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Experimental Test of Pion Exchange and PCAC in Proton-Nucleus

Charge Exchange Reactions at 144 MeV*

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MASTER

ABSTRACT

We have measured the differential cross sections of the (p,n) reaction on ${}^6\text{Li}$, ${}^{12}\text{C}$, and ${}^{14}\text{N}$ to the ground state of the final nuclei at $E_p = 144$ and $0 < \theta_{\text{LAB}} < 20^\circ$. The quantum numbers of these transitions, like those in Gamow-Teller beta decay, have $\Delta J = 1$, $\Delta T = 1$, and $\Delta P = 0$, which is consistent with the one-pion exchange mechanism. Differential cross sections were calculated with the absorption modified one-pion exchange model. The nucleus was treated as an elementary particle, as first suggested by Kim and Primakoff. Using the Partially Conserved Axial Vector Current Hypothesis (PCAC) the magnitude and momentum dependence of the initial nucleus-pion-final nucleus coupling constant, $g_{\text{NN}\pi}$, was obtained from beta decay and inelastic electron scattering, respectively. The calculations, which had no free parameters, were found to agree with the data for ${}^6\text{Li}$ and ${}^{12}\text{C}$, but not for ${}^{14}\text{N}$.

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It is an interesting common feature of the reactions

that the isospin (I), angular momentum (J), and parity (P) change of the target nuclei are the same: $\Delta I = \Delta T = 1$, $\Delta P = 0$. These quantum number changes are like those in Gamow-Teller beta decay, which are consistent with the one-pion exchange mechanism. We therefore conjectured that the reactions (1)-(3) are dominated by one-pion exchange¹ for 144 MeV protons and small scattering angles. Then, accepting the suggestion of Kim and Primakoff that the nuclei be treated as elementary particles,² the interactions were described in terms of the nucleus-pion-nucleus coupling constant $(g_{N\pi N}, (q^2))$ in the framework of the Absorption Model.³ The advantage of this procedure is that PCAC, beta decay, and inelastic electron scattering can be used to predict both the magnitude of $g_{N\pi N}$ at $q^2 = 0$ and its q^2 dependence, respectively. The variable q^2 and the symbols N and N' are defined in Ref. 10. Thus, the applicability of one-pion exchange and PCAC to nuclei can be tested.

The experiment was carried out at the Indiana University Cyclotron Facility (IUCF) with an incident proton beam of 144 MeV. The experimental setup is shown in Fig. 1. The targets were placed in a vacuum chamber located between the pole faces of a dipole bending magnet, which swept the noninteracting primary beam into a beam dump. The only obstructions between the target and the detector were a 0.003" kapton window and air. However, the scattered neutrons had to pass through movable narrow collimators of lead and concrete in order to reach the detector.

The detector consisted of a charged particle identifier followed by two plastic scintillator timing rods⁴ and a liquid scintillator vat⁵ used for setting the energy threshold. A valid neutron event consisted of a coincidence between one of the rods and the vat, with no signal in the charged particle identifier. A phototube was placed on each end of the rods, and the time difference between each phototube pulse and the subsequent cyclotron RF peak pulse was stored. The neutron time-of-flight spectrum for each rod was then obtained by adding the time differences from each of its ends. Sample spectra are shown in Figs. 2a-2c.

The efficiency of the detector was determined by comparing its yield to that of another detector,⁶ which was calibrated in an independent experiment with a tagged neutron beam using the reaction

The details of this experiment will be reported elsewhere.

The differential cross sections are plotted in Figs. 3a-3c. The absolute normalization, including the efficiency and target thickness, has an accuracy of about 12%.

The theoretical calculations were done relying on the absorption modified pion exchange model (APEM). In APEM the pion exchange Born helicity amplitudes for the charge exchange reactions (Eqs. 1-3) were modified to account for distortions of the projectile's wave function by the elastic scattering potential of the target. The general form of an absorption modified helicity amplitude is then

$$H^{\text{absorption}} = \sum_j \frac{1}{2} (\eta_{j,j+\frac{1}{2}} S_{j,j+\frac{1}{2}} + \eta_{j,j-\frac{1}{2}} S_{j,j-\frac{1}{2}}) H_j^{\text{Born}} \quad (5)$$

$S_{j\ell}$ is determined from an elastic scattering optical model code and $\eta_{j\ell}$

accounts for the fact that the range of the inelastic scattering potential is comparable to that of the elastic scattering potential. This differs from the original model, which sets $\eta_{j\ell} = 1$ and calculates $S_{j\ell}$ from the Eikonal approximation.

The magnitudes of the six independent Born helicity amplitudes for a $\Delta J = \Delta T = 1$, $\Delta P = 0$ transition via one-pion exchange can be written as⁷

$$H_{++}^0 = g_{p\pi\pi}(q^2)g_{N\pi N}(q^2)b \xi_- \frac{E_1}{M_1} \left[-d_{\frac{1}{2},\frac{1}{2}}^{\frac{1}{2}} + (z-\gamma) \frac{\cos \frac{\theta}{2}}{z-\cos \theta} \right] \quad (6a)$$

$$H_{+-}^0 = g_{p\pi\pi}(q^2)g_{N\pi N}(q^2)b \xi_+ \frac{E_1}{M_1} \left[-d_{-\frac{1}{2},\frac{1}{2}}^{\frac{1}{2}} + (z-\gamma) \frac{\sin \frac{\theta}{2}}{z-\cos \theta} \right] \quad (6b)$$

$$H_{++}^1 = g_{p\pi\pi}(q^2)g_{N\pi N}(q^2) \frac{b}{\sqrt{2}} \xi_- \left[-d_{\frac{1}{2},\frac{1}{2}}^{\frac{1}{2}} + (z+1) \frac{\sin \frac{\theta}{2}}{z-\cos \theta} \right] \quad (6c)$$

$$H_{--}^1 = -g_{p\pi\pi}(q^2)g_{N\pi N}(q^2) \frac{b}{\sqrt{2}} \xi_- \frac{\sin \frac{\theta}{2} \cos^2 \frac{\theta}{2}}{z-\cos \theta} \quad (6d)$$

$$H_{+-}^1 = g_{p\pi\pi}(q^2)g_{N\pi N}(q^2) \frac{b}{\sqrt{2}} \xi_+ \left[+d_{\frac{1}{2},\frac{1}{2}}^{\frac{1}{2}} + (1-z) \frac{\cos \frac{\theta}{2}}{z-\cos \theta} \right] \quad (6e)$$

$$H_{-+}^1 = g_{p\pi\pi}(q^2)g_{N\pi N}(q^2) \frac{b}{2} \xi_+ \frac{\sin^2 \frac{\theta}{2} \cos \frac{\theta}{2}}{z-\cos \theta} \quad (6f)$$

They can then be expanded in terms of total angular momentum, J , following the procedure of Hogaasen and Hogaasen.⁸ However, the terms that involve the $d_{\alpha,\beta}^J$ functions need no further expansion in J ; they are referred to as the extraordinary terms. The variables used in Eqs. 6 are defined in Refs. 9 and 10.

Treating the nucleus as an elementary particle, Kim and Primakoff² generalized the Goldberger-Treiman relation¹¹ to nuclei. With the help of the Partially Conserved Axial Vector Current (PCAC) hypothesis¹² they

related the nucleus-pion-nucleus coupling constant to the axial vector form factor $F_A(q^2)$ at $q^2 = 0$ as;

$$g_{N\pi N'}(0) = \frac{\sqrt{M_N M_{N'}}}{a_\pi} F_A(0) \quad (7)$$

The symbols M_N , $M_{N'}$, and a_π denote the masses of nuclei N, N' and the pion decay constants, respectively. Since Eqs. 6 apply to the physical region ($q^2 > 0$), the q^2 dependence of $g_{p\pi n}(q^2)$ and $g_{N\pi N'}(q^2)$ must be determined. From nuclear beta decay Kim¹³ obtained the expressions;

$$g_{p\pi n}(q^2) = \frac{g_{p\pi n}(0)}{(1+0.04143 q^2/m_\pi^2)^2} \quad (8)$$

at the proton, virtual pion, neutron vertex of all three reactions (Eqs. 1-3). From inelastic electron nucleus experiments Kim and Towsand¹⁴ derived an expression

$$g_{N\pi N'}(q^2) = \frac{(1-0.04 q^2/m_\pi^2)}{(1+q^2/M_A^2)^2} g_{N\pi N'}(0) \quad (9)$$

for reactions 1 and 2 (Eqs. 1 and 2) and another expression^{15,16}

$$g_{N\pi N'}(q^2) = (1-1.04 \frac{q^2}{m_\pi^2+q^2}) (1+1.01 \frac{q^2}{m_\pi^2}) \exp(-0.384 \frac{q^2}{m_\pi^2}) g_{N\pi N'}(0) \quad (10)$$

for reactions 3 (Eq. 3). The parameter M_A^2 in Eq. 9 has the values¹³ $2m_\pi^2$ and $2.6m_\pi^2$ for Eqs. 1 and 2, respectively. In order to obtain the input value of $g_{N\pi N'}(q^2)$ for Eqs. 6 from Eqs. 7, $F_A(0)$ must be determined. The relation between the ft value and $F_A(0)$ in nuclear beta decay of the N' nucleus has been found¹⁷;

$$F_A(0)^2 = \frac{2\pi^3 \ln 2 (2J_{N'}+1)}{3G^2 \cos^2 \theta_c m_e^5 \text{ ft}} \quad (11)$$

The symbols are defined in Ref. 10. From Eqs. 7 and 11, it follows in Born approximation that the zero degree differential cross sections ($q^2 \approx 0$ at $\theta = 0$), for the reactions (1)-(3) should be inversely proportional to the corresponding ft values. The experimentally measured zero degree differential cross sections are given in Table 1. An inspection of Table 1 shows that the ratio of the zero degree differential cross sections of Eq. 1 to Eq. 2 are in rough agreement with that of the corresponding ratios of the $g_{NN}^2(0)$ values while this is not the case for Eq. 3. Next we used the detailed APEN with the coupling constants described in Eqs. 7-11. The resulting theoretical predictions are shown in Figs. 3a-3c. It is gratifying that this well defined, but complex, relationship between weak interaction and pion exchange holds for reactions 1 and 2. The solid curves were obtained when the extraordinary terms in Eqs. 6 were removed. The theoretical predictions for Eq. 3 had to be multiplied by 323 to bring it in agreement with our experimental data. Although no theoretical calculation was done on this problem, the work of Goulard et al.¹⁶ bears on this question. The nucleons only impulse approximation calculation gives a small $^{14}\text{O} \rightarrow ^{14}\text{N} + e^+ + \nu_e$ amplitude. Inclusion of meson current terms operating among the nucleons gives a term of about equal in magnitude but opposite in size to the nucleon terms only. Since the two amplitudes add coherently, the beta decay amplitude is further reduced. However, if the meson current effects are removed from the nuclear beta decay coupling constant,¹⁶ then the predicted one pion exchange cross sections are still a factor of three lower than the data, and the shape is still different. A further complication is that the first order axial vector β decay matrix element between ^{14}O and the $L = 2$ wave functions of ^{14}N is exactly zero. If the ^{14}N wave function ($\psi_N = N_2^1 \psi_{L=2}^{S=1} + N_1^1 \psi_{L=1}^{S=0} + N_0^1 \psi_{L=0}^{S=1}$) is dominantly

in the $L = 2$ state then this state dominates the charge exchange process. Since the $L = 2$ state is not accessible to β decay there can be no relation between the charge exchange cross section and β decay in the $L = 2$ state.

In conclusion we find that one-pion exchange and the elementary particle treatment of nuclei, when combined with PCAC, provide a good description of Gamow-Teller type transitions for Reactions 1 and 2, but not for Reaction 3.

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3. K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 5783 (1964).
L. Durand and Y. T. Chiu, Phys. Rev. 139, 646 (1965).
4. The rods were made from NE-102 plastic scintillator, had a diameter of 1.75", and were 18" long. Their time resolution was better than 400 picoseconds.
5. The vat was 20" long x 10" high x 6" deep, with walls of 0.125" thick pyrex. It was filled with NE-213 liquid scintillator and viewed with two phototubes.
6. This detector was 5" long and had a 5" diameter, was filled with NE-213 liquid scintillator, and had one phototube.
7. L. J. Gutay, F. T. Meiere, D. D. Carmony, F. J. Loeffler and P. L. Csonka, Nuclear Physics B12, 31 (1969).
8. H. Hogaasen and J. Hogaasen, Nuovo Cimento 39, 941 (1965).
9. The variables used in Eqs. 6 are defined below:

$H_{\lambda, \lambda}^{\mu}$ = helicity amplitude for reactions 1-3.

μ = helicity of the target or recoil nucleus. ($\mu = 1, 0, -1$)

λ', λ = helicity of the initial and final state neutron. ($\lambda' = \pm \frac{1}{2}, \lambda = \pm \frac{1}{2}$)

$g_{p\pi n}(q^2)$ = the strength of the proton-pion-neutron coupling constant at momentum transfer q^2 .

$g_{N\pi N'}(q^2)$ = the strength of the target nucleon-pion-recoil nucleon coupling constant of momentum transfer q^2 .

$d_{\pm \frac{1}{2}, \pm \frac{1}{2}}^{\pm \frac{1}{2}}(\theta)$ = the matrix element of the rotation operator.

θ = center of mass scattering angle.

E_1 = center of mass energy of spin-1 nucleus.

E_0 = center of mass energy of spin-0 nucleus.

E = center of mass energy of the proton.

E' = center of mass energy of the neutron.

p = initial center of mass momentum.

p' = final center of mass momentum.

M_1 = mass of spin-1 nucleus.

m_p = mass of the proton.

m_n = mass of the neutron.

m_{π} = mass of the pion.

$$z = \frac{2EE' + m_n^2 - m_p^2 - m_{\pi}^2}{2pp'}$$

$$\xi_{\pm} = \sqrt{(E + m_p)(E' + m_n)} \left(\frac{p}{E + m_p} \pm \frac{p'}{E' + m_n} \right)$$

ΔE = mass difference between the initial target and final state recoil nucleus.

$(J^P T)$ = the spin, parity, isotopic spin of the initial state target (i) and final state recoil (f) nucleus.

$(1^+; 0)_i \rightarrow (0^+; 1)_f = {}^6\text{Li} \rightarrow {}^6\text{Be}$ transition that can be induced by off mass shell pion absorption in $\ell = 1$ state. $\Delta E = 3.5$ MeV.

$(0^+; 0)_i \rightarrow (1^+; 1)_f = {}^{12}\text{C} \rightarrow {}^{12}\text{N}$ transition that can be induced by off mass shell pion absorption in $\ell = 1$ state. $\Delta E = 17.3$ MeV.

$(1^+; 0)_i \rightarrow (0^+; 1)_f = {}^{14}\text{N} \rightarrow {}^{14}\text{O}$ transition that can be induced by off mass shell pion absorption in $\ell = 1$ state. $\Delta E = 2.4$ MeV.

Transitions	$0^+ \rightarrow 1^+$	$1^+ \rightarrow 0$
Values of b are	$1/p'$	$1/p$
Values of γ are	$p'E_0/pE_1$	$pE_0/p'E_1$

10. $p_{\mu} = (\vec{p}; E)$, the four vector of the incident proton.

$p'_{\mu} = (\vec{p}'; E')$, the four vector of the scattered neutron.

$q^2 = (p - p')^2 = (E - E')^2$, the common four momentum transfer squared to both the proton and the target.

N = the subscript that denotes the target nucleus.

N' = the subscript that denotes the recoil nucleus.

m_{π}, m_e = the mass of on mass shell pion and electron, respectively.

$a_{\pi} = 0.131$ GeV, the pion decay coupling constant.

$F_A(0)$ = weak axial-vector form factor for $N' \rightarrow N$ beta decay.

$G = 1.026 \times 10^{-5}/m_p^2$, the weak coupling constant.

$\theta_c = 0.257$, the Cabbibo angle.

$J_{N'}$ = spin of the θ decaying N' nucleus.

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J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring 17, 757 (1960).

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$$\text{PCAC relation; } F_A(q^2) + q^2 \frac{F_p(q^2)}{m_{\pi}^2} = \frac{a_{\pi} m_{\pi}^2 g_{N\pi N'}(q^2)}{\sqrt{M_N M_{N'}} (q^2 + m_{\pi}^2)}$$

$$\text{with the impulse approximation result } \frac{F_p(q^2)}{F_A(q^2)} = \frac{g_p(q^2)}{g_A(q^2)} \approx -1.04 \frac{m_{\pi}^2}{m_{\pi}^2 + q^2}$$

(see Ref. 13, Eq. 56 and Ref. 14, Eq. 16). The q^2 dependence of $F_A(q^2)$ was taken from Eqs. 59 and 60 of Ref. 13.

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16. Just as in Ref. 14, the PCAC relation, the result $F_P(q^2)/F_A(q^2) = -1.04 m_\pi^2 / (m_\pi^2 + q^2)$, and $F_A(0) = a_{\pi N N'}(0) / \sqrt{M_N M_{N'}}$ give;
 $g_{\pi N N'}(q^2) = (1 - 0.04 q^2 / m_\pi^2) F_A(q^2)$. In this case the q^2 dependence of $F_A(q^2)$ was taken from Eq. 12b of Ref. 15 and Eq. 84 of Ref. 13.
17. If the β decaying state has spin $J_{N'}$, Eq. 7a of Ref. 15 has to be multiplied by $2J_{N'} + 1$.

Table 1 Experimental I_0 values, form factor charge exchange cross sections and the threshold, nucleon-pion+nucleus² coupling constants for Eqs. 1-3.

Target	I_0	I_0^2	I_0^3
Si(ncc)	815	1.33×10^6	2.0×10^3
$\frac{d\sigma(\sigma)}{d\Omega}$ (mb/SR)	11.7	5.7	0.13
$g(\sigma)_{\text{h-n}}$	72.1	56.4	1.91

Fig. 1. Experimental layout.

Fig. 2. Sample neutron time-of-flight spectra at zero degrees from 144-MeV protons scattering on (A) ${}^6\text{Li}$, (B) ${}^{12}\text{C}$, and (C) ${}^{14}\text{N}$.

Fig. 3. Differential cross sections. The solid curves are the theoretical calculations with no extraordinary terms included, and the dashed curves are theoretical calculations that include the extraordinary terms.