FUSION RESEARCH CENTER

THE UNIVERSITY OF TEXAS

Austin, Texas

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Fusion Research Center

Theory Program

MASTER

PROGRESS REPORT

January 1982

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University of Texas
Fusion Research Center Theory Program

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PROGRESS REPORT

January 1982

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I. Summary

The Texas FRC theory program is directed primarily toward understanding the initiation, heating, and confinement of tokamak plasmas. It supports and complements the experimental programs on the TEXT and PRETEXT devices, as well as providing information generally applicable to the national tokamak program. A significant fraction of the Center's work has been carried out in collaboration with, or as a part of, the program of the Institute for Fusion Studies (IFS). During the past twelve months, 14 FRC theory reports and 12 IFS reports with partial FRC support have been issued (Appendices A and B).

The PRETEXT program is furthest along experimentally. Studies of breakdown and current penetration, runaway induced cyclotron emission, and Alfvén wave heating have been successfully pursued in close collaboration with the experimentalists. One of the most exciting results is the recent observation of density fluctuations associated with Alfvén wave absorption, which had been predicted theoretically.

TEXT is now routinely operating at magnetic fields of 28kG, densities ranging from 2 to 8×10^{13} , and plasma currents of 300kA. With crucial diagnostics to be installed during the next several months, preparations for theoretical collaboration are continuing. The emphasis is on computational and analytical studies of transport and scaling, in an effort to identify and quantify important features of impurity flow and energy balance. With regard to the latter subject, processes which lead to critical temperatures or critical radii for macroscopic instabilities and processes which transfer energy from electrons to ions will prove important, in addition to anomalies in the electron and ion thermal conductivities.

These topics are closely related to studies of instabilities and nonlinear phenomena being pursued in the FRC and IFS. However, as emphasized in the U.S.-Japan workshop on Drift Wave Turbulence, held in Austin, January, 1982, a considerable gap remains between theory and experiment. Thus, in Sec. III, below, a rather arbitrary division between basic theory and studies directly applicable to experiment has been made. In addition to tokamak work, FRC members have participated in IFS studies of equilibrium and stability in non-axisymmetric devices, including stellarators and tandem mirrors.

In the coming year, PRETEXT studies on current penetration and Alfvén heating will concentrate on quantitative comparison with experiment. An effort will be made to identify tearing modes induced near the minimum q surface during current penetration, and to associate observed Alfvén absorption phenomena with particular modes described by the theory. In the TEXT program, numerical studies of impurity transport will begin, and further clarification of controversial issues in energy balance and scaling will be sought. Finally, in work recently performed in the IFS, the possibility of explaining the broad frequency spectra observed in turbulent density

fluctuation measurements has arisen. The FRC will continue to monitor this work and to participate in these and other fundamental studies of drift wave turbulence and nonlinear phenomena.

II. Organization and Activities

A. Personnel

1. Permanent Faculty and Staff

William E. Drummond, Director*

David W. Ross, Assistant Director**

Richard D. Hazeltine

Daniel A. Hitchcock (currently at DOE headquarters, Germantown)

Fred L. Hinton (currently on leave of absence)

Alan B. Macmahon*

Swadesh M. Mahajan

Alan A. Ware

James C. Wiley

H. Vernon Wong***

*supported in part by TEXT

**supported in part by IFS

***supported in part by Texas Atomic Energy Research Foundation

The contract now supports 6.75 man-years per year of Ph.D. level staff. Vernon Wong has filled the slot currently vacated by Fred Hinton.

2. Students

The contract supported eight graduate students (4 man-years) during the past twelve months. Of these, four received the Ph.D. degree. Paul Terry has joined the IFS as a post-doctoral Research Fellow, Gwo-Liang Chen is continuing fusion research at the Oak Ridge National Laboratory, while Curtis Bolton and David Brock have accepted non-fusion positions.

3. Visitors

Visitors brought to the FRC specifically to advise and contribute to the PRETEXT and TEXT programs have been Cliff Surko (Bell Labs), Bruno Coppi (MIT), and Ron Waltz (GA). Their discussions of fluctuation measurements, scaling laws, and transport have been extremely helpful. The FRC has also benefited from the many visitors to the IFS. In particular, Paulette Liewer (Cal Tech) reviewed fluctuation measurements, Dieter Sigmar (ORNL) described experiments on ISX-B, and Ira Bernstein (Yale) began a variational study of Alfvén wave heating which has resulted in a paper submitted to Physics of Fluids.

B. Study Groups and Experimental Support

1. A TEXT support group has been formed and has had several preliminary meetings with the experimentalists to map out areas of active collaboration. The group consists of A. A. Ware, K. Gentle, D. W. Ross, S. M. Mahajan, and J. C. Wiley. The TEXT tokamak is now operating routinely with magnetic fields of 28kG and currents of 300kA. Diagnostics are being installed in preparation for the experimental program. During the next twelve months the principal concern will be to establish good spatial resolution in the measurement of impurity concentrations and to extract transport rates. As more sophisticated diagnostics are installed, the research program will gradually turn to the study of energy balance, fluctuations, and anomalous electron thermal transport. To help interpret data from TEXT and PRETEXT, Jim Wiley has developed a transport code, capable of dealing with many impurity species and arbitrary transport coefficients. A strategy for using this code to study TEXT experiments is being developed.
2. A tokamak anomalous energy loss study group under Alan Ware's direction has been formed in preparation for TEXT experimental studies. So far, Dave Ross and Bruno Coppi have reviewed scaling laws, and Alan Ware has reviewed the experimental data and outlined alternative mechanisms for electron-ion transfer as well as transport of various species.
3. Dave Ross is conducting a Turbulence Theory group, which has been focusing on the Direct Interaction approximation as applied to fluids and plasmas. This is a joint IFS-FRC venture. Dave Ross, Philippe Similon, and Pat Diamond have led discussions on Kraichnan's work and the Kolmogorov hypothesis. Ravi Sudan has visited the IFS and discussed applications of the DIA to plasma turbulence with this group.

C. Collaboration with the Institute for Fusion Studies

Most of the staff listed in Sec. A, above, and supported by the theory program, are also members of the IFS and participate actively in the IFS research program. This is reflected in the list of reports given in Appendix B. The majority of this work is motivated by a desire to understand and control tokamak confinement and is thus ultimately applicable to the TEXT program. In work which will soon appear in a series of IFS reports, P. Diamond, J. Meiss, M. N. Rosenbluth, P. Similon, K. Swartz, and P. Terry (all of IFS) and R. D. Hazeltine and S. M. Mahajan (of FRC and IFS) have undertaken a systematic study of the renormalized turbulence theory of trapped electron drift waves. While far from complete, this work has recently indicated the possibility of explaining the broad frequency spectra observed in scattering experiments.

FRC members are also contributing to the theory of non-axisymmetric systems such as stellarators and tandem mirrors. Similarly, some IFS members are participating directly in the experimental support work. Some of this work is described in Sec. III.B., below.

III. Technical Progress and Future Plans

A. Experimental Support and Applied Tokamak Physics

1. PRETEXT studies

a) Current Penetration on PRETEXT (S. M. Mahajan, R. D. Hazeltine, J. C. Wiley, A. Bhattacharjee (IFS))

The PRETEXT experiment provides detailed information concerning the early, skin current phase of a tokamak discharge. The relatively cold plasma is collision dominated and plasma currents would be expected to diffuse inward due to classical (Spitzer) resistivity. In fact, as in most tokamak experiments, PRETEXT current penetration occurs at a rate orders of magnitude faster than resistive diffusion.

The skin phase is characterized by a safety-factor (q) profile which has a minimum a few centimeters inside the limiter. Such profiles are typically unstable to collisional tearing modes, which grow near rational values of q on either side of its minimum. The nonlinear overlap of islands associated with such "double-tearing modes" is often used to explain the anomalous penetration rate. In reexamining this problem, we begin by studying the linear stability of electromagnetic perturbations for the case of a parabolic q -profile. This global approach differs from standard treatments, which consider only the locally linear (in minor radius) variation of q . Thus new factors of the linear stability problem are uncovered, including a novel electromagnetic instability which is localized near the q -minimum, between the two rational flux surfaces. The growth rate of this "centered" mode, being proportional to the square-root of the resistivity, is larger than that of conventional tearing modes. However, for geometrical reasons it does not tear the magnetic field lines.

Future plans

Nonlinearly, we expect the centered mode to interact with the usual double tearing mode in significant ways. Specifically, an enhanced current penetration rate, beyond that for the case of the double tearing mode alone, is anticipated. Such effects will be investigated both analytically and numerically; a detailed match to the experimental profile dynamics is planned. However, the nonlinear treatment of this problem has now only just begun.

b) Enhanced Cyclotron Emission and Relaxation Oscillations in Tokamaks* (D. A. Hitchcock and S. M. Mahajan)

We have proposed a new mechanism to explain the observed enhanced cyclotron emission and relaxation oscillations of the runaway electrons. Previous work had shown that the elongation of the electron distribution function by a d.c. parallel electric field (runaway distribution) can destabilize the obliquely propagating magnetized plasma oscillations. We propose an alternative and stronger mechanism for enhanced emission at $n\omega_{ce}$ (in particular, at $2\omega_{ce}$); a mechanism which implies exponential growth of the signal. We show that the crescent shaped distribution function, which results from the quasilinear effects of magnetized plasma oscillations, destabilizes the $n=2$ electromagnetic finite $k_{\parallel}$ electron Bernstein mode. The growth rate and other properties of the mode are calculated using the relativistic Vlasov equation. We also compute the quasilinear saturation time, and suggest a mechanism for eventual damping of these waves. The theoretically calculated growth rate and saturation times are in good agreement with the experiment.

*See FRC Theory Newsletter, #2, Dec. 1981, and Reports FRC #229 and #231.

c) Alfven Wave Heating in Tokamaks* (David W. Ross, Gwo-Liang Chen, and Swadesh M. Mahajan)

Plasma heating by absorption of electromagnetic radiation at frequencies well below the ion cyclotron frequency has been proposed by a number of authors as an attractive alternative to other methods of auxiliary heating. In this method, the energy enters the plasma via a cutoff compressional Alfven mode. Near the resonance layer, a flux surface defined by $\omega = k_{\parallel} v_A$, the mode is converted to a propagating shear Alfven wave which is then damped, delivering its energy to the particles. At the University of Texas, experimental and theoretical studies are continuing in an effort to demonstrate the feasibility of Alfven wave absorption as a heating method and its usefulness as a diagnostic aid.

Theoretical and Numerical Studies

We have studied the propagation and absorption of waves driven by an idealized antenna in cylindrical geometry by solving equations for the perturbed field components derived from kinetic theory. These go beyond the usual MHD formulations, in that they include the effects of the ion gyroradius and electron inertia (which modify the field structures), electron Landau damping (the principal absorption mechanism), and finite values of ω/ω_{ci} (which affects the mode coupling and, hence, the plasma impedance). At the same time, important MHD effects, in particular the contribution

of the equilibrium current, are retained.

*See FRC Newsletter #1, Sept., 1981, and FRC Reports #220, #223, and #227.

The results are quite encouraging, but we note that practical resistances are expected only if the physical antenna has the correct helicity to drive the particular mode (ℓ, m) of interest. We are able to compute in detail profiles of such quantities as the energy deposition, $E(r) \cdot j(r)$ and the density fluctuations, $\delta n(r)$. These tend to be broadly localized in the neighborhood of the Alfvén resonance layer; their spatial extent is determined by kinetic effects. On the one hand this provides a means of controlling the heating profile. On the other hand, we have shown that the equilibrium current profile, $j_0(r)$, can be inferred from a measurement which locates the peak of $\delta n(r)$.

Experiments on PRETEXT (R. E. Bengtson, M. E. Oakes, S. -H. Lin, R. Michie, T. E. Evans, Y. M. Li, and C. M. Surko† (Bell Labs))

Experiments at low power to demonstrate plasma loading have been conducted in the frequency range $f=1$ to 7MHz with magnetic field range $B_0 = 7-10$ kG and peak densities $n_0 = 0.6-3.0 \times 10^{13} \text{ cm}^{-3}$. With two poloidal antenna loops in series, spaced 90° or 180° around the torus, or with a single toroidal segment extending $1/8$ or $1/4$ of the distance around the torus, plasma resistances ranging from 0.1 to 4Ω have been measured. These exhibit broad peaks as functions of frequency, which have been tentatively identified as Alfvén absorption at $\omega = k_{\parallel} v_A$ for small toroidal mode numbers, $\ell = \pm 1, \pm 2$. By means of CO_2 laser interferometry, density fluctuations at the driven frequency have been observed with peak values $\tilde{n}/n$ exceeding the noise by a factor of ten. During the steady state phase of the discharge ($T_e \approx 200$ eV), the scattered signal is largest at chords positioned well inside the plasma, indicating absorption of the RF power at Alfvén resonant surfaces. During the current and density rise phase of the discharge, density fluctuations are also observed, appearing as spikes when the plasma density passes through particular values.

Future plans

Encouraged by the qualitative agreement between theory and experiment, and in particular by the observation of density fluctuations, we plan to devote the coming year to making the comparison quantitative. This is complicated by the need to Fourier analyse the antenna structures employed, to include important ω/ω_{ci} effects, and to estimate the degree of toroidal coupling. The possible contribution of impurity cyclotron absorption should also be considered.

†The work of C. M. Surko is not supported by the D.O.E.

2. TEXT studies

a) Study of Edge Plasma Screening of Impurities
(W. L. Rowan and J. C. Wiley)

- presented Topical Conference on Atomic Processes in High Temperature Plasmas, Feb. 1981. The migration of impurities through the edge plasma region and the evolution of low ionization stages against a fixed background was studied using a computer model. The work was performed to facilitate the design and analysis of an experiment for TEXT.

b) Transport Computations (J. C. Wiley)

In order to help interpret the experimental data from TEXT and PRETEXT a new transport code was designed and is being built. The code which is designed to run on our local VAX solves the equations for both the background hydrogen plasma and the various charge states of the impurities considered. The transport code uses a fully implicit technique to solve the partial differential equations. The problem is divided into modules. The plasma is first solved over a partial time step holding the impurities and neutrals fixed. Then each impurity species is advanced holding the plasma and other impurities fixed. The procedure is then iterated until the fractional change in any quantity is below a given limit. All functions are expanded in terms of Hermite cubic splines. The breakpoints which define the spline space divide the radius into seven zones, one of which defines the limiter shadow region. Each zone may be further refined as necessary. One unique feature of the code is that the support for any function can extend over one or more zones. This feature is especially useful for the impurities since a particular charge state may exist over a rather narrow radial region. By reducing the support of a given impurity the linear system which must be solved at each time step is significantly reduced.

To date, the module which solves the impurity equations for one impurity has been written and debugged as well as the hydrogen plasma module. The neutral hydrogen transport routine, ANTIC, by S. Tamor, has also been included. The code is now being applied to data obtained by S. McCool and coworkers from the PRETEXT experiment.

Future plans

Plans for the transport code for 1982 include interfacing the code to the experimental data archive, adding additional atomic and kitchen physics processes to model the boundary zone, and working toward an interactive system which allows the user to interact with both the experimental data and the transport parameters.

3. Tokamak Transport and Scaling

a) The Critical Temperature for Loss of Poloidal Equilibrium and Tokamak Containment Scaling* (A. A. Ware)

The theory of sawtooth disruptions based on multiple equilibria with different poloidal rotation speeds was published in May.¹ This theory predicts a critical central electron temperature (T_{ec}) above which the normal equilibrium with low poloidal rotation is lost. An $m=0, n=0$ mode occurs involving poloidal acceleration towards an alternate equilibrium with substantially higher poloidal rotation and greater non-uniformity on a magnetic surface, followed by a transition back to the original equilibrium because of enhanced electron cooling and local decrease of the current. The effect of this relaxation oscillation is to limit the central electron temperature to the value T_{ec} . For ohmically heated discharges T_{ec} is given by the approximate formula

$$\frac{T_{ec}^2}{T_{io}} \left(\frac{\sigma_{||}}{T_e^{3/2}} \right) E_T = C \left(\frac{m_i}{m_e} \right)^{1/2} n_{io} v_{Tz} \quad (1)$$

where $\sigma_{||}$ is the Spitzer conductivity, E_T the toroidal electric field, n_{io} the central hydrogen ion density and v_{Tz} is the impurity ion thermal velocity. C is an approximate constant with value 0.47 for impurities such as oxygen or carbon.

To compare Eq. (1) with the empirical scaling for ohmically heated Tokamaks² a further relationship is needed since Eq.(1) contains parameters such as T_{io} , n_{io} , E_T which are not considered in the experimental scaling. The relationship taken is to assume the radial ion heat flux (q_i) is approximately equal to the incoming ohmic power (the Poynting Vector), thus

$$q_i = E_T B_\theta / 4\pi \quad (2)$$

The case for this assumption is that there are many processes which transfer energy from the electrons to the ions³ so that at least 50% of the Ohmic heat is conducted out by the ions. Using Eqs.(1) and (2), the scaling obtained for T_{eo} is⁴

$$T_{eo} = \exp(-14.1) a^{-5/4} R I Z_{eff}^{1/2} (n_e^0 B^0 A_i^0) \quad (3)$$

which agrees accurately with the experimental scaling obtained by Pfeiffer and Waltz² for all seven parameters. Also using the replacement time to give the electron energy containment time (τ_{Ee}) Eq.(3) leads to the predicted scaling

*See also FRC Theory Newsletter #1, Sept. 1981.

$$\tau_{Ee} \sim n_{eo} a^{7/8} R^{5/2} Z_{eff}^{1/4} I^{1/2} \quad (4)$$

which is also in close agreement with the experimental scaling.²

1. A. A. Ware and J. C. Wiley, Phys. Fluids 24, 936(1981).
2. W. Pfeiffer and R. E. Waltz, Nucl. Fusion, 19, 51(1979).
3. A. A. Ware, Nucl. Fusion, 21, 623(1981) and 21, 631(1981).
4. A. A. Ware, Institute for Fusion Studies Report IFSR#8, 1981.

Future plans

A theory for the multiple equilibria in low-impurity discharges is needed to complement the theory referred to above. Not only is one of the assumptions of that theory ($n_z Z^2 \gg n_i$) no longer valid but since the experimentally observed size of the sawtooth region in low-impurity discharges (r_s) is comparable with the poloidal Larmor radius, normal neoclassical formulas are not valid.

b) Ion Heat Conduction in Tokamaks (C. Bolton and A. A. Ware)

In this work¹ the neoclassical and Pfirsch-Schluter components of the ion heat conduction for a pure hydrogen Tokamak plasma were recalculated without assuming large aspect ratio and without neglecting energy scattering collisions. Using a model collision operator the conductivity was determined numerically for various collision frequencies and aspect ratios. An approximate algebraic expression was fitted to the results. Even for comparatively large aspect ratios (~ 10) energy scattering increases the conductivity in the banana regime by about 50% and for small aspect ratios (~ 3.3) the Pfirsch-Schluter contribution causes a further increase of approximately 60%.

1. C. Bolton and A. A. Ware, Fusion Research Center Report FRCR#239, 1981.

Future plans

In the outer regions of discharges with high ion temperatures, the ions in the thermal tail diffuse radially too rapidly to thermalize with incoming cold ions. A two-component model - a hot ion plasma diffusing through an incoming cold ion plasma - is being developed.

B. Basic Theory and IFS Collaboration

1. MHD Equilibrium (A. Bhattacharjee (IFS), and J. C. Wiley)

A computer code was developed which solves the equations for magnetostatic equilibria in toroidal geometry using a variant of the inverse formulation of L. L. Lao, S. P. Hirshman, and R. M. Wieland, (ORNL/TM-7616). Various versions of this code have contributed to the following works:

1. A. Bhattacharjee, J. C. Wiley, R. L. Dewar, Variational Method for the Three-Dimensional Inverse Equilibrium Problem in Toroids, Report IFSR#48, 1981.
2. A. Bhattacharjee and J. C. Wiley, Minimum Energy Toroidal Equilibria, presented at the 1981 Annual Controlled Fusion Theory Conference, Report IFSR #52 (in preparation).
3. A. Bhattacharjee, R. L. Dewar, A. H. Glasser, M. S. Chance, and J. C. Wiley, Bulletin Amer. Phy. Soc. 26, 1981, Report IFSR#51 (in preparation).

Future plans

So far, two dimensional equilibria have been computed numerically and checked for accuracy. Plans include extending the techniques to three dimensions and computing stellarator equilibria.

2. Kinetic Theory - Collisionless and Collisional
 - a) Hamiltonian Formulation of Guiding Center Motion and of the Linear and Nonlinear Gyrokinetic Equation* (H. Vernon Wong)

A derivation of the gyrokinetic equation based on the Hamiltonian formulation of the guiding center motion is given. The transformation to guiding center variables is accomplished by a sequence of canonical transformations. The kinetic equation is determined by Liouville's equation. This equation is then ordered in accordance with the gyrokinetic ordering, and analyzed order by order to obtain the linear and nonlinear gyrokinetic equations.

*See Report FRCR#230, 1981.

- b) Renormalization in Toroidal Geometry* (R. D. Hazeltine)

A one-point version of the Direct Interaction Approximation (DIA) is used to incorporate, into the Vlasov equation, effects of turbulent fluctuations. This problem has been analyzed previously for the case of electrostatic fluctuations in slab or cylindrical geometry. The present treatment is electromagnetic and, more importantly, fully toroidal. Thus the effects of, for example, particles trapped in the poloidal variation of the equilibrium magnetic field are included. The analysis is based on the use of action-angle variables as phase-space coordinates. These make application of the DIA quite transparent and yield a relatively compact form for the renormalized kinetic equation.

Future plans

The results of this analysis have been used in more recent, quite extensive, studies of the renormalized response of trapped electrons to electrostatic drift-wave fluctuations

*See Report IFSR#18, 1981.

(with P. H. Diamond and K. Swartz (IFS), and S. M. Mahajan). A mini-maximal variational principle for solving the renormalized kinetic equation has been derived. This work will continue in the coming year.

c) Impurity Flow Reversal in Tokamaks Without Momentum Input* (F. L. Hinton)

A new method for impurity flow reversal is suggested, which does not depend upon momentum input. The method uses neutral-beam injection or radio-frequency fields to drive the Pfirsch-Schluter ion current required for toroidal equilibrium. The necessary alteration of the momentum transfer between the hydrogen ions and impurity ions can cause the former to diffuse into the plasma and the impurities to diffuse out.

*See Report IFSR#43, 1981.

d) Stabilization of Axisymmetric Mirror Plasmas by Energetic Ion Injection* (F. L. Hinton, M. N. Rosenbluth)

Plasmas in axisymmetric mirror devices can be made stable to MHD interchange modes by injecting energetic ions which contribute significantly to the pressure and spend a sufficiently large fraction of a bounce time in regions of favorable curvature. Pitch-angle scattering adversely affects the method by reducing this fraction. The ions must be sufficiently energetic that pitch-angle scattering is not detrimental for that part of a slowing-down time during which they contribute significantly to the pressure. We have solved the bounce-averaged Fokker-Planck equation, including drag and pitch-angle scattering, and calculated the energetic ion contribution to the stability integral. With specially tailored magnetic fields, the required injection energy and power drain are found to be reasonable.

*See Report IFSR#42, 1981.

3. Plasma Instabilities and Anomalous Transport

a) Vlasov Stability of Ion Pressure Gradient-Driven Drift Modes (P. Terry, W. Anderson, D. Brock, and W. Horton (IFS)).

Linear Theory. The threshold of instability of the ion pressure gradient-driven toroidal drift mode is investigated from the drift-kinetic equation. The analysis involves study of the analytical properties of a new special function that we call the guiding center dispersion function. The linear plasma response includes the grad-B and curvature drifts of the ions and the finite ion gyroradius averaging of the wave field. With the asymptotic expansion of large aspect ratio, we show precisely the relationship between the kinetic and fluid descriptions of the modes.¹

1. P. W. Terry, W. C. Anderson, W. Horton, Report IFSR#36, 1981, submitted to Nuclear Fusion.

Nonlinear Theory. In a large toroidal plasma with a high ion pressure due either to auxiliary heating or alpha particle heating, the linear micro-instability theory predicts the onset of slow drift waves. The growth rate is proportional to the geometric mean of the ion pressure gradient and the unfavorable toroidal curvature. Below the critical beta for the onset of MHD ballooning, the instabilities are basically electrostatic in polarization. To determine the effects of these instabilities, we developed a reduced nonlinear model describing the nonlinear $E \times B$ convection of the plasma in the turbulent state.

Extensive simulations on MFE-CRAY were carried out on the toroidal-ion-pressure gradient driven instability for the two-fold purpose of determining numerically the saturation and comparing the k, ω spectral features with previously published predictions¹ derived from renormalized turbulence theory. The nonlinear model describes locally, in the poloidal angle, the two-dimensional structure of the electrostatic potential $\phi(x, y, t)$ and the pressure fluctuation $\delta p(x, y, t)$ in the plane perpendicular to the magnetic field. The background gradients are frozen so that all quasi-linear effects are analytically eliminated from the model. The turbulent fields are thus characterized by their root-mean-square values of ϕ_0 , δp_0 and their k, ω spectra. The model describes the dimensionless fields in the limit of vanishing ρ_i/a and contains two independent parameters $\alpha = 2r_{ii} \cos \theta_0 / R$ (inverse aspect ratio) and $\beta = dp_0/dx$ (the dimensionless pressure gradient). Special numerical techniques used in atmospheric studies are used to preserve the invariants of the Poisson bracket coupling for the $E \times B$ convection.

Many fascinating phenomena were observed in the simulations. Here, we only mention the most important and better understood results. The exponential (linear) growth phase determines when the $(2k_x, k_y = 0)$ mode becomes comparable with the fastest-growing (k_x, k_y) mode. When the system is strongly unstable, $\gamma_k/\omega_k \sim 1$, the k_x spectrum contains many components at $2nk_x$. Finally, parametric decays occur in which $2nk_x \rightarrow k_x + k_x$ with the lowest harmonic k_x having the largest component. This final spectrum is dominated by k_x , and the saturation level is given by $k_x \delta p_0 = C_1 |\beta|$ where C_1 is a constant and $\beta = dp_0/dx$. Thus, for this particular model, we understand why the phenomenological mixing length theory for the root-mean-square fluctuation level applies. The details of these results will appear in Plasma Physics.

1. W. Horton, D. I. Choi, and W. M. Tang, Phys. Fluids 24, 1077(1981).
2. David Bröck, Report FRCR#235, 1981 (Ph.D. dissertation).

b) Nonlinear Fluid Dynamics Near the Magnetic Axis of a Tokamak Plasma (J. D. Meiss (IFS), and R. D. Hazeltine)

A novel method has been found for studying the nonlinear dynamics of the inner, near-axis region of a tokamak discharge. In this region, fluid ("saw-tooth") oscillations are observed to limit the maximum central temperature.

Our approach, like several previous studies, begins with the resistive, reduced MHD equations. Complicated radial dependence, especially in the (three) nonlinear terms near the $q=1$ surface, precludes any straightforward analysis, so that progress in solving the equations has depended upon numerical computation. However, we observe that the most important linear instabilities are resistive tearing and twisting modes, whose radial eigenfunctions are known. We thus use an effectively complete set of such eigenfunctions, and their adjoints, to construct an interaction representation for the nonlinear system. The interaction representation describes the evolution of amplitudes which depend only on time, and whose coupling reflect all the nonlinearities of the original system. Radial information is summarized by coupling coefficients, which are evaluated analytically in terms of integrals over eigenfunction (and adjoint eigenfunction) triplets.

Future plans

As a system of ordinary differential equations, the interaction representation of reduced MHD appears relatively tractable. However, the nonlinear terms are likely to require an at least partially numerical treatment. Without having yet attempted to solve the system, we note that it resembles three-wave coupling systems, some of which have been shown to process stochastic solutions.

c) Effects of Equilibrium Currents and Current Gradients on Tearing Instabilities (G. Craddock, S. M. Mahajan, D. A. Hitchcock, and R. D. Hazeltine)

Recent studies of linear electromagnetic instabilities, especially at small collision frequency, have relied on several simplifications. Probably the most important concerns the equilibrium plasma current and its radial gradient, the direct effects of which are usually neglected. Both the current and its gradient modify the electron response in a similar way, in that both break a radial symmetry (parity) of the standard electromagnetic eigenmode equations. The symmetry breaking can be appreciable in some cases (including that of radio-frequency

current drive schemes); in the drift-wave case, it has been shown to significantly modify the dispersion relation.

Future plans

An investigation of current-induced symmetry breaking on tearing instabilities has begun. Derivation of the generalized linear eigenmode equations is nearly complete. The major remaining task is incorporation of both the modified electron response and the asymmetrized boundary conditions into numerical codes for solving the eigenvalue problem.

d) Integral Equation Formulation of Electromagnetic Mode Equations* (S. M. Mahajan)

The coupled set of differential equations describing the electromagnetic perturbations in Tokamak plasmas is reduced to a single simple integral equation with a symmetric kernel. Obvious analytical and computational advantages are obtained.

Future plans

Variational solutions will be obtained and compared with conventional ones. Implementation of the method numerically will depend on availability of student help.

*See Report IFSR#47, 1981.

e) Anomalous Transport From Drift Modes Driven by Temperature Gradients and Trapped Electrons* (S. M. Mahajan)

We propose that the drift modes driven unstable by the collisional detrapping and/or magnetic drifts of the trapped electrons in the presence of normal temperature gradients $n_e = d\ln T_e / d\ln n_e > 0$ should be considered as a basis for a theory of anomalous transport. We show that the radial diffusion of passing particles resulting from this instability can quench the instability giving rise to a saturation diffusion coefficient which is found to be in the experimentally observed range.

Future plans

This work is part of the ongoing nonlinear theory program of the FRC and IFS.

*See Report IFSR#31, 1981.

f) Growth Rates for the Magnetic Trapped Particle Mode (H. Vernon Wong and M. N. Rosenbluth (IFS))

In plasma confinement systems such as tandem mirrors which contain plasma trapped in regions of unfavorable curvature, low frequency collisionless magnetic trapped particle modes become unstable when the diamagnetic pressure is sufficient to reverse particle drift velocities.¹ This mode is magnetic at marginal stability, but for finite growth rates, it becomes predominantly electrostatic, and may be regarded as a finite β modified electrostatic trapped particle mode.²

The growth rate γ can be obtained from a quadratic variational form.² We employ an analytically tractable model for the variation of the equilibrium magnetic field B with distance along the field line; $B = B_0(1 + \ell^2/L^2)$, and a trial function $\tilde{\phi} = 1 + \alpha\mu/\epsilon$. The curvature is taken to be an arbitrary function of ℓ . This could be achieved in practice for an axisymmetric annular plasma by appropriately shaped inner and outer conductors, and serves to model quadrupole geometry.

The effect of electron collisions is stabilizing. In the limit $\omega_{be} > v_e \gg v_i$, the electron response is somewhat different from the ion response, and the purely growing mode is transformed into an oscillatory, electrostatic mode ($\omega \sim \omega_*$). However, the oscillatory mode is weakly unstable, and is analogous to the unstable trapped ion mode³ predicted to be present in Tokamaks. Using the pitch angle collision operator of Ref. 3 to model the electron and ion collisions, a quadratic variational form can again be derived, and an estimate of the frequency ω and growth rate γ obtained.

1. M. N. Rosenbluth, Phys. Rev. Letters 46, 1525(1981).
2. M. N. Rosenbluth and M. L. Sloan, Phys. Fluids 14, 1725(1971).
3. M. N. Rosenbluth, D. W. Ross, and D. P. Kostomarov, Nucl. Fusion 12, 3(1972).

Future plans

The investigation of the magnetic trapped particle mode will be extended to include the effects of finite Larmor radius.

Appendix A

FRC THEORY REPORTS AND EXPERIMENTAL REPORTS WITH
THEORETICAL COLLABORATION SINCE FEBRUARY, 1981

- #220 D. W. Ross, G. L. Chen, and S. M. Mahajan, "Kinetic Theory of Alfven Wave Heating", February, 1981 (presented at the Fourth Topical Conference on RF Heating, Austin, 1981).
- #221 D. A. Hitchcock and S. M. Mahajan, "Currents Driven by Electrostatic and Electromagnetic Waves", February, 1981.
- #223 S. M. Mahajan, D. W. Ross, G. L. Chen, and R. D. Bengtson, "A New Diagnostic for $B_p(r)$ and $q(r)$ ", April, 1981, submitted to Phys. Fluids.
- #225 C. W. Bolton, "The Ion Thermal Conductivity for a Pure Tokamak Plasma", June, 1981 (dissertation).
- #227 D. W. Ross, G. L. Chen, and S. M. Mahajan, "Kinetic Description of Alfven of Alfven Wave Heating", June, 1981 (to be published in Phys. Fluids).
- #229 D. A. Hitchcock and S. M. Mahajan, "Destabilization of Electron Bernstein Modes by Runaway Electrons", June, 1981 (submitted to Phys. Fluids).
- #229A Paul Terry, "Theoretical Aspects of the Nonlinear interaction of Drift-Type Instabilities in a Plasma", July, 1981 (dissertation).
- #230 H. V. Wong, "Hamiltonian Formulation of the Guiding Center Motion of the Linear and Nonlinear Gyrokinetic Equation", July, 1981.
- #231 R. F. Gandy, D. A. Hitchcock, S. A. Eckstrand, R. D. Bengtson, and S. M. Mahajan, "Runaway Induced Relaxation Oscillations in the PRETEXT Tokamak", August, 1981 (submitted to Phys. Rev. Lett.).
- #233 R. F. Gandy, D. A. Hitchcock, S. M. Mahajan, and R. D. Bengtson, "Runaway Electron Instability in High Density Low Electric Field Regime", August, 1981 (submitted to Phys. Fluids).
- #235 David Brock, "Simulations of Nonlinear Problems in Plasma Physics", August, 1981 (dissertation).
- #236 J. F. Benesch, R. D. Bengtson, M. E. Oakes, and S. M. Mahajan, "Breakdown in the PRETEXT Tokamak", November, 1981 (submitted to Nucl. Fusion).
- #239 C. Bolton and A. Ware, "Ion Thermal Conductivity for a Pure Tokamak Plasma", October, 1981 (submitted to Phys. Fluids).

#240 Gwo-Liang Chen, "Kinetic Theory of Alfven Wave Heating in Tokamaks", December, 1981 (dissertation).

Appendix B

IFS REPORTS WITH PARTIAL FRC SUPPORT

- #1 W. Horton, "Anomalous Ion Conduction from Toroidal Drift Modes", October, 1980 (to be published in Plasma Physics).
- #3 A. A. Ware, "Neoclassical Energy Transfer Between Electrons and Ions in a Tokamak", October, 1980 (Nucl. Fusion 21, 631(1981)).
- #4 D. A. Hitchcock, R. D. Hazeltine, and S. M. Mahajan, "Unified Kinetic Theory in Toroidal Systems", December 1980.
- #8 A. A. Ware, "Electron Temperature Limit for Poloidal Equilibrium and Tokamak Containment Scaling", February, 1981.
- #17 S. M. Mahajan, R. D. Hazeltine, and D. A. Hitchcock, "Quasi-linear Momentum Transport", March, 1981.
- #18 R. D. Hazeltine, "Renormalization of Plasma Turbulence in Toroidal Geometry", April 1981.
- #20 P. Terry and W. Horton, "Stochasticity and the Random Phase Approximation for Three Electron Waves", May, 1981 (submitted to Phys. Fluids).
- #26 J. D. Meiss, R. D. Hazeltine, P. H. Diamond and S. M. Mahajan, "Effect of Turbulent Diffusion on Collisionless Tearing Instabilities", June, 1981 (submitted to Phys. Fluids).
- #31 S. M. Mahajan, "Anomalous Transport from Drift Modes Driven by Temperature Gradients and Trapped Electrons", July, 1981 (submitted to Phys. Rev. Letters).
- #32 W. Horton and D. Brock, "Evolution of Toroidal Drift-Wave Fluctuations", August, 1981.
- #36 P. Terry, W. Anderson, and W. Horton, "Kinetic Effects on the Toroidal Ion Pressure Gradient Drift Mode", October, 1981 (submitted to Nucl. Fusion).
- #42 F. L. Hinton and M. N. Rosenbluth, "Stabilization of Axisymmetric Mirror Plasmas by Energetic Ion Injection", October, 1981 (submitted to Nucl. Fusion).
- #43 F. L. Hinton, "Impurity Flow Reversal in Tokamaks Without Momentum Input", October, 1981 (submitted to Phys. Fluids).
- #47 S. M. Mahajan, "Integral Equation Formulation of Electromagnetic Mode Equations", November, 1981.

- #48 A. Bhattacharjee, J. C. Wiley, and R. L. Dewar, "Variational Method for the Three-Dimensional Inverse Equilibrium Problem in Toroids", December, 1981.