \ll \ell$. We neglect the ordinary mode, whose intensity is decreased from the extraordinary mode by a factor $\sim v_{\perp t}^2 / c^2$. From Eq. (11) we can find the frequencies of the harmonic peaks in the $j_\perp$ spectrum by setting $dj_\perp / dx = 0$. This gives

$$4 x_\ell^4 - x_\ell^3 + 2 \mu \ell x_\ell^2 + 3 \ell^2 x_\ell - 2 \ell^3 \mu = 0. \quad (12)$$

We wish to find $x(\mu)$. Since $x \sim \ell \sim \mu \gg 1$, the second and fourth terms of Eq. (12) are somewhat smaller than the rest, and may be neglected. The resulting quadratic equation gives

$$x_\ell^2 = \frac{1}{2} (\sqrt{\mu^2 \ell^2 + 8 \mu \ell^3} - \mu \ell). \quad (13)$$

The accuracy of these approximations can be checked by comparing x_ℓ as calculated by numerical integration of Eq. (1) with the result of Eq. (13). The code calculation is for both modes - extraordinary and ordinary - and therefore calculates the total emission coefficient. The comparison is displayed in Table 1, which shows Eq. (13) to be accurate to 2%. Inverting Eq. (13) we find the relationship between the measured x_ℓ and $T_\perp$ to be

$$\frac{k_B T_\perp}{m c^2} = \frac{\ell(\ell^2 - x_\ell^2)}{2 x_\ell^4}. \quad (14)$$

Equation (14) can also be checked against the code calculations, using the calculations as simulated experimental data. The

results, displayed in Table 1, show that Eq. (14) can be used to measure $T_{\perp}$ fairly accurately.

In the above discussion, results of analytical theory were compared to code calculations of $j_{\perp}$. However, what is measured in an experiment is the intensity, not the emission coefficient, and the intensity involves an integration of $j_{\perp}$ over the ray path in the plasma. Both θ and $|\vec{B}|$ will vary along the ray path (though in TMX-Upgrade the variation of $|\vec{B}|$ in the thermal barrier region is small) and it is necessary to ask whether these changes in the direction and magnitude of $\vec{B}$ will change the dependence of the harmonic peak frequencies on $T_{\perp}$. The computer code described in Section IV was used to check this. Again $T_{\perp}$ was not allowed to vary spatially, and the observation direction was that of the PECE diagnostic. The results are shown in Fig. 5. (Note that the shape of the spectrum at low harmonics in Fig. 5 is due to self-absorption.) It can be seen from this figure that the peaks in the $j_{\perp}$ spectrum occur at the same frequencies as those in the expected experimental spectrum. Thus Eq. (14) is expected to be useful in the more complicated system of a plasma in the experimental magnetic field profile.

Equation (14) will cease to be useful for high harmonics, where overlapping of harmonics makes individual harmonic peaks indiscernible. For 70 keV, for instance, this occurs for $\ell > 4$. From Fig. 5, however, one can see that for optically gray harmonics Eq. (14) is still useful, so that for mildly

relativistic $T_{\perp}$ there should always be a few harmonics which can be used to measure $T_{\perp}$.

Equation (12) can actually be easily solved for $T_{\perp}(x_{\ell})$ without dropping the terms neglected above. The resulting formula gives $T_{\perp}$ quite accurately when only the contribution of one harmonic to the intensity is calculated, but Eq. (14) is more accurate for the temperature range of interest (10 - 70 keV), where harmonic overlap is non-negligible. Apparently the slight effects of neglecting the smaller terms in Eq. (12) has a similar effect to the small correction for harmonic overlap, making Eq. (14) more accurate than would be expected.

Finally, it is to be noted that Eq. (11) with $\mu = m_0 c^2 / k_B T_{\perp}$ models the extraordinary mode bimaxwellian emission coefficient well. A similar formula in reference 12 describes the ordinary mode. These can therefore be used for a quicker calculation of the emitted intensity than can be done by numerically integrating Eq. (1).

B. Optically Thick Frequencies

We can calculate the source function, and therefore the intensity emitted by a homogeneous plasma at optically thick frequencies, by numerically integrating Eq. (1) and Eq. (3) and taking the ratio of the two. The result for a bimaxwellian at $\theta = \pi/2$ is shown in Fig. 6. It is evident that the perpendicular source function is the same for a bimaxwellian as for a

Maxwellian distribution with $T_{\text{Max}} = T_{\perp}$. Therefore the source function is

$$S = \frac{\omega^2 k_B T_{\perp}}{8\pi^3 c^2} \quad (15)$$

Measuring the intensity at low harmonics, then, one can easily infer $T_{\perp}$.

This can be shown analytically using Eqs. (1) and (3). For the relativistic bimaxwellian we find, using Eq. (3), that

$$\alpha = \frac{8\pi^3 c^2}{\omega^2 k_B T_{\perp}} \int n f(\vec{p}) \frac{\gamma}{\sqrt{1 + n p_{\parallel}^2 + p_{\perp}^2}} d^3 p \quad (16)$$

The electrons which most contribute to the cyclotron emission or absorption (those with the largest emissivity, n_{ST}) are those with large $p_{\perp}$. But at a given frequency the relativistic γ of the radiation particles is fixed by the cyclotron resonance condition, $\omega = \ell \omega_c / \omega$. Therefore these high $p_{\perp}$ particles have low $p_{\parallel}$. Thus the $n p_{\parallel}$ term in Eq. (16) is de-emphasized. For low n , $\gamma \approx \sqrt{1 + n p_{\parallel}^2 + p_{\perp}^2}$, and the equality is much better for low $p_{\parallel}$ particles. Therefore $\alpha \approx j 8\pi^3 c^2 / (\omega^2 k_B T_{\perp})$, which is just Eq. (15). For a non-relativistic bimaxwellian it can be shown in the same way that Eq. (15) is exact.

II. Diagnosing the Bimaxwellian with a "Loss Cone"

A. Optically Thin Frequencies

Figure 7 shows the results of code calculations of $j_{\perp}$ for the distribution function of Eq. (9), with $\xi = 0, 1, 2$. Note that $T_{\perp}$, n , and n_e are kept constant while ξ varies. All curves shown are for $n = 3$, but results are similar for other values of n . As ξ increases, in order that the density be kept constant the number of high $p_{\perp}$ electrons must increase. Or, to say it differently, since the average perpendicular energy, $E_{\perp}$, is $(\xi + 1)k_B T_{\perp}$, as ξ increases, $E_{\perp}$ must increase. Therefore, as shown in Fig. 7, increasing ξ causes the harmonic peak frequencies to decrease, $j_{\perp}(\ell + 1)/j_{\perp}(\ell)$ to increase, and the absolute magnitude of $j_{\perp}$ to increase (by as much as an order of magnitude), causing more harmonic overlap. From this one can see that the harmonic peak frequencies, x_{ℓ} , depend on both ξ and $T_{\perp}$, and cannot be used to measure $T_{\perp}$ if ξ is unknown. For instance, harmonic peak frequencies for $T_{\perp} = 62$ keV, $\xi = 0, n = 3$ and $T_{\perp} = 30$ keV, $\xi = 2, n = 3$ are nearly identical. Since the frequency of emission for a given electron depends only on its energy, one might expect that x_{ℓ} is constant for constant average energy, $(\xi + 1)k_B T_{\perp} + \frac{1}{2}k_B T_{\parallel}$, but this is not the case, as shown by the example just given.

In order to measure the two parameters, ξ and $T_{\perp}$, two measurements must be made. Table II shows values of x_4 and

$j_{\perp}(4)/j_{\perp}(5)$ taken from numerically calculated spectra for $j_{\perp}$. Both of these parameters are sensitive to $T_{\perp}$ and ξ . For the same length of radiating plasma, $I_4/I_5 = j_4 \Delta x / j_5 \Delta x = j_4/j_5$. So measured values of x_4 and I_4/I_5 can be compared with those in Table II to fit the parameters ξ and $T_{\perp}$.

B. Optically Thick Frequencies

The distribution function of eq. (9) is not monotonically decreasing with increasing $p_{\perp}$ for $\xi \neq 0$. This can lead to a negative absorption coefficient -i.e., to instability. (A similar instability for $\partial f / \partial E > 0$ is discussed in reference 12.) From Eq. (3) we can see, however, that for a given frequency, while all electrons in resonance contribute to the absorption coefficient, only those resonant electrons from the positive-slope part of the distribution function ($\partial f / \partial p_{\perp} > 0$) give energy to the wave. Resonant electrons which have $\partial f(\vec{p}) / \partial p_{\perp} < 0$ will damp the wave. Thus at the harmonic peaks and low frequency side of the harmonics where the large $p_{\perp}$ particles are radiating, we expect net damping and not instability. On the high frequency edges of the lines low $p_{\perp}$ particles are resonant and α will be negative. This is shown in Figure 8. Note that this feature will have an effect on the efficiency of electron cyclotron resonance heating of the plasma if the distribution function has $\partial f / \partial p_{\perp} > 0$. Or the instability may try to fill in the "loss cone", leading to enhanced electron loss from the thermal barrier.

At higher harmonics, though the absorption coefficient is decreased on the high frequency side of the line by particles which feed the wave, harmonic overlap provides enough resonant electrons with higher $p_{\perp}$ that the net result is damping. The possibility for instability exists only at lower harmonics on the high frequency side of the line. We expect that for lower $T_{\perp}$, then, there is possibility for instability and decreased absorption at higher harmonics. Conversely, at higher $T_{\perp}$ harmonic overlap will prevent the occurrence of a negative absorption coefficient. This is shown in Figure 10, where for $T_{\perp} = 30$ keV, α is always positive.

The source function will be enhanced where α is decreased, and this is shown in Figure 10 for the same parameters as Figure 9. At optically thick frequencies when $\xi \neq 0$ we do not expect the monotonic behavior of the spectrum seen with the bimaxwellian - instead we would see enhanced emission, with peaks in the spectrum of S . The intensity spectrum from an actual experiment will not in general be monotonic at low frequencies even for a bimaxwellian. The range of frequencies at which emission occurs is determined, of course, by the magnitude of the magnetic field strength at the location of the plasma. Except at the edges of these spectral lines, however, we can expect $I \propto \omega^2 T_{\perp}$ for perpendicular observation of a bimaxwellian. The presence of a peaked spectrum instead would be indication of $\xi \neq 0$ in a fit to the distribution function of Eq. (9). More generally, any distribution function with $\partial f(\vec{p}) / \partial p_{\perp} > 0$ for some range of $p_{\perp}$

would produce a similar effect. To summarize then, the spectrum at optically thick frequencies is not simply related to $T_{\perp}$ when $\xi \neq 0$, and any non monotonically-increasing behavior of the intensity with frequency in this frequency range can be an indication that $\partial f(p_{\perp})/\partial p_{\perp} > 0$ in the distribution function. This work has not determined whether different features of other distribution functions could cause a similar effect, but it is unlikely, given the mechanism need to produce this feature.

VIII. Summary

The purpose of this work is to explore the diagnostic possibilities of perpendicular electron cyclotron emission from thermal barriers in tandem mirrors. The theory discussed in this paper was done using the "single particle" theory, and neglects the dielectric effects of the plasma on the optical constants. The distribution function used was a relativistic bimaxwellian, with an analytical model of a loss cone. It has been shown that the perpendicular emission from such a distribution is insensitive to the mean parallel energy of the distribution. The emission spectrum from a bimaxwellian, then, is well approximated by that of a Maxwellian distribution whose temperature is equal to $T_{\perp}$ of the bimaxwellian. At optically gray or thin frequencies a simple formula has been derived (Eq. 14) for calculating $T_{\perp}$ from the frequencies of the intensity maxima of a measured bimaxwellian spectrum. The source function for $\theta = \pi/2$ for the bimaxwellian has been shown to be $\omega^2 k_B T_{\perp} / 8\pi^3 c^2$ in the single

particle limit, so that a measurement of the intensity from a bimaxwellian at optically thick frequencies is just a measure of $T_{\perp}$.

For $\xi \neq 0$, the intensity has been seen to increase with ξ ($T_{\perp}$, n_e , and n constant), while the harmonic peak frequencies decrease. This is due to the increase in the average perpendicular energy of the electrons. For $\xi \neq 0$, measurement of the frequency of a harmonic peak and of the ratio of the intensities at two harmonic peaks at optically thin frequencies will determine ξ and $T_{\perp}$ through comparison to calculated values. An example of such calculations is provided in Table II. Finally, at optically thick frequencies for $\xi \neq 0$ the absorption coefficient will be negative for $T_{\perp} \lesssim 10$ keV. This occurs at low harmonics, near minima of the emission spectra. This may affect electron cyclotron resonance heating rates if some mechanism maintains a positive slope in $f(p_{\perp})$. The emission spectrum at optically thick frequencies will indicate this positive slope by having peaks rather than the monotonic increase of intensity with frequency found in the bimaxwellian case.

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TABLE I.

$T_{\perp}$	ℓ	x_{theory}	x_{code}	$T_{\text{theory},\perp}$
10 keV	3	2.85	2.84	11 keV
	4	3.75	3.75	10
	5	4.63	4.60	11
30	3	2.66	2.62	35
	4	3.44	3.42	32
	5	4.20	4.20	30
50	3	2.52	2.48	58
	4	3.25	3.26	49
	5	3.94	4.00	45
70	3	2.42	2.40	75
	4	3.10	3.16	62
	5	3.76	none	--

TABLE II

<u>T₁</u>	<u>ξ</u>	<u>x₄</u>	$\frac{j_4}{j_5}$
10 keV	0	3.75	4.32
	1	3.68	3.63
	2	3.62	3.15
30	0	3.42	2.09
	1	3.30	1.78
	2	3.16	1.56
50	0	3.26	1.59
	1	3.10	1.36
	2	2.96	1.22

$$\eta = 3$$

Fig.1 Specific intensity v.s. frequency emitted by a Maxwellian plasma in the magnetic field of Tmx-Upgrade. $T = 30$ keV, $n_{e0} 3 \times 10^{11} \text{cm}^{-3}$. Observation is in the same direction as the PECE diagnostic. ω is normalized to ω_c at 5 kG.
 $I_{BB} = \omega^2 kT / 8\pi^3 c^2$.

Fig.2 Specific intensity v.s. frequency emitted by a Maxwellian plasma in the magnetic field of Tmx-Upgrade. $T = 30$ keV, $n_{e0} = 3 \times 10^{12} \text{ cm}^{-3}$. Observation is in the same direction as the PECE diagnostic. ω is normalized to ω_c at 5kG. $I_{BB} = \omega^2 k T / 8\pi^3 c^2$.

Fig. 3 $j_{\perp}$ v.s. ω/ω_C for $\eta = 1, 3, 10$. $T_{\perp} = 30$ keV, $\xi = 1$, $B = 5$ kG, $n_e = 3 \times 10^{11} \text{ cm}^{-3}$.

Fig. 4 $j_{\perp}$ v.s. ω/ω_c for a bimaxwellian. $\eta = 3$, $B = 5\text{kG}$, $n_e = 3 \times 10^{11} \text{ cm}^{-3}$

Fig. 5 $j_{\perp}$ and specific intensity v.s. ω/ω_c . $n_e = 3 \times 10^{11} \text{ cm}^{-3}$, $B = 5 \text{ kG}$. The intensity is computed using the Tmx-Upgrade magnetic field profiles and the observation direction of the Tmx diagnostic. Both modes are included.

Fig.6 The source function for the extraordinary mode v.s. ω/ω_C for a bimaxwellian. The dependence on $T_{||}$ is negligible. $T_{\perp} = 50$ keV; $B = 5$ kG.

Fig.7 $j_{\perp}$ v.s. ω/ω_C for $\eta = 3$, $B = 5 \text{ kG}$, $n_e = 3 \times 10^{11} \text{ cm}^{-3}$. In each graph the lowest curve is $\xi = 0$, then $\xi = 1$, then $\xi = 2$.

Fig. 8 Absorption coefficient for the extraordinary mode v.s. ω/ω_c for $B = 5\text{kG}$, $n_e = 3 \times 10^{11} \text{ cm}^{-3}$. A region of negative absorption coefficient is indicated.

Fig. 9 Absorption coefficient for the extraordinary mode v.s. ω/ω_c for $B = 5\text{kG}$, $n_e = 3 \times 10^{11} \text{ cm}^{-3}$. Harmonic overlap prevents any occurrence of negative absorption coefficient, but α is decreased as ξ increases at the high frequency side of the lower harmonics.

Fig.10 Extraordinary mode source function v.s. ω/ω_c . $\eta=3$, $B=5\text{kG}$.