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CONTRACT ACTIVITY

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I. ABSTRACT

The purpose of this study is to make a second order model of a synchronous generator that retains the electromechanical eigenvalues of the detailed machine model. Unfortunately, the parameters of the second order model generated depend on the machine's operating point (real and reactive power generated) as well as the external reactance connected to the machine. Therefore, a second order model with constant parameters will not retain the electromechanical eigenvalues of the machine. The variation of the parameters of the second order model with operating point and external reactance is presented graphically as the major result of this study.

II. INTRODUCTION

This project is a part of a larger project wherein the goal is to identify power system dynamic models from time series data. In such identification work, a simple model for the synchronous machine is desirable for several reasons. These include simplifying theoretical analysis, reducing the magnitude of the problem of gathering system data, and lowering computational requirements to reasonable levels. Considerations of this type will become ever more important as the application of system identification techniques to large power systems increases, and the need for a simple machine model will grow accordingly.

A simple machine model must be developed from a detailed model. The detailed model used in this paper is an eighth-order nonlinear differential equation model which represents the machine in the rotor reference frame. This model can be linearized about an operating point, and then characterized by the eigenvalues of the resulting linear system. Of the eight eigenvalues produced, the two electromechanical eigenvalues have the most effect on the system as a whole, and are most easily measured. Therefore, it is appropriate to use them as a basis for an attempt at a simple machine model.

In this paper, the eighth order model for the synchronous machine is presented first, along with the second order model to be derived from its eigenvalues. These models are then linearized about an operating point. A digital programming method is developed for extracting the electromechanical eigenvalues from the complex model, and forming the simple machine model from them. Next, an analog simulation of the detailed model of the synchronous machine is developed to verify the accuracy of the digital study, and the agreement of these methods is discussed. Finally, the second order model generated is studied to determine the variation of its parameters with

respect to the operating point of the detailed machine (defined as the real and reactive power it is producing) and the value of the transmission line reactance connected to it. The ability of the simple model, determined at one operating point and value of transmission line reactance, to predict the electromechanical eigenvalues of the detailed model under varying operating conditions is then discussed. The conclusion of this paper is a judgement of the usefulness of this second order model in system identification work.

III. MACHINE MODELS

All of the work in this paper is based upon the system shown in Fig. 1. This is a one-machine infinite bus system consisting of a synchronous generator connected through a transmission line to an infinite bus. A small, balanced, resistive load is connected to the terminals of the generator for simulation convenience. It is small enough, however, that it does not significantly affect the machine. The machine is represented in the rotor reference frame using d-q axis variables as described in Ref. 1. Since the machine is a generator, positive current is assumed to be as of the machine. The parameters and variables used are all in the per unit system.

A. The Eighth Order Model

This model is a slightly expanded version of the one presented in Ref.

2. It uses the following notation for system parameters:

r_s = armature resistance

x_{ls} = armature leakage reactance

r_{d1} = direct-axis field winding resistance

x_{d1} = direct-axis field winding leakage reactance

r_{d2} = direct-axis damper winding resistance

x_{d2} = direct-axis damper winding leakage reactance

r_{q1} = quadrature-axis field winding resistance

x_{q1} = quadrature-axis field winding leakage reactance

r_{q2} = quadrature-axis damper winding resistance

x_{q2} = quadrature-axis damper winding leakage reactance

x_{md} = direct-axis magnetizing reactance

x_{mq} = quadrature-axis magnetizing reactance

r_t = transmission line resistance

x_t = transmission line reactance

$$r_s^* = r_s + r_t$$

$$x_{ls}^* = x_{ls} + x_t$$

P = real power produced by the machine

Q = reactive power produced by the machine

Note, r_s^* and x_{ls}^* are sums of armature and transmission line variables. This is because those parameters were lumped together for convenience in the digital eigenvalue analysis.

Throughout the entire paper, the following notation holds:

$$p = \frac{d}{dt}$$

ω_e = synchronous speed (377 radians/second)

δ = the angle between the quadrature axis of the machine and the quadrature axis of the infinite bus

ω_r = the angular velocity of the rotor

V_t = the magnitude of the line to neutral voltage of the infinite bus (1 per unit)

H = the inertial constant of the machine

T_{pm} = the prime mover torque applied to the machine

Variables with the subscript 1 (eg. r_{d1} or ψ_{q1}) refer to field winding variables, those with the subscript 2 refer to damper winding variables, and those with the subscript t are transmission line variables.

Given this notation, the differential equations which make up the detailed synchronous machine model are as follows.

In the quadrature axis the equations are:

$$p\psi_q/\omega_e = V_t \cos(\delta) - \omega_r \psi_d/\omega_e + r_s i_q \quad (1)$$

$$p\psi_{q2}/\omega_e = -r_{q2}i_{q2} \quad (2)$$

$$p\psi_{q1}/\omega_e = -r_{q1}i_{q1} \quad (3)$$

referring to the stator, rotor damper, and rotor field windings respectively. Note also that $V_t \cos(\delta) = V_q$, the magnitude of the q-axis voltage.

In the direct axis the equations are:

$$p\psi_d/\omega_e = v_t \sin(\delta) + \omega_r \psi_q/\omega_e + r_s i_d \quad (4)$$

$$p\psi_{d2}/\omega_e = -r_{d2}i_{d2} \quad (5)$$

$$p\psi_{d1}/\omega_e = v_{d1} - r_{d1}i_{d1} \quad (6)$$

Again referring to stator, damper, and field windings. Also $V_t \sin(\delta) = V_d$, the magnitude of the d-axis voltage.

The mechanical equations are:

$$p\left(\frac{\omega_r}{\omega_e}\right) = (T_{pm} - \psi_d i_q + \psi_q i_d)/(2H) \quad (7)$$

$$p(\delta) = \omega_e \left(\frac{\omega_r}{\omega_e}\right) \quad (8)$$

referring to the dynamics of the rotor and its angular velocity.

This type of machine representation is developed in Ref. 3 and Ref. 4. The equations can be applied using data readily measurable in actual machines. Thus they make a good model for detailed studies in the area of system identification.

B. Second Order Model

The simple model, which is to be created from the detailed model and compared against it, is described by the equations:

$$p\left(\frac{\omega_r}{\omega_e}\right) = (T_{pm} - \psi_d i_q + \psi_q i_d - D\frac{\omega_r}{\omega_e}) / (2H) \quad (9)$$

$$p\delta = \omega_e \left(\frac{\omega_r}{\omega_e}\right) \quad (10)$$

These equations are the same as equations (7) and (8) except for the damping term $D \frac{\omega_r}{\omega_e}$. This term is added to approximate the electrical system effects, many of which are damping effects. The physical parameters which describe this model are: H the same parameter used in the detailed model, D a damping factor found from the eigenvalues, and X an internal reactance also found from the eigenvalues.

The main advantage of this model is its simplicity which will make its application to large system analysis practicable. The question this report is to resolve is whether this model can reliably predict the eighth order small displacement electromechanical mode frequency given an operating point and external reactance.

C. Parameters of System Chosen for Study

To perform meaningful studies using these models it is necessary to use parameters from a machine which is representative of its type. Table 1 contains the parameters of the machine used for this paper. It is a steam-turbine powered, high-speed generator of the type found in many generating facilities. This table also contains the range of values over which X_t (the transmission line reactance) was varied and the value used for r_t (the

transmission line resistance).

The range of values of X_t was chosen because this range of values extends from 0% to 16% of the machines magnetizing reactance, thus covering a large set of operating conditions.

Parameter	Value
r_s	.0032
x_{ls}	.123
r_{d1}	.001062
x_{d1}	.0896
r_{d2}	.01205
x_{d2}	.0723
r_{q1}	.007936
x_{q1}	.2983
r_{q2}	.02191
x_{q2}	.0463
x_{md}	1.507
x_{mq}	1.437
H	2.44
x_t	0.0-.25
r_t	.009

Table 1. Physical parameters of the synchronous machine and transmission line used for this paper. All values are measured in per unit except H whose units are seconds.

IV. DIGITAL COMPUTER STUDY

The digital computer program, whose development is described in this section, produces the results from which the conclusions of this paper are drawn. It takes the linearized eighth order system model, extracts the electromechanical eigenvalues, and calculates the second order system parameters from them. It does this for several operating points (amounts of real and reactive power produced) at a fixed value of transmission line reactance and so shows how the simple model produced varies with operating point.

A. Linearization of the Eighth Order Model

Before the eigenvalues can be extracted from this system, it must be placed in linearized state variable form. Therefore, a linearized version of equations (1) to (8) are used in a state variable representation where the state variable vector is:

$$\tilde{x}^T = \left[\Delta i_{q1}, \Delta i_{q2}, \Delta i_{q1}, \Delta i_d, \Delta i_{d2}, \Delta i_{d1}, \Delta \frac{\omega_r}{\omega_e}, \Delta \delta \right] \quad (13)$$

The equations as they stand have derivatives contained in the vector:

$$\tilde{\psi}^T = \left[\psi_{q1}, \psi_{q2}, \psi_{q1}, \psi_d, \psi_{d2}, \psi_{d1}, \frac{\omega_r}{\omega_e}, \Delta \delta \right] \quad (14)$$

To linearize this system a relation between the currents and the flux linkages is necessary. Such a relation is found in Ref. 2, and is:

$$\Delta \tilde{\psi} = \begin{bmatrix} -\tilde{x}_{ls}^* - x_{mq} & x_{mq} & x_{mq} & 0 & 0 & 0 & 0 & 0 \\ -x_{mq} & x_{q2} + x_{mq} & x_{mq} & 0 & 0 & 0 & 0 & 0 \\ -x_{mq} & x_{mq} & x_{q1} + x_{mq} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\tilde{x}_{ls}^* - x_{md} & x_{md} & x_{md} & 0 & 0 \\ 0 & 0 & 0 & -x_{md} & x_{d2} + x_{md} & x_{md} & 0 & 0 \\ 0 & 0 & 0 & -x_{md} & x_{md} & x_{d1} + x_{md} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tilde{x}^T$$

or

$$\Delta \tilde{\psi} = \tilde{M} \tilde{x}^T \quad (16)$$

Given these relations, the system can be linearized to the form:

$$p\tilde{\psi} = A^* \tilde{x}^T \quad (17)$$

where A^* is the matrix

$$\begin{array}{ccccccc}
 \omega_e r_s^* & 0 & 0 & \omega_e (X_{ls}^* + X_{md}) & -\omega_e X_{md} & -\omega_e X_{md} & -\omega_e \psi_{d0} & -\omega_e V_t \sin \delta \\
 0 & -\omega_e r_{q2} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & -\omega_e r_{q1} & 0 & 0 & 0 & 0 & 0 \\
 -\omega_e (X_{ls}^* + X_{mq}) & \omega_e X_{mq} & \omega_e X_{mq} & \omega_e r_s^* & 0 & 0 & \omega_e \psi_{q0} & \omega_e V_t \cos \delta \\
 0 & 0 & 0 & 0 & -\omega_e r_{d2} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & -\omega_e r_{d1} & 0 & 0 \\
 (-i_{d0} (X_{ls}^* + X_{mq}) & \frac{i_{d0} X_{mq}}{2H} & \frac{i_{d0} X_{mq}}{2H} & (i_{q0} (X_{ls}^* + & \frac{-i_{q0} X_{md}}{2H} & \frac{-i_{q0} X_{md}}{2H} & 0 & 0 \\
 -\psi_{d0})/2H & & & X_{md}) + \psi_{q0})/2H & & & & \\
 0 & 0 & 0 & 0 & 0 & 0 & \omega_e & 0
 \end{array}$$

In A^* , all variables which have the subscript 0 are steady state operating point values.

To put this into true state variable form ($\dot{p}\tilde{X} = \tilde{A}\tilde{X}^T$) equation (17) is premultiplied on both sides by $\tilde{M}^{-1}$ which is defined in equation (16).

The matrix $\tilde{A}$ is the one that the machine extracts the eigenvalues from. Thus the matrices $\tilde{M}$ and A^* must be filled. The variables ω_e , V_t , H , and the reactances are either known or given. The others depend upon operating point and the fact that the infinite bus voltage is assumed to be one per unit at zero angle.

The remaining unknown quantities in A^* can be found by using equations (18) to (21) from Ref. 5 where $f = V$, i , or ψ :

$$f_t = f_q^e - j f_d^e \quad (18)$$

$$f_q^r = f_q^e \cos \delta - f_d^e \sin \delta \quad (19)$$

$$f_d^r = f_q^e \sin \delta + f_d^e \cos \delta \quad (20)$$

$$V_d^r = -r_s i_d^r + (X_{ls}^* + X_{mq}) i_q^r \quad (21)$$

Equation (18) relates transmission line variables to their q and d axis components in the synchronous reference frame (denoted by superscript e). Equations (19) and (20) relate variables in the rotor reference frame (denoted by superscript r) to those in the synchronous. Equation (21) is an expression for the d-axis voltage of the machine.

Given these equations, the remaining unknowns in A^* are found as follows:

$$i_t = \left(\frac{P + jQ}{V_t} \right)^* = i_q^e - j i_d^e = \text{Re}(i_t) + j \text{Im}(i_t). \quad (22)$$

Substituting equations (19) and (20) into (21), collecting terms, and solving for δ gives

$$\delta = \text{TAN}^{-1} \frac{(X_{ls}^* + X_{mq}) i_q^e - r_s^* i_d^e - V_d^e}{V_q^e + r_s i_q^e + (X_{ls}^* + X_{mq}) i_d^e} \quad (23)$$

Substituting from equations (18), and (22), and knowing

$$V_t = 1 = V_q^e - j V_d^e \text{ so } V_d^e = 0, \text{ this is}$$

$$\delta = \text{TAN}^{-1} \frac{(X_{ls}^* + X_{mq})(\text{Re}(i_t)) + r_s (\text{Im}(i_t))}{V_t + r_s (\text{Re}(i_t)) - (X_{ls}^* + X_{mq})(\text{Im}(i_t))} \quad (24)$$

which defines δ . This δ is actually a steady state value and will be denoted henceforth as δ_0 .

The currents i_{d0} and i_{q0} in A^* are in the machine so they are actually i_{d0}^r and i_{q0}^r respectively. Thus, by equations (19), (20), and (22)

$$i_{d0} = \text{Re}(i_{t0})\sin\delta - \text{Im}(i_{t0})\cos\delta \quad (25)$$

$$i_{q0} = \text{Re}(i_{t0})\cos\delta + \text{Im}(i_{t0})\sin\delta \quad (26)$$

The flux linkages ψ_{d0} and ψ_{q0} can be found from equations (1) and (4) respectively and the fact that the machine is in steady state. In this case the derivatives are zero and

$$\psi_{d0} = V_t \cos\delta + r_s^* i_{q0} \quad (27)$$

$$\psi_{q0} = -V_t \sin\delta - r_s^* i_{d0} \quad (28)$$

This is the last unknown in A^* so the detailed system is now linearized and in a form suitable for analysis by digital computer.

B. Linearization of the Second Order Model

The electrical portion of this model is shown in Fig. 2. It includes the transmission line reactance with its internal reactance just as the complex model does.

The currents and voltage in this model are related by equation (29).

$$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{X_d} \\ \frac{1}{X_d} & 0 \end{bmatrix} \begin{bmatrix} V_q - E_x \\ V_d \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{X_d} \\ \frac{1}{X_d} & 0 \end{bmatrix} \begin{bmatrix} V_t \cos \delta - E_x \\ V_t \sin \delta \end{bmatrix} \quad (29)$$

Using this the differential equations describing this model (equations (9) and (10)) may be linearized as follows:

From Ref. 1

$$\psi_d i_q - \psi_q i_d = T_e = V_q i_q + V_d i_d \quad (30)$$

substituting from (29) gives

$$T_e = \frac{1}{X_d} (V_t^2 \sin \delta \cos \delta) - \frac{1}{X_d} (V_t \cos \delta - E_x) V_t \sin \delta \quad (31)$$

$$= \frac{E_x V_t \sin \delta}{X_d}$$

Substituting this into equation (9) gives

$$p \left(\frac{\omega_r}{\omega_e} \right) = \frac{1}{2H} \left[T_{pm} - \frac{V_t E_x}{X_d} \sin \delta - D \left(\frac{\omega_r}{\omega_e} \right) \right] \quad (32)$$

Writing this and equation (10) in terms of small displacement variables and putting them in matrix form gives:

$$P \begin{bmatrix} \Delta & \frac{\omega_r}{\omega_e} \\ \Delta & \delta \end{bmatrix} = \begin{bmatrix} \frac{D}{2H} & \frac{1}{2H} \left(\frac{-V_t E_x}{X_d} \right) \cos \delta & 0 \\ \omega_e & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta & \frac{\omega_r}{\omega_e} \\ \Delta & \delta \end{bmatrix} \quad (33)$$

This completes the linearization of the second order system. The unknown variables in equation (33) (D , E_x , X_d , and δ) will be determined from the P and Q operating point and the electromechanical eigenvalues of the detailed model.

C. Determining Second Order Model

To determine the parameters of the second order model, the electromechanical eigenvalues of the detailed model must first be determined. The elements of $\tilde{A}$ are all known, so a digital computer can form that matrix, and find its eigenvalues. The eigenvalues found will resemble the following list.

$$-1.5 \pm j .377.$$

$$-50.$$

$$-30.$$

$$-5.$$

$$-.2$$

$$-3. \pm j 7.$$

These numbers, of course, only show the order of magnitude to be expected for each eigenvalue.

The last eigenvalue pair is the electromechanical eigenvalue pair. Its imaginary part is distinctive and the computer can separate it from the others. Call it $Z \pm j \omega_n$.

Now the parameters of the simple model must be determined using P , Q , Z , and ω_n . The equations for doing this are derived as follows.

First find the eigenvalues of the second order system (equation (33)). These are found by solving $\det|\tilde{S}\tilde{I} - \tilde{A}| = 0$ to give

$$\det \begin{vmatrix} s + \frac{D}{2H} & \frac{V_t E_x \cos \delta_0}{2HX_d} \\ -\omega_e & s \end{vmatrix} = 0 \quad (34)$$

or

$$s^2 + \frac{D}{2H} s + \frac{\omega_e V_t E_x \cos \delta_0}{2HX_d} = 0 \quad (35)$$

$$s = \frac{D}{4H} \pm \frac{1}{2} \sqrt{\frac{D^2}{4H^2} - \frac{2\omega_e V_t E_x \cos \delta_0}{HX_d}} \quad (36)$$

These should be the same as the electromechanical eigenvalues so by equating real parts it is seen that

$$Z = \frac{-D}{4H} \quad (37)$$

$$D = -4ZH \quad (38)$$

and the first of the parameters of the simple model is defined.

From Fig. 2 it is easily determined that

$$P = \frac{V_t E_x}{X_d} \sin \delta_0 \quad (39)$$

$$Q = \frac{V_t^2}{X_d} - \frac{V_t E_x}{X_d} \cos \delta_0 \quad (40)$$

Since the radical in equation (36) must be negative,

$$\omega_n = \frac{1}{2} \sqrt{\frac{2\omega_e V_t E_x \cos \delta_0}{H X_d} - \frac{D^2}{4H^2}} \quad (41)$$

Substituting from equation (38)

$$\omega_n = \frac{1}{2} \sqrt{\frac{2\omega_e V_t E_x \cos \delta_0}{H X_d} - 4Z^2} \quad (42)$$

solving for X_d gives

$$X_d = \frac{2\omega_e V_t E_x \cos \delta_0}{H(4\omega_n^2 + 4Z^2)} \quad (43)$$

Substituting this into equation (39) and solving for δ leads to

$$\delta_0 = \tan^{-1} \left[\frac{P\omega_e}{H(2\omega_n^2 + 2Z^2)} \right] \quad (44)$$

The second unknown in the simple model.

To find E_x for this model, divide equation (39) by (40) and solve for E_x to give

$$E_x = \frac{P V_t}{(Q \sin \delta_0 + P \cos \delta)} \quad (45)$$

X_d , the final unknown, can now be determined by substituting the calculated values into equation (43). Then

$$X = X_d - X_t \quad (46)$$

as shown in Fig. 2.

This completes the determination of the second order model parameters.

D. Implementation of Digital Study

The equations derived in this section were directly implemented in a digital computer program. The matrices A^* and M were generated using system parameters and then the product $\tilde{A} = M^{-1}A^*$ was formed using subroutines available on most computing systems. Using other system routines, the eigenvalues of $\tilde{A}$ were determined, and the electromechanical eigenvalue pair split off. At this point the parameters of the simple model were determined using the equations derived.

This sequence of calculations was followed for various P and Q operating points to show how the parameters D and X in the second order model must vary to predict the electromechanical eigenvalues. Based on these results, the ability of this model to predict the eighth order electromechanical eigenvalues was evaluated.

V. ANALOG SIMULATION

A. Purpose of the Analog Simulation

The digital computer program, the development of which was presented in the last section, was the source of the data from which the conclusions of this paper are drawn. As such, it was imperative that independent means be used to assure its accuracy. The final output of the program could only be checked by carefully searching the program for errors. However, the determination of the electromechanical eigenvalues (which comprised the majority of the program) was readily amenable to verification. The analog simulation of a synchronous generator presented in this section was the tool used in this capacity.

B. Presentation of Simulation

The analog simulation used in this study is based on the simulation developed in Ref. 2. It represents the synchronous machine in the rotor reference frame, and uses the same differential equations to describe the machine that are set forth in Eq. 1 to Eq. 8 of this paper. Therefore, it is ideally suited for verification of the digital version of that model.

The simulation is broken up into four interconnected segments whose block diagrams are present in Figs. 3-6. These segments develop voltages representing direct-axis quantities, quadrature-axis quantities, torque and power angle variables, and transmission line variables respectively. The equations leading to this simulation and the variables used in it have all been presented except for the transmission line equations which are

$$V_q = r_t i_{qt} + \frac{X_t}{\omega_b} p_{qt} + \frac{X_t}{\omega_b} i_{dt} p\theta + V_t \cos\delta \quad (47)$$

$$V_d = r_t i_{dt} + \frac{X_t}{\omega_b} p_{dt} - \frac{X_t}{\omega_b} i_{qt} p\theta + V_t \sin\delta \quad (48)$$

$$V_q = R(i_q - i_{qt}) \quad (49)$$

$$V_d = R(i_d - i_{dt}) \quad (50)$$

where θ is the angle of the rotor, and the variables:

$$X_{ad} = 1 / \left(\frac{1}{X_{md}} + \frac{1}{X_{ls}} + \frac{1}{X_{d1}} + \frac{1}{X_{d2}} \right) \quad (51)$$

$$X_{aq} = 1 / \left(\frac{1}{X_{mq}} + \frac{1}{X_{ls}} + \frac{1}{X_{q1}} + \frac{1}{X_{q2}} \right) \quad (52)$$

The inputs to this system (E_x and T_{pm}) are also unspecified. They are set by means of variable potentiometers when the system is in operation, and determine the operating point of the machine.

C. Determination of Electromechanical Eigenvalues

A system described by differential equations, and simulated by analog methods, has eigenvalues which characterize it. All of the eigenvalues affect each system variable to some degree, but most of them have a variable which they affect much more strongly than any other. In this simulation, the variable $\frac{\Delta\omega}{\omega_b}$ very strongly reflects the electromechanical eigenvalue. Thus it is the one used to study that eigenvalue in this case.

To extract the eigenvalue at a specific operating point, the simulation is first allowed to reach steady state with the proper excitation for that operating point, but a T_{pm} (prime mover torque) fractionally incorrect. Then T_{pm} is stepped to the correct value, the signal representing $\frac{\Delta\omega}{\omega_b}$ is

greatly amplified, and it is recorded on a strip chart recorder.

The main component of the signal on the strip chart recording (Fig. 7a) is a sine wave multiplied by a decaying exponential. The frequency of the sine wave and the decay constant of the exponential are the imaginary and real parts of the electromechanical eigenvalue respectively. Unfortunately, there is another signal added to the one described which is also a decaying exponential. This makes direct measurement of the exponential constant impossible, even though the frequency of the sine wave may be determined by measuring peak to peak distance. To solve this problem, the envelope of the damped sinusoid is approximated using a french curve (Fig. 7b). Measurements of this envelope were used to find the constant of the exponential and hence the real part of the electromechanical eigenvalue.

VI. COMPARISON OF RESULTS OF DIGITAL AND ANALOG METHODS

A. Extent of Comparison of Results

The great majority of the program used in this, or any similar, digital study is that part which is involved in calculating the electromechanical eigenvalues. The remainder is a short series of relatively simple algebraic statements which may be readily checked for accuracy. Therefore, a comparison of the eigenvalues calculated by the program and those measured from the analog simulation of the same machine at the same operating point, if they agree, constitute sufficient verification of the accuracy of both.

The process of confirming the accuracy of the program was carried out with the object of reducing experimental error as much as possible in mind. Once the analog simulation of the machine under investigation was patched and debugged, the electromechanical eigenvalues were measured five times at each of seventeen operating points. The operating points were rotated so that a fresh adjustment of operating point was made before each measurement. Then for each operating point the five values measured for the real and imaginary parts of the eigenvalue were averaged. The resulting values were tabulated and compared against those calculated by the program. This was done using as small a value of transmission line reactance as was practicable. The results of this procedure are presented in table 2. The error percentages, both average and absolute, are so small that they are well within the range of unavoidable experimental error.

This validates both the program and the simulation at that value of X_t and over the range of operating points. To attempt to validate both constructions over a wide range of X_t values, this parameter was increased to a level as large as was practicable, and measurements were taken as described above. Due to time constraints only five operating points were tested, but

they were widely separated to make them as significant as possible. The results of these measurements are shown in Table 3. These error levels are not as low as the initial set because the size of the sample space they came from was considerably smaller. These levels were still within the range of unavoidable experimental error, however.

Operating Point		Real Part			Imaginary Part		
P	Q	Measured value	Calculated value	% error	Measured value	Calculated value	% error
1.0	.4	-1.78	-1.83	2.73	16.45	16.36	-.55
1.0	.2	-1.69	-1.70	.59	16.28	16.15	-.80
1.0	-.2	-1.41	-1.36	-3.68	16.03	15.84	-1.20
.8	.2	-1.86	-1.92	3.13	15.87	15.81	-.38
.8	0.0	-1.79	-1.73	-3.47	15.79	15.67	-.77
.6	.4	-2.32	-2.29	-1.31	15.59	15.68	.57
.6	0.0	-2.16	-2.02	-6.93	15.14	15.23	.59
.6	-.2	-1.86	-1.72	-8.14	15.28	15.22	-.39
.4	.4	-2.63	-2.56	-2.73	15.32	15.32	0
.4	.2	-2.58	-2.55	-1.18	15.18	14.94	-1.61
.4	0.0	-2.54	-2.45	-3.67	14.71	14.67	-.27
.2	0.0	-3.00	-2.94	-2.04	14.54	14.1	-3.12
.2	-.2	-3.03	-2.88	-5.21	13.96	13.77	-1.38
0.0	.4	-2.8	-2.86	2.10	14.78	14.9	.81
0.0	.2	-2.96	-3.02	1.99	14.25	14.4	1.04
0.0	0.0	-3.15	-3.19	1.25	13.43	13.82	2.82
0.0	-.2	-3.45	-3.39	-1.77	13.01	13.26	1.89
average error = 1.67				average error = -.16			
absolute average error = 3.06				absolute average error = 1.07			

Table 2. Measured value, calculated value, and % error

$$(\% \text{ error} = \frac{\text{calculated value} - \text{measured value}}{\text{calculated value}} \times 100)$$

at various operating points and $X_d = .09$ per unit of the real and imaginary parts of the electromechanical eigenvalues.

Operating Point		Real Part			Imaginary Part		
P	Q	Measured value	Calculated value	% error	Measured value	Calculated value	% error
1.0	.4	-.47	-.58	18.96	10.76	11.68	7.88
.6	.2	-.67	-.74	9.46	10.68	10.83	1.39
.2	0.0	-1.06	-1.06	0.0	10.0	9.81	-1.94
0.0	.4	-.90	-.91	1.1	10.96	11.24	2.49
0.0	-.2	-1.4	-1.39	-.72	8.72	8.87	1.69
average error = 5.67				average error = 2.30			
absolute average error = 6.05				absolute average error = 3.08			

Table 3. Measured value, calculated value, and % error

$$\left(= \frac{\text{calculated value} - \text{measured value}}{\text{calculated value}} \times 100 \right)$$

at various operating points and $X_d = .5$ per unit of the real and imaginary parts of the electromechanical eigenvalues.

B. Discussion of Agreement of Methods

The error limits for this study are in the order of ten percent for the real part of the eigenvalue and three percent for the imaginary part. These tolerance estimates came from error inherent in using a french curve to draw the exponential envelopes and from error which is unavoidable when reading values from strip chart recordings, and are at least as stringent as the actual tolerances.

The data presented in Table 2 and Table 3 shows that while a few of the eigenvalues measured differ widely from their calculated counterparts, the majority match closely. Thus the average error over all the eigenvalues measured is within acceptable limits. This, and the fact that the digital program was developed independently of the simulation, indicates that both are valid sources of data about the machine under study. Therefore, the results generated using that data, which are presented later in this report, are also valid.

VII. STUDY OF SECOND ORDER MODEL GENERATED

A. Variation of Parameters

The parameters which describe the second order machine, D and X , are dependent on more than just the parameters of the machine. They also depend on the P and Q operating point of the machine and the external reactance connected to it. This dependence is due to the fact that the electromechanical eigenvalues vary with these factors. Therefore an analytical description of the behavior of X and D is not available. Instead, their behavior is characterized graphically in Fig. 8 through Fig. 19.

The behavior of X is characterized in Figs. 8, 9, and 10. These graphs are in order of increasing external reactance, and show X versus P with Q as a parameter. They reveal X to be at least cubic in nature, even though linearity of the plots improves as the values of P and Q increase. The graphs might even be approximated by straight lines, if accuracy tolerances are low enough. It is not possible, however, to represent this parameter as a constant and achieve reasonable accuracy of results, unless the work being done is of the small signal about an operating point type.

Figures 11, 12, and 13 contain the same information as the three preceding figures. The difference is that X is plotted against Q with P as a parameter. The comments to be made about these graphs are exactly the same as those made about the three before them.

Figures 14, 15, and 16 characterize the behavior of D . Again, the graphs are in order of increasing external reactance. They show D versus P with Q as a parameter, and exhibit characteristics that are at least cubic. Linearization of these curves would only be practicable for high Q curves, and, as with X , a single value of D in the machine would be useless except for work requiring extremely low accuracy or only dealing with operating

point type studies.

Figures 17, 18, and 19 also characterize the behavior of D . They show D plotted against Q with P as a parameter. The comments in the preceeding paragraph apply equally well to them.

The effect of the external reactance in all of these plots is either to move the curves as a block up or down the vertical axis or to spread the curves out with respect to this axis. It doesn't seem to affect the nature of the curves themselves in any significant fashion. Therefore, while it must be taken into account for completeness, it increases the complexity of the situation only by degree. It does not change the situation itself, especially since it would be a constant in most practical applications of this type of study.

VIII. CONCLUSION

A detailed model of a synchronous machine was presented and linearized about an operating point. An eigenvalue analysis was performed on it and the electromechanical eigenvalues were used to find parameters describing a second order model. The variation of these parameters with operating point and external reactance was studied. It was found that these parameters could not be represented by constants. Therefore one second order machine model with constant parameters can not predict the electromechanical eigenvalues of the detailed model at varying operating conditions with reasonable accuracy, and the model as described is not useful for system identification work, unless it is in operating point type studies.

IX. REFERENCES

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X. FIGURES

Figure 1 The system under investigation.

Figure 2 The electrical system represented by the second order model.

Figure 3 The simulation diagram of the direct-axis of the synchronous machine.

Figure 4 The simulation diagram of the quadrature-axis and speed voltages of the synchronous machines.

Figure 5 The simulation diagram of the torque and power angle of the synchronous machine.

Figure 6 The simulation diagram of the transmission line connected to the synchronous machine.

Figure 7a A strip chart recording of $\frac{\Delta\omega}{\omega_b}$ showing the response of the synchronous machine to a step change in prime mover torque.

Figure 7b A strip chart recording like the one in Fig. 7a., but with the envelop drawn in so the exponential decay may be measured.

Figure 8 Curves of per unit internal reactance of the simple machine model (X) versus per unit power (P) with reactive per unit power (Q) as the third parameter and per unit transmission line reactance (X_t) equal to zero.

Figure 9 Curves of X versus P with Q as the third parameter and X_t equal to .09.

Figure 10 Curves of X versus P with Q as the third parameter and X_t equal to .25.

Figure 11 Curves of X versus Q with P as the third parameter and X_t equal to zero.

Figure 12 Curves of X versus Q with P as the third parameter and X_t equal to .09.

Figure 13 Curves of X versus Q with P as the third parameter and X_t equal to .25.

Figure 14 Curves of per unit damping factor (D) versus P with Q as the third parameter and X_t equal to zero.

Figure 15 Curves of D versus P with Q as the third parameter and X_c equal to .09.

Figure 16 Curves of D versus P with Q as the third parameter and X_t equal to .25.

Figure 17 Curves of D versus Q with P as the third parameter and X_t equal to 0.

Figure 18 Curves of D versus Q with P as the third parameter and X_c equal to .09.

Figure 19 Curves of D versus Q with P as the third parameter and X_c equal to .25.