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DOE RESEARCH AND
DEVELOPMENT REPORT**

**A MODEL FOR INCORPORATING FUEL
SWELLING AND CLAD SHRINKAGE EFFECTS
IN DIFFUSION THEORY CALCULATIONS
(LWBR Development Program)**

MARCH 1980

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A MODEL FOR INCORPORATING FUEL SWELLING
AND CLAD SHRINKAGE EFFECTS IN DIFFUSION
THEORY CALCULATIONS
(LWBR Development Program)

W. C. Schick, Jr.
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E. Duncombe

March 1980

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FOREWORD

The Shippingport Atomic Power Station located in Shippingport, Pennsylvania was the first large-scale, central-station nuclear power plant in the United States and the first plant of such size in the world operated solely to produce electric power. This program was started in 1953 to confirm the practical application of nuclear power for large-scale electric power generation. It has provided much of the technology being used for design and operation of the commercial, central-station nuclear power plants now in use.

Subsequent to development and successful operation of the Pressurized Water Reactor in the Department of Energy (DOE) owned reactor plant at the Shippingport Atomic Power Station, the Atomic Energy Commission in 1965 undertook a research and development program to design and build a Light Water Breeder Reactor core for operation in the Shippingport Station.

The objective of the Light Water Breeder Reactor (LWBR) program has been to develop a technology that would significantly improve the utilization of the nation's nuclear fuel resources employing the well-established water reactor technology. To achieve this objective, work has been directed toward analysis, design, component tests, and fabrication of a water-cooled, thorium oxide-uranium oxide fuel cycle breeder reactor for installation and operation at the Shippingport Station. The LWBR core started operation in the Shippingport Station in the Fall of 1977 and is expected to be operated for about 4 to 5 years. At the end of this period, the core will be removed and the spent fuel shipped to the Naval Reactors Expended Core Facility for a detailed examination to verify core performance including an evaluation of breeding characteristics.

In 1976, with fabrication of the Shippingport LWBR core nearing completion, the Energy Research and Development Administration, now DOE, established the Advanced Water Breeder Applications (AWBA) program to develop and disseminate technical information which would assist U.S. industry in evaluating the LWBR concept for commercial-scale applications. The program is exploring some of the problems that would be faced by industry in adapting technology confirmed in the LWBR program. Information being developed includes concepts for commercial-scale prebreeder cores which would produce uranium-233 for light water breeder cores while producing electric power, improvements for breeder cores based on the technology developed to fabricate and operate the Shippingport LWBR core, and other information and technology to aid in evaluating commercial-scale application of the LWBR concept.

All three development programs (Pressurized Water Reactor, Light Water Breeder Reactor, and Advanced Water Breeder Applications) are under the technical direction of the Division of Naval Reactors of DOE. They have the goal of developing practical improvements in the utilization of nuclear fuel resources for generation of electrical energy using water-cooled nuclear reactors.

Technical information developed under the Shippingport, LWBR, and AWBA programs has been and will continue to be published in technical memoranda, one of which is this present report.

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A model has been devised for incorporating into the thermal feedback procedure of the PDQ few-group diffusion theory computer program the explicit calculation of depletion and temperature dependent fuel-rod shrinkage and swelling at each mesh point. The model determines the effect on reactivity of the change in hydrogen concentration caused by the variation in coolant channel area as the rods contract and expand. The calculation of fuel temperature, and hence of Doppler-broadened cross sections, is improved by correcting the heat transfer coefficient of the fuel-clad gap for the effects of clad creep, fuel densification and swelling, and release of fission-product gases into the gap. An approximate calculation of clad stress is also included in the model.

The model has been tested by comparing calculated fuel temperatures and clad radii with results obtained using the CYGRO-4 fuel rod analysis program. The history of a number of Light Water Breeder Reactor (LWBR) fuel rods was followed over core lifetime using both codes. The calculated fuel temperatures were in agreement within 40 F° or better and the clad radii within 0.3 mil or better in nearly all cases. The improvement made by including the model in PDQ may be as great as several tenths of one percent in eigenvalue.

A MODEL FOR INCORPORATING FUEL SWELLING AND CLAD SHRINKAGE EFFECTS IN DIFFUSION THEORY CALCULATIONS

(LWBR Development Program)

W. C. Schick, Jr.
S. Milani
E. Duncombe

I. INTRODUCTION

The Light Water Breeder Reactor (LWBR) Program is developing the technology to breed fissile material in a light water reactor in order to use nuclear fuel significantly more effectively in light water thermal reactors. To achieve this objective, technology is being developed through the design and fabrication of a breeder reactor core that has been installed and is operating in the existing Department of Energy (DOE) owned pressurized water

reactor plant at Shippingport, Pennsylvania. The design and operating environment of the LWBR reactor are described in Reference (1). This report presents the results of work performed in support of this program.

The calculational model used in the analysis of nuclear performance of LWBR is presented in Reference (2). (Section V.G of Reference (2) is a summary of the model which is described in this report.) As explained in this reference, neutron diffusion-depletion calculations are performed on the LWBR core using the PDQ computer program (Reference (3)). The thermal feedback (TFB) option in PDQ (Reference (4)) provides a means for including certain thermal effects in conjunction with the nuclear calculations. These effects include Doppler broadening of resonances in the nuclear cross sections and changes in the H and O atomic number densities due to thermal expansion of the moderator. The purpose of the model described in this report is to extend the TFB nuclear performance calculations in PDQ to account for the following physical processes:

1. Radial contraction of the fuel rods due to fast-neutron and pressure induced creep in the cladding;
2. Radial expansion of the fuel inside the fuel rods due to fission-product swelling and thermal expansion;
3. Radial contraction of the fuel inside the fuel rods due to pellet densification - i.e., the closing of pores in the sintered oxide fuel;
4. Changes in the thermal conductivity of the gas in the gap between the fuel and the cladding due to the release of noble-gas fission products;
5. Induced circumferential stress in the cladding where the combined effects of clad contraction and fuel expansion have caused the fuel-clad gap to disappear, so that the fuel presses outward on the cladding.

Process 1 affects the calculation of the H and O atomic number densities, since in a closely-spaced core like LWBR the fuel-rod shrinkage can change the cross-sectional area of the coolant channel by as much as a few percent. Processes 1, 2, and 3 all affect the calculation of fuel temperature (which in turn determines the amount of Doppler broadening), since the temperature drop across the fuel-clad gap depends in part on the size of the gap. Process 4 also affects the temperature drop across the gap. Process 5 does not actually affect the TFB calculations; the consideration of this process was included primarily to aid in fuel rod analysis, especially in locating potentially limiting rods.

The model does not explicitly consider axial (as opposed to radial) expansion or contraction of the fuel. The principal consequence of axial expansion or contraction is to change slightly the number densities of fuel atoms. This is expected to have a smaller effect on the nuclear performance of the LWBR core than the radial processes described in this report. A second consequence of axial expansion is to hasten the radial contraction of the cladding due to fuel-clad interaction. This effect is included indirectly in the model, as discussed in Section III.G below.

The approach used in this model is based in part on methods used in the FIGRO (References (5), (6)) and CYGRO-4 (Reference (7)) fuel rod analysis codes. These programs deal, however, with only a single segment of a single fuel rod whose power history is provided as input, whereas PDQ deals with all segments of all rods at a given time in life. Accordingly, considerable simplification and modification of the FIGRO and CYGRO-4 methods were required to provide a model which could be incorporated into the PDQ coding without significantly increasing either the execution time or the computer memory requirements. Although the model was devised specifically for analysis of the LWBR core, the approach is sufficiently general that it could be applied to other water-moderated reactors with rod-type fuel for which PDQ analysis with thermal feedback is performed.

A summary of the relevant aspects of the TFB nuclear performance calculations in PDQ is given in Section II. Details of the model are

described in Section III. The qualification of the model by comparison with FIGRO and CYGRO-4 calculations is discussed in Section IV. The conclusions and summary are given in Section V. Appendix A gives a table of symbols. Appendix B details those input data items required to execute the modified PDQ which are not described in the main body of the report.

II. SUMMARY OF PDQ THERMAL FEEDBACK CALCULATIONS

A. General Considerations

A detailed description of the TFB calculations is given in Reference (4). Only the relevant aspects of these calculations will be summarized here.

The reactor core is divided into a series of thermal mesh blocks. Each mesh block consists of an axial segment of one or more fuel rods and the surrounding water channel. (Structural elements may also be included in mesh blocks, but these will not be discussed.) The different fuel rods contained in a given mesh block are assumed to have identical thermal properties, so it is possible to treat each block as if it were represented by a single rod.

Before any thermal calculations are carried out, a nuclear calculation is performed using cross sections and number densities based on average fuel and moderator temperatures supplied by the requestor. The heat generation rate, Q (in units of BTU/hr), and the depletion, f (in units of 10^{20} fissions/cm³), are determined at each mesh block. A preliminary thermal pass through the core is then made, in which Q is used to calculate several quantities at every mesh block, including the enthalpy of the coolant and the heat flux at the surface of the fuel rod, ϕ (BTU/hr-ft²). Thereafter, one or more passes through the core are carried out in which the fuel average temperature and the moderator temperature and density are calculated from ϕ using a procedure outlined in the following paragraphs. The nuclear calculation is repeated after each of these passes, using Doppler-broadened cross sections appropriate for the fuel average temperature at each mesh

block, and using the hydrogen number density appropriate for the calculated values of moderator temperature and density. In this manner, a covered, consistent nuclear and thermal description of the core is obtained. A block diagram of the TFB calculations is given in Figure 1.

B. Fuel Temperature - Preliminary Calculations

The geometry used in calculating the fuel temperature is illustrated in Figure 2. The (cylindrical) fuel rod is taken to have an outer surface radius r_s . The clad inner radius is r_1 . The outer radius of the fuel is r_2 . Provision is made in the coding to accommodate annular fuel pellets with a central void of radius r_3 ; however, since the LWBR fuel has no central void, r_3 is always set to zero. In the previous PDQ model r_s , r_1 , and r_2 were assumed to have fixed values, with a single value of each parameter being applied throughout each region of the LWBR core - seed, standard blanket, power flattening blanket, and reflector. (See Reference (1) or (2) for a description of these core regions.) The model described here, however, treats r_s , r_1 , and r_2 as variables whose values must be calculated at each mesh block.

The temperatures corresponding to r_s , r_1 , r_2 , and r_3 are T_s , T_1 , T_2 , and T_3 , respectively, where T_3 is the fuel centerline temperature in LWBR applications. The clad and fuel have thermal conductivities of k_1 and k_3 (BTU/hr-ft-F°), respectively. The quantity k_1 is taken to be constant throughout the core, while k_3 is supplied in tabular form as a function of both depletion and fuel temperature. The fuel-clad gap is characterized by a heat transfer coefficient h_2 (BTU/hr-ft²-F°). In the previous TFB calculations h_2 was also supplied in tabular form, as a function of both depletion and heat flux. An important feature of the model described here is that h_2 is now calculated explicitly at each mesh block, taking into account the effects of clad shrinkage, fuel swelling and contraction, and release of fission-product gases into the gap. Because h_2 depends on the thickness of the gap, which in turn depends on the thermal expansion and hence on the temperature of the fuel, and since the temperature of the fuel depends upon h_2 , an iterative procedure is used to obtain consistent values for h_2 , T_2 and T_3 . This iterative procedure is described in Section II.C below.

FIGURE 1
BLOCK DIAGRAM OF THERMAL FEEDBACK PROCEDURE

CLAD THERMAL CONDUCTIVITY = k_1
 GAP HEAT TRANSFER COEFFICIENT = h_2
 FUEL THERMAL CONDUCTIVITY = k_3
 VOLUMETRIC AVERAGE FUEL TEMPERATURE = T_F

FIGURE 2
 FUEL ROD GEOMETRY

As noted above, the preliminary thermal pass through the core provides values for the enthalpy and heat flux at each mesh block. The calculation of fuel temperature then begins with the determination of T_s from these two quantities. The details of this determination are not relevant to the present discussion; the reader is directed to Reference (4) for further information. Using this value of T_s , the program obtains T_1 from

$$T_1 = T_s + \frac{\phi r_s}{12 k_1} \ln \frac{r_s}{r_1} \quad (1)$$

where the factor of 12 in the denominator arises from the fact that r_s is measured in inches, while the unit of length in ϕ and k_1 is feet.

C. Iterative Procedure

The iterative procedure for h_2 is diagramed in Figure 3. The first step is to calculate T_2 from

$$T_2 = T_1 + \frac{\phi r_s}{h_2 r_1} \quad (2)$$

As an initial estimate for h_2 , a value is used which is appropriate for the case where the gap has disappeared and $r_1 = r_2$, as described in Section III.C below. T_3 may now be determined by making use of the fact that, for a uniform heat generation rate (which is a good approximation for LWBR fuel),

$$\int_{T_2}^{T_3} k_3(T) dT = \frac{1}{2} \phi \frac{r_s}{12} \quad (3)$$

(Equation (3) is a special case of the more general equation derived in Reference (4) that applies where $r_3 \neq 0$.) The input table of k_3 versus temperature and depletion is used to generate a second table, giving $\int k_3 dT$ versus temperature and depletion. By linear interpolation in this table, the value of T_3 is found for which the integral on the left side of Equation (3) equals the right side. Finally, the volumetric average fuel temperature T_F is found using the approximation that the fuel temperature varies as a parabolic

FIGURE 3
BLOCK DIAGRAM OF ITERATIVE PROCEDURE

function of radial distance from the centerline, which is the case if $r_3 = 0$ and the heat generation rate is the same at all radii. This leads to

$$T_F = \frac{(T_2 + T_3)}{2} . \quad (4)$$

Equation (4) is strictly valid only where k_3 is independent of temperature, but the correction to T_F due to the fact that k_3 changes over the temperature range from T_2 to T_3 is small, and therefore is neglected.

With the fuel temperature known, it is possible to calculate a corrected value for the fuel radius, r_2 , taking into account the thermal expansion of the fuel as well as other swelling and contraction effects. The procedure for doing this is described below in Sections III.D and III.E. A corrected value for the clad inner radius, r_1 , taking into account clad creep and the effect of fuel-clad interaction as discussed in Sections III.F and III.G, is also available. With r_1 and r_2 known, the thickness of the fuel-clad gap is established and a better value of h_2 than that used in Equation (2) is obtained by the procedure described in Section III.C.

With the calculation of the improved value of h_2 , the first iteration is complete. It is now possible to recalculate T_2 by using Equation (2) again, and to keep repeating the whole procedure until consistent values of h_2 , T_2 , and T_3 are obtained. It has been found that when this is done, the process converges in an oscillatory manner; that is, h_2 alternately increases and decreases on successive iterations. To dampen these oscillations and hasten convergence, Equation (2) is replaced by

$$T_2 \text{ (new)} = 0.75 \left(T_1 + \frac{\phi r_s}{h_2 r_1} \right) + 0.25 T_2 \text{ (old)} \quad (5)$$

on all iterations after the first. Here, $T_2 \text{ (old)}$ is the value of T_2 obtained in the previous iteration. In effect, the correction actually applied to T_2 is only three-fourths as great as the calculated value. Convergence is defined to occur when the magnitude of the difference between $T_2 \text{ (new)}$ and $T_2 \text{ (old)}$ is less than 2 F°. In nearly all test cases studied, convergence was found to take place in five iterations or fewer. Rapid convergence is a desirable feature of the model since the iterative procedure is applied at every mesh block on each thermal pass through the core.

D. Final Calculations

If the calculated value of r_2 ever exceeds the calculated value of r_1 , this is an indication that the gap has disappeared and the fuel is pressing outward on the clad. In this case r_1 is set equal to the calculated value of r_2 .

After the final values of r_1 and r_2 have been obtained, a corrected value of r_s is calculated under the assumption that the clad thickness ($r_s - r_1$) is a constant. This corrected value of r_s defines a corrected cross-sectional area for the fuel rod. The change in rod cross-sectional area from the nominal value is of course accompanied by a change of equal magnitude but opposite sign in the cross-sectional area of the coolant channel. The hydrogen number density is adjusted at each mesh block to correspond to this change in coolant channel area.

On the last thermal pass through the core in the TFB procedure, a clad stress calculation is carried out as indicated in Section III.H below. Since these stress calculations do not affect any other aspects of the TFB procedure, they may conveniently be performed at the end of the procedure.

III. DETAILED DESCRIPTION OF THE MODEL

A. Release of Gaseous Fission Products

LWBR fuel rods are initially charged internally with one atmosphere of He gas. During core life, however, some of the noble-gas fission-product Kr and Xe will migrate from the fuel pellet into the gap contained inside the fuel rod and will mix with the He in the fuel-clad gap and plenum. Since the thermal conductivities of Kr and Xe are lower than that of He by a factor of about 20, this will affect the temperature drop across the fuel-clad gap. It is necessary, therefore, to estimate how much fission gas has been released. Once the composition of the He-Kr-Xe mixture has been determined, the conductivity of the mixture can be determined by the method described in the following section.

The model used to calculate fission gas release is based on the methods of Reference (8). The fraction of gas released, R_F , is assumed to be given by

$$R_F = C_1 + C_2 f + C_3 F_V \quad (6)$$

where C_1 , C_2 , and C_3 are constants, f is depletion, and F_V is the volume fraction of fuel above a certain temperature known as the release temperature T_{rel} . T_{rel} is obtained by approximating the intermediate temperature thorium curve in Figure 7 of Reference (8) by

$$T_{rel} (^{\circ}\text{F}) = 2900 - 230 \log_{10} t \quad (7)$$

where t is time in thousands of effective full-power hours (EFPH). For $t \leq 0.1$, T_{rel} is taken to be 3130°F .

The C_1 and C_2 terms in Equation (6) represent the fraction of fission gas released at low temperatures due, in part, to recoil of the fission fragments. Constants C_1 and C_2 depend on the percent of theoretical density of the sintered oxide fuel. The values of both C_1 and C_2 used for analysis of LWBR lie between about 5.5×10^{-4} and 7.5×10^{-4} , depending on the density of fuel in different regions of the core.

The C_3 term in Equation (6) represents fission gas bubble diffusion and migration at higher temperatures. A single value of 0.05 is used for C_3 . Only a very small fraction of the LWBR fuel ever gets above T_{rel} . The volume fraction, F_V , is calculated under the assumption of a parabolic variation of fuel temperature with distance from the fuel centerline. The result is

$$F_V = \frac{T_3 - T_{rel}}{2(T_3 - T_F)} \quad (8)$$

when $T_3 > T_{rel}$; otherwise $F_V = 0$. This calculation is performed during the preliminary thermal pass through the core. Since T_3 and T_F have not yet been calculated when the preliminary thermal pass is carried out, values are used

for these quantities which were determined previously for earlier times in life. T_F in Equation (8) is the value obtained for the volumetric average fuel temperature at the most recent previous time in life for which PDQ with thermal feedback was performed. T_3 in Equation (8) is the highest value obtained for the fuel centerline temperature at any previous time in life.* These values of T_F and T_3 are retained in the concentration file as pseudo-nuclide "concentrations". The pseudo-nuclide concentrations are discussed further in Reference (3).

The reason for performing the fission gas release calculations during the preliminary thermal pass, rather than as part of the iterative procedure discussed in Section II.C, is that the gas released from all parts of a rod will mix in the gap and plenum. It is therefore necessary to perform the calculations for all thermal mesh blocks along the length of a rod before the composition of the gas in the rod can be determined. Only then can the thermal conductivity of the gas (which is needed in the iterative procedure) be evaluated. Little inaccuracy is introduced by the fact that the temperatures used in Equation (8) are appropriate for earlier times in life because, as noted above, R_F is independent of temperature below T_{rel} , and only a small fraction of the core goes above this temperature.

B. Conductivity of Gas in the Gap

The number of fission gas atoms released from the fuel in the segment of a fuel rod contained in one thermal mesh block is given by

$$N_{FG} = \alpha_{FG} f R_F V_F \quad (9)$$

where α_{FG} is the average number of noble gas atoms produced per fission, f is depletion, R_F is obtained from Equation (6), and V_F is the volume of fuel in the segment. The gas released is assumed to consist of 84.7% Xe and 15.3%

*This approach to temperature-dependent fission gas release is based on a specific temperature history and might require some adjustment, based on appropriate test data, if applied to a different history pattern.

Kr. In addition, the release of ternary fission-product He is also considered. The number of ternary He atoms released from the fuel in the segment is given by

$$N_T = \alpha_T f R_T V_F \quad (10)$$

where α_T is the average number of ternary He atoms produced per fission and R_T is the fraction of He atoms which are released from the fuel. Because α_T is approximately two orders of magnitude smaller than α_{FG} , the effect of ternary He is small; it is nevertheless included in the calculations because of the large thermal conductivity of He. R_T in Equation (10) is an input constant. The values used for α_{FG} , α_T , and R_T are 0.33, 0.004, and 0.5, respectively.

N_{FG} and N_T are summed over the whole length of the fuel rod. These sums are combined with the known amount of fill gas in the gap and plenum (one atmosphere of He for the LWBR fuel rods) to determine the resulting gas composition. The thermal conductivity of this mixture of gases is calculated as

$$k = \sum_i w_i a_i T^{n_i} \quad (11)$$

where the subscript i labels the individual gases, w_i is a weighting factor which is proportional to the mole fraction of gas i times the cube root of its molecular weight, T is absolute temperature, and the constants a_i and n_i are properties of the individual gases. The values of n_i adopted in Reference (6) are 0.721, 0.791, and 0.954 for He, Xe, and Kr respectively, while the corresponding values for a_i are 14.4×10^{-4} , 0.38×10^{-4} , and 0.22×10^{-4} for k in units of BTU/hr-ft-F°. In order to simplify the calculations, a single value of 0.721 (the value appropriate for He, which provides the dominant contribution to k) was assumed for each n_i . The corresponding values of a_i for Xe and Kr were adjusted to give the best fit over the temperature range from 600° to 1500°F. It can be shown that the maximum error in k introduced by this procedure is under 1% for LWBR fuel rods. With this simplification, Equation (11) may be rewritten

$$k = S T^{0.721} \quad (12)$$

where the value of S is established for each rod when the composition of the gas in the gap is determined during the preliminary thermal pass.

C. Gap Heat Transfer Coefficient

The heat transfer coefficient of the gap, h_2 , is calculated in the iterative procedure at each mesh block by a simplification of the procedure used in program FIGRO, Reference (5). The coefficient is taken to be the sum of a term due to gas conduction plus, for sufficiently small gaps, a term for conduction through solid points of contact. Radiation is neglected.

The gas conduction term is

$$h_g = \frac{k}{C_g + t} \quad (13)$$

where k is calculated by Equation (12) with the gas temperature taken to be the average of T_1 and T_2 , C_g is an input parameter, and t is the radial gap thickness (i.e., $t = r_1 - r_2$). C_g replaces a rather complicated expression in the FIGRO model, which depends on quantities such as surface roughness and "temperature jump distance" (Reference (5)). It was found that a single value of 4×10^{-4} inch for C_g gave close agreement when fuel temperature calculations made with the present model were compared with FIGRO/CYGRO-4 calculations in all regions of the core. The exact value of C_g is not required for the determination of fuel temperatures. The reason is that in the seed the gap thickness is considerably greater than C_g , so that h_g is relatively insensitive to C_g ; whereas in the blanket, although the gap thickness may go to zero and cause h_g to be strongly dependent on C_g , this happens only when the temperature drop across the gap is small, so that the fuel temperature is insensitive to the value of h_g . Thus, the simple expression in Equation (13) proves to be adequate.

When t goes to zero and the gap disappears, a second term is added to h_g to account for conduction through solid points of contact. This term is

$$h_s = C_s \frac{k_1 k_3}{k_1 + k_3} \quad (14)$$

where C_s is an input parameter, k_1 is the thermal conductivity of the clad, and k_3 is the thermal conductivity of the fuel at its surface. Although C_s should properly be a function of the contact pressure between the fuel and clad, it was found that a single constant value of 206 ft^{-1} (corresponding to a contact pressure of 2000 psi) provides sufficient accuracy for fuel temperature calculations.

Where the gap thickness is greater than zero but less than some value Δ , the program calculates h_2 by linear interpolation between the values appropriate for $t = 0$ and $t = \Delta$. This is in accordance with the procedure used in FIGRO. A value for Δ of 5×10^{-4} inch, the same value as is used in FIGRO, was found to give the best agreement.

D. Fuel Thermal Expansion

The thermal expansion of the fuel is determined as follows:

$$\frac{\Delta r_2}{r_2} = \beta_0 + \beta_1 T_F + \beta_2 T_F^2 \quad (15)$$

where Δr_2 = radius of fuel pellets at reference temperature (68°F),
 r_2 = change in fuel radius from reference temperature to T_F ,
 T_F = volumetric average fuel temperature calculated by Equation (4).

The input parameters β_0 , β_1 , and β_2 are linear functions of the weight percent of UO_2 in the binary thorium-uranium fuel.

E. Fuel Densification and Swelling

The change in fuel radius due to irradiation effects is calculated using a modification of the expression suggested in Reference (9):

$$\frac{\Delta r_2}{r_2} = -B_1 (1 - e^{-B_2 f}) + B_3 f \quad (16)$$

where r_2 is the initial (beginning-of-life) fuel pellet radius at the reference temperature, Δr_2 is the change in fuel pellet radius, f is depletion in units of 10^{20} fissions/cm³, and B_1 , B_2 , and B_3 are input parameters. B_1 is a coefficient describing fuel contraction or densification due to shrinkage of pores. An expression was used for B_1 in terms of fuel grain size that is based on the approach of Reference (9). The actual LWBR fuel grain sizes in different regions of the core were used for the calculations. The values obtained for B_1 range from 0.0019 to 0.0033.

B_2 in Equation (16) is the "delay constant" for the densification. A value of 1.27 for B_2 was found to give good agreement with CYGRO-4 calculations for test cases from all parts of the LWBR core except the reflector. In the LWBR reflector, a larger value of 5.0 for B_2 was found to be necessary. The depletion is generally much smaller as a function of time in the reflector than in other regions of the core, so the larger value of B_2 is apparently necessary to account for the actual rate of densification.

B_3 in Equation (16) is the coefficient which accounts for fission product swelling. The values used in the present model were determined by trial and error in comparing the model with CYGRO-4 calculations. The values of B_3 appropriate for the different regions of the core were found to be nearly equal; namely, 0.0013 for the seed and 0.0014 for the blanket and reflector.

F. Clad Shrinkage

Before the fuel-clad gap disappears, the clad shrinks under the influence of the external coolant pressure due to fast neutron induced creep. The assumption is made that the rate of creep is proportional to the applied stress and to the fast (> 1 MeV) neutron flux. That is;

$$\frac{\delta r_1}{r_1} = A \sigma \psi \Delta t \quad (17)$$

where δr_1 is the change that has taken place in the clad inner radius in the time Δt , A is an input parameter, σ is the circumferential stress (in psi)

due to the coolant, and ψ is the fast flux at the present time (in neutrons/cm²-sec). (Parameter A in Equation (17) is denoted A_1 in Reference (3).) The value determined for r_1 at each mesh block is retained in the concentration file as a pseudo-nuclide "concentration" at the end of the TFB procedure for a given time in life. The quantity Δt is the time (in EFPH) since the last TFB calculation was performed, and δr_1 is the change in clad inner radius taking place in this time. Note that this definition of δr_1 is different from that of Δr_2 in Equation (16), which is the total change in r_2 since beginning-of-life.

The value of A in Equation (17) was adjusted to give the best comparison with CYGRO calculations. A value of 1.6×10^{-25} was obtained for seed rods and 8.0×10^{-25} for blanket and reflector rods, in units of (psi)⁻¹ (neutrons/cm²-sec)⁻¹ (hr)⁻¹.

In Equation (17) and elsewhere in this report, the usual sign convention for circumferential stress is followed: σ is positive for a net outward force on the clad, giving a tensile stress, and negative for a net inward force, giving a compressive stress.

The thermal expansion of the clad is not calculated in this model. The program accepts hot, pressurized values of clad inner and outer radii as input. Variations in the clad radii from these input values due to differences in clad temperature throughout the core are negligible (< 0.1 mil) during normal operation.

G. Fuel-Clad Interaction

An additional degree of flexibility has been added to the calculation of the clad radius in order to take into account an interaction between the fuel and clad. This interaction arises in blanket and reflector fuel from the fact that the clad tends to assume a slightly oval or elliptical shape as the gap becomes small. The clad and fuel first come into contact along the narrowest part of the oval. As the fuel expands isotropically due to fission product swelling, the clad is stretched axially as well as radially

at the narrow part of the oval. The elastic properties of the clad make it tend to contract radially along the wide part of the oval in response to the axial stretching along the narrow part. The net result is that the (average) clad radius decreases more rapidly after the initial contact has been made between the clad and fuel.

The interaction is represented in the model by the calculation of an additional change in the clad inner radius using the following fitted relationship:

$$\Delta r_1 = - \Delta r_2 \frac{R}{1 + R}, \quad (18)$$

where Δr_2 is the total change in fuel radius from Equations (15) and (16), and R is an input parameter. Note that Δr_1 is the total change in r_1 due to fuel-clad interaction since beginning-of-life, unlike δr_1 in Equation (17) which applies only to the time period Δt . Thus, the total overall change in r_1 since beginning-of-life is equal to Δr_1 plus the sum of the δr_1 values for all time periods Δt up to the present time.

It has been found appropriate to treat R as a linear function of time. In the binary part of the blanket R is set equal to $0.1 \times \text{time}$ (in thousands of EFPH), while in the thoria part of the blanket and in the reflector region R is equal to $0.05 \times \text{time}$. These values of R significantly improve the agreement with CYGRO-4 values of clad radius where the gap has become small. The input parameter R is set to zero in the seed since the gap is not expected to disappear in most seed rods (although the gap may disappear in the hottest seed rods at end of life).

H. Clad Stress Calculational Model

In much of the blanket region of LWBR, the combined effects of clad contraction and fuel expansion eventually cause the gap between fuel and clad to disappear. Subsequently, the outward pressure of the fuel expanding inside the clad exceeds the inward pressure of the coolant so that the clad is subjected to a positive circumferential stress. An approximate calculation of

the stress has been included as part of the TFB procedure. This stress calculation is of necessity much simpler than those performed in programs such as CYGRO-4, because highly detailed calculations could not be included in PDQ without a prohibitive increase in execution time.

Before the stress is calculated, it is necessary to determine whether the fuel-clad gap has disappeared. This is accomplished by comparing the value of r_1 calculated from Equations (17) and (18) with the value of r_2 calculated from Equations (15) and (16). If r_1 is calculated to be less than r_2 , the gap has disappeared and so the value of r_1 is set equal to the calculated value of r_2 .

The circumferential strain in the clad, ϵ , is essentially equal to the fractional change in r_1 . After the gap has disappeared, the circumferential stress in the clad, σ , is calculated from ϵ under the assumption that there are two contributions to the strain - an elastic and a plastic contribution. The elastic strain is proportional to the stress:

$$\epsilon_{el} = \frac{\sigma}{E} \quad (19)$$

where E is an elastic constant depending on Young's modulus and Poisson's ratio. The plastic strain is assumed to obey the relationship

$$\dot{\epsilon}_{pl} = \text{Max} \left[A \dot{\psi} \sigma, B \dot{\psi} \sigma^4 \right] \quad (20)$$

where the dot means differentiation with respect to time, A and B are input constants, and the maximum of the two quantities in brackets is taken.

(Constants A and B are denoted A_1 and A_4 , respectively, in Reference (3).) Equation (20) is a simplification of the more general relationship between plastic strain and stress. Let us define σ_C as the value of stress for which the quantities in brackets in Equation (20) are equal. Then we have

$$\sigma_C = \sqrt[3]{\frac{A}{B}} \quad (21)$$

We may now combine Equations (19), (20), and (21) to obtain the total strain:

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + A \psi \sigma \quad \text{for } \sigma \leq \sigma_C \quad (22a)$$

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + B \psi \sigma^4 \quad \text{for } \sigma > \sigma_C \quad (22b)$$

In order to solve Equations (22) for σ , we will assume that both r_1 and r_2 vary linearly with time during the interval Δt between successive times in life for which PDQ analysis with TFB is performed. Under this assumption, ϵ is a constant and is simply equal to the change in r_1 over the interval Δt , divided by Δt . As noted above, the current value of r_1 is retained in a pseudo-nuclide concentration file at the end of the TFB procedure for each PDQ execution, so the change in r_1 between executions (i.e., over the time interval Δt) is available. If the gap has disappeared during the interval Δt , then linear interpolation of the values of r_1 and r_2 is used to determine the time when the gap disappeared (i.e., when r_1 became equal to r_2), and ϵ is calculated only over the remainder of the time interval.

With $\dot{\epsilon}$ known and constant, Equation (22a) is readily solved to obtain σ at the end of the interval Δt . The solution is

$$\sigma = \sigma_A + (\sigma_0 - \sigma_A) e^{-\alpha \Delta t} \quad (23)$$

where $\sigma_A = \frac{\dot{\epsilon}}{A \psi} \quad (24)$

and $\alpha = AE \psi, \quad (25)$

and σ_0 is the value of σ at the beginning of the time interval.

Equation (22b) does not have a simple analytic solution even when $\dot{\epsilon}$ is constant. An approximate solution is obtained from

$$\sigma = \sigma_B + (\sigma_0 - \sigma_B) e^{-\beta \Delta t} \quad (26)$$

where $\sigma_B = \sqrt[4]{\frac{\dot{\epsilon}}{B \psi}} \quad (27)$

and $\beta = BE \psi \frac{\sigma_0^4 - \sigma_B^4}{\sigma_0 - \sigma_B} \quad (28)$

Equation (26) is correct to first order for $\beta \Delta t < 1$ and can be shown to provide the correct asymptotic solution for $\beta \Delta t \gg 1$.

The code checks to determine if $\sigma_0 \leq \sigma_c$ or $\sigma_0 > \sigma_c$, then determines σ from Equation (23) or (26), as appropriate. It then checks to see if σ passed through the value σ_c during the time interval Δt and, if so, determines σ by using a modification of Equation (26) or (23) for the remaining time after σ became equal to σ_c .

It is clear that a high degree of accuracy cannot be guaranteed from this procedure in view of all the approximations made. This is especially evident when one observes that the time intervals used between PDQ executions are typically hundreds or thousands of hours, whereas a more precise calculation would require considerably smaller values for Δt . Nevertheless, the determinations of clad stress obtained through this procedure are adequate to provide a reasonable indication of the general trends which occur throughout the whole core. These results are obtained with very little additional input and with almost no increase in execution time.

IV. QUALIFICATION OF THE MODEL

A. General Procedure

In order to test the adequacy of the model and to obtain best estimate values for certain of the input parameters, the model calculations were first encoded in a manner that permitted direct comparison with programs FIGRO and CYGRO-4. These two programs follow the time history of a single segment of a fuel rod over core lifetime. They require that the heat flux, fast neutron flux, depletion, and rod average depletion be provided as input data. In addition, the fraction of fission gas released from the fuel is specified as a function of lifetime for both programs.

The thermal calculations of the model were first compared with FIGRO calculations for a number of LWBR rod segments. When it was determined that good agreement could be obtained, comparison was then made with the

complete mechanical and thermal calculations of the CYGRO-4 code (Reference (7)), which incorporates the thermal analysis capability of FIGRO. Best-estimate values were used, along with as-built average fuel grain size and percent of theoretical density. The input data for heat flux, fast neutron flux, depletion, and rod average depletion were obtained from files of nominal values obtained from previous PDQ executions.

Comparisons of the model calculations with CYGRO-4 were made for various segments of at least two rods from each region of the LWBR core (seed, standard blanket, power flattening blanket, and reflector). At least one "hot" or potentially limiting rod and one "average" or typical rod were included from each region. About 35 CYGRO-4 runs were performed in all.

Six input quantities in the model were considered to be adjustable parameters with values to be determined by best fit to the CYGRO-4 results. These quantities are C_g , Equation (13); C_s , Equation (14); B_2 and B_3 , Equation (16); A , Equation (17); and R , Equation (18). It was found that a single value was sufficient for each of the first two of these quantities, while two values were necessary in different regions of the core for each of the last four quantities. Several other parameters were assigned values based either on other work (such as B_1 in Equation (16)) or on the values assigned to analogous parameters in FIGRO/CYGRO (such as Δ described in the last paragraph of Section III.C). The effect of changing the values of these parameters was investigated, and in all cases the results were not strongly dependent on the value of the quantity. Thus, the model has been successfully qualified with only a limited amount of parameter fitting.

B. Temperature and Radius Calculations - Seed

A comparison of the PDQ model calculations with CYGRO-4 for T_F in a "hot" seed rod is shown in Figure 4. Segment 3104 is in the upper portion of the rod, segment 3108 is near the middle, and segment 3111 is in the lower portion. The worst disagreement is about 40 F°, but in general the agreement is very much better than this. The significance of this agreement is discussed below in Section IV.F.

A similar plot for an "average" seed rod is shown in Figure 5. Again, the worst disagreement is about 40 F°.

A comparison of clad radius calculations is given in Figure 6. The change in radius of seed rods is rather gradual compared to that of blanket rods, due to the smaller ratio of outer diameter to clad thickness in the seed rods (a ratio of 14 for seed rods versus about 20 for blanket rods).

C. Temperature and Radius Calculations - Blanket and Reflector

A comparison of the PDQ model calculations with CYGRO-4 for T_F in a "hot" standard blanket rod is shown in Figure 7. As before, the lowest-numbered segment is in the upper portion of the rod and the highest-numbered segment is in the lower portion. Although the temperature history is quite different from that for the seed rods, the agreement is still quite good. The worst disagreement is again about 40 F°.

The temperature comparison for an "average" blanket rod is shown in Figure 8. By way of contrast, Figure 9 shows temperatures calculated with the new PDQ model for the same rod segments as Figure 8 plotted with temperatures calculated as in the previous TFB procedure of PDQ, where h_2 was provided in an input table rather than being calculated. At beginning-of-life, when the actual gap thickness is close to the value to which the table applies, the agreement is reasonable. However, as the gap shrinks, the temperatures calculated using the table for h_2 rapidly become too high.

The temperature comparison for an "average" power flattening blanket rod is shown in Figure 10. It may be observed that usually the worst disagreement for blanket rods comes when the gap has just disappeared according to CYGRO, since the model tends to predict a slower gap closure and hence a higher fuel temperature.

FIGURE 4
COMPARISON OF PDQ AND CYGRO-4 VOLUMETRIC AVERAGE FUEL TEMPERATURE
CALCULATIONS FOR A "HOT" SEED ROD

FIGURE 5
COMPARISON OF PDQ AND CYGRO-4 VOLUMETRIC AVERAGE FUEL TEMPERATURE
CALCULATIONS FOR AN "AVERAGE" SEED ROD

FIGURE 6
COMPARISON OF PDQ AND CYGRO-4 RADIUS CALCULATIONS
FOR A "HOT" SEED ROD

FIGURE 7
 COMPARISON OF PDQ AND CYGRO-4 VOLUMETRIC AVERAGE FUEL TEMPERATURE CALCULATIONS
 FOR A "HOT" STANDARD BLANKET ROD

FIGURE 8
COMPARISON OF PDQ AND CYGRO-4 VOLUMETRIC AVERAGE FUEL TEMPERATURE
CALCULATIONS FOR AN "AVERAGE" STANDARD BLANKET ROD

FIGURE 9
COMPARISON OF NEW PDQ MODEL WITH PREVIOUS PDQ VOLUMETRIC AVERAGE FUEL TEMPERATURE
CALCULATIONS FOR AN "AVERAGE" STANDARD BLANKET ROD

FIGURE 10
COMPARISON OF PDQ AND CYGRO-4 VOLUMETRIC AVERAGE FUEL TEMPERATURE CALCULATIONS
FOR AN "AVERAGE" POWER FLATTENING BLANKET ROD

Clad outer radius calculations for "hot" and "average" standard blanket rods are shown in Figures 11 and 12, respectively, and for an "average" power flattening blanket rod in Figure 13. It may be seen that the agreement with CYGRO-4 is consistently within 0.3 mil, and usually much closer.

Comparisons between the model and CYGRO-4 calculations for both temperature and clad radius of a reflector rod segment are shown in Figure 14. This is the hottest segment of a "hot" reflector rod.

D. Clad Stress Calculations

Comparisons between the model calculations and CYGRO-4 for clad stress are shown in Figures 15 and 16 for "hot" and "average" blanket rods, respectively. It may be seen that while the agreement is not exact, the model does provide a reasonable representation of the variations in clad stress over core life.

E. Discussion of Results

The significance of the results described in the previous paragraphs may be understood by examining the effect which some parameters determined by the model have on an eigenvalue calculation. These parameters include clad radius, fuel radius, and thermal conductivity of the gas in the fuel-clad gap (which depends on the release of fission-product gases). Changes in the clad radius affect the eigenvalue by changing the H and O atomic number densities. Changes in clad radius, fuel radius, and gas thermal conductivity affect the eigenvalue by changing the gap heat transfer coefficient, which determines the fuel temperature and consequently the amount of Doppler broadening.

A calculation was performed in which the clad radius was decreased by 1.5 mil throughout the binary sections of the blanket and by 2.0 mil in the thorium sections of the blanket, while the heat transfer coefficient throughout the blanket was increased to 2.5 times the values obtained from the standard

input tables previously used in PDQ. (These changes are comparable to the effect of eliminating the fuel-clad gap in the whole blanket region.) The resulting change in eigenvalue was approximately 0.7%, with slightly more than half of this change resulting from the decrease in clad radius and the remainder from the increase in heat transfer coefficient.*

Since an eigenvalue change of this magnitude is rather large, it is to be expected that a satisfactory calculation of clad radius should be considerably more accurate than 1.5 - 2.0 mils on the average, and a satisfactory calculation of fuel temperature should be more accurate than 150-200 F°, which is the temperature change typically resulting from an increase in h_2 by a factor of 2.5 in the blanket region. The preceding paragraphs show that the new model satisfies these accuracy criteria.

F. Incorporation of the Model Into PDQ

The model coding was verified by numerous hand calculations. This coding was then incorporated into the existing PDQ program. Subsections of the PDQ program including the new coding were executed for several cases of input data. The results were verified to be in agreement with those obtained when the model coding was executed by itself.

The modified PDQ is used routinely for analysis of the LWBR core throughout life. The performance has been adequate in all respects. The increase in execution time caused by the addition of the new model is negligible since the whole thermal feedback procedure typically consumes less than 2% of the total execution time.

The model has been incorporated into PDQ in a manner which permits the user to continue using the old TFB procedure without making use of the new model, if he so desires.

*The authors are indebted to H. C. Hecker for supplying this information.

FIGURE 11
COMPARISON OF PDQ AND CYGRO-4 CLAD RADIUS CALCULATIONS
FOR A "HOT" STANDARD BLANKET ROD

FIGURE 12
COMPARISON OF PDQ AND CYGRO-4 CLAD RADIUS CALCULATIONS
FOR AN "AVERAGE" STANDARD BLANKET ROD

FIGURE 13
COMPARISON OF PDQ AND CYGRO-4 CLAD RADIUS CALCULATIONS
FOR AN "AVERAGE" POWER FLATTENING BLANKET ROD

FIGURE 14
COMPARISON OF PDQ AND CYGRO-4 CALCULATIONS
FOR A "HOT" REFLECTOR ROD

FIGURE 15
COMPARISON OF PDQ AND CYGRO-4 CLAD STRESS CALCULATIONS
FOR A "HOT" STANDARD BLANKET ROD

FIGURE 16
COMPARISON OF PDQ AND CYGRO-4 CLAD STRESS CALCULATIONS
FOR AN "AVERAGE" STANDARD BLANKET ROD

V. SUMMARY AND CONCLUSIONS

A model has been developed for incorporating into the PDQ computer program the explicit calculation of depletion-dependent fuel-rod shrinkage and swelling at each mesh point. The model takes into account the following physical processes:

1. Clad contraction due to fast neutron induced creep;
2. Fuel expansion due to fission-product swelling and thermal expansion;
3. Fuel contraction due to pellet densification;
4. Changes in the thermal conductivity of the gas in the fuel-clad gap due to the release of noble gas fission products;
5. Development of tensile stress in the clad where the above effects have caused the fuel-clad gap to disappear, so that the fuel presses outward on the clad.

One purpose of the model is to improve the thermal feedback procedure of PDQ by including the effect on reactivity and other nuclear characteristics of the changes in hydrogen concentration caused by the variations in coolant channel area as the rods contract and expand. A second purpose is to provide more accurate calculations of fuel temperature, which determines the Doppler broadening of the nuclear cross sections, and which also affects predicted core nuclear characteristics. The model explicitly calculates the heat transfer coefficient of the fuel-clad gap, h_2 , at each mesh point; previously, the user was required to supply input values of h_2 as a function of heat flux and depletion. Finally, the model includes an approximate determination of clad stress as an aid in fuel rod analysis.

The model was devised and is used for analysis of the Light Water Breeder Reactor (LWBR) at Shippingport, Pa. The approach used is based in part on methods employed in the FIGRO and CYGRO-4 fuel rod analysis codes.

These programs deal, however, with only a single segment of a single fuel rod whose power history is provided as input, whereas PDQ deals with the core as a whole. The methods of FIGRO and CYGRO-4 were sufficiently simplified to permit the model to accomplish its purpose without significantly increasing the PDQ execution time.

The model first evaluates the total amount of fission gas released into the fuel-clad gap along the length of each rod. The thermal conductivity, k , of the mixture of fission and fill gases is determined as a function of temperature. The heat transfer coefficient of the gap is calculated at each mesh point as given by Equations (13) and (14). The fuel radius is corrected for thermal expansion, for fission product swelling, and for fuel densification. The clad radius is corrected for creep, which is assumed to progress at a rate proportional to both the applied stress and the fast neutron flux. When the gap is small, an additional contribution is added to the clad creep to account for the effect of fuel-clad interaction. An iterative process is used to obtain consistent values of gap heat transfer coefficient, gap thickness (determined by the fuel and clad radii), and fuel temperature.

The model has been tested by comparison with the more extensive calculations of the CYGRO-4 code for various segments of individual fuel rods throughout the LWBR core. The segments analyzed cover a wide range of power histories, depletions, rod diameters, and values of weight percent UO_2 in thoria-based fuel. The fuel temperature calculations of the two codes agree within 40°F or better in nearly all cases. Figure 5 shows the comparison for three segments of a typical LWBR seed rod. The agreement in calculations of clad radius is within 0.3 mil in nearly all cases. The significance of these results is illustrated by the fact that a decrease in clad radius throughout the LWBR core blanket region by 1.5 to 2.0 mils coupled with a decrease in fuel temperatures of $150\text{--}200^\circ\text{F}$ changes the eigenvalue by approximately 0.7%. Since an eigenvalue change of this magnitude is quite large, it is concluded that the new model makes a significant improvement in the accuracy of the PDQ predictions of the nuclear characteristics of the Light Water Breeder Reactor.

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A number of people have contributed to the work described in this report. Dr. L. B. Freeman supplied the initial instruction in the operation of PDQ and related programs. Dr. J. B. Newman also advised the authors in the original methods of approach to the problem. Dr. P. L. Pfennigwerth advised in the application of the approach which he developed for treating fuel shrinkage and swelling. Mr. J. F. Giovengo worked closely with the authors in running the FIGRO and CYGRO computer codes, and in devising the proper input for use with LWBR fuel. Mr. I. Goldberg also provided advice on the proper treatment of LWBR fuel. Mr. H. C. Hecker supplied the authors with various pieces of information concerning the physics of the LWBR core. Mr. C. J. Pfeifer performed the actual incorporation of the model into PDQ. Finally, one of the authors would like to acknowledge the able supervision of Dr. M. Natelson in the early part of the work and Ms. J. A. Mitchell in the later part.

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APPENDIX A

TABLE OF SYMBOLS

SYMBOL	DEFINITION	EQUATION	UNITS
A	Coefficient in expression for clad creep	(17),(20)	$(\text{psi})^{-1} (\text{neutrons}/\text{cm}^2\text{-sec})^{-1}$
a_i	Coefficient in expression for thermal conductivity of gas i	(11)	$\text{BTU}/\text{hr-ft}-(^{\circ}\text{K})^{n+1}$
B	Coefficient in expression for clad creep (high stress term)	(20)	$(\text{psi})^{-4} (\text{neutrons}/\text{cm}^2\text{-sec})^{-1}$
B_1	Coefficient describing amount of fuel densification	(16)	--
B_2	Coefficient describing rate of fuel densification	(16)	$(10^{20} \text{ fissions}/\text{cm}^3)^{-1}$
B_3	Coefficient describing rate of fuel swelling	(16)	$(10^{20} \text{ fissions}/\text{cm}^3)^{-1}$
C_g	Parameter in expression for h_g	(13)	inches
C_s	Parameter in expression for h_s	(14)	ft^{-1}
$\left. \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} \right\}$	Coefficients in expression for gas release fraction	(6)	$\left\{ \begin{matrix} -- \\ (10^{20} \text{ fissions}/\text{cm}^3)^{-1} \\ -- \end{matrix} \right.$
E	Clad elastic constant	(19)	psi
F_v	Volume fraction of fuel above gas release temperature	(6),(8)	--
f	Depletion	(Various)	$10^{20} \text{ fissions}/\text{cm}^3$
h_g	Gas heat transfer coefficient	(13)	$\text{BTU}/\text{hr-ft}^2\text{-F}^{\circ}$
h_s	Heat transfer coefficient for solid points of contact	(14)	$\text{BTU}/\text{hr-ft}^2\text{-F}^{\circ}$
h_2	Total heat transfer coefficient of gap	(2)	$\text{BTU}/\text{hr-ft}^2\text{-F}^{\circ}$
k	Conductivity of gas mixture in gap	(11),(12)	$\text{BTU}/\text{hr-ft-F}^{\circ}$
k_1	Conductivity of clad	(1),(14)	$\text{BTU}/\text{hr-ft-F}^{\circ}$

SYMBOL	DEFINITION	EQUATION	UNITS
k_3	Conductivity of fuel	(3),(14)	BTU/hr-ft-F°
N_{FG}	Number of fission gas atoms released from fuel	(9)	--
N_T	Number of ternary fission He atoms released from fuel	(10)	--
n_i	Exponent in expression for thermal conductivity of gas i	(11)	--
Q	Heat generation rate	--	BTU/hr
R	Parameter in expression for fuel-clad interaction	(18)	--
R_F	Fraction of fission gas released from fuel	(6),(9)	--
R_T	Fraction of ternary fission He released from fuel	(10)	--
r_s	Clad surface radius	(Various)	inches
r_1	Clad inner radius	(Various)	inches
Δr_1	Change in r_1 since BOL due to fuel-clad interaction	(18)	inches
δr_1	Change in r_1 during time Δt due to clad creep	(17)	inches
r_2	Fuel radius	(Various)	inches
Δr_2	Change in r_2 due to thermal expansion, densification, swelling	(15),(16)	inches
r_3	Radius of central void in fuel (zero for LWBR)	--	inches
S	Coefficient in expression for conductivity of gap in gap	(12)	BTU/hr-ft-(°R) ^{1.721}
T_F	Volumetric average temperature of fuel	(4),(8), (15)	°F
T_{rel}	Gas release temperature	(7),(8)	°F

SYMBOL	DEFINITION	EQUATION	UNITS
T_s	Temperature at clad outer surface	(1)	°F
T_1	Temperature at clad inner surface	(1),(2)	°F
T_2	Temperature at fuel surface	(2) - (5)	°F
T_3	Temperature at fuel centerline	(3),(4),(8)	°F
t	Gap thickness	(13)	inches
Δt	Time since last TFB calculation was performed	(17),(23), (26)	EFPH
V_F	Volume of fuel in segment	(9),(10)	cm ³
w_i	Weight of ith gas in expression for conductivity of gas mixture	(11)	--
α	Stress relaxation constant	(23),(25)	(EFPH) ⁻¹
α_{FG}	Average number of noble gas atoms produced per fission	(9)	--
α_T	Average number of ternary He atoms produced per fission	(10)	--
β	Stress relaxation constant	(26),(28)	(EFPH) ⁻¹
$\left. \begin{matrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{matrix} \right\}$	Coefficients in expression for fuel thermal expansion	(15)	$\left\{ \begin{matrix} -- \\ (\text{°F})^{-1} \\ (\text{°F})^{-2} \end{matrix} \right.$
Δ	Value of gap thickness below which linear interpolation is used for h_2	--	inches
ϵ_{el}	Elastic strain	(19)	--
ϵ_{pl}	Plastic strain	(20)	--
σ	Clad circumferential stress	(Various)	psi
σ_A	Asymptotic stress	(23),(24)	psi
σ_B	Asymptotic stress	(25)-(27)	psi
σ_C	Value of stress for which two expressions for ϵ_{pl} are equal	(21),(22)	psi

<u>SYMBOL</u>	<u>DEFINITION</u>	<u>EQUATION</u>	<u>UNITS</u>
σ_0	Value of stress at time of previous TFB calculation	(23), (26), (28)	psi
ϕ	Heat flux at rod surface	(1)-(5)	BTU/hr-ft ²
ψ	Fast (< 1 Mev) neutron flux	(Various)	neutrons/cm ² -sec

APPENDIX B

MISCELLANEOUS PDQ INPUT DATA

The input data required to execute PDQ with the modified TFB procedure is described in Reference (3). Most of the data items have been discussed in the main body of this report. There are a few items, however, which have not been discussed elsewhere and so are described in this Appendix. These input data items are as follows:

D_1 , multiplier of the group 1 flux. This is the fraction of the flux in PDQ energy group 1 which is over 1 MeV in energy. It is used in determining ψ .

κ , energy (in BTU) released per fission. This is used to calculate the depletion rate, which is used only in the stress calculations.

A_r/A_w , ratio of rod cross-sectional area to water channel area. This is used to relate the fractional change in clad surface radius to the change in hydrogen number density.

P_g , pressure of fill gas in gap and plenum. This is used in the calculation of clad stress.

V_R , ratio of volume of gap plus plenum to volume of fuel. This is used in determining the composition of the gas in the gap.

E_1, E_2, E_3, E_4 , and E_5 , parameters used in the conductivity of the gas in the gap. The conductivity is calculated using the method outlined in Section III.B to obtain

$$k = \frac{E_1 x + E_2 (1-x)}{E_3 - E_4 x} T^{E_5} \times 10^{-4} \quad (B1)$$

where x is the mole fraction of fill gas present and hence $1-x$ is the mole fraction of fission gas. Assuming the fission gas to be 84.7% Xe and 15.3% Kr, and the fill gas to be pure He, one deduces values for E_1 through E_5 of 22.95, 3.280, 4.975, 3.387, and 0.721, respectively. The value of E_1 may be reduced slightly to account for the release of volatile adsorbed gases from the fuel into the gap; a value of 22.268 is used for LWBR applications. With the gas temperature T measured in degrees Kelvin, these values give k in units of BTU/hr-in-F°. The program actually uses T in degrees Rankine and k in BTU/hr-ft-F°, so a factor of $12 \times (5/9)^{0.721} = 7.855$ is used for conversion of units.

Finally it should be noted that the coefficients A and B in the expressions for clad creep, Equations (17) and (20), are denoted A_1 and A_4 , respectively, in Reference (3).