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DEVICES FOR LAUNCHING 0.1-g PROJECTILES TO 150 km/s OR MORE TO INITIATE FUSION: PART 1, MAGNETIC-GRADIENT AND ELECTROSTATIC ACCELERATORS

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OR MORE TO INITIATE FUSION:

PART 1, MAGNETIC-GRADIENT AND ELECTROSTATIC ACCELERATORS

ABSTRACT

I studied the feasibility of using magnetic-gradient and electrostatic accelerators to launch a 0.1-g projectile to hypervelocities (150 km/s or more). Such hypervelocity projectiles could be used to ignite deuterium-tritium fuel pellets in a fusion reactor. For the magnetic-gradient accelerator, I explored several types of projectile: shielded and unshielded copper, ferromagnetic, and superconducting. My calculations revealed the superconducting projectile to be the best of those materials I studied. It would require a 3.2-km-long magnetic-gradient accelerator and achieve a 92% efficiency. Of the cases I studied, this accelerator-projectile combination would be the one most likely to launch a 0.1-g projectile to 150 km/s or more. Its components would cost \$58.9 million. I found the electrostatic accelerator to be impractical because of its excessive length of 23 km.

1. INTRODUCTION

The promise of abundant energy has inspired many approaches to controlled thermonuclear fusion, the premise of which is thermonuclear burn of small deuterium-tritium pellets. Pellets weighing less than a gram, ignited one per second, could be the energy source for power plants producing billions of watts of electrical power. To achieve ignition it is believed necessary to deliver about 1 MJ of energy in about 10 ns into a volume less than 1 cm^3 . Candidate igniters have included lasers, electron beams, and high-energy heavy-ion beams. Another approach would be to impact the pellet with tiny projectiles or "darts" weighing about 0.1 g and accelerated to hypervelocities (150 km/s or more) by magnetic accelerators.¹⁻¹⁰ The advantage here is that the energy is naturally concentrated into a small volume. If the dart moves rapidly enough to contain the energy, then it is easy to deliver the energy in the required 10 ns by making the dart short (a few millimeters).

This study explores two proposed accelerators: magnetic gradient and electrostatic. Of these, electrostatic accelerators are too long to be practical. This report describes the critical factors seen as limits on accelerator design and operation. Using these limits I calculate the probable performance. From this work I conclude that although the magnetic-gradient accelerator is a long device, it can accelerate 0.1-g projectiles to hypervelocities and deserves future study.

Sections 2 and 3 present analyses of magnetic-gradient and electrostatic accelerators, respectively. Section 4 summarizes this work and offers my conclusions. Section 5 recommends future studies involving these accelerators.

2. MAGNETIC-GRADIENT ACCELERATOR

A magnetic-gradient accelerator is a device that uses a high-gradient magnetic field to push a projectile along. Figure 1 conceptualizes such an accelerator by showing a cylinder-shaped projectile being pushed by a magnetic field. The accelerator's magnetic field motion is assumed to be synchronized with the projectile.

FIG. 1. Magnetic-gradient accelerator, whose essence is a projectile in a high-gradient field. The field's motion must be synchronized with the projectile.

This chapter begins by presenting details on the design of a simple magnetic-gradient accelerator. Then, a first-order model of the accelerator is used to study the performance of three types of projectiles: shielded and unshielded copper, ferromagnetic, and superconducting.

As a result I found that because of eddy-current resistive heating, a 0.1-g copper projectile will melt long before it achieves a velocity of 150 km/s. Also, to accelerate a copper projectile to such a velocity, one might cover it with a sacrificial copper sheath that is allowed to melt and fall behind in the accelerator. The sheathed projectile required a 31-km long accelerator to reach 150 km/s, and the efficiency of converting electrical energy into payload kinetic energy was 0.3%.

For ferromagnetic projectiles I found that an accelerator length of 12.4 km was required and an efficiency of 75% could be expected.

The superconducting projectile would require an accelerator length of 3.2 km and have an efficiency of 92%.

2.1 ACCELERATOR DESIGN

2.1.1 Principles of Operation

Force on Projectiles. On a nonferromagnetic metal projectile that excludes the magnetic field, the force,

$$F = \left(B^2 / 2\mu_0 \right) A , \quad (1)$$

where B , A , and μ_0 are the magnetic-field intensity of the traveling wave, the rear surface area of the projectile, and the permeability of free space. The force on a dipole moment, $\vec{\mu}$, is

$$F = \vec{\mu} \cdot (\nabla \vec{B}) , \quad (2)$$

where $(\nabla \vec{B})$ is the gradient of the magnetic field. The dipole moment can be created by a ferromagnetic material or by a persistent current excited in a superconducting loop. In these cases the magnetic field is driven in synchronism with the projectile to maintain projectile acceleration.

FIG. 2. Magnetic-gradient accelerator, whose current-carrying coils are loaded with capacitors. A projectile passes through these coils. The accelerator's capacitor-inductor circuit acts as a transmission line for the current pulse. This pulse moves down the accelerator just behind the projectile, allowing the magnetic field to drive the projectile.

Traveling-Wave Transmission Line. In this report I discuss the type of magnetic-gradient accelerator previously suggested by Winterburg⁷ and by Garwin, Muller, and Richter⁹ (Fig. 2). The accelerator is a set of coils periodically loaded with capacitors. Because the coils and capacitors act as a transmission line, once a current pulse is started down the line it continues to propagate.

Self-inductance of a one-turn coil is

$$L = 2\pi\mu_0 r_a^2, \quad (3)$$

where r_a is the radius of the coil. The necessary value of capacitance at each point along the accelerator is determined by the transmission-line equation,

$$v(x) = 1/[L_u C_u(x)]^{1/2}, \quad (4)$$

where $v(x)$ is the velocity of the projectile at any given point x along the accelerator and where L_u and $C_u(x)$ are the distributed inductance and capacitance, respectively. The characteristic impedance, $Z_0(x)$, for the transmission line can be written as

$$Z_0(x) = [L_u/C_u(x)]^{1/2} . \quad (5)$$

To find the required coil current, I , to produce a required magnetic-field intensity, B , at the coil's center, I use the expression

$$I = 2Br_a/\mu_0 . \quad (6)$$

Initial Input Current. I will consider in some detail the excitation of the transmission-line accelerator. Assume that at the beginning of the transmission line a storage capacitor, C_s , resistor, R_s , and switch, S_1 , are connected in series to act as a current source. This circuit is shown in Fig. 3(a). In this type of accelerator the consecutive capacitors change rather slowly; so the excitation capacitor will see only the characteristic impedance of

$$Z_0(0) = [L_u/C_u(0)]^{1/2} = (L/C_1)^{1/2} , \quad (7)$$

where C_1 is the capacitance of the first accelerator capacitor. The equivalent circuit for the source and transmission line are shown in Fig. 3(b). This source will generate a current,

$$I_s(t) = I_0 \exp (-t/\tau_s) , \quad (8)$$

where I_0 is the initial current, t is time, and τ_s is the time constant of the circuit, which is given by

$$\tau_s = C_s [R_s + Z_0(0)] . \quad (9)$$

Distributed Energy Input. If the transmission line is lossless the current pulse propagates down the accelerator unchanged. However, the energy transfer to the projectile results in a reduced current-pulse magnitude. So there must

a) Source circuit

b) Equivalent circuit

FIG. 3. (a) A series-capacitance circuit connected to the end of a magnetic-gradient accelerator. This shows a method to excite a current pulse into the accelerator's end. (b) The accelerator's equivalent circuit and current values.

be a method to excite the accelerator as the projectile moves. One such scheme is shown in Fig. 4. Here, charged capacitors are switched into the circuit as the projectile moves along the accelerator.

To understand how this supplemental current-source works, I will investigate the circuit with one capacitor switched into the accelerator and no accelerator current present. Figure 5 shows the equivalent circuit. A charged capacitor causes two exponential current pulses, I_r and I_f , to propagate in opposite directions. These current pulses can be described by the expressions

FIG. 4. Magnetic-gradient accelerator, showing a method to add energy to its current pulse. Its capacitors are charged before the experiment. As a projectile moves down the accelerator, the switch of each supplemental energy source closes at the proper time.

FIG. 5. Equivalent circuit of a supplemental energy source exciting the magnetic-gradient accelerator. The charged capacitor generates two exponential pulses propagating in opposite directions on the accelerator.

$$I_r = I_f = I_m \exp(-t/\tau_m), \quad (10a)$$

$$I_m = V_m / [Z_0(x_m)], \quad (10b)$$

and

$$\tau_m = (1/2) C_m^s [Z_0(x_m)]. \quad (10c)$$

The quantities x_m , V_m , and C_m^s are the points where the m th power circuit connects to the accelerator, transmission line voltage, and source capacitance used to store the energy, respectively.

In this equivalent circuit I assume that the characteristic impedance of the transmission line is identical, looking up and down the line. This is possible only because the capacitors in the transmission line slowly vary, proceeding along the line. Now that one sees how a single capacitor creates a current pulse, one can investigate the effects of a charged capacitor in the presence of a traveling pulse.

Assume that to drive the projectile one needs a current pulse I_m^+ , expressed as

$$I_m^+(t) = I_m^+ \exp(-t/\tau_m), \quad (11)$$

where $\tau_m = 1/2 [Z_0(x_m)C_m^S]$ but has reduced to a current expressed as

$$I_m^-(t) = I_m^- \exp(-t/\tau_m), \quad (12)$$

owing to losses. Also assume that one desires to use a capacitor charged with a voltage and a switch to raise the current amplitude. To accomplish this I will use the method outlined in Fig. 6.

Figure 6(a) shows the current pulse $I_m^-(t)$ propagating down the line just before the traveling pulse arrives at the point where the accelerator is to be excited. The transmission-line voltage V_m^- can be written as

$$V_m^-(t) = [I_m^-/Z_0(x_m)] \exp(-t/\tau_m). \quad (13)$$

At this time the switch is open. The switch is closed just as the leading edge of the pulse arrives at the point x_m on the accelerator [Fig. 6(b)]. If the voltage V_m is greater than $I_m^-/Z_0(x_m)$, the two current pulses will be generated. One current pulse,

$$(I_m^- + \Delta I_m) \exp(-t/\tau_m), \quad (14)$$

where

$$\Delta I_m = (I_m^+ - I_m^-)$$

propagates in the positive direction along with the projectile, and the second,

$$(\Delta I_m) \exp(-t/\tau_m), \quad (15)$$

propagates in the opposite direction, as shown in Fig. 6(c).

FIG. 6. Charged capacitors, which can raise the amplitude of an exponential current pulse moving down the magnetic-gradient accelerator. (a) The switch is open and the current pulse has not quite arrived at the charged capacitor's terminals. (b) The switch has just been closed and the current pulse has just arrived at the terminals. (c) The capacitor has been drained of its charge, creating two current pulses. The first pulse raises the current magnitude of the forward-moving pulse; the second pulse moves opposite the first.

The current pulse moving opposite the projectile represents a loss in energy if nothing is done to collect this energy. In this consideration I will assume that capacitors are used to collect and store the energy for future use. One way to trap this energy is to connect several series capacitor-diode circuits along the line and to switch these circuits across the line after the projectile has passed.

2.1.2 Design Consideration

Up to now I have been discussing some of the operational features of a traveling-wave, magnetic-gradient accelerator. Next I will turn to a specific accelerator of this form and investigate all the parameters needed to evaluate the accelerator's performance. Figure 7 shows the particular accelerator that will be used for this purpose. The helical accelerator coil has a radius of 4 mm, a wire radius of 1 mm, and a pitch of 10 mm. The current return is accomplished by means of a wire that runs parallel to the axis and is placed outside the helix through which the projectile travels.

A capacitor is connected between each coil turn and the return wire. The accelerator, including the coil, return wire, and capacitors, would be placed in a stainless steel cylinder serving as a vacuum container. Wires that penetrate the vacuum case connect to the coils and current returns to supply the accelerator with supplemental current. Since the magnetic forces are large, the coil capacitors and excitation cables must be encapsulated in place.

Several of the sections in Fig. 7(a) are shown in Fig. 7(b) connected together, with the necessary circuits to supply the supplemental current pulses. In Fig. 7(b) a command cable is needed to sequentially trigger the switches that connect the supplemental storage capacitors to the accelerator. An integral part of the command cable is a delay circuit that keeps the command pulse in pace with the projectile. A typical source circuit is shown in Fig. 7(c).

Coil Design. Turning to a design for a specific accelerator, let the maximum magnetic-field intensity in the accelerator be 10 T. Let the coils have a 5-mm radius, a 10-mm pitch, and be wound from copper wire with a 1-mm radius as shown in Fig. 7(a). From Eqs. (3) and (6) one finds that the inductance of a single turn of the coil is $9.9(10^9)$ H and the current of 80 kA would generate 10 T. The energy stored in each turn of the coil is

a) Cross-sectional view of accelerator

b) Accelerator source block diagram

c) Source circuits -- S_p^m

FIG. 7. Possible design of a magnetic-gradient accelerator. (a) Cross-section of accelerator, showing a coil, capacitors, and return line in a stainless-steel vacuum case. The coil's center is free of material to allow projectile movement down the accelerator. Periodically along the accelerator an excitation wire that penetrates the vacuum case connects to the coil and return wire. (b) Accelerator source block, showing that several sections of accelerator in (a) are connected, together with supplemental energy sources, to make the accelerator function. (c) Source circuits, showing detail of energy source circuits, S_p^m , of (b).

$$E_L = 1/2 (LI)^2 . \quad (16)$$

For this case, 31.2 J would be stored in each turn at peak field.

Resistive Energy Loss. The resistance of the coil is

$$R = \rho_c (\ell/A) , \quad (17)$$

where ℓ is the length of the wire and A is the cross-sectional area of the current flow. The depth to which the current penetrates into the wire, commonly called the skin depth, is

$$\delta = \left[\frac{2\rho_c}{\omega \mu_0} \right]^{1/2} , \quad (18)$$

where ρ_c and ω are the resistivity of copper and the apparent frequency of the current pulse. The resistivity of copper at 20°C is $1.72(10^{-8}) \Omega\text{m}$. The radian frequency for this case is $4.7(10^7)$ rad/s. So the skin depth is $2.4(10^{-5})$ m.

Using Eq. (17) and the skin depth, one finds that the resistance of a single-wire loop at the point where the projectile velocity is 75.0 km/s is $3.6(10^{-3}) \Omega$. The current pulse on the accelerator can be expressed by Eq. (10a); so one can calculate the energy loss from the equation

$$\int_0^{\infty} [I(t)]^2 R dt , \quad (19)$$

which reduces to

$$I_m^2 R_c (\tau_m/2) \text{ J/coil-turns.} \quad (20)$$

If one assumes that τ_m is about one-fifth the time it takes the projectile to move between two coils, one gets a resistive energy loss of 0.3 J/coil-turn. This energy loss is small compared with both the energy needed to generate the magnetic field and the energy delivered to the projectile.

Magnetic-Field Gradient. The performance of the accelerator is proportional to the maximum field-gradient that can be propagated. The gradient of the magnetic field of the accelerator shown in Fig. 7(a) can be approximated by a rule of thumb that states that the magnetic field will be negligibly small at a distance of one coil-radius away from the coil. If one assumes that the coil radius is 4 mm and the magnetic field of 10 T varies from maximum to zero in this length, one gets a field of gradient of $2.5(10^3)$ T/m.

Number of Turns. In Sect. 2.2, it will be shown that the shortest accelerator capable of accelerating a projectile to 150 km/s is 3.2-km long. Since this accelerator has a coil every 10 mm, the accelerator will have a 320 000-turn helical coil. The energy gain by the projectile in the accelerator is 1.1 J/coil turn.

Excitation Circuit. Since the initial energy input to the accelerator is 31 J and the kinetic energy of the 0.1-g projectile at 150 km/s is 1.1 MJ, most of the energy will be supplied by the distributed energy sources. To minimize the number of capacitors and switches, without allowing current and acceleration to diminish appreciably, I will use one excitation circuit for every two coil-turns. The excitation of every second coil will require 150 000 excitation circuits of the type shown in Figs. 7(b) and (c).

In that part of the accelerator where the projectile velocity is 75 km/s, the required current for a maximum magnetic field is about 80 kA and the transmission-line voltage 5890 V. If the loss per coil-turn is 3.12 J and the coil inductance is $9.86(10^{-9})$ H/turn, one finds that current reduces to about 70.6 kA and the transmission-line voltage reduces to 5180 V at the end of two turns. The excitation voltage needed to restore the transmission-line voltage is 5890 V. I pointed out in Sect. 2.1.1 that two equal oppositely traveling current pulses are generated by the switching of a charged capacitor onto the transmission line. So twice the energy delivered to the projectile is needed in each excitation circuit, and the excitation capacitor must have a stored energy of 13.72 J and a capacitance of $7.9(10^{-7})$ F. Such a capacitor will create two 9.4-kA current pulses moving in opposite directions on the transmission line.

2.1.3 Estimated Cost

The estimated cost of all components plus fabrication of each 2-m-long accelerator section shown in Fig. 7(a) is about \$1500. For a 3.2-km-long accelerator 1600 sections would be needed at a total cost of about \$2.4 million. Each excitation circuit, including spark gap, trigger circuit, and capacitor, would be about \$350. The estimated cost of the needed 160 000 circuits would be about \$56 million. So the basic elements of the accelerator would total about \$58.4 million. This estimate does not include such items as

- Vacuum system cost.
- Ground and shelter cost.
- Instrumentation cost.
- Trigger system and delay circuits cost.
- Component connection cost.

2.2 PROJECTILE CALCULATIONS

Here I describe calculations for three types of projectiles: (1) copper with and without expendable sheaths, (2) ferromagnetic, and (3) superconducting.

2.2.1 Unshielded Copper Projectile

Here I discuss the use of a magnetic-induction accelerator to accelerate a single copper projectile of 0.1-g mass to 150 km/s. To achieve this I assume the use of a cylinder as a projectile (Fig. 8). For this study, let the cylinder's radius r_p be 1.35 mm and length ℓ_p be 2 mm.

FIG. 8. Copper-cylinder projectile accelerated by magnetic-field pressure on cylinder's end.

If the diffusion time of a magnetic field into the conducting projectile is long compared with acceleration time, the force on the cylinder can be written as

$$F = \frac{B^2}{2\mu_0} (\pi r_p^2) \quad (21)$$

In Eq. (21) I assume that the gradient of the magnetic field is large enough to have no net force on the other end of the cylinder. If the magnetic-field gradient were not sufficiently large to guarantee this, the force on the cylinder would be less.

The cylinder's acceleration from this force is

$$a = \frac{B^2 (\pi r_p^2)}{2\mu_0 M^T} \quad (22)$$

where M^T is the total mass of the cylinder. The time, t_p , it takes to accelerate the cylinder to a velocity, v , is

$$t_p = \frac{2M^T \mu_0 v}{B^2 (\pi r_p^2)} \quad (23)$$

To calculate the thermal heating of the cylinder, one must know the current, I_p , on the surface of the cylinder when the accelerating magnetic field is applied to the end of cylinder. Since the magnetic field inside the cylinder is zero, a good estimate for this current is that of a single loop of wire (with the same radius as the cylinder) that creates a magnetic field equal and opposite to the applied magnetic field. This current is

$$I_p = \frac{2r_p B}{\mu_0} \quad (24)$$

If one assumes that the current flows uniformly throughout the cylinder, then the electrical resistance,

$$R^P = \rho_c \left(\frac{2\pi (r_p/2)}{r_p \ell_p} \right) = \rho_c \left(\frac{\pi}{\ell_p} \right), \quad (25)$$

where ρ_c is the copper's resistivity. Electrical power loss during acceleration of the projectile is

$$P^P = R^P (I_p)^2, \quad (26)$$

where electrical energy loss,

$$E^E = R^P (I_p)^2 t_p. \quad (27)$$

Substituting Eqs. (23) and (24) into Eq. (27), one finds

$$E^E = \frac{8 R^P v_f^T M^T}{\pi \mu_0}. \quad (28)$$

Equation (28) represents electrical heating energy loss in the copper cylinder as it is accelerated to the final velocity, v_f .

The electrical heating of the copper cylinder will continue until the projectile either reaches its final velocity or melts. The thermal energy in the cylinder is

$$E^h = M^T (C_v \Delta T + H^C), \quad (29)$$

where C_v , ΔT , and H^C represent the specific heat initial to melting, temperature change, and latent heat. Starting with copper at the temperature of liquid nitrogen, 77 K, $\Delta T = 1279$ K, $C_v = 377$ J/kg K, $H^C = 2.05 (10^5)$ J/kg, and density is about $9.0 (10^3)$ kg/(m)³. The cylinder mass is 0.103 J, and it takes 71 J to melt the copper cylinder.

Using room temperature resistivity as an approximate value between values at liquid nitrogen and the melting temperature together with Eqs. (25) and (28), one finds

$$R^P = 2.7 (10^{-5}) \Omega$$

and

$$E^e = 846 \text{ J} .$$

Electrical heat generated in the projectile is much greater than the heat it takes to melt the 0.1-g projectile, which melts before it reaches 150 km/s. So accelerating a single copper projectile will not work. The next section describes the projectile covered with an outer sheath of copper that melts away as the projectile accelerates.

2.2.2 Shielded Copper Projectile

Because of the eddy current in the projectile, a 1-g copper cylinder melts before reaching a velocity of 150 km/s. In this section I consider a 0.1-g copper cylinder covered with an outer copper sheath. This sheath is allowed to completely melt as the projectile accelerates. The copper cylinder sheath comprises five layers (Fig. 9). I assumed that each layer is separated from the next by an infinitesimally thin, thermally insulating surface, and that all the current is in the outermost surface until each is melted. I also assumed that the thin insulating layer falls away when the outermost copper layer is melted away. The advantage of constructing the cylinder with several outer sheaths instead of just one is that the extra mass added to the projectile is not required to be accelerated the full length of the accelerator. I have found that the added mass needed to protect the final 0.1-g copper payload from melting before it reaches the end of the accelerator requires a very long and inefficient accelerator.

Before discussing the behavior of the multilayer copper projectile in an accelerator, I will consider the reasons for designing the projectile (Fig. 9). In a high-gradient magnetic field most of the current would flow on the back side of the projectile; but here the projectile is evenly covered with copper sheaths. A good first-order estimate can be found by assuming that the current in the projectile flows evenly throughout the outer sheath. In the next section I discuss the fact that the magnetic field will diffuse into the copper projectile unless prevented, which can be done by switching the magnetic field off and on as the projectile moves down the accelerator. Periodic switching of the magnetic field creates an electrical skin depth that determines the thickness of the sheaths.

FIG. 9. Five cylindrical copper sheaths, electrically and thermally isolated from each other. These sheaths cover the projectile so it can attain the desired velocity without melting.

Mechanical Motion. The force on the cylinder can be written as

$$F_n = \frac{B^2}{2\mu_0} (\pi r_n^2) , \quad (30)$$

where r_n is the radius of the outer sheath. I have performed this calculation by first letting the 0.1-g projectile, with its five outer covers, start from rest and then accelerate to the velocity at which the outermost cover melts. Then the 0.1-g projectile, with its four remaining outer covers,

begins with the initial velocity at which the previous outer cover melted and is again accelerated until its fourth sheath is melted away. This procedure is continued until only the 0.1-g projectile remains.

Again I assume a magnetic-field gradient large enough to guarantee a maximum field on one end of the cylinder and a negligible field on the other. The cylinder's acceleration is

$$a_n = \frac{B^2 (\pi r_n^2)}{2\mu_0 M_n^T}, \quad (31)$$

where M_n^T is the total mass of the cylinder. The time it takes to accelerate the cylinder from the initial velocity, v_n^i , for each sheath to the final velocity, v_n^f , for that sheath can be written as

$$\Delta t_n = \frac{2M_n^T \mu_0 (v_n^f - v_n^i)}{B^2 (\pi r_n^2)}. \quad (32)$$

The current, I_n^S , circulating in the outer sheath is

$$I_n^S = \frac{2r_n B}{\mu_0}. \quad (33)$$

I have assumed that current uniformly flows in the outermost sheath and that its electrical resistance is R_n . The electrical power dissipated in each sheath is

$$P_n = R_n (I_n^S)^2, \quad (34)$$

and the energy is

$$E_n^e = R_n (I_n^S)^2 \Delta t_n. \quad (35)$$

By substituting Eqs. (32) and (33) into Eq. (35), one obtains

$$E_n^e = \frac{8R_n (v_n^f - v_n^i) M_n^T}{\pi \mu_0}. \quad (36)$$

The energy required to melt each sheath is

$$E_n^h = M_n^s (C_v \Delta T + H_l^c) , \quad (37)$$

where M_n^s is the corresponding mass of the sheath.

Equating Eqs. (36) and (37) and solving for the velocity at which each sheath melts, one finds

$$v_n^f = v_n^i + \frac{E_n^h \mu_0}{8R_n M_n^T} . \quad (38)$$

The distance, d_n , the projectile travels with each sheath is

$$\Delta d_n = \frac{a_n}{2} (\Delta t_n)^2 + v_n^i (\Delta t_n) . \quad (39)$$

Sheath Resistance. I have used the electrical resistance of the cylinder's outer surface to calculate the surface heating. The outer sheath can be considered as two disks and a cylindrical shell (Fig. 10). I have assumed that the current uniformly flowed around the axis of rotation. The electrical resistance of each end-disk is

$$R_n^e = \frac{\rho_c 2\pi \left(\frac{r_n}{2}\right)}{d_n^e r_n} , \quad (40)$$

FIG. 10. Dimensions used to calculate effective resistance for outer shell of copper-sheath projectile.

where d_n^e is the disk thickness. The electrical resistance of the cylindrical shell is

$$R_n^e = \frac{\rho_c 2\pi r_n}{\ell_n d_n^e}, \quad (41)$$

where ℓ_n and d_n^0 are the length and thickness of the cylindrical shell.

The total current in the sheath, I_n^T , is the sum of the current, I_n^e , in each end-disk and the current I_n^P in the cylinder, where

$$I_n^e = \frac{I_n^T}{A_n^T} A_n^e, \quad (42a)$$

$$I_n^{\ell} = \frac{I_n^T}{A_n^T} A_n^{\ell}, \quad (42b)$$

and

$$A_n^T = A_n^{\ell} + 2A_n^e. \quad (43)$$

The terms A_n^{ℓ} and A_n^e are the cross-section area through which the current flows in the cylindrical shell and end caps, respectively. Power dissipated in the sheath is

$$P = (I_n^{\ell})^2 R_n^{\ell} + (I_n^e)^2 R_n^e + (I_n^e)^2 R_n^e. \quad (44)$$

When Eq. (42) is substituted into Eq. (44), one finds

$$P = (I_n^S)^2 \left\{ \left(\frac{1}{A_n^T} \right)^2 \left[2(A_n^e)^2 R_n^e + (A_n^{\ell})^2 R_n^{\ell} \right] \right\} \quad (45a)$$

$$= (I_n^S)^2 R_n, \quad (45b)$$

where

$$R_n = \left\{ \left(\frac{1}{A_n^T} \right)^2 \left[2(A_n^e)^2 R_n^e + (A_n^{\ell})^2 R_n^{\ell} \right] \right\} \quad (46)$$

Accelerator Length. The equations developed in Sect. 2.2.2 were used to calculate the performance of a projectile with five sheaths. I assumed that each sheath remained intact until the energy dissipated was adequate to melt it. At that point the mass of that sheath was completely melted and the next sheath became active. Table 1 shows the pertinent results of these calculations. From the tabulated data one finds that the accelerator must be at least 31.3-km long.

Efficiency. As the sheathed projectiles move along, the accelerator part of the projectiles is left behind due to projectile melting. The process of accelerating mass, which is left in the accelerator, leads to an inefficient device. By summing all the energy losses in Table 1 one finds that 380 MJ of kinetic energy is imparted to the projectile. This energy is later lost, leaving only 1.12 MJ of kinetic energy in the projectile. The result is a 0.3% efficiency for the projectile, without even considering the accelerator's losses.

Diffusion of the Magnetic Field into Projectiles. As the copper projectile moves down the accelerator, the magnetic field diffuses into the projectile. This diffusion reduces the amount of force on the projectile. To see if diffusion of the magnetic field into the projectile was a problem, I used the calculations of H. S. Carslaw and J. C. Jaeger¹¹ for diffusion of a magnetic-field impulse into a cylinder of infinite length and radius, r_p . From their work I found that it requires a time of 1.7 ms for the magnetic

TABLE 1. Results for a five-sheath projectile^a (shown in Fig. 9) accelerated to a velocity of 150 km/s by a 10-T/m traveling magnetic-field gradient.

Sheath	Sheath (g)	Projectile (g)	Sheath electrical resistance ($10^{-5} \Omega$)	Electrical energy to melt sheath (kJ)	Initial velocity (km/s)	Velocity when surface vaporizes (km/s)	Acceleration of Projectile (10^5 m/s^2)	Time for sheath to melt (s)	Distance traveled (km)	Kinetic energy loss (MJ)
1	120	200	0.573	82.238	0	35.4	1.5	0.240	4.3	75
2	59	80	0.609	40.352	35.4	76.2	2.0	0.203	11.3	170
3	17	21	0.852	11.748	76.2	108	3.2	0.101	9.3	100
4	3.7	4	1.351	2.511	108	129	5.5	0.039	4.6	31
5	0.52	0.62	2.702	0.358	129	140	11	0.009	1.2	5
Pro- jectile		0.103	2.702		140	150	22.105	0.005	0.64	

^aProjectile's final temperature = 1083°C.

field at the center for a cylinder with 0.01535-m radius to reach 15% of the surface field. Since the projectile described in Table 1 must be accelerated for 0.59 s, the magnetic field must be switched off to prevent the field from completely diffusing into the projectile. To avoid significant magnetic-field diffusion into the copper projectile, acceleration time must be less than 1.0 ms. So the accelerator's field must be cycled off and on at a rate of about 10^{+3} pulses/s. The skin depth at this frequency is 2.1 mm, which compares to the sheath thickness that supports the original assumption that current is uniform.

2.2.3 Ferromagnetic Projectile

I now turn to projectiles fabricated from ferromagnetic materials. Again, a propagating magnetic-field gradient is used to accelerate the projectile. I find that the magnetic force on a ferromagnetic projectile is small, requiring a rather long accelerator to achieve 150 km/s for a 0.1-g projectile. This accelerator will be 12.4-m long, with a 75% efficiency.

Accelerator Length. The accelerating force is

$$F = \mathcal{M}_f V \left[\mu_0 \left(\frac{\partial H_x}{\partial x} \right) \right] , \quad (47)$$

where $\mathcal{M}_f$ and V are the magnetization (dipole moment per unit volume) of the ferromagnetic material and the volume. Magnetization is given by

$$\mathcal{M}_f = \frac{1}{\mu_0} B_f - H_x , \quad (48)$$

where B_f is the magnetic induction in the material. The distance it takes to accelerate the projectile is

$$d = 1/2 \left[\frac{M (v_f)^2 D_f}{\mathcal{M}_f \left(\mu_0 \frac{\partial}{\partial x} H_x \right)} \right] , \quad (49)$$

where v_f and D_f are the final velocity and density of the projectile.

As one increases the magnetic field on a ferromagnetic material, the magnetic induction in the material increases until all atomic dipoles are

aligned with the magnetic field. The state where all atomic dipoles are aligned is called saturation of the magnetization. If the magnetic field is increased beyond the saturation point, there is no further increase in magnetization of the material. In ferromagnetic materials, $H_x \ll B_f$ and the magnetic induction of the best ferromagnetic materials saturates at about 4 T. Using a field gradient of $2.5(10^3)$ T/m, one finds that the accelerator must be 12.4-km long.

Efficiency. The efficiency of a magnetic-induction accelerator with a ferromagnetic projectile will now be discussed. A ferromagnetic projectile does not have an eddy current. So there is no resistive energy loss nor is there need to provide energy to accelerate material that will be melted from the projectile. The only loss is that of the accelerator.

In Sect. 2.1.2 I designed an accelerator that had an energy loss of 0.3 J/coil-turn. The 12.4-km-long accelerator had $1.24(10^6)$ turns. So the energy loss of the accelerator is 373 kJ. The kinetic energy delivered to the projectile is 1.13 MJ; the net efficiency of the accelerator could, in principle, be as high as 75%.

2.2.2 Superconducting Projectile

I have studied projectiles made of copper and ferromagnetic materials and now I will investigate the use of a superconducting material. I assume that the superconducting projectile is shaped as a single-turn coil with a current, I , induced into the coil as it is cooled to its superconducting temperature. The axis of the accelerator is perpendicular to the coil (Fig. 11). The Lorentz force of the accelerator's magnetic field together with the current in the coil accelerates the projectile.

Accelerator Length. The accelerating force is

$$F = \mu_s \left(\frac{\partial B}{\partial x} \right) , \quad (50)$$

where μ_s is the dipole moment of the coil. The dipole moment of a single-turn current loop is

$$\mu_s = \pi r_c^2 I , \quad (51)$$

FIG. 11. Superconducting coil accelerated by magnetic field. A current, I , is assumed to be excited in the coil.

where r_c is the coil's radius. By using the above equations one finds that the required accelerator length to achieve a final velocity, v_f , is

$$d = 1/2 \left[\frac{(v_f)^2 M_c}{\mu_s \frac{\partial B}{\partial x}} \right], \quad (52)$$

where M_c is projectile mass, which in this analysis is the mass of the superconducting coil. The coil's mass can be written as

$$M_c \approx 2 \pi r_c l_c t_c D_s, \quad (53)$$

where D_s is the density, t_c is the length, and l_c is the thickness of the coil. On substituting Eqs. (51) and (53) into Eq. (52), one finds

$$d = \frac{(v_f)^2 D_s}{r_c J_c \left(\frac{\partial B}{\partial x} \right)}, \quad (54)$$

where J_c is the density in the coil.

I now consider the one of the better superconducting materials available, niobium germanium (Nb_3Ge). Since Nb_3Ge has a density of $8.57(10^3) \text{ g/m}^3$, a

0.1-g coil could have $r_c = 2$ mm and $l_c = t_c = 0.96$ mm. All superconducting materials have a critical magnetic field at which the material goes from a superconductor to a normal conductor. At this critical field the coil has large resistive losses and the current rapidly decays. So one must keep the magnetic field below the critical field. Type II superconductors have a much higher critical magnetic field than Type I superconductors. Nb_3Ge is a Type II superconductor with a critical field of 10 T at $0^\circ K$ and a current density of $1.2(10^{10})$ A/m². So to accelerate a 0.1-g coil projectile made of Nb_3Ge to 150 km/s, one needs an accelerator length of 3.2 km.

When one examines Eq. (54) one notes that the accelerator length can be reduced if the gradient of the magnetic field can be increased. One might suggest that the magnetic-field gradient can be increased by increasing the maximum magnetic field in the accelerator. But increasing the magnetic field to shorten the accelerator is a futile gesture. As one increases the magnetic field on a superconductor, the maximum current the superconductor can sustain diminishes; so the product of the current and field gradient in Eq. (54) at best remains the same or at worst decreases.

Efficiency. Since current in the projectile experiences no energy loss, the only energy given to the projectile is kinetic energy. So accelerator losses represent the only losses in the device. In the second section of this chapter, I designed an accelerator that had an energy loss of 0.31 J/turn. A 3.2-km accelerator would have $3.2(10^5)$ turns, and the accelerator efficiency for a superconducting coil in a magnetic-gradient accelerator could, in principle, be as high as 92%.

Time Variation of Magnetic Gradient. It should be pointed out that, for Type II superconductors, large fluctuations in the accelerator's magnetic field can cause superconductor heating and cause it to become a normal conductor. If the superconductor goes normal it melts or ceases to be accelerated. So care must be taken when building the accelerator that the magnetic field in the projectile's vicinity does not vary with time as it propagates down the accelerator. This can be accomplished by constructing the accelerator coils with great tolerance.

Care in construction may not be a sufficient guarantee, however, since current pulse will vary as it propagates along the accelerator. This variation is a result of energy transfer to the projectile and of periodic energy

sources. In the magnetic gradient accelerator design presented in this chapter, current pulse, and thus, magnetic field, vary by more than 10.0% between two consecutive energy source points. This variation may be enough to cause the superconductor projectile to go normal.

Future work should include a calculation to see if magnetic-field source variation drives the superconductor to the normal state.

2.3 SUMMARY

In this chapter, I calculated the feasibility of using a magnetic-gradient accelerator to accelerate a 0.1-g projectile made of copper, ferromagnetic, and superconducting materials to 150 km/s.

The 0.1-g copper projectile with no sheath would melt because of eddy currents in the projectile. To accelerate a 0.1-g copper projectile to 150 km/s would require covering the projectile with copper sheaths that melt away as the projectile moves down the accelerator. Acceleration of the material lost in the accelerator would require a long and inefficient accelerator. The copper-covered projectile studied in this chapter would require an accelerator length of 31.0 km and a 0.3% efficiency.

The 0.1-g ferromagnetic projectile would require an accelerator length of 12.4 km, but it could have an efficiency as high as 75%.

The best type of projectile studied was the 0.1-g superconducting coil, which would require a 3.2-km accelerator with a possible efficiency of up to 92%.

The magnetic-gradient accelerator, designed to accelerate projectiles in the shortest practical distance would use a magnetic field of 10 T and a field gradient of $2.5 (10^3)$ T/m. This accelerator would accelerate a 0.1-g superconducting Nb_3Ge projectile to 150 km/s in 3.2 km. The energy stored in the current pulse traveling in the accelerator would be 31.2 J; the energy delivered to the projectile as it passed each coil-turn would be 3.1 J.

To sustain the current in the accelerator, energy must be supplied to the coil as the projectile passes. I chose to supplement the current at every second accelerator coil. Since the coils are 10 mm apart, a 3.2-km long accelerator must have 160 000 excitation circuits distributed along the accelerator length. These excitation currents can be built from charged capacitors and spark gaps. I estimated that the parts for the main body of the accelerator and the excitation circuits would cost a minimum of \$58.9 million. This

estimate did not include such items as vacuum system, shelter, trigger system, instrumentation, and component connection.

3. ELECTROSTATIC ACCELERATOR

Here I studied the feasibility of using a propagating electrostatic field to accelerate 0.1-g projectiles to 150 km/s. From this study I found that the electrostatic field accelerator is too long to be practical.

An electrostatic accelerator is a long set of parallel conducting plates with a hole large enough for the projectile to pass through (Fig. 12). A voltage is placed across consecutive plates to create an electrostatic field that accelerates the projectile. Now assume that the projectile is a metallic cylinder (Fig. 13) and that the electric field is parallel to the cylinder's axis. The force on a metallic projectile arises because the electric field is excluded from inside the projectile. Because the electric field is not excluded from the interior of a dielectric material, the dielectric projectile

FIG. 12. Possible electrostatic accelerator. A conducting projectile moves through a series of holes cut in a set of metallic plates. The voltages V_1 , V_2 , V_3 , and V_4 cause the projectile to accelerate.

FIG. 13. Cylindrical projectile pushed by an electrical field.

will not experience a force in an electrostatic accelerator. The pressure exerted by the electrostatic field on the cylinder is

$$P_e = 1/2 \left(\epsilon_0 E_0^2 \right), \quad (55)$$

where ϵ_0 is permittivity of free space. In Eq. (55) I assumed that the gradient of the electric field is sufficient to be maximum on the back end of the cylinder and negligible on the leading end. If the gradient is not sufficient to achieve this, the presence of the electric field on the front side of the cylinder will reduce the net accelerator force.

The maximum electrostatic field that can be placed on the same surface is 10^8 V/m. The maximum electrostatic field might be increased an order-of-magnitude if the surfaces were positively charged. Using a maximum of 10^9 V/m, one finds the maximum possible electrostatic pressure is $P_e = 8.9 (10^6) \text{ N/m}^2$.

I will now investigate the same copper projectile used in Sect. 2. This 0.103-g copper cylinder has radius of 1.35 mm and a length of 2 mm. Using the maximum possible electrical pressure of $8.9 (10^6) \text{ N/m}^2$, I find the projectile's acceleration is 490 km/s^2 . This projectile will take 23 km to reach the final velocity of 150 km/s. An accelerator length of 23 km appears to be too long as a practical device for accelerating large projectiles to hyper-velocities.

4. CONCLUSIONS

I studied the feasibility of using magnetic-gradient and electrostatic accelerators to launch a 0.1-g projectile to 150 km/s. The first-order design of the magnetic-gradient accelerator was presented. This analysis led to the following conclusions.

Regarding the magnetic-gradient accelerator, which had a magnetic field of 10 T and a field gradient of $2.5(10^3)$ T/m, I concluded

- The superconducting projectile requires a 3.2-km-long accelerator and could achieve an efficiency as high as 92%.
- The ferromagnetic projectile requires a 12.4-km-long accelerator and could achieve an efficiency as high as 75%.
- An ablatable copper-sheathed projectile requires a 31-km-long accelerator and could achieve an efficiency as high as 0.3%.
- An unsheathed copper projectile would melt before achieving the desired velocity.

I estimated cost of components for the magnetic-gradient accelerator used to launch the superconducting projectile to be \$58.9 M. This estimate did not include such items as vacuum system, shelter, trigger system, instrumentation, and component connection.

The electrostatic accelerator would need a 23-km length, which is too long to be practical.

5. RECOMMENDATION

I concluded that of the two accelerators studied in this report the magnetic-gradient with superconducting projectile is the most likely to launch a 0.1-g projectile to 150 km/s. To realize this potential it needs more study both numerically and experimentally. Both studies must try to obtain a better understanding of the interaction between superconducting projectile and magnetic field.

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REFERENCES

1. E. R. Harrison, "Alternative Approach to the Problem of Producing Conducting Thermonuclear Power," *Phys. Rev. Lett.* 11, 535 (1963).
2. F. Winterberg, *Bull. Am. Phys. Soc.* 9, 309 (1964).
3. F. Winterberg, "On the Attainability of Fusion Temperatures Under High Densities by Impact Shock Waves of Small Solid Particles Accelerated to Hypervelocities," *Z. Naturforsch* 19a, 231 (1964).
4. C. Maisonnier, "Macroparticle Accelerators and Thermonuclear Fusion," *Nuovo Cimento* 42B(2), 332 (1966).
5. J. G. Linhart, in *Proc. Intern. School Enrico Fermi* (Academic Press, New York City, 1971), p. 151.
6. F. Winterberg, "Nuclear Fusion by Magnetic Acceleration of a Superconducting Solenoid," *Nucl. Fusion Lett.* 6, 152 (1966).
7. F. Winterberg, "Magnetic Acceleration of a Superconducting Solenoid to Hypervelocities," *Plasma Phys.--J. Nucl. Energy, Part C* 8, 541 (1966).
8. J. P. Barber, *The Acceleration of Macroparticles and a Hypervelocity Electromagnetic Accelerator*, unpublished PhD thesis, Australian National University, Canberra, Australia (1972).
9. R. L. Garwin, R. A. Muller, and B. Richter, *Magnetic-Gun Ignition for Controlled Thermonuclear Fusion*, SRI International, Arlington, Virginia, JSN-77-20 (1978).
10. H. H. Kolm, "Electromagnetic Guns, Launchers, and Reaction Engines," presented at *JANNAF 1979 Propulsion Meeting, Anaheim, Calif., 1979* (Francis Bitter National Magnetic Laboratory, Cambridge, MA 02139).
11. H. Carslaw and J. Jaeger, *Conduction of Heat in Solids* (Oxford University Press, New York City, 1959), 2nd ed., pp. 199-200.