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MASTER

ABSTRACT

A computer model was developed which predicts the axial temperature profile and heat flux at the outer surface of the test section of the Sodium Boiling Test Facility constructed by the Engineering Technology Division at ORNL. The model was in agreement with observed temperature profiles at furnace power levels representative of single phase, dual phase, and dry-out operations. A parametric study demonstrated the effect of sodium flow rate and surface emissivities on the predicted temperature profile. It was concluded that axial conduction in the Hastelloy tube and sodium must be incorporated into the model to improve accuracy.

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1. SUMMARY

The Sodium Boiling Test Facility (SBTF) is a single-channel test loop designed to match the hydraulic characteristics of a typical Liquid Metal Fast Breeder Reactor (LMFBR) coolant channel. Accident scenarios have defined free-convective sodium flow as a worst case for dissipating heat generated in the reactor core. The purpose of this work was to develop a model of the SBTF, which is capable of correlating the observed test section temperature profile and heat loading.

A model was developed for calculating the net heat flow to the test section by application of a radiation heat balance. This heat flux is used to compute the axial temperature profile at the wall of the test section.

Four SBTF runs, representing typical single-phase, dual-phase, and dry-out flow conditions were simulated by this model. In each case, the predicted temperature profile agreed well with actual test-section data. However, in each case the calculated test section heat-loading was greater than that observed. Accounting for axial conduction in the model should resolve this discrepancy. Results from the model quantify the effect on the predicted temperature profile of uncertain sodium flow rate and target and reflector emissivity.

2. INTRODUCTION

2.1 Background

An important aspect governing the safe operation of a Liquid Metal Fast Breeder Reactor (LMFBR) is the design of a heat exchanger system capable of removing the heat generated in the reactor core. Under normal conditions, the maximum coolant temperature in a sodium-cooled breeder reactor is approximately 300°C below the coolant boiling point. In spite of this margin, several hypothetical accidents that could result in coolant boiling have been postulated. One such accident is initiated by a major rupture of a coolant pipe. As a result, the coolant flow through the core would be reduced and liquid sodium saturation temperatures would be achieved. The coolant flow through the boiling flow channels would then be facilitated by the differential pressure head between the boiling and non-boiling flow channels. This free-convection mechanism could significantly delay dry-out and fuel failure for certain accident conditions. The Sodium Boiling Test Facility (SBTF) was developed at ORNL to investigate these and related effects.

The SBTF (Fig. 1) is a single-channel test loop whose hydraulic characteristics were selected to simulate a typical LMFBR coolant channel (6). The test section is a 3.0-meter (118 in.) Hastelloy X tube with a 3.25-mm

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SCHEMATIC DIAGRAM OF
SODIUM BOILING TEST FACILITY

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FIG.
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(0.128 in.) I.D., and a 2.90-mm (0.114 in.) wall. A 16-kW quadelliptical reflector radiant furnace (10) supplies heat to a 0.97-m (38 in.) zone of the test section to simulate core heat loads (see Appendix 8.4 for other dimensions). Sodium flows up through the radiant furnace into an adiabatic section and is recirculated back to the furnace through a downcomer leg. Because the average temperature in the downcomer leg is lower than that at the furnace, a single-phase thermosiphon effect forces sodium up through the furnace at power levels lower than those required for boiling inception. Such natural convective flow simulates the heat exchanger capacity of liquid sodium in a LMFBR under conditions of pump failure or loss of pipe integrity.

The SBTF is also equipped with an electromagnetic pump to simulate normal operation of a LMFBR. The apparatus contains a data-acquisition system which monitors the axial-temperature profile of the Hastelloy tube, system pressures, the power supplied to the furnace, the flow rate of the sodium, and the inlet and outlet temperature and flow rate of cooling water in the walls of the furnace.

2.2 Objective and Approach

The flow characteristics of a boiling channel are dependent on the quality distributions and hence the axial heat flux distribution. The objective was to develop a steady-state model of the heat transfer to the test section in the radiant furnace that would define this distribution. By neglecting axial conduction in the test-section tube and back conduction in the sodium, this model can be formulated by solving a radiation energy balance in the furnace, coupled with an energy balance over the Hastelloy tube and liquid sodium. The three-body radiation problem describing radiative exchange between the radiation source, the reflector walls, and the Hastelloy target tube, should account for the specular nature of the aluminum reflector, grey body and diffusely reflected radiation, and end effects in the furnace. From the net heat flux at the target, the flow rate and inlet temperature of the sodium may be combined to generate an axial sodium temperature profile.

3. THEORY

The model was developed by considering the radiation balance between the various surfaces, sodium sensible heat effects, and Hastelloy wall temperature drops. These will be considered independently and will then be integrated into a complete algorithm.

3.1 Radiation Balance at a Specular Diffuse Surface

The total flux leaving a specular-diffuse surface (Fig. 2) can be expressed in terms of three contributions (1, 2, 4, 7, 11, 13, 14): (1) grey-

$$A_j H_j = \sum_i W_{D_i} (s_i s_j) s$$

$$W_{D_j} = \epsilon_j + \rho_{D_j} H_j$$

$$Q_{NET} = E_j \epsilon_j - H_j \epsilon_j$$

$$E_j = \sigma T_j^4$$

$$\alpha_j = \epsilon_j = 1 - (\rho_{D_j} + \rho_{S_j})$$

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RADIATION BALANCE AT A
SPECULAR-DIFFUSE SURFACE
HOTTEL-SAROFIM MODEL

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body radiation which depends only on the temperature and emissivity of the surface, (2) diffusely reflected radiation which depends on the intensity of the radiation incident to the surface and the diffuse reflectivity of the surface, and (3) specularly reflected radiation which depends on the intensity and angle of incidence of incoming radiation and the specular reflectivity of the surface.

In an enclosure, the radiation leaving one body must be accounted for by the radiation incident to it and the other surfaces. A total exchange area (11) between body i and body j $(s_i s_j)_s$ can be defined as the area of i A_i times the fraction of radiation leaving i , which reaches j both directly and after multiple specular reflection from all other surfaces, as given below:

$$\sum_j (1 - \rho_{s_j})(s_i s_j)_s = A_i \quad (1)$$

and by reciprocity:

$$(s_i s_j)_s = (s_j s_i)_s \quad (2)$$

Note that for imperfect reflectors ($\rho_s < 1.0$), the intensity of a reflected ray will be diminished by the specular reflectivity of each surface from which the ray is reflected. Furthermore, any ray leaving i and reaching j could also be the path for a ray leaving j and reaching i . The total exchange area $(s_i s_j)_s$ is the weighting factor by which the diffuse component of radiation W_{d_i} leaving each surface must be multiplied before being summed to equal the incident radiation H_j on surface j . With reference to Fig. 2,

$$\sum_j W_{d_i}(s_i s_j)_s = A_j H_j \quad (3)$$

and

$$W_{d_j} = \rho_{d_j} H_j + \epsilon_j E_j \quad (4)$$

By setting $i = 1$, eliminating H_j , and collecting terms in W_{d_1} ,

$$[(s_1 s_1)_s - \frac{A_1}{\rho_{d_1}}] W_{d_1} + (s_1 s_2)_s W_{d_2} + (s_1 s_3)_s W_{d_3} = - \frac{A_1 \epsilon_1}{\rho_{d_1}} E_1 \quad (5)$$

Equation (5) and its counterparts for surfaces 2 and 3 can easily be represented in matrix form as shown below:

Transfer Matrix	Response Vector	Excitation Vector	
$\begin{bmatrix} (s_1 s_1)_s - \frac{A_1}{\rho_{d1}} & (s_1 s_2)_s & (s_1 s_3)_s \\ (s_s s_1)_s & (s_2 s_2)_s - \frac{A_2}{\rho_{d2}} & (s_2 s_3)_s \\ (s_3 s_1)_s & (s_3 s_2)_s & (s_3 s_3)_s - \frac{A_3}{\rho_{d3}} \end{bmatrix}$	$\begin{bmatrix} W_{d1} \\ W_{d2} \\ W_{d3} \end{bmatrix}$	$= \begin{bmatrix} -\frac{A_1 \epsilon_1}{\rho_{d1}} E_1 \\ -\frac{A_2 \epsilon_2}{\rho_{d2}} E_2 \\ -\frac{A_3 \epsilon_3}{\rho_{d3}} E_3 \end{bmatrix}$	(6)

The response vector is determined from the product of the excitation vector and the inverse of the transfer matrix. The net flux at surface j , by inspection of Fig. 2, is then given by:

$$Q_{netj} = \epsilon_j E_j - \epsilon_j H_j$$

Or, eliminating H_j using Eq. (4),

$$Q_{netj} = \frac{\epsilon_j}{\rho_{dj}} [E_j (1 - \rho_{sj}) - W_{dj}] \quad (7)$$

Since all the terms in the transfer matrix and excitation vector are only functions of the system geometry and surface properties, the response vector can be solved for a given set of W_{dj} values which specify the net heat flux at each surface by Eq. (7).

To evaluate the response vector for the problem under consideration, the exchange areas in the transfer matrix must be determined. The configuration of the radiant furnace is shown in Fig. 3. The radiant furnace consists of an aluminum quadelliptical reflector with the target at the common focus and high intensity lamps at the four remaining foci. Let the tungsten filament be body 1, the Hastelloy target be body 2, and the aluminum reflector be body 3. An inspection of a cross-section of the quadelliptical furnace reveals that only the Hastelloy tube cannot see an image of itself in the aluminum reflector. Therefore, the $(s_2 s_2)_s$ exchange area alone would be zero.

If only the filament-to-filament $(s_1 s_1)_s$ and filament-to-target $(s_1 s_2)_s$ exchange areas were known, the conservation of flux relation [Eq. (1)] would give the filament-to-wall $(s_1 s_3)_s$ exchange area by:

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CROSS SECTION OF
QUADELLIPTICAL FURNACE

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$$(s_1 s_3)_s = \frac{1}{1 - \rho_{s3}} [A_1 - (s_1 s_1)_s - (s_1 s_2)_s] \quad (8)$$

By the reciprocity relation [Eq. (2)], $(s_2 s_1)_s = (s_1 s_2)_s$. Since $(s_2 s_2) = 0$, conservation of flux may again be used to solve for $(s_2 s_3)_s$. Similarly, $(s_3 s_3)_s$ can be related to $(s_3 s_1)_s$ and $(s_3 s_2)_s$, which are equal to $(s_1 s_3)_s$ and $(s_2 s_3)_s$, respectively, by reciprocity.

As discussed above, knowledge of the filament-to-filament and filament-to-target exchange areas determined all the other exchange areas in the quadelliptical furnace.

In the foregoing analysis the quadelliptical furnace was assumed to be an enclosure, when in fact its open ends account for 1.3% of the total wall surface area. Since the length of the furnace is finite, the fraction of rays that will leave surface i and reach surface j vary as a function of the distance z along the furnace. Utilizing symmetry, the exchange area can then be expressed as that fraction of the surface area of a hemisphere through which effective rays must pass. The filament-to-filament exchange area (see Fig. 4) is:

$$(s_1 s_1)_s = A_1 \rho_{s3} \left[\frac{1 - \cos[\tan^{-1}(\frac{R_f}{2(a-f)})]}{2\pi} [\tan^{-1}(\frac{z}{2(a-f)}) + \tan^{-1}(\frac{L-z}{2(a-f)})] \right] \quad (9)$$

Note that the exchange area has been diminished by the reflectivity of aluminum to account for the attenuation of reflected rays. The filament-to-target exchange area (Fig. 4) has both a direct and reflected component and may be expressed as:

$$(s_1 s_2)_s = (s_1 s_2)_{\text{direct}} + \rho_{s3} (s_1 s_2)_{\text{reflected}}$$

where:

$$(s_1 s_2)_{\text{direct}} = A_1 \left[\frac{1 - \cos[\tan^{-1}(\frac{R_{t0}}{2f})]}{2\pi} [\tan^{-1}(\frac{z}{2f}) + \tan^{-1}(\frac{L-z}{2f})] \right] \quad (10)$$

$$(s_1 s_2)_{\text{reflected}} = A_1 \left[\frac{1 - \cos[\pi - \tan^{-1}(\frac{y}{x})]}{2\pi} [\tan^{-1}(\frac{z}{2a}) + \tan^{-1}(\frac{L-z}{2a})] \right] \quad (11)$$

See Appendix 8.1 for details. These exchange areas, combined with Eqs. (1), (2), (6), and (7), define the net flux at the Hastelloy target.

3.2 Heat Effects in the Hastelloy Target

As liquid sodium moves up in the test section, its temperature increases proportionally with the amount of heat it absorbs. For an incremental height Δz (see Fig. 4), the energy balance yields:

$$\Delta T_{Na} = \frac{Q_{net}(z)}{M C_{pNa}} \frac{A_2}{L} \Delta z \quad (12)$$

where A_2 is the outer surface area of the tube, L is the length of the furnace, $Q_{net}(z)$ is the local net heat flux to the outer surface of the tube, and M is the sodium flow rate. Therefore, if the inlet temperature of sodium and $Q_{net}(z)$ are known, a temperature profile of the liquid sodium can be calculated.

Because Hastelloy has a moderate thermal conductivity, the temperature drop across the walls of the tube must be accounted for (3). This is given by:

$$\Delta T_{wall}(z) = - \frac{Q_{net}(z) R_{t0}}{K_{t0}} \ln\left(\frac{R_{t0}}{R_{ti}}\right) \quad (13)$$

where R_{t0} and R_{ti} are the outside and inside ratio of the Hastelloy tube.

Equations (6), (7), (12), and (13) can now be combined to predict the wall temperature profiles. Inputs to the model include the total power input to the furnace which is assumed to be uniformly distributed over its length, the reflector temperature and properties, sodium inlet temperature, and flow rate properties of the filament and target.

Since the power input to the filament is known rather than the filament temperature, the transfer matrix and excitation vector must be modified by eliminating E_1 in Eq. (7). Thus Eq. (6) becomes

$$\begin{bmatrix} (s_1 s_1)_s - A_1 & (s_1 s_2)_s & (s_1 s_3)_s \\ (s_2 s_1)_s & -\frac{A_2}{\rho_{d2}} & (s_2 s_3)_s \\ (s_3 s_1)_s & (s_3 s_2)_s & (s_3 s_3)_s - \frac{A_3}{\rho_{d3}} \end{bmatrix} \begin{bmatrix} W_{d1} \\ W_{d2} \\ W_{d3} \end{bmatrix} = \begin{bmatrix} -A_1 Q_{net1} \\ -\frac{A_2 \epsilon_2}{\rho_{d2}} E_2 \\ -\frac{A_3 \epsilon_3}{\rho_{d3}} E_3 \end{bmatrix} \quad (14)$$

where $(s_2 s_2)_s$ has been neglected, as discussed above. Each element in the transfer matrix in Eq. (14) is now defined by the input parameters.

If Δz is sufficiently small to assume that the target temperature remains essentially constant, then E_2 can be evaluated. Since Q_{net1} and $E_3 (= \sigma T_3^4)$ are known and are constant throughout the furnace, Eq. (14) can be solved for the response vector. Using Eq. (7), Q_{net2} is determined from E_2 and W_{d2} . Finally, Eq. (12) is used to calculate the sodium temperature at $z = \Delta z$, while the target surface temperature at $z = \Delta z$ is found from Eq. (13). This procedure can then be repeated until the end of the furnace is reached.

4. RESULTS AND DISCUSSION

Based on the theoretical development presented, a computer code was developed to compute heat fluxes and temperature profiles of the test section (see Appendix 8.2). This model was then used to correlate single-phase and dual-phase SBTF runs. The model was also used to investigate the effect of various conditions on the predicted axial temperature profiles in the test section.

4.1 Experimental Conditions

In Table 1 the actual conditions of 4 SBTF runs are given. The

Table 1. Summary of SBTF Runs

Test	Supplied Furnace Power (KW)	$\dot{M}C_{pNa}$ (3) (W/K)	Furnace Efficiency (%)	Comments
T108R2	5.10	1.0±0.7	9.8	single phase
T120R1	7.50	2.7±0.7	20.5	2-phase, high flow, low quality
T119R1	8.55	1.0±0.7	18.2	2-phase, low flow, high quality
T128R1	10.00	1.0±0.7	16.7	dry-out

supplied power refers to the power input to the quadrupole furnace, while furnace efficiency is the ratio of power consumed in the test section to the power supplied to the furnace. Quality is defined as the vapor mass flow rate over the liquid mass flow rate. The accuracy of sodium flow measurement $\dot{M}$ in $\dot{M}_{Co}$ is limited by the sensitivity of the electromagnetic flow meter to monitor oscillatory flow at low velocities.

In Table 2 estimated surface properties for each component of the radiant furnace are presented. Test runs were simulated using the nominal values of the properties, while in the sensitivity analysis the effects of the indicated ranges of uncertainty are investigated.

Table 2. Properties of Materials in the Furnace

Surface	Property	Nominal Value	Range of Uncertainty
Hastelloy	emissivity	0.6 (8)	0.2
Tungsten	emissivity	0.25 (5)	0.02
Aluminum	emissivity	0.2 (15)	0.1
Aluminum	specular component of reflectivity	0.9 (15)	0.6 to 0.95

4.2 Axial Temperature Profiles

Figures 5 and 6 compare the measured and simulated axial temperature profiles at the outer surface of the target tube for runs given in Table 1. Good agreement was noted between the measured and predicted temperatures in the upper 40 cm of the furnace. Also shown in these figures is the calculated net heat flux along the target. The decreasing heat flux towards the upper third of the furnace, where the target tube temperature is constant, can be partially attributed to the view factors, which account for furnace end effects. The peculiar shape of the heat flux distribution at the furnace inlet is due to competing heat transfer effects. Since the increasing target temperature reduces the driving force for radiative heat transfer, the view factor is increasing the fraction of filament power which impinges on the target.

In these figures the ratios of the calculated furnace efficiency to the measured furnace efficiency, η_m/η_o , are also presented. This ratio varied from 1.17 to 1.54. To reconcile the departure of this ratio from 1.0, three factors must be considered. First, as shown in Fig. 7, the ratio can be strongly effected by modifying the surface properties in the furnace. Decreasing the reflective characteristics of aluminum reduces the heat transfer to the target. Second, axial conduction was neglected in

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SIMULATED AXIAL TEMPERATURE AND
HEAT FLUX PROFILES
(NOMINAL PARAMETERS)

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SIMULATED AXIAL TEMPERATURE AND HEAT FLUX PROFILES (NOMINAL PARAMETERS)			
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REFLECTOR EMISSIVITY = 0.3
 SPECULAR-DIFFUSE PARTITION = 0.8

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SIMULATED AXIAL TEMPERATURE AND
 HEAT FLUX PROFILES
 (SELECTED PARAMETERS)

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both the Hastelloy and sodium. The literature (12) indicates that back-conduction in a convective system cannot be neglected for Peclet numbers between 0 and 100, while conditions typical of the run cited have Peclet numbers of about 6. If back-conduction in the sodium and Hastelloy were incorporated into the model, it would account for heat removed from the test section, which is currently assumed to remain there. Thus, axial conduction would reduce the net heat flux at the target tube and would consequently bring the furnace efficiency closer to the observed value. Third, the energy balance that was invoked to determine direct exchange areas [Eq. (1)] assumes a complete enclosure, when in fact the aluminum reflector has two open ends which account for 1.3% of the total surface area. Though it appears that the energy lost through the ends of the furnace were accounted for in the $(s_1s_3)_s$ term [since it was determined by considering $(s_1s_1)_s$ and $(s_1s_2)_s$], the net effect may not be so simple. The artificially large $(s_1s_3)_s$ leads to a large $(s_3s_1)_s$, which subsequently affects $(s_3s_3)_s$. Because this last effect only indirectly influences the target, it is probably of secondary importance.

4.3 Parameteric Study

To quantify the effect of the uncertainties in the parameters presented in Tables 1 and 2 for the simulated 7.5-kW axial temperature profile, each parameter was independently varied over the range of uncertainty while holding all other parameters at their nominal value. The results of this analysis are presented in Figs. 8 through 11. The actual test data for T120R1 appear on each graph to illustrate the effect of changing parameters. In each case the deviations of simulated temperature profiles seem reasonable. Increasing the sodium flow rate cools the test section, although saturation temperatures are eventually reached in each case. Decreasing the emissivity, and therefore the absorptivity of the Hastelloy, causes the test section temperature to drop. Likewise, increasing the emissivity of the reflector, which physically corresponds to degrading the surface from a highly polished case to a rougher aluminum oxide surface, also reduces the temperature of the test section as expected. Finally, the test section appears to cool when the reflector's specular component of reflectivity is decreased, which can be explained by the effective reduction of the fraction of rays emanating from the filament which are focused by the ellipse on the target.

5. CONCLUSIONS

1. SBTF data can be predicted by the proposed model over a wide range of conditions within an acceptable accuracy.
2. The predicted furnace efficiency which agrees with a correlation of the observed axial temperature profiles are consistently higher than those observed. This discrepancy might be resolved by incorporating axial conduction into the model.

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SENSITIVITY OF AXIAL TEMPERATURE
 PROFILE WITH SODIUM SENSIBLE HEAT

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FIG.
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SENSITIVITY OF AXIAL TEMPERATURE PROFILE WITH HASTELLOY EMISSIVITY			
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SENSITIVITY OF AXIAL TEMPERATURE
PROFILE WITH REFLECTOR EMISSIVITY

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SENSITIVITY OF AXIAL TEMPERATURE
PROFILE WITH SPECULAR-DIFFUSE
PARTITION OF REFLECTOR

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FIG.
11

3. Variation of the furnace parameters yields temperature profiles consistent with the actual physical phenomena which the model simulates.

6. RECOMMENDATIONS

1. The model should be modified to include axial conduction in the Hastelloy and liquid sodium.

2. The surface properties of the Hastelloy and aluminum should be determined with more accuracy to improve the reliability of computer predictions.

3. An instrumentation development program should be initiated to extend the state-of-the-art of electromagnetic flow meters. Increasing the velocity of sodium through these flow meters, without significantly increasing the flow rate or pressure drop, could also increase the precision of these measurements.

7. ACKNOWLEDGMENT

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8. APPENDIX

8.1 Derivation of Direct Exchange Areas

The $(s_i s_j)_s$ direct exchange area is equal to the area of surface i times the $(s_i s_j)_s$ fraction of radiation leaving i which reaches j both directly and after multiple specular reflections off any other surfaces in the enclosure. When i represents the radiant lamp, the evaluation of this fraction can be simplified by assuming the filament as a line source. Then a hemisphere of unit radius can be placed around any point along the filament. The fraction of rays reaching surface j is given by the fractional surface area through which rays hitting j must pass. In spherical coordinates this area is (Fig. 12):

$$A_0 = \int_0^\delta \int_{\frac{\pi}{2}-\beta}^{\frac{\pi}{2}+\alpha} \sin\theta \, d\phi \, d\theta \quad (15)$$

If α and β are independent of θ , and δ is constant, this expression leads to:

$$A_0 = (\alpha + \beta)(1 - \cos \delta) \quad (16)$$

Dividing by the surface area of a hemisphere of unit radius and multiplying by the area of surface i yields the total exchange area $(s_i s_j)_s$:

$$(s_i s_j)_s = A_i \left(\frac{1 - \cos \delta}{2\pi} \right) (\alpha + \beta) \quad (17)$$

If the path from i to j contains a reflection off surface k , the right side of Eq. (17) must be diminished by the specular reflectivity ρ_{sk} ,

$$(s_i s_j)_s = A_i \rho_{sk} \left(\frac{1 - \cos \delta}{2\pi} \right) (\alpha + \beta) \quad (18)$$

Nomenclature for the filament-filament (filament to reflector to filament) direction exchange area is given in Fig. 13. By inspection,

$$\delta = \tan^{-1} \left[\frac{R_f}{2(a-f)} \right], \quad \alpha = \tan^{-1} \left[\frac{z}{2(a-f)} \right] \text{ and } \beta = \tan^{-1} \left[\frac{L-z}{2(a-f)} \right]$$

$$A_0 = \int_0^\delta \int_{\frac{\pi}{2}-\beta}^{\frac{\pi}{2}+\alpha} \sin \theta \, d\phi \, d\theta$$

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HEMISPHERICAL SURFACE INTEGRAL

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a) Cross-Sectional Plane

b) Axial Plane

a = HALF OF MAJOR ELLIPSE AXIS

R_f = FILAMENT RADIUS

L = FILAMENT LENGTH

z = AXIAL DISTANCE

f = ONE HALF FOCUS - FOCUS DISTANCE

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SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY

FILAMENT-FILAMENT TOTAL EXCHANGE AREA

DATE

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FIG.

13

where f is half the focus-to-focus distance, $f = \sqrt{a^2 - b^2}$. The filament-filament total exchange area is determined by substitution into Eq. (13):

$$(s_1 s_1)_s = A_1 \rho s_3 \frac{[1 - \cos[\tan^{-1}(\frac{R_f}{2(a-f)})]]}{2\pi} \left[\tan^{-1}(\frac{z}{2(a-f)}) + \tan^{-1}(\frac{L-z}{2(a-f)}) \right]$$

It can be seen from Fig. 4 that the filament-target total exchange area contains both a direct and a reflected component. By inspection, the angles for the direct component are:

$$\delta = \tan^{-1} \frac{R_t}{2f}, \quad \alpha = \tan^{-1} \frac{z}{2f}, \quad \beta = \tan^{-1} \left(\frac{L-z}{2f} \right)$$

Substitution into Eq. (17) defines the direct component of the filament-target total exchange area:

$$(s_1 s_2)_{\text{direct}} = \frac{A_1}{2\pi} \left[1 - \cos[\tan^{-1}(\frac{R_t}{2f})] \right] \left[\tan^{-1}(\frac{z}{2f}) + \tan^{-1}(\frac{L-z}{2f}) \right]$$

The reflected component of the $(s_1 s_2)_s$ total exchange area must be considered in terms of projections of the ray geometry onto the cross-sectional (Fig. 4) and axial planes (Fig. 14). Again, by inspection

$$\delta = \tan^{-1} \left(\frac{y}{f+x} \right)$$

where x is supplied by the manufacturer, and y is determined from the ellipse equation:

$$y = b \sqrt{1 - (x^2/a^2)}$$

By inspection of any ray in the axial plane from Fig. 14:

$$\alpha = \tan^{-1}(r/\ell) \quad \beta = \tan^{-1}(s/\ell) \quad (19)$$

The law of corresponding parts of similar triangles dictates that:

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PROJECTION OF FILAMENT-TARGET
REFLECTED RAYS ON AXIAL PLANE

DATE 2-25-79	DRAWN BY FEA	FILE NO. X-285	FIG. 1a
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$$\frac{r}{\ell} = \frac{z - r}{2a - \ell} \quad \frac{s}{\ell} = \frac{L - z - s}{2a - \ell} \quad (20)$$

Solving for r and s yields,

$$r = \frac{\ell z}{2a} \quad s = \frac{\ell(L - z)}{2a} \quad (21)$$

Eliminating r and s between Eqs. (19), (20), and (21) gives

$$\alpha = \tan^{-1}\left(\frac{z}{2a}\right) \quad \beta = \tan^{-1}\left(\frac{L - z}{2a}\right)$$

Finally, when substituting for α and β into Eq. (18), the reflected component of the filament target direct exchange area becomes:

$$(s_1 s_2)_{\text{reflected}} = \frac{A_1}{2\pi} \{1 - \cos[\tan^{-1}\left(\frac{y}{f+x}\right)]\} [\tan^{-1}\left(\frac{z}{2a}\right) + \tan^{-1}\left(\frac{L - z}{2a}\right)]$$

3.2 Computer Code

The computer code referenced in the text consists of a main program and four subroutines. The parameters given below are input variables for the main program:

POWER	supplied furnace power (W)
WCP	heat capacity times mass flow rate sodium (W/K)
XL	heated length, T/S (M)
TSO	inlet sodium temperature equal to inlet wall temperature (K)
TR	reflector temperature (K)
TSAT	boiling point of sodium (K)
SPECR	specular reflectivity/emissivity aluminum
EMA	half major ellipse axis (M)
EMI	half minor ellipse axis (M)
EDX	ellipse overhang (M)

AREA(I) total surface area, Body I (M^2)

E(I) emissivity, Body I

DERIV is called from the main program to calculate $Q_{\text{net}}(\Delta z)$, $T_{\text{Na}}(\Delta z)$, and $\Delta T_{\text{wall}}(\Delta z)$ in Eqs. (7), (12), and (13), respectively. DERIV in turn calls AMAT to establish the total exchange area matrix. The matrix is inverted by a call to INVERT and is returned to DERIV through AMAT. Finally, DERIV calls RKAUTO to integrate both $\Delta T_{\text{Na}}(\Delta z)$ and $\Delta T_{\text{wall}}(\Delta z)$ along the test section as the entire procedure is repeated iteratively.

Note the current form of the main program performs a sensitivity analysis. For nominal runs, lines 7, 18-31, and 90-93 should be eliminated from the following program listing.

* In SUBROUTINE DERIV (p. 34), the program value of 0.9249 in. for $R_{t0} \ln(\frac{R_{t0}}{R_{ti}})$ appearing in line six should be 0.4624 in., to correctly simulate the experimental apparatus; however, this minimally changes the results presented, because the total contribution of ΔT_{wall} [Eq. (13)] is negligible.

```

154 0002      COMMON /C1/ AREA(3),E(3),FLUX(3),RHOD(1),RHOS3
154 0003      COMMON /C2/ TR,WCP,XL,TW,TSAT
154 0004      COMMON /C3/ EMA,ENI,EDX
154 0005      DIMENSION T(4),OG(4),ER(4),X1(4),X2(4),X3(4),X4(4),X5(4),X6(4),
154 0006      X7(4),X8(4),X9(4)
154 0007      EXTERNAL DERIV
154 0008      DATA WCP1,WCP2,E3,E2,SPEC/,337.,.675.,.1.,.6.,.6/
154 0009      15 FORMAT (5E15.8)
154 0010      25 FORMAT (14-1)
154 0011      35 FORMAT (7F10.0)
154 0012      44 FORMAT (2E15.4)
154 0013      45 FORMAT (3E15.4)
154 0014      46 FORMAT (4E15.4)
154 0015      READ 35, POWER,WCP,XL,TSO,TR,TSAT,SPECR
154 0016      DO 2 I=1,3
154 0017      2 READ 35,AREA(I),E(I)
154 0018      READ 35, EMA,ENI,EDX
154 0019      DO 120 JJ=1,3
154 0020      JJJ=JJ-1
154 0021      WCP=WCP1+(FLOAT(JJJ)*WCP2)
154 0022      DO 110 LL=1,3
154 0023      LL=LL-1
154 0024      F(3)=E3+(FLOAT(LL)*E3)
154 0025      DO 100 MM=1,3
154 0026      MM=MM-1
154 0027      E(2)=E2+(FLOAT(MM)*E2)*E3)
154 0028      DO 90 KK=1,4
154 0029      IF (KK.EQ.2) GO TO 90
154 0030      KN=KK-1
154 0031      SPECR=SPECR+(FLOAT(KN)*E3)
154 0032      WRITE (6,25)
154 0033      WRITE (6,25)
154 0034      WRITE (6,15) POWER,WCP,XL,TR,TSAT,SPECR
154 0035      WRITE (6,25)
154 0036      WRITE (6,44) (AREA(I),E(I),I=1,3)
154 0037      WRITE (6,25)
154 0038      TW=TSO
154 0039      DO 3 I=1,3
154 0040      FLUX(I)=0.0
154 0041      3 RHOD(I)=1.-F(I)
154 0042      RHOS3 = SPECR/RHOD(3)
154 0043      RHOD(3)=RHOD(3)-RHOS3
154 0044      WRITE (6,44) (RHOD(I),I=1,3),RHOS3
154 0045      WRITE (6,25)
154 0046      WRITE (6,15) EMA,ENI,EDX
154 0047      WRITE (6,25)
154 0048      FLUX(I) = POWER/AREA(I)
154 0049      77 MIN = 1.E-05
154 0050      40 Z = 0.
154 0051      77 = .01
154 0052      DO 20 I=1,4
154 0053      G(I) = 0.
154 0054      20 ER(I) = 1.E-05
154 0055      G(1) = TSO
154 0056      ISTOP = 0
154 0057      ISTEP = 0
154 0058      WRITE (6,25)
154 0059      CALL DERIV (4,Z,DZ,G,DG,DERIV,DZMIN,ER,-1,X1,X2,X3,X4,X5,X6,X7,
154 0060      X8,X9)
154 0061      TS=TS
154 0062      DELT=TW-TS
154 0063      WRITE (6,15) Z,TW,TS,FLUX(2),FLUX(3),DZ,DELT
154 0064      50 CALL 3KAUTO (4,Z,DZ,G,DG,DERIV,DZMIN,ER,-1,X1,X2,X3,X4,X5,X6,X7,
154 0065      X8,X9)
154 0066      ISTEP = ISTEP+1
154 0067      TS = G(1)
154 0068      DELT=TW-TS
154 0069      WRITE (6,15) Z,TW,TS,FLUX(2),FLUX(3),DZ,DELT
154 0070      IF (ISTOP.EQ.1) GO TO 65
154 0071      60 IF ((Z+2.*(DZ+DZMIN)).GT.ZPR) DZ=(ZPR-Z)/2.
154 0072      IF ((Z+2.*(DZ+DZMIN)).GT.XL) DZ=(XL-Z)/2.
154 0073      IF ((Z+2.*(DZ+DZMIN)).GT.XL) ISTOP=1
154 0074      IF ((Z+2.*(DZ+DZMIN)).GT.ZPR) OR ((Z+2.*(DZ+DZMIN)).GT.XL) IPR=1
154 0075      GO TO 50
154 0076      65 OT2 = WCP*(TS-TSO)
154 0077      OT1 = G(2)
154 0078      OR = G(3)
154 0079      OF = G(4)
154 0080      FTA=OT1/POWER
154 0081      FAL = (OT1+OR)/POWER
154 0082      XSTEP = FLOAT(ISTEP)
154 0083      WRITE (6,25)
154 0084      WRITE (6,15) POWER,OT2,OT1,OR,FTA,FAL,OF,XSTEP
154 0085      70 CONTINUE
154 0086      90 CONTINUE
154 0087      100 CONTINUE
154 0088      110 CONTINUE
154 0089      120 CONTINUE
154 0090      STOP
154 0091      END

```



```

150 0002      SUBROUTINE DERIV (N,Z,G,DG)
150 0003      COMMON /C1/ AREA(3),E(3),FLUX(3),RHOD(3),RHOS3
150 0004      COMMON /C2/ TR,WCP,XL,TW,TSAT
150 0005      DIMENSION G(4),DG(4),T(3),C(3),XJ(3),A(3,3),H(3,3)
150 0006      X(2)=G(1)+(-1.)*FLUX(2)*.9249/(2.104*TW+246.8)
150 0007      T(1) = TR
150 0008      SIGMA = 4.47E-08
150 0009      PI = 3.1415
150 0010      C(1)=AREA(1)*FLUX(1)+(-1.)
150 0011      C(2)=AREA(2)*SIGMAT(2)*.044.*E(2)/RHOD(2)+(-1.)
150 0012      C(3)=AREA(3)*SIGMAT(3)*.044.*E(3)/RHOD(3)+(-1.)
150 0013      CALL ANAT (Z,A)
150 0014      DO 20 I=1,3
150 0015          XJ(I) = 0.
150 0016      ON 20 J=1,3
20  XJ(I) = XJ(I)+C(J)*A(I,J)
150 0017      FLUX(2)=(E(2)/RHOD(2))*SIGMAT(2)*.044.-XJ(2)
150 0018      FLUX(3)=(E(3)/RHOD(3))*SIGMAT(3)*.044.*(1.-RHOS3)-XJ(3)
150 0019      DG(1) = -FLUX(2)*WCP*AREA(2)/XL
150 0020      DG(2) = -FLUX(2)*AREA(2)/XL
150 0021      DG(3) = -FLUX(3)*AREA(3)/XL
150 0022      DG(4) = -FLUX(1)*AREA(1)/XL
150 0023      TW=T(2)
150 0024      IF (G(1).GT.TSAT)DG(1)=0.
150 0025      RETURN
150 0026      END

150 0002      SUBROUTINE ANAT (Z,A)
150 0003      COMMON /C1/ AREA(3),E(3),FLUX(3),RHOD(3),RHOS3
150 0004      COMMON /C2/ TR,WCP,XL,TW,TSAT
150 0005      COMMON /C3/ EMA,ENI,ENK
150 0006      DIMENSION A(3,3),H(3,3)
150 0007      XE=SQRT(EMA*EMA-ENI*ENI)
150 0008      PI=3.1415
150 0009      RFI1=AREA(1)/(XL*PI*.08.)
150 0010      RTAR=AREA(2)/(XL*PI*.02.)
150 0011      DY=SQRT(ENI*ENI*(1.-EDX*EDX/(EMA*FMA)))
150 0012      XFF=(1.-COS(ATAN(RFI1/(2.*EMA-XF))))/(2.*PI)
150 0013      XFT=(1.-COS(ATAN(RTAR/(2.*XF))))/(2.*PI)
150 0014      XFI=(1.-COS(PI-ATAN(DY/(XF*EDX))))/(2.*PI)
150 0015      FFI=XFF*(ATAN(Z/(2.*EMA-XF))+ATAN((XL-Z)/(2.*EMA-XF)))
150 0016      FFI=XFT*(ATAN(Z/(2.*XF))+ATAN((XL-Z)/(2.*XF)))
150 0017      FFI=XFI*(ATAN(Z/(2.*EMA))+ATAN((XL-Z)/(2.*EMA)))
150 0018      H(1,1)=FFI*RHOS3+AREA(1)
150 0019      H(1,2)=AREA(1)*(FTI*FTI/RHOS3)
150 0020      H(1,3)=(AREA(1)-H(1,1)-H(1,2))/(1.-RHOS3)
150 0021      H(2,1)=H(1,2)
150 0022      H(2,2)=0.0
150 0023      H(2,3)=(AREA(2)-H(2,1))/(1.-RHOS3)
150 0024      H(3,1)=H(1,3)
150 0025      H(3,2)=H(2,3)
150 0026      H(3,3)=(AREA(3)-H(3,1)-H(3,2))/(1.-RHOS3)
150 0027      DO 10 I=1,3
150 0028      DO 10 J=1,3
150 0029      10 A(I,J)=H(I,J)
150 0030      A(1,1)=H(1,1)-AREA(1)
150 0031      A(2,2)=H(2,2)-(AREA(2)/RHOD(2))
150 0032      A(3,3)=H(3,3)-AREA(3)/RHOD(3)
150 0033      CALL INVERT(A,3,3,SING)
150 0034      RETURN
150 0035      END

```

```

ISN 0002      SUBROUTINE RK4AUTO(N,T,M,Y,YPRIME,DERIV,HLOW,ERLMT,RL,
ISN 0003      I,X,YTL,TY,DELTA,R1,R2,R3,R4,R5)
ISN 0004      DIMENSION Y(N),YPRIME(N),ERLMT(N),X(N),YTL(N),TY(N),DELTA(N),
ISN 0005      R1(N),R2(N),R3(N),R4(N),R5(N)
ISN 0006      DO 2 I=1,N
ISN 0007      2 TY(I)=Y(I)
ISN 0008      DT=M
ISN 0009      TT=T
ISN 0010      1 IFLAG=0
ISN 0011      4 CALL DERIV(N,T,Y,R1)
ISN 0012      DO 5 I=1,N
ISN 0013      5 XI(I)=Y(I)+R1(I)*M/2.
ISN 0014      CALL DERIV(N,T+M/2.,XI,R2)
ISN 0015      DO 6 I=1,N
ISN 0016      6 XI(I)=Y(I)+R2(I)*M/2.
ISN 0017      CALL DERIV(N,T+M/2.,XI,R3)
ISN 0018      DO 7 I=1,N
ISN 0019      7 XI(I)=Y(I)+R3(I)*M
ISN 0020      CALL DERIV(N,T+M,XI,R4)
ISN 0021      DO 8 I=1,N
ISN 0022      8 Y(I)=Y(I)+4*(R1(I)+2.*(R2(I)+R3(I))+R4(I))/6.
ISN 0023      T=T+M
ISN 0024      IF(ILL)26,99,99
ISN 0025      9 IF(FLAG)14,9,14
ISN 0026      9 IFLAG=1
ISN 0027      DO 11 I=1,N
ISN 0028      11 IF(ERLMT(I))111,111,10
ISN 0029      111 XI(I)=0.0
ISN 0030      GO TO 11
ISN 0031      10 XI(I)=TY(I)+2.*M*(R1(I)-2.*(R3(I)+R4(I)))
ISN 0032      11 CONTINUE
ISN 0033      CALL DERIV(N,TY+2.*M,XI,R5)
ISN 0034      DO 13 I=1,N
ISN 0035      13 IF(ERLMT(I))13,13,12
ISN 0036      12 YTL(I)=TY(I)+4*(R1(I)+4.*R2(I)+R5(I))/3.
ISN 0037      13 CONTINUE
ISN 0038      GO TO 4
ISN 0039      14 IF(MP1=0.0)
ISN 0040      KKK=0
ISN 0041      DO 17 I=1,N
ISN 0042      17 IF(ERLMT(I))17,17,15
ISN 0043      15 DELTA(I)=ABS(YTL(I)-Y(I))
ISN 0044      IF(DELTA(I))17,17,19
ISN 0045      19 TEMP2=DELTA(I)/(ERLMT(I)*ABS(Y(I)))
ISN 0046      IF(TEMP1-TEMP2)20,17,17
ISN 0047      20 KKK=1
ISN 0048      IF(MP1=TEMP2)
ISN 0049      17 CONTINUE
ISN 0050      IF(KKK)11,32,31
ISN 0051      31 IF(MP4)5(3),ERLMT(KKK)*Y(KKK)/DELTA(KKK))99,(2)
ISN 0052      32 TEMP5=A15(T)-ABS(HLOW)
ISN 0053      IF(TEMP5)22,23,23
ISN 0054      22 DT=HLOW
ISN 0055      23 IF(ILL)24,27,24
ISN 0056      24 IF(M-T)27,27,25
ISN 0057      25 T=TT
ISN 0058      DO 26 I=1,N
ISN 0059      26 Y(I)=TY(I)
ISN 0060      M=DT
ISN 0061      WRITE(6,30) T,M
ISN 0062      30 FORMAT(9H RECYCLED,2E17.8)
ISN 0063      GO TO 3
ISN 0064      27 M=DT
ISN 0065      28 CALL DERIV(N,T,Y,YPRIME)
ISN 0066      RETURN
ISN 0067      END

```

```

154 0002      SUBROUTINE INVERT(A,ORDER,DINA,SING)
C             SUBROUTINE INVERTS A MATRIX IN ITS OWN SPACE USING THE GAUSS-
C             JORDAN METHOD AS ARRANGED BY H. RITSHAUSER WITH FULL PIVOTING
154 0003      INTEGER ORDER,DINA
154 0004      INTEGER IP(10),IO(10)
154 0005      LOGICAL SING
154 0006      DIMENSION A(DINA,1),B(10),C(10)
154 0007      N=ORDER
154 0008      DO 1 K=1,N
C             DETERMINATION OF PIVOT ELEMENT
154 0009      PIVOT=0.
154 0010      DO 100 I=K,N
154 0011      DO 100 J=K,N
154 0012      IF (ABS(A(I,J))-ABS(PIVOT)) 100,100,101
154 0013      101 PIVOT=A(I,J)
154 0014      IP(K)=I
154 0015      IO(K)=J
154 0016      100 CONTINUE
154 0017      IF (PIVOT) 102,900,102
154 0018      102 IPK=IP(K)
154 0019      IOK=IO(K)
C             EXCHANGE OF THE PIVOTAL ROW WITH THE KTH ROW
154 0020      IF (IPK-K) 200,299,200
154 0021      200 DO 201 J=1,N
154 0022      Z=A(IPK,J)
154 0023      A(IPK,J)=A(K,J)
154 0024      201 A(K,J)=Z
154 0025      299 CONTINUE
C             EXCHANGE OF THE PIVOTAL COLUMN WITH THE KTH COLUMN
154 0026      IF (IOK-K) 300,399,300
154 0027      300 DO 301 I=1,N
154 0028      Z=A(I,IOK)
154 0029      A(I,IOK)=A(I,K)
154 0030      301 A(I,K)=Z
154 0031      399 CONTINUE
C             JORDAN ELIMINATION STEP
154 0032      DO 400 J=1,N
154 0033      IF (J-K) 402,402,403
154 0034      402 B(J)=1./PIVOT
154 0035      C(J)=1.
154 0036      GO TO 404
154 0037      403 B(J)=-A(K,J)/PIVOT
154 0038      C(J)=A(J,K)
154 0039      404 A(K,J)=0.
154 0040      400 A(J,K)=0.
154 0041      DO 405 I=1,N
154 0042      DO 405 J=1,N
154 0043      405 A(I,J)=A(I,J)+C(I)*B(J)
154 0044      1 CONTINUE
C             REORDERING THE MATRIX
154 0045      K=N
154 0046      DO 500 KJUM=1,N
154 0047      IPK=IP(K)
154 0048      IOK=IO(K)
154 0049      IF (IPK-K) 501,502,501
154 0050      501 DO 503 I=1,N
154 0051      Z=A(I,IPK)
154 0052      A(I,IPK)=A(I,K)
154 0053      503 A(I,K)=Z
154 0054      502 IF (IOK-K) 504,500,504
154 0055      504 DO 506 J=1,N
154 0056      Z=A(IOK,J)
154 0057      A(IOK,J)=A(K,J)
154 0058      506 A(K,J)=Z
154 0059      500 K=K-1
154 0060      SING=.FALSE.
154 0061      IF TURN
154 0062      900 SING=.TRUE.
154 0063      IF TURN
154 0064      END

```

8.3 Hastelloy-X Thermal Conductivity

From the literature (3), the thermal conductivity of Hastelloy-X can be expressed as a function of temperature:

$$K = 2.104 \times 10^{-4} T (K) + 0.02468 \quad \text{W-cm/cm}^2\text{-K}$$

8.4 Dimensions of Quadelliptical Furnace

(See Fig. 4)

<u>Dimension</u>	<u>Length (m)</u>
Test section height, L	9.7×10^{-1}
Filament radius, R_f	7.937×10^{-4}
Outer target radius, R_{t0}	4.521×10^{-3}
Inner target radius, R_{ti}	1.625×10^{-3}
Ellipse center to focus, f	3.405×10^{-2}
Half major axis, a	4.445×10^{-2}
Half minor axis, b	2.857×10^{-2}
Ellipse extension, x	4.762×10^{-3}
Ellipse depth, y	2.842×10^{-2}

8.5 Nomenclature

a	half of major ellipse axis, m
A_i	area of i th body, m^2
A_0	see Fig. 12, m^2
b	half of minor ellipse axis, m
C_p	heat capacity, J/kg-K
E_i	black body radiation from surface j , W/m

f	half of distance between foci, m
$F_i(z)$	point at height z on i th surface
H_i	incident flux density on surface j , W/m
k	thermal conductivity, W-cm/cm ² -K
x	variable filament to wall distance, m
L	length of filament, m
$\dot{M}$	flow rate of sodium, kg/s
$Q_{\text{net},i}$	net radiation incident to surface i
r	see Fig. 14, m
R_f	filament radius, m
R_t	Hastelloy target radius, m
$(s_i s_j)_{\text{direct}}$	exchange area of direct component
$(s_i s_j)_{\text{reflected}}$	exchange area of reflected component
$(s_i s_j)_s$	total exchange area from i to j
s	see Fig. 14, m
T_i	temperature of i th body, K
$W_{d,i}$	total diffuse flux leaving surface i , W/m
x	ellipse overhang, m
y	depth of ellipse, m
z	axial distance from base of filament, m
α	azimuthal angle down from horizon bounding effective rays, radians
α_i	adsorptivity of surface i
β	azimuthal angle up from horizon bounding effective rays, radians
δ	equatorial angle bounding effective rays, radians
ϵ_i	grey body emissivity of body i
η_m	furnace efficiency predicted by model

η_0	observed furnace efficiency
ρ_{di}	diffuse reflectivity of surface i
ρ_{si}	specular reflectivity of surface i
σ	Stefan-Boltzman constant, 5.67×10^{-8} , $W(m^2K^4)^{-1}$
θ	equitorial angle of effective rays, radians
ϕ	azimuthal angle of effective rays, radians

Subscripts

Na	sodium
1	filament
2	target
3	reflector wall
s	specular
d	diffuse
i	inner
o	outer

8.6 Literature Cited

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