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CHIRAL TRANSITION IN LATTICE GAUGE THEORIES

WITH HIGGS AND FERMIONS*

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ABSTRACT

Analytic and numerical studies of the lattice gauge theories with both Higgs and fermion fields are reported. A chiral transition is found in a wide class of such theories, both abelian and non-abelian. This transition separates each phase diagram into two regions, one with spontaneous chiral symmetry breaking and the other with explicit chiral symmetry. The transition is second order where probed. In models where Yukawa couplings are present, there exists a critical value of this coupling below which chiral transition still occurs. Some physical implications are noted.

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As a non-perturbative approach to investigating the Higgs sector of the standard model, lattice gauge theories with scalar fields have been a subject of extensive study during the past few years. Since fermions do not play a direct role in the Higgs mechanism, one might think that the inclusion of fermions in the lattice gauge-Higgs theories would not have any important effect on these theories. However, a numerical simulation of the $SU(2)$ lattice gauge theory with a Higgs field in the fundamental representation ($I_h = 1/2$) and a fermion field with $I_f = 1/2$ or $I_f = 1$ revealed the existence of a new line of phase transitions which separate the phase diagram into two regions, characterized by whether or not chiral symmetry is spontaneously broken.¹ This contrasts with the situation in the absence of fermions where the confinement and Higgs phases are analytically connected. Because the new transition involves the spontaneous breaking of chiral symmetry, it may be called the chiral transition.

It is worthwhile to investigate this chiral transition more closely for several reasons. First of all, although in ref. 1 we only considered a vectorlike theory (where the left- and right-handed fermions transform the same way under the gauge transformations), while the $SU(2)_L$ sector of the standard model is a chiral theory (with left- and right-handed fermions transforming differently), it can be shown that, because the representations of $SU(2)$ are real, a chiral $SU(2)_L$ gauge theory with $2N_f$ left-handed fermion doublets can be expressed as a vectorlike $SU(2)$ gauge theory with N_f left- and N_f right-handed fermion doublets.^{2,3} Therefore, the chiral transition found in ref. 1 is relevant to the $SU(2)_L$ sector of the lattice electroweak theory.⁴ Furthermore, the existence of a new line of phase transitions, reopens the question of a continuum limit away from the Gaussian fixed point.

To further investigate this new transition, we carried out analytic studies⁴⁻⁷ at strong gauge coupling limit and demonstrated that chiral transition occurs in a wide class of lattice gauge theories with a Higgs field in the fundamental representation and with fermions in various representations.

For simplicity, let us first concentrate on one of the simplest of such theories, namely, the $U(1)$ gauge theory with a unit-charged Higgs field and a fermion field with charge q .^{4,5}

The Euclidean action of this theory can be written as $S = S_G + S_H + S_F$, where

$$S_G = \beta_g \sum_{\text{plaq.}} [1 - \text{Re}(U_{\text{plaq.}})] \quad (1)$$

$$S_H = -2\beta_h \sum_{n,\mu} \text{Re}[\phi_n^\dagger U_{n,\mu} \phi_{n+e_\mu}] \quad (2)$$

$$S_F = \frac{1}{2} \sum_{n,\mu} \eta_{n,\mu} \bar{\chi}_n [(U_{n,\mu})^{q_f} \chi_{n+e_\mu} - (U_{n-e_\mu,\mu}^\dagger)^{q_f} \chi_{n-e_\mu}] \\ + m \sum_n \bar{\chi}_n \chi_n \quad (3)$$

with $\beta_g = 1/g^2$. The sites n lie on a d -dimensional hypercubic lattice. (The lattice spacing a is set to one unless otherwise stated.) The Higgs fields satisfy $\phi_n^\dagger \phi_n = 1$. As is well known, this “fixed-length” condition on the lattice Higgs fields does not imply that the continuum Higgs fields are of fixed length. With no loss of generality, we take $\beta_h \geq 0$. The χ_n and $\bar{\chi}_n$ are one-component, anticommuting, staggered fermion fields⁸ assigned to each lattice site n ; and the $\eta_{n,\mu} = \{1 \text{ for } \mu = 1; (-1)^{n_1+\dots+n_{\mu-1}} \text{ for } 2 \leq \mu \leq d\}$.⁹ The partition function is then

$$Z = \int \prod_{n,\mu} dU_{n,\mu} d\phi_n d\phi_n^\dagger d\chi_n d\bar{\chi}_n e^{-S} \quad (4)$$

For technical simplicity, we consider the strong coupling limit $\beta_g = 0$. Integrating out the gauge and Higgs fields, we obtain $Z = [I_0(2\beta_h)]^{N_l} \int \prod_n d\chi_n d\bar{\chi}_n e^{-S'}$, where N_l is the number of links in the lattice, and

$$S' = \frac{1}{2} r \sum_{n,\mu} \eta_{n,\mu} (\bar{\chi}_n \chi_{n+e_\mu} - \bar{\chi}_{n+e_\mu} \chi_n) + m \sum_n \bar{\chi}_n \chi_n \\ - \frac{1}{4} (1 - r^2) \sum_{n,\mu} \bar{\chi}_n \chi_n \bar{\chi}_{n+e_\mu} \chi_{n+e_\mu} \quad (5)$$

with $r = I_q(2\beta_h)/I_0(2\beta_h)$, where $I_n(x)$ denotes the modified Bessel function. r increases monotonically from zero to one as β_h increases from zero to infinity. It is clear from eq. (5) that the theory becomes a free fermion theory at $r = 1$ (i.e. $\beta_h = \infty$).

Next, we decompose the four-fermion terms in eq. (5) into fermion bilinears, integrate out the fermion fields, and obtain an equation for the chiral condensate, $\langle \bar{\chi} \chi \rangle$.

This can be done either by introducing auxiliary scalar fields and using the saddle point approximation or by the mean field approximation. We have checked that both methods give the same consistency equation

$$\langle \bar{\chi}\chi \rangle = \frac{d}{2}(1 - r^2) \langle \bar{\chi}\chi \rangle J \quad (6)$$

where $J = \int (dp) \{ [(d/2)(1 - r^2) \langle \bar{\chi}\chi \rangle]^2 + r^2 s \}^{-1}$ with $\int (dp) = (2\pi)^{-d} \int d^d p$ and $s = \sum_{\mu=1}^d \sin^2 p_{\mu}$. Note in passing that $\langle \bar{\chi}\chi \rangle = 0$ is always a solution; however, a non-zero solution, if exists, minimizes the free energy and is therefore the physical solution.

Eq. (6) can be solved, and the results for $d = 3, 4$, and ∞ are plotted in Fig. 1, where $M \equiv (d/2)^{\frac{1}{2}} \langle \bar{\chi}\chi \rangle$. It is clear from Fig. 1 that, for a given $d (\geq 3)$, a continuous chiral transition occurs at a critical point r_c , which separates a region ($r < r_c$) where chiral symmetry is spontaneously broken from a chirally symmetric region ($r > r_c$). For $d \rightarrow \infty$, the solution can be obtained in closed form,

$$M^2 = \frac{(1 - 2r^2)}{(1 - r^2)^2} \quad \text{for } d \rightarrow \infty \quad (7)$$

It implies that the critical point $r_c^2 = 1/2$, and the critical exponent $\beta = 1/2$. Since r is a monotonically decreasing function of q_f , the fermion charge, for any β_h , the critical $\beta_{h,c}$ increases monotonically with q_f . This can be understood because the fermion-anti-fermion condensate forms through the interaction between the fermions and the gauge fields, which increases in strength as q_f increases.

Although we have used the mean field approximation in the above derivation, some effect of the dynamical fermions still shows up. One way of seeing this is to calculate a quantity which involves only the bosonic fields, e.g., $\langle L \rangle \equiv \langle \text{Re}(\phi_n^\dagger U_{n,\mu} \phi_{n+e_\mu}) \rangle$. In the absence of fermions, $\langle L \rangle (\beta_g = 0)$ can be calculated, and one finds that, for $\beta_g = 0$,

$$\langle L \rangle |_{\text{bosonic}} = I_1(2\beta_h)/I_0(2\beta_h) \quad (8)$$

When fermions are present, we obtain from our analytic calculation with mean field approximation that

$$\begin{aligned} \langle L \rangle &= \langle L \rangle |_{\text{bosonic}} \\ &+ \frac{1}{rd} r' (2\beta_h) \left[1 - \frac{d}{2} (1 + r^2) \langle \bar{\chi}\chi \rangle^2 \right] \end{aligned} \quad (9)$$

Eq. (9) shows that, in the presence of fermions, $\langle L \rangle$ picks up an additional contribution which is a function of $\langle \bar{\chi}\chi \rangle$. Since $\langle \bar{\chi}\chi \rangle$ is non-analytic at r_c , $\langle L \rangle$ develops a non-analyticity at r_c also. This non-analyticity in $\langle L \rangle$, which does not exist in the absence of fermions, will never show up in a calculation with the quenched approximation. It implies, therefore, that our analytic results go beyond the quenched approximation.

Apart from a redefinition of r as a function of β_h , most of what was derived above could be applied to the gauge group Z_N as well as $U(1)$.⁴ A similar, though more complicated, analysis has been completed recently for theories with non-abelian gauge groups.⁷

It is interesting to compare the above analytic results with the measurements from numerical simulations. A simulation of the four-dimensional $U(1)$ gauge theory with a unit-charge Higgs field and a fermion field with charge $q_f = 1$ or 2 was performed in the quenched approximation.⁵ $\langle \bar{\chi}\chi \rangle_m$ was measured on a 8^4 lattice with bare fermion mass $m = 0.08, 0.06, 0.04$ and 0.02 . The chiral condensate in the thermodynamic limit, $\langle \bar{\chi}\chi \rangle$, was obtained by extrapolating the $\langle \bar{\chi}\chi \rangle_m$ values to their zero-bare mass limits. Fig. 2 shows the dependence of $\langle \bar{\chi}\chi \rangle$ on β_h along the line $\beta_g = 0$ for both fermions with $q_f = 1$ and $q_f = 2$. The data indicate the existence of a continuous chiral transition at $\beta_{h,c}(\beta_g = 0) = 1.25 \pm 0.15$ and 4.0 ± 0.5 for $q_f = 1$ and 2, respectively. These values agree very well with the analytic predictions ($\beta_{h,c}(\beta_g = 0) = 1.15 \pm 0.01$ and 3.60 ± 0.05 , respectively.)

We next go one step further to investigate the chiral transition in the interior of the phase diagram numerically. In particular, we would like to know whether the new chiral phase boundary coincides with the partial confinement-Higgs transition phase boundary where the latter exists. Measurements of $\langle \bar{\chi}\chi \rangle$ were made along an oblique line defined by $\beta_h = 0.38667\beta_g$, and the results are shown in Fig. 3. The chiral transition is found to occur at $(\beta_g, \beta_h)_c = (0.80 \pm 0.04, 0.31 \pm 0.02)$ for $q_f = 1$, and at $(1.2 \pm 0.1, 0.46 \pm 0.04)$ for $q_f = 2$. To see where these points stand relative to the confinement-Higgs (CFH) phase boundary along this oblique line, we also measured the quantities $\langle L \rangle$ (as defined above) and $\langle P \rangle \equiv \langle \text{Re}(U_{Flaq.}) \rangle$. The confinement-Higgs transition is found to be at $(\beta_g, \beta_h)_{CFH} = (0.78 \pm 0.02, 0.302 \pm 0.008)$. It appears that, in the quenched approximation, the chiral transition of the $q_f = 1$ fermions coincides with the confinement-Higgs transition

(where the latter exists), while, for the $q_f = 2$ fermions, the chiral transition lies beyond the confinement-Higgs transition (further in the Higgs phase). Similar results were also found in the $SU(2)$ case.¹ From these measurements, we inferred the phase diagram of the $U(1)$ gauge theory with unit-charged Higgs and fermion fields as shown in Fig. 4. The schematic phase diagram of the $SU(2)$ gauge theory with $I = 1/2$ Higgs and fermion fields is shown in Fig. 5.

In summary, several general features have emerged from our analytic and numerical studies of a class of lattice gauge theories with Higgs fields in the fundamental representation and fermion fields in general representations. First, there exists a chiral phase boundary separating the phase diagram into a chirally symmetric region and a region where chiral symmetry is spontaneously broken. Second, at a fixed β_g , the higher the fermion representation (or charge) is, the larger the critical point of the chiral transition $\beta_{h,c}$ is. Third, it seems that where the confinement-Higgs phase boundary exists, the chiral phase boundary of the fundamental representation (or unit-charge) fermion coincides with the CFH boundary. The first two features are supported by both the analytic mean field studies and the numerical quenched simulations, and it is expected that they will still hold in the full theory with dynamical fermions. The third one has only been observed in the simulations with the quenched approximation; it will be interesting to see whether it will remain true in the presence of dynamical fermions.

This work yields some important physical implications. As was mentioned in the beginning, this phase diagram can be regarded as describing the $SU(2)$ sector of the standard electroweak theory on the lattice. Since the known fermions are light compared to the electroweak scale, wherever the continuum limit is, it has to be approached from within the chirally symmetric phase in order to keep the fermions light. This may, indeed, explain¹⁰ why the known fermions have left-handed components with $I = 1/2$. Furthermore, the mass spectra of the two phases separated by the chiral phase boundary are quite different when fermions are present.¹

The above discussion has dealt with models in which no Yukawa coupling is present. An extension of the $U(1)$ gauge theory discussed above to include the Yukawa coupling has been studied analytically in the strong gauge coupling limit.⁷ It is found that a chiral

transition still occurs for small Yukawa couplings. The critical point, $\beta_{h,c}$, decreases as the Yukawa coupling increases from zero to a critical value, above which the whole line at $\beta_g = 0$ becomes analytic.

Thus, the inclusion of fermions has introduced important new changes in the understanding of gauge-Higgs models. Many interesting questions remain to be investigated.

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FIGURE CAPTIONS

Fig. 1. Plot of solution to eq. (6), $M \equiv (d/2)^{\frac{1}{2}} < \bar{\chi}\chi >$ as a function of r^2 for $d = 3$ (dotted curve), $d = 4$ (dot-dashed curve), and $d = \infty$ (solid curve).

Fig. 2. Numerical measurements of $< \bar{\chi}\chi > a^3$ as a function of β_h for $\beta_g = 0$ and fermion charge $q_f = 1$ ($\bullet$); 2 ($\circ$).

Fig. 3. Numerical measurements of $< \bar{\chi}\chi > a^3$ as a function of β_h along the oblique line $\beta_h = 0.38667\beta_g$ for fermion charge $q_f = 1$ ($\bullet$); 2 ($\circ$).

Fig. 4. Phase diagram of four-dimensional U(1) gauge theory with $g_h = 1$ Higgs field and $q_f = 1$ quenched fermion fields. The dashed lines are the phase boundaries present in

the pure gauge-Higgs theory (which remain invariant for quenched fermions); the dotted line is the chiral boundary. The location of the chiral boundary between the measured points P and Q is only shown schematically. The arrows indicate that the chiral boundary continues on toward lower β_h , completely separating the chirally symmetric and broken phases.

Fig. 5. Schematic phase diagram of four-dimensional SU(2) gauge theory with $I_h = 1/2$ Higgs field and $I_f = 1/2$ quenched fermion fields. The dashed curve denotes the confinement-Higgs (CFH) phase boundary present in the pure gauge-Higgs theory (which remain invariant for quenched fermions); the dotted line is the chiral boundary.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

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