ELECTROMAGNETIC WAVE SCATTERING FROM MAGNETIC FLUCTUATIONS IN TOKAMAKS

PPPL--2778

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Abstract

Cross sections are calculated for electromagnetic wave scattering and mode transformation from magnetic and density fluctuations in the vicinity of the principal electron cutoffs and resonances in a homogeneous plasma. For the special case of scattering perpendicular to the magnetic field, density fluctuations scatter ordinary to ordinary and extraordinary to extraordinary modes - but cannot transform these modes. On the other hand, magnetic fluctuations perpendicular to the field can transform modes but cannot scatter on a single branch. For incident frequencies on the order of the electron plasma frequency or gyrofrequency, the cross sections for scattering and transformation due to field and density fluctuations have a similar value. Estimates are given for scattering

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in a tokamak plasma with special emphasis on the question of how to detect and localize magnetic field fluctuations. Toroidal effects are handled by ray tracing. Ray tracing calculations and estimates of practical limitations on detection technique show that magnetic fluctuations can be detected and localized by this method - even when the magnetic fluctuation levels are orders of magnitude lower than the density fluctuations.

PACS: 52.25 , 52.70, 52.35

I. Introduction

It is important to develop diagnostics that can detect magnetic fluctuations in a fusion plasma, especially because of the interconnection between magnetic fluctuations and transport¹. Such a diagnostic for tokamaks must overcome a universal problem: the level of magnetic fluctuations is much lower than that of the electron density fluctuations. Typically, the density fluctuations arise from electron drift wave fluctuations²⁻⁴ with

$$\frac{\delta n_e}{n_{0e}} \approx \frac{\rho_s}{L_n} \quad (1)$$

where $\rho_s = r_{ce} (T_e/T_i)^{1/2}$, $L_n = -n_{e0}/[\partial n_{e0}/\partial r]$ is the density gradient length scale and r_{ce} is the electron gyroradius. In particular, Liewer³ has considered the dependence of the experimentally-determined $\delta n_e/n_{0e}$ on the drift theoretical parameter ρ_s/L_n for various tokamaks. Typically one can consider $\delta n_e/n_{0e} \approx 10^{-3}$. On the other hand, magnetic field fluctuations are due to magnetohydrodynamic (MHD) processes which have a low frequency, long wavelength coherent component and a high frequency, short wavelength incoherent part. Scattering experiments typically measure large poloidal mode number ($m \approx 100$) fluctuations. These magnetic fluctuations have been measured in the edge region, with Mirnov loops⁵ and, in very small tokamak devices, with interior magnetic probes⁶. High frequency probe measurements give $\delta B/B_0 \approx 10^{-4}$ - 10^{-5} in the interior of Microtor for $\omega/2\pi \leq 30$ kHz, and $\delta B/B_0 \approx 10^{-5}$ - 10^{-6} at the edge of TFTR in the range $100 \text{ kHz} \leq \omega/2\pi \leq 150 \text{ kHz}$. Thus, for the interior of TFTR³ one might expect a relative fluctuation level

$$\left[\frac{\langle \delta B_{\perp}^2 \rangle / B_0^2}{\langle \delta n_e^2 \rangle / n_{0e}^2} \right]_{m \gg 1} \approx 10^{-4} - 10^{-5} \quad (2)$$

The question that is posed here is : are there any magnetic fluctuation effects on wave scattering in tokamaks which are experimentally observable? [In ignited plasmas, it is expected that super Alfvénic alphas will drive various instabilities that can possibly raise this level of magnetic to density fluctuations.]

Typically, the scattering of electromagnetic waves in a plasma has been thought to be caused exclusively by electron density fluctuations. This is certainly the case for sufficiently high frequency waves, $\omega_i \gg \omega_{pe}$ (where $\omega_{pe}^2 = 4\pi n_{oe}e^2/m_e$), since the plasma dielectric tensor ϵ reduces to the identity tensor : $\epsilon_{ij} \approx \delta_{ij}$. For lower frequency waves it is no longer a good approximation to take $\epsilon_{ij} \approx \delta_{ij}$ and now the effects of transverse modes and magnetic fluctuations can be important. Consider, for example, the scattering of microwave radiation using the extraordinary (X) mode propagating perpendicular to the magnetic field B with frequency $\omega \approx \omega_{pe} \approx \omega_{ce} = eB/m_e c$. The scattering cross section for forward scattering can be roughly estimated from the Landau & Lifshitz⁷ variational expression

$$\sigma \approx \left\langle \left[\int \delta \epsilon_{ij} d^3x \right]^2 \right\rangle \quad (3)$$

where the cold plasma dielectric tensor

$$\epsilon = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \quad (4)$$

Here

$$S = 1 - \frac{\alpha}{1 - \beta}, \quad D = -\frac{\alpha \beta^{1/2}}{1 - \beta}, \quad P = 1 - \alpha \quad (5)$$

with

$$\alpha = \frac{\omega_{pe}^2}{\omega^2}, \quad \beta = \frac{\omega_{ce}^2}{\omega^2} \quad (6)$$

From Eq. (4), one can readily derive the cold dispersion relation for the waves propagating in the plasma. In particular, for perpendicular wave propagation, the refractive index $N = ck/\omega$ preserves the block diagonal structure of ϵ , Eq. (4) : from the ϵ_{33} element one obtains the linearly polarized ordinary (O) mode refractive index

$$N_0^2 = 1 - \alpha \quad (7)$$

while from the 2×2 submatrix one obtains the elliptically polarized X-mode index

$$N_X^2 = 1 - \frac{\alpha(1-\alpha)}{1-\alpha-\beta} \quad (8)$$

Thus, for perpendicular X-mode scattering, the 2×2 submatrix $\epsilon_{ij} = N_X^2 \delta_{ij}$. We can now estimate the variation

$$\begin{aligned} \delta\epsilon_{\perp} &= \alpha \left[1 + \frac{\beta(1-\beta)}{(1-\alpha-\beta)^2} \right] \frac{\delta n_e}{n_{oe}} + \left[\frac{2\alpha\beta(1-\alpha)}{(1-\alpha-\beta)^2} \right] \frac{\delta B}{B_0} \\ &= \delta\epsilon_n \frac{\delta n_e}{n_{oe}} + \delta\epsilon_B \frac{\delta B}{B_0} \end{aligned} \quad (9)$$

Hence the relative contribution of the magnetic to density fluctuations to the scattering cross section

$$\frac{\delta\epsilon_B}{\delta\epsilon_n} = \frac{2\beta(1-\alpha)}{(1-\beta)(1-2\alpha) + \alpha^2} \approx O(1) \quad (10)$$

Thus if the fluctuation level of $\delta B/B_0$ is comparable to $\delta n_e/n_{oe}$ then one might expect to see magnetic field fluctuations causing a significant part of the scattering. From Eq. (2), this seems to rule out the detection of scattering from magnetic fluctuations. However, as will become apparent later, this result relates only to forward scattering from density and parallel magnetic fluctuations. Furthermore, scattering of the same order of magnitude can produce mode conversion from perpendicular magnetic fluctuations. It should be noted that

there have been some recent experimental attempts^{4,8-11} at X-mode scattering from density fluctuations in TFTR.

Consider, now, O $\rightarrow$ X mode scattering in an inhomogeneous plasma with incident frequency $\omega < \omega_{ce}$. An incident O-mode will approach a cutoff near the local plasma frequency ($\alpha \rightarrow 1$, $N_0^2 \rightarrow 0$) and be reflected. An X-mode will propagate through this cutoff layer (at which $N_x^2 \approx 1$) and through the plasma and emerge on the other side provided the plasma density remains below the X-mode cutoff (e.g., $\alpha \rightarrow 1 + \beta^{1/2} \approx 3$ for the TFTR 60 GHz scattering system at 5 T). Now for mode propagation perpendicular to **B**, the O-mode is linearly polarized along **B**, while the X-mode is elliptically polarized in a plane perpendicular to **B**. Since density fluctuations are scalar in nature, they cannot force this O $\rightarrow$ X mode conversion. The only scattering process which can produce this mode conversion is through the tensorial magnetic fluctuations and several experiments^{8,12,13} have proposed this O $\rightarrow$ X scattering as a possible magnetic fluctuation diagnostic.

These conclusions are strictly valid only for exactly perpendicular propagation to **B**. But all radiation from antenna sources have finite beamwidths as well as angular and polarization resolution limitations. Those components of the incident O-mode beam which are not exactly perpendicular to **B** can mode convert to an X-mode due to density fluctuations alone. Since density fluctuation levels are so much higher than the magnetic fluctuations, Eq. (2), we need to examine whether this density fluctuation scattering will mask that from magnetic fluctuations. This finite angular width effect is considered in Sec. III.

Typically, microwave or millimeter wave transmitting and receiving antennas can resolve polarizations to a resolution better than 10^{-3} . With special attention to wall reflections and antenna design much higher resolution can be obtained. It is important to be able to distinguish the elliptically polarized X-mode scattered by magnetic fluctuations from the linearly polarized O-mode scattered by density

fluctuations. This effect is considered in Sec. V for large angle scattering - for small angle scattering one can readily avoid the polarization resolution problem by choosing cross sectional launch sites such that the O-mode will encounter its cutoff layer and be deflected away from the detector.

Another non-ideal effect that can cloud the desired signal is the polarization mismatch of the incident O-mode with the magnetic field at the plasma edge. In particular there are two issues here: (i) the launch polarization can be slightly mismatched with the field normal resulting in the unwanted generation of the wrong mode, and (ii) the shear in the magnetic field at low densities results directly in mode conversion because the wave polarization does not rotate with the field direction. In either case, this polarization mismatch can yield some $O \rightarrow X$ mode conversion due to density fluctuations near the edge of the plasma. This X-mode can then propagate through the O-mode cutoff layer and reach the detector. This effect is considered in Sec. VI. Plasma inhomogeneity effects are discussed in Sec. IV and are handled by ray tracing techniques. The differential scattering cross section is derived in Sec. II.

In Sec. VII we consider the complimentary problem of incident X-mode scattering and its possible use as a diagnostic for magnetic fluctuations. Finally, in Sec. VIII, we summarize the results of our investigation.

II. SCATTERING THEORY

A. Introduction

While scattering theory is a fairly well-developed subject^{14,15}, recent theoretical work¹⁶⁻²² has been concerned with the possibility of using electromagnetic scattering to determine the alpha particle distribution function in a fusion plasma. These theories¹⁶⁻²² dealt with collective scattering from density

fluctuations in a homogeneous infinite plasma in the electrostatic approximation. Chiu²³ has considered electromagnetic effects on density fluctuations while Aamodt and Russell²⁴ have included scattering from magnetic fluctuations. However, all these alpha particle theories still assumed a homogeneous, infinite plasma. Here, we will extend these theories to handle toroidal inhomogeneities through ray tracing to and from the scattering volume and concentrate on the possibilities of microwave scattering to detect magnetic fluctuations. The distinction^{25-27,20} between scattering into the detector ray solid angle rather than into the wave vector solid angle is also considered explicitly here. This gives rise to certain geometric factors not present in the Sitenko¹⁴ formulation which is concerned with scattering into the wave vector solid angle.

B. Derivation of the differential scattering cross section

In the Sitenko¹⁴ formalism for scattering from fluctuations in a locally homogeneous magnetized plasma, the incident wave is treated as a monochromatic plane wave satisfying

$$\nabla \times \nabla \times \mathbf{E}_i + \frac{1}{c^2} \boldsymbol{\epsilon}_i \frac{\partial^2 \mathbf{E}_i}{\partial t^2} = 0 \quad (11)$$

where $\boldsymbol{\epsilon}_i$ is the plasma dielectric tensor. The scattered field $\mathbf{E}_s$ satisfies

$$\nabla \times \nabla \times \mathbf{E}_s + \frac{1}{c^2} \boldsymbol{\epsilon}_s \frac{\partial^2 \mathbf{E}_s}{\partial t^2} = - \frac{4\pi}{c^2} \frac{\partial \mathbf{J}_s}{\partial t} \quad (12)$$

where $\mathbf{J}_s$ is the current due to the interaction of the incident plane wave $(\mathbf{k}_i, \omega_i)$ with the plasma density, velocity and magnetic fluctuations $\delta n(\mathbf{k}, \omega)$, $\delta \mathbf{v}(\mathbf{k}, \omega)$, $\delta \mathbf{B}(\mathbf{k}, \omega)$. From the conservation of momentum and energy, the scattered wave number and frequency are given by

$$\mathbf{k}_s = \mathbf{k}_i + \mathbf{k} \quad , \quad \omega_s = \omega_i + \omega \quad (13)$$

Fourier transforming Eq. (11), and using the MHD equations to determine the scattering current J_s , one obtains

$$\begin{aligned} & \left[\frac{c^2 k_s^2}{\omega_s^2} \left(\frac{k_{s,\alpha} k_{s,\beta}}{k_s^2} - \delta_{\alpha\beta} \right) + \epsilon_{\alpha\beta}(\mathbf{k}_s, \omega_s) \right] E_{s,\beta}(\mathbf{k}_s, \omega_s) = E_{i,\beta}(\mathbf{k}_i, \omega_i) \times \\ & \times \left[\frac{\omega_i}{\omega_s} \{ \delta_{\alpha\beta} - \epsilon_{\alpha\beta}(\mathbf{k}_i, \omega_i) \} \frac{\delta n_e(\mathbf{k}, \omega)}{n_{oe}} + \frac{ie}{m_e c} \frac{\omega_i}{\omega_s^2} \{ \delta_{\alpha\gamma} - \epsilon_{\alpha\gamma}(\mathbf{k}_s, \omega_s) \} \epsilon_{\zeta\eta} \{ \delta_{\zeta\beta} - \epsilon_{\zeta\beta}(\mathbf{k}_i, \omega_i) \} \delta B_\eta(\mathbf{k}, \omega) \right. \\ & + \left\{ \frac{1}{\omega_i} \{ \delta_{\alpha\zeta} - \epsilon_{\alpha\zeta}(\mathbf{k}_s, \omega_s) \} [k_{i,\mu} \delta_{\beta\zeta} - k_{i,\zeta} \delta_{\mu\beta}] + \frac{\omega_i^2}{\omega_s \omega_{pe}^2} [\delta_{\eta\beta} - \epsilon_{\eta\beta}(\mathbf{k}_i, \omega_i)] [k_\eta \delta_{\zeta\mu} + k_{i,\mu} \delta_{\eta\zeta}] + \right. \\ & \left. \left. + [\delta_{\zeta\beta} - \epsilon_{\zeta\beta}(\mathbf{k}_i, \omega_i)] k_{i,\zeta} \delta_{\alpha\mu} \right\} \delta v_\mu(\mathbf{k}, \omega) \right] \end{aligned} \quad (14)$$

where $\epsilon_{\zeta\eta}$ is the standard Levi-Civita symbol, n_{oe} is the local background electron density. [summation over repeated Greek subscripts is understood].

The radiation field at the receiver antenna location $\mathbf{r}_s$ is thus given by²⁷

$$E_s(\mathbf{r}_s, t) \approx \frac{1}{r_s} \sum_{\mathbf{k}_s} \frac{(\mathbf{J}_s \cdot \mathbf{e}_s) \mathbf{e}_s}{|\nabla \Lambda| |\mathbf{K}|^{1/2}} \exp[i(\mathbf{k}_s \cdot \mathbf{r}_s - i\omega_s t)] + \dots \quad (15)$$

where the summation is over all wave numbers $\mathbf{k}_s$ that lie on the surface $\Lambda = 0$ such that the group velocity (in the direction $\nabla \Lambda$) is parallel to the observer position $\mathbf{r}_s$. $\mathbf{e}_s$ is the normalized polarization of the scattered electric field, Λ is defined by

$$\Lambda = \det \left[\frac{c^2 k_s^2}{\omega_s^2} \left(\frac{k_{s,\alpha} k_{s,\beta}}{k_s^2} - \delta_{\alpha\beta} \right) + \epsilon_{\alpha\beta}(\mathbf{k}_s, \omega_s) \right] \quad (16)$$

with K the Gaussian curvature of the surface $\Lambda = 0$ at the points $\mathbf{k}_s$. (An explicit expression for K is given later in Eq. (20).) Lighthill²⁷ has examined the special case when the Gaussian curvature is zero at these $\mathbf{k}_s$. In these cases, the Fourier inversion of Eq. (14) exhibits poles that come from inflection points (points of zero

curvature) on the dispersion surface $\Lambda = 0$. This results²⁷ in a focusing effect which yields a cuspidal edge such that the electric field E_s has a far field asymptotic decay $\approx r_s^{-5/6}$. For a non-zero solid angle around this cuspidal edge direction, the Fourier inversion leads to an Airy integral from which one can determine a continuous variation between this singular cuspidal edge field decay $\approx r_s^{-5/6}$ and the spherical wave decay $\approx r^{-1}$. For perpendicular propagation and away from the O-mode cutoff layer ($\alpha = 1$), the Gaussian curvature is always non-zero and so we do not discuss these effects further here.

On calculating the average power at the antenna position r_s , one can determine the differential scattering cross section per unit ray solid angle

$$\begin{aligned} \frac{d^2\sigma}{d\Omega_r d\omega_s} = & \frac{1}{2\pi} \left(\frac{e^2}{m_e c^2} \right)^2 \frac{\omega_i^2 \omega_s^2}{\omega_{pe}^4} \frac{R \cos \delta_i \cos \delta_s}{K} n_{oe}^2 \left[|\xi|^2 \frac{\langle \delta n^2 \rangle_{k,\omega}}{n_{oe}^2} + \right. \\ & + \frac{\omega_{ce}^2 \omega_s^2}{\omega_{pe}^4} a_\alpha a_\beta^* \frac{\langle \delta B_\alpha \delta B_\beta \rangle_{k,\omega}}{B_0^2} + \frac{2\omega_{ce}\omega_s}{\omega_{pe}^2} \operatorname{Im} \left(\xi a_\alpha^* \frac{\langle \delta n \delta B_\alpha \rangle_{k,\omega}}{n_{oe} B_0} \right) + \\ & + \frac{\omega_s^2}{\omega_i^2} b_\alpha b_\beta^* \frac{\langle \delta v_\alpha \delta v_\beta \rangle_{k,\omega}}{c^2} + \frac{2\omega_s}{\omega_i} \operatorname{Re} \left(\xi b_\alpha^* \frac{\langle \delta n \delta v_\alpha \rangle_{k,\omega}}{n_{oe} c} \right) - \\ & \left. - \frac{2\omega_{ce}\omega_s^2}{\omega_i \omega_{pe}^2} \operatorname{Im} \left(a_\alpha b_\beta^* \frac{\langle \delta B_\alpha \delta v_\beta \rangle_{k,\omega}}{B_0 c} \right) \right] \end{aligned} \quad (17)$$

where

$$\begin{aligned} \xi &= e_{s,\alpha}^* [\epsilon_{\alpha\beta}(k_i, \omega_i) - \delta_{\alpha\beta}] e_{i,\beta} , \\ a_\alpha &= \epsilon_{\beta\gamma} e_{s,\zeta}^* [\epsilon_{\zeta\beta}(k_s, \omega_s) - \delta_{\zeta\beta}] [\epsilon_{\gamma\rho}(k_i, \omega_i) - \delta_{\gamma\rho}] e_{i,\rho} , \\ b_\alpha &= \frac{c}{\omega_i} \left(\{ \epsilon_{\beta\gamma}(k_s, \omega_s) - \delta_{\beta\gamma} \} \left[k_{i,\gamma} \delta_{\alpha\zeta} - k_{i,\alpha} \delta_{\zeta\gamma} - \frac{\omega_i^2}{\omega_{pe}^2} \{ \epsilon_{\eta\zeta}(k_i, \omega_i) - \delta_{\eta\zeta} \} \{ k_{\eta} \delta_{\alpha\gamma} + k_{i,\alpha} \delta_{\eta\gamma} \} \right] + \right. \\ & \left. + \frac{\omega_i}{\omega_s} \{ \epsilon_{\gamma\zeta}(k_i, \omega_i) - \delta_{\gamma\zeta} \} k_{i,\gamma} \delta_{\alpha\beta} \right) e_{i,\zeta} e_{s,\beta}^* \end{aligned} \quad (18)$$

and e_i is the incident wave polarization. The factor

$$R = \frac{N_s^3 N_i}{[e_{s,\alpha}^* \epsilon_{\alpha\beta}(\mathbf{k}_s, \omega_s) e_{s,\beta}] [e_{i,\gamma}^* \epsilon_{\gamma\eta}(\mathbf{k}_i, \omega_i) e_{i,\eta}]} \quad (19)$$

where $N_i = ck_i/\omega_i$ and $N_s = ck_s/\omega_s$ are the refractive indices for the incident and scattered waves, and the Gaussian curvature K

$$K = \left[1 + \sin^2 \delta_s - \frac{\cos^2 \delta_s}{N_s} \frac{d^2 N_s}{d\theta_s^2} \right] \frac{\sin \rho_s \cos \delta_s}{\sin \theta_s} \quad (20)$$

with θ_s being the angle between B_0 and $\mathbf{k}_s$, and ρ_s is the angle between B_0 and $\mathbf{r}_s$ (see Fig. 1). δ_s is the angle between the wave vector $\mathbf{k}_s$ and the ray direction $\mathbf{r}_s$ and is given $\tan \delta_s = -\partial N_s / \partial \theta_s$.

In evaluating the geometric form factors in eq. (17), it can be noted from Eqs. (18) - (20) that only the incident and scattered frequencies, ω_i and ω_s , occur. Moreover, since we stay away from resonances ω_i/k and $\omega_s/k \gg$ electron thermal speed. Hence it is permissible to use the cold plasma dielectric tensor approximation. This result is confirmed by the mildly relativistic calculations of Bindslev²³ on scattering from density fluctuations.

C. Special case - propagation perpendicular to B_0

Consider the special case of propagation perpendicular to the magnetic field B_0

$$\theta_i = 90^\circ = \theta_s \quad (21)$$

$\omega_s \approx \omega_i$ since the scattering is from low frequency fluctuations. Moreover, to leading order, the effect of velocity fluctuations on the differential scattering cross section can be neglected since these are basically $O(v^2/c^2)$. For propagation perpendicular to the magnetic field, the contribution of the cross correlations $\langle \delta n \delta B \rangle$ - form factors in Eq. (17) is found to be lower than that from the magnetic fluctuations by a few orders of magnitude (even if the corresponding fluctuation levels are of similar magnitudes). Thus, the cross-correlations terms are also neglected.

For perpendicular scattering, Eq. (21), the ray $\mathbf{r}_s$ and the scattered wave vector $\mathbf{k}_s$ are parallel since the angle between these vectors δ_s

$$\delta_s = \pm \tan^{-1} \left[- \frac{(1-N^2) \beta \cos \theta_s \sin \theta_s}{\{\beta^2 \sin^4 \theta_s + 4\beta (1-\alpha)^2 \cos^2 \theta_s\}^{1/2}} \right] = 0 \quad (22)$$

where the upper sign (lower sign) is for the O (X) mode. The Gaussian curvature K for the scattered mode, Eq. (20), reduces to

$$\begin{aligned} \text{O-mode: } K_0 &= N_0^2 \\ \text{X-mode: } K_X &= 2 - N_X^2 \end{aligned} \quad (23)$$

Note that in the limit $\alpha \rightarrow 1$ [i.e., $\omega \rightarrow \omega_{pe}$], $N_0 \rightarrow 0$ so that the Gaussian curvature for the C-mode $K_0 \rightarrow 0$. This special case of $\alpha = 1$, with its singular cuspidal effects and the required matching of the wave amplitudes across this singularity, will be discussed elsewhere.

The density fluctuation factors in the cross section are given by

$$R_{00} |\xi_{00}|^2 = \alpha^2$$

$$R_{XX} |\xi_{XX}|^2 = \frac{\alpha^2}{(1-\alpha-\beta)^2} [\beta(1-\beta-\alpha^2) \sin^2 \phi + \{(1-\beta)(1-2\alpha) + \alpha^2\}^2 \cos^2 \phi]$$

$$R_{XO} |\xi_{XO}|^2 = 0$$

$$R_{OX} |\xi_{OX}|^2 = 0 \quad (24)$$

where ϕ is the scattering angle, the angle between $\mathbf{k}_i$ and $\mathbf{k}_s$. The subscript notation OO, XX, XO, OX refers to $O \rightarrow O$, $X \rightarrow X$, $X \rightarrow O$, $O \rightarrow X$ mode conversion, respectively.

If the magnetic spectrum $\langle \delta B_y \delta B_y \rangle$ is diagonal, then the contribution of the magnetic fluctuation factors to the cross section is

$$R_{00} [a_y a_z^* \langle \delta B_y \delta B_z \rangle]_{00} = 0$$

$$R_{xx} [a_y a_z^* \langle \delta B_y \delta B_z \rangle]_{xx} = \left[\frac{\alpha}{1-\alpha-\beta} \right]^4 [(\beta + (1-\alpha)^2)^2 \sin^2 \phi + 4\beta (1-\alpha)^2 \cos^2 \phi] \langle \delta B_{\parallel}^2 \rangle$$

$$R_{x0} [a_y a_z^* \langle \delta B_y \delta B_z \rangle]_{x0} = \frac{N_0}{N_x} \frac{\alpha^4}{(1-\alpha-\beta)^2} [(1-\alpha)^2 \langle \delta B_1^2 \rangle + \beta \langle \delta B_2^2 \rangle]$$

$$R_{0x} [a_y a_z^* \langle \delta B_y \delta B_z \rangle]_{0x} = \frac{N_x}{N_0} \frac{\alpha^4}{(1-\alpha-\beta)^2} [(\beta \sin^2 \phi + (1-\alpha)^2 \cos^2 \phi) \langle \delta B_1^2 \rangle + \\ + ((1-\alpha)^2 \sin^2 \phi + \beta \cos^2 \phi) \langle \delta B_2^2 \rangle] \quad (25)$$

where $\langle \delta B_1^2 \rangle$ is the magnetic spectrum in the propagation direction, $\langle \delta B_2^2 \rangle$ is the spectrum in the polarization direction, and $\langle \delta B_{\parallel}^2 \rangle$ the spectrum in the magnetic field direction. If the perpendicular components are equal, $\langle \delta B_1^2 \rangle = \langle \delta B_2^2 \rangle = \langle \delta B_{\perp}^2 \rangle$, then Eq. (25) simplifies to

$$R_{x0} [a_y a_z^* \langle \delta B_y \delta B_z \rangle]_{x0} = \frac{N_0}{N_x} \frac{\alpha^4}{(1-\alpha-\beta)^2} [\beta + (1-\alpha)^2] \langle \delta B_{\perp}^2 \rangle \\ R_{0x} [a_y a_z^* \langle \delta B_y \delta B_z \rangle]_{0x} = \frac{N_x}{N_0} \frac{\alpha^4}{(1-\alpha-\beta)^2} [\beta + (1-\alpha)^2] \langle \delta B_{\perp}^2 \rangle \quad (26)$$

For simplicity, we shall assume for the rest of this calculation that the perpendicular magnetic components are equal. It is interesting to note that for forward scattering $\phi \sim 0$, the ratio of magnetic to density form factors for $X \rightarrow X$ scattering reduces to that derived from the simple Landau-Lifshitz⁷ formalism, Eq. (10).

We can immediately conclude from Eq. (24), that there is no $O \rightarrow X$ or $X \rightarrow O$ mode conversion due to scattering from density fluctuations. We had deduced this earlier in Sec. I, based on physical arguments on the impossibility of polarization changes due to scalar fluctuating quantities. On the other hand, from Eq. (26), mode conversion $O \rightarrow X$ or $X \rightarrow O$ has a non-zero cross section because of magnetic fluctuations.

Now the scattering volume, V_{sc} , is the region in which the incident beam and the scattered beam intersect within the plasma. For millimeter radiation this scattering volume can be quite localized ($V_{sc} \approx$ several centimeters³) if the scattering angle defined by $\cos^{-1}(\mathbf{k}_i \cdot \mathbf{k}_s / k_i k_s)$ is large. Thus, from Eq. (24), if one chooses V_{sc} to encompass the O-mode cutoff layer then the scattering cross section for $O \rightarrow X$ is considerably higher than that for $X \rightarrow O$ mode conversion, since within V_{sc} the refractive index $N_0 \ll 1$ while $N_x \approx 1$. In particular for an incident O-mode propagating perpendicular to B_0 in an inhomogeneous plasma, transformation scattering by magnetic fluctuations in front of the O-mode cutoff layer will generate an X-mode that can penetrate through this cutoff and be detected. The incident O-mode, however, will be reflected back out of the inhomogeneous plasma. Hence this $O \rightarrow X$ mode scattering appears to be a viable diagnostic^{8,14} for magnetic fluctuations.

III. EFFECTS OF FINITE BEAMWIDTHS ON PERPENDICULAR $O \rightarrow X$ MODE SCATTERING

We must now consider the effects of finite beamwidths on both the incident and scattered beams, since only for exactly perpendicular propagation to B_0 is the $O \rightarrow X$ mode transition possible only from magnetic fluctuations. In particular, for finite beamwidths, $R_{Ox} | \xi_{Ox} |^2 \neq 0$ but small. This will allow for some $O \rightarrow X$ mode conversion by density fluctuations and so possibly mask the signal from the mode conversion scattering caused by magnetic fluctuations.

The beamwidths considered here have a total angular spread Δ about the mean wave vector angles $\langle \theta_i \rangle = 90^\circ = \langle \theta_s \rangle$. Thus, from Fig. 1, $\langle \phi \rangle$ is the average scattering angle (between the wave vectors) for the finite beamwidth microwaves. For incident perpendicular O-mode propagation, the average angle $\langle \delta_i \rangle$ between the

wave vector $\mathbf{k}_i$ and ray $\mathbf{r}_i$ is non-zero only because of the finite beamwidth. $\langle \delta_i \rangle$ increases as $\omega_i \rightarrow \omega_{pe}$: $\langle \delta_i \rangle \approx -1.2^\circ$ for $\alpha = \omega_{pe}^2/\omega_i^2 = 0.95$, while $\langle \delta_i \rangle \approx -0.1^\circ$ for $\alpha = 0.5$. For the scattered perpendicular X-mode, the corresponding angle $\langle \delta_s \rangle$ is typically an order of magnitude lower than $\langle \delta_i \rangle$. For notation simplicity, we will now suppress the average notation, $\langle \dots \rangle$, on the angles.

In Fig. 2, the magnetic form factor in the $O \rightarrow X$ differential scattering cross section, Eq. (17),

$$F(\delta B_{\perp}^2)_{OX} = \frac{R_{OX} \cos \delta_i \cos \delta_s}{K_X} \frac{\beta}{\alpha^2} a_i a_i^* \quad (27)$$

is plotted as a function of $\alpha = \omega_{pe}^2/\omega_i^2$. Here, we model the magnetic fluctuation tensor $\langle \delta B_{\alpha} \delta B_{\beta} \rangle$ to be diagonal with the components perpendicular to $\mathbf{B}_0$ being equal. It is found that this magnetic form factor $F(\delta B_{\perp}^2)_{OX}$ is basically independent of the microwave beamwidths Δ , and has a weak dependence on the scattering angle ϕ . As $\omega_i \rightarrow \omega_{pe}$, there is a significant increase in $F(\delta B_{\perp}^2)_{OX}$. Thus the power scattered from magnetic fluctuations and reaching the detector is significantly increased if in the scattering volume V_{sc} the incident microwave frequency ω_i is close to the O-mode cutoff layer. There is a slight decrease in the magnetic form factor as B_0 increases - this is due to the geometric factor R_{OX}/K_X for the $O \rightarrow X$ scattering.

The effects of microwave beamwidth on the corresponding density form factor in the $O \rightarrow X$ cross section, Eq. (17),

$$F(\delta n^2)_{OX} = \frac{R_{OX} \cos \delta_i \cos \delta_s}{K_X} |\xi_{OX}|^2 \quad (28)$$

is shown in a semi-log plot in Fig. 3. As expected, the density form factor $\rightarrow 0$ as the beamwidth $\Delta \rightarrow 0$. Moreover the density form factor decreases as the scattering angle ϕ decreases from 90° to 0° but the rate of this decrease is strongly dependent on α .

The contribution from magnetic fluctuations to the scattered power over that coming from density fluctuations is, from Eqs. (17), (27) and (28),

$$\frac{d\sigma(\delta B_{\perp}^2)_{0x}}{d\sigma(\delta n^2)_{0x}} = \frac{F(\delta B_{\perp}^2)_{0x}}{F(\delta n^2)_{0x}} \frac{\langle \delta B_{\perp}^2 \rangle / B_0^2}{\langle \delta n^2 \rangle / n_{0e}^2} \quad (29)$$

In Figs. 4 and 5 the form factor ratio $F(\delta B_{\perp}^2)_{0x}/F(\delta n^2)_{0x}$ is considered under various conditions. The semi-log plot in Fig. 4 examines this form factor ratio for various microwave beamwidths and magnetic field strengths assuming forward scattering ($\phi = 0^\circ$), while in Fig. 5 this ratio is considered for various scattering angles ϕ at fixed beamwidth and magnetic field. The ratio $F(\delta B_{\perp}^2)_{0x}/F(\delta n^2)_{0x}$ is optimized by choosing higher β (i.e., higher magnetic fields B_0) and smaller microwave beamwidths. On the other hand, this form factor ratio gradually decreases as $\alpha \rightarrow 1$, with the decrease for $\beta = 2.25$ being a factor of 1.1 faster than that for $\beta = 4.0$. However, from Figs. 2 and 3, one notes that the power scattered increases by an order of magnitude as $\alpha = 0.5$ increases to $\alpha = 0.9$ while the contribution from magnetic fluctuations to that signal decreases by a factor of 2. These opposing effects must be weighed against the problem of obtaining an adequate signal at the receiver versus the expected level of magnetic to density fluctuations. For values of α very close to the O-mode cutoff, one must resort to amplitude matching across this cutoff layer. Some attempt²⁹ has been made to perform this amplitude matching as it appears to give the possibility that even for forward scattering the measurement of $\langle \delta B^2 \rangle$ can be localized to some degree by the local increase in amplitude of the O-mode near its cutoff ($\alpha = 1$). This effect is currently under investigation and the results will be reported elsewhere.

IV. RAY TRACING FOR PERPENDICULAR PROPAGATION TO B.

Plasma inhomogeneities are considered in the propagation of the microwave beams to and from the scattering volume by using the TORCH³⁰ ray tracing code in a TFTR plasma. In particular, we consider refractive effects on a bundle of incident O-modes and a bundle of scattered X-modes, both with a total beamwidth $\Delta = 5^\circ$. For perpendicular propagation of the O-mode ($\theta_i = 90^\circ$) and the scattered X-mode ($\theta_s = 90^\circ$) relative to the magnetic field B, these modes retain their identity as they propagate through a slowly varying inhomogeneous plasma, i.e., the polarization of these modes remains invariant relative to the local magnetic field in the eikonal approximation. This is also consistent with the observation³¹ of tokamak emission at $2\omega_{ce}$ in which the perpendicularly polarized mode retained its orientation relative to local field B in the emitting layer as well as to the magnetic field B at the plasma edge. A more general calculation of Boyd³¹ also addresses this question.

In Fig. 6, a bundle of O-modes at $\omega_i/2\pi = 45$ GHz and beamwidth $\Delta = 5^\circ$ are incident from the low magnetic field side with $Z > 0$. They are reflected at the O-mode cutoff layer at $r = r^*$, where $\omega_i = \omega_{pe}(r^*)$, and then refracted back out of the plasma. Refractive effects on a bundle of X-modes with beamwidth $\Delta = 5^\circ$ are shown in Figs. 6(a)-(c) for various scattering angles ϕ as they emanate from a given scattering volume V_{sc} . In Fig. 7, the bundle of O-modes are incident from the high field side and oriented to pass through the plasma center, while in Fig. 8 the O-mode bundle is incident from below in the vertical direction. These are representative TFTR ray tracing plots and no optimization (e.g., on the incident O-mode frequency, ...) is attempted here.

We restrict ourselves to several comments on these ray tracing results:

- (1) Since the O-mode cutoff layer acts as a filter for the incident O beam, forward scattering is particularly attractive from an experimental point of view. Refractive

effects on the emergent X-beam can be significant if the scattered beam traverses a large poloidal cross section,

(2) Beam and viewing dumps should be able to be utilized to avoid signal detection contamination between the incident and scattered beams. In Fig. 8(c), the case of perpendicular scattering $\phi = 90^\circ$, the reflected-refracted O-mode beam could be distinguished from the scattered X-mode bundle because of the frequency shift ω in the scattered signal ($\omega_s = \omega_i + \omega$). Moreover, by choosing V_{sc} to lie sufficiently below the O-mode cutoff layer (and by optimizing the incident O-mode frequency ω_i), the emergent O-modes and X-modes can be spatially non-overlapping, with the O-mode power not contaminating the X-mode signal at the detector.

V. POLARIZATION RESOLUTION FOR LARGE ANGLE SCATTERING

For large angle scattering, one needs to consider the effects of $O \rightarrow O$ mode scattering from V_{sc} by density fluctuations and how this signal at the receiver can be distinguished from the $O \rightarrow X$ scattering from magnetic fluctuations. The respective scattered modes have different polarizations which can be resolved if the relative scattered power from these two processes are around 10^{-3} , easily within the resolution of typical millimeter and microwave antennas. For small scattering angles ϕ one can easily avoid this polarization resolution problem by choosing incident cross sectional positions such that the O-mode will encounter the O-mode cutoff layer and be deflected from the detector location.

The ratio of the cross section for $O \rightarrow X$ scattering from magnetic fluctuations to that for $O \rightarrow O$ scattering from density fluctuations is

$$\frac{d\sigma(\delta B^2)_{OX}}{d\sigma(\delta n^2)_{OO}} = \frac{F(\delta B^2)_{OX}}{F(\delta n^2)_{OO}} \frac{\langle \delta B^2 \rangle / B_0^2}{\langle \delta n^2 \rangle / n_e^2} \quad (30)$$

where the ratio $F(\delta B^2)_{0g}/F(\delta n^2)_{00}$ for large angle scattering is shown in Fig. 9, where $F(\delta n^2)_{00}$ is defined by

$$F(\delta n^2)_{00} = \frac{R_{00} \cos \delta_i \cos \delta_s}{K_0} |\xi_{00}|^2$$

This form factor ratio is insensitive to $\beta = \omega_c^2/\omega^2$. If the relative levels of magnetic to density fluctuation levels are on the order of

$$\frac{\langle \delta B^2 \rangle / B_0^2}{\langle \delta n^2 \rangle / n_{0e}^2} \approx 5 \times 10^{-4} \quad (31)$$

then, from Fig. 9, it appears that choices of lower α may be needed for polarization resolution of the scattered X mode (from magnetic fluctuations) from the scattered O mode (from density fluctuations) if one wishes to use large angle scattering. For small angle scattering, the polarization resolution problem can be easily avoided. It should also be remembered that Eq. (31) is a fluctuation estimate for magnetic and density fluctuations within a specific frequency range $\omega = \omega_s - \omega_i$.

Because of possible polarization resolution problems for large angle scattering, the complementary scattering case of $X \rightarrow O$ conversion (due to magnetic fluctuations) and $X \rightarrow X$ scattering (due to density fluctuations) is discussed in Sec. VII.

VI. POLARIZATION MISMATCH AND SHEAR AT THE PLASMA EDGE

Another nonideal effect that needs to be considered is that of the polarization mismatch of the incident O-mode with the magnetic field at the plasma edge. It arises from two distinct process: (i) an imprecise knowledge of the magnetic field direction at the plasma edge, and (ii) shear induced mismatch, which predominantly occurs also at the plasma edge^{32,33}. These edge mismatches will result in some generation of X-modes. This edge X-mode can then scatter from

density fluctuations ($X \rightarrow X$) into the detector. It is important that the power scattered by this $O \rightarrow X \rightarrow X$ mismatch-density fluctuation process be significantly less than the power scattered by magnetic fluctuations in the $O \rightarrow X$ conversion.

However, we find that these effects at the plasma edge will not mask the scattering from magnetic fluctuations in a low shear plasma. Indeed, from the full wave calculations of Brambilla and Moresco³³ for a tokamak, one finds that the power fraction of the incident O-mode that is converted to an X-mode due to polarization mismatch at the edge is $\approx 10^{-5}$. The corresponding cross section ratio is

$$\frac{d\sigma(\delta B_{\perp}^2)_{O \rightarrow X}}{d\sigma(\delta n^2)_{O \rightarrow X \rightarrow X}} = \frac{F(\delta B^2)_{OX}}{F(\delta n^2)_{XX}} \frac{\langle \delta B_{\perp}^2 \rangle / B_0^2}{\langle \delta n^2 \rangle / n_{oe}^2} \times 10^5 \quad (32)$$

where

$$F(\delta n^2)_{XX} = \frac{R_{XX} \cos \delta_i \cos \delta_s}{K_X} |\xi_{XX}|^2$$

For small forward scattering angles ϕ , and for values $\alpha = 0.8$ and $\beta = 4.0$, one finds

$$\frac{F(\delta B^2)_{OX}}{F(\delta n^2)_{XX}} \approx 85 \quad (33)$$

while for lower $\alpha = \omega_{pe}^2 / \omega_i^2 = 0.5$, this ratio $\rightarrow 425$. This high ratio for small forward scattering angles at $\alpha = 0.5$ can be readily understood by noting that for exactly perpendicular scattering with $\phi = 0^\circ$ one finds $F(\delta n^2)_{XX} = 0$. For $\beta > 2$, there will thus be a sharp maximum in the ratio Eq. (33) for $\alpha = [\beta(\beta-1)]^{1/2} - (\beta-1) \approx 0.4 - 0.5$. For perpendicular scattering $\phi = 90^\circ$ and $\alpha = 0.8$, this ratio $\rightarrow 15$.

Thus, it appears that launch polarization mismatch as well as magnetic shear induced mismatch at the plasma edge will not mask the effect of magnetic fluctuation scattering even if the magnetic to density fluctuation ratios within V_{sc} (at the same frequency $\omega = \omega_s - \omega_i$) are as low as 10^{-6} .

VII. INCIDENT X-MODE SCATTERING

Here we consider the case of $X \rightarrow O$ scattering by magnetic fluctuations. The relative contributions to the scattered power from magnetic and density fluctuations in an $X \rightarrow O$ mode conversion

$$\frac{d\sigma(\delta B^2)_{XO}}{d\sigma(\delta n^2)_{XO}} = \frac{F(\delta B^2)_{XO}}{F(\delta n^2)_{XO}} \frac{\langle \delta B^2 \rangle / B_0^2}{\langle \delta n^2 \rangle / n_{0e}^2} \quad (A1)$$

where the form factor ratio $F(\delta B^2)_{XO}/F(\delta n^2)_{XO}$ with

$$F(\delta n^2)_{XO} = \frac{R_{XO} \cos \delta_i \cos \delta_s}{K_0} |\xi_{XO}|^2$$

$$F(\delta B^2)_{XO} = \frac{R_{XO} \cos \delta_i \cos \delta_s}{K_0} \frac{\beta}{\alpha^2} a_\gamma a_\gamma^*$$

has a very similar dependence on the angular beamwidth and scattering angle ϕ as in the $O \rightarrow X$ mode case [cf., Figs. 4 and 5]. In particular, in Fig. 10 we plot this ratio for $\beta = 2.25$ and a narrow angular width of $\Delta = 1^\circ$. On comparing this to Fig. 5 for the $O \rightarrow X$ case, we see that forward scattering is more advantageous for $X \rightarrow O$ scattering, while there is basically no difference in these corresponding form factor ratios for large angle scattering.

Plasma inhomogeneities effects are very different from that of $O \rightarrow X$ scattering and are shown in Fig. 11 for an incident beam propagating from below. The incident X-mode will penetrate through the plasma cross section (provided the plasma density is not too large: typically $\alpha < 5$). Again, the scattering volume V_{sc} is to be chosen to lie in front of the O-mode cutoff layer - otherwise the scattered O-mode will be evanescent and undetectable. Depending on the location of V_{sc} and for small forward scattering angles, the scattered O-mode can reach the O-mode cutoff layer and then be reflected/refracted out of the plasma. In Fig. 11(a), for a scattering angle of $\phi = 90^\circ$, V_{sc} is so located that the scattered O-modes do not reach the O-mode cutoff layer. However, with this same V_{sc} volume, all O-modes

with scattering angle $\phi < 50^\circ$ reach the cutoff layer and can be refracted into the same localized detector region. This is shown in Fig. 11(b). This has the clear advantage of a much enhanced signal at the detector. By lowering the scattering volume V_{sc} , the scattered O-modes at $\phi = 90^\circ$ no longer encounter the O-mode cutoff layer and are just refracted out from the plasma.

For incident X-modes, unlike the case for incident O-modes, there is now little difficulty in the polarization resolution of the O-mode scattered by magnetic fluctuations from the X-mode scattered by density fluctuations. The corresponding form factor ratio also increases as $\alpha \rightarrow 1$, Fig. 12. This result should be contrasted with that for incident O-modes, Fig. 9, which is insensitive to $\beta = \omega_{ce}^2/\omega_i^2$.

Hence it appears that incident microwave X-modes can be a very powerful tool for detecting magnetic fluctuations.

VIII. SUMMARY

Here we have examined the possibilities of using microwave scattering perpendicular to the magnetic field to detect magnetic fluctuations whose fluctuation levels are orders of magnitude below that of the density fluctuations. The scattering cross section is calculated for either incident O-modes or incident X-modes from both magnetic and density fluctuations in a scattering volume V_{sc} , where V_{sc} is defined by the intersection of the incident beam with the scattered beam. For sufficiently localized V_{sc} , this cross section can be determined from the locally homogeneous formalism of Sitenko while the toroidal effects on the incident and scattered waves are included by the use of ray tracing techniques.

This is the first calculation that examines both ray tracing and possible experimental difficulties with polarization resolution for the important special case of scattering perpendicular to the magnetic field. From ray tracing, it is found that the O-mode cutoff layer at the local plasma frequency can be exploited both for

incident O-modes (in which the cutoff layer acts as a filter) and for incident X-modes (in which the cutoff layer acts as a focusing reflector). A more careful treatment is required if the scattering volume V_{sc} is so chosen that scattering occurs at the O-mode cutoff layer. This calculation is now underway and will be reported elsewhere (see also Ref. 28). As regards experimental polarization limitations we have considered nonideal effects such as the polarization resolution of the detector, the polarization mismatch of the incident microwaves with the magnetic field at the plasma edge as well as the generation of unwanted modes due to mode conversion due to magnetic shear effects. It is found that polarization mismatch at the plasma edge with the direction of the edge magnetic field presents no problems for a low shear tokamak plasma. However, the polarization resolution of the detector antenna could pose some difficulties, especially for large angle scattering of incident O-modes. For small forward scattering angles there is no such resolution problem for incident O-modes. On the other hand, for incident X-modes and arbitrary scattering angles, it is found that there are no significant polarization resolution problems.

We conclude that magnetic fluctuations in a tokamak plasma can be detected by microwave scattering :

- (1) for incident O-modes, polarization resolution considerations are important and these can be satisfied for small forward scattering angles;
- (2) for incident X-modes, clever localization of the scattering volume and scattering angles can lead to a strong scattering signal from magnetic fluctuations.

ACKNOWLEDGMENTS

This work was supported by the Department of Energy at Old Dominion University under grant no. DE-FG05-90ER54093, at William & Mary under grant no. DE-FG05-84ER53176, and at Princeton under grant no. DE-AC02-76-CHO-3073.

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Figure Captions

Fig. 1 The scattering geometry. The magnetic field is in the z-direction. The incident wave vector $\mathbf{k}_i$ is in the x-z plane, with θ_i the angle between $\mathbf{k}_i$ and $\mathbf{B}_0$. The incident ray $\mathbf{r}_i$ is also in the x-z plane and makes an angle δ_i with $\mathbf{k}_i$. Similarly the scattered wave vector $\mathbf{k}_s$ and the scattered ray $\mathbf{r}_s$ are in the same plane with the corresponding angles θ_s and δ_s . ϕ is the angle between the $\mathbf{k}_i$ - $\mathbf{B}_0$ plane and the $\mathbf{k}_s$ - $\mathbf{B}_0$ plane.

Fig. 2 The variation of the magnetic form factor, Eq. (25), for $O \rightarrow X$ mode conversion for several magnetic fields B_0 . $\beta = \omega_{ce}^2/\omega_i^2$ and $\alpha = \omega_{pe}^2/\omega_i^2$. This form factor is insensitive to the angular beamwidths and has a weak dependence on the scattering angle ϕ . It increases significantly as the incident frequency $\omega_i \rightarrow \omega_{pe}$, the O-mode cutoff frequency and is somewhat higher for lower B_0 because of the geometric factors.

Fig. 3 The variation of the density form factor, Eq. (26), for $O \rightarrow X$ mode scattering with $\beta = \omega_{ce}^2/\omega_i^2 = 4.0$ for various angular widths:

- (i) angular width = 5° - with the upper curve is for perpendicular scattering $\phi = 90^\circ$ and the lower curve is for forward scattering $\phi = 0^\circ$;
- (ii) angular width = 2° - upper curve is for $\phi = 90^\circ$, and the lower curve for $\phi = 0^\circ$;
- (iii) angular width = 1° - upper curve is for $\phi = 90^\circ$, and the lower curve for $\phi = 0^\circ$.

Fig. 4 The dependence of the magnetic/density form factor on B_0 as a function of $\alpha = \omega_{pe}^2/\omega_i^2$ for various angular widths Δ at fixed forward scattering angle $\phi = 0$. The upper curve for $\Delta = 1$ is for $\beta = \omega_{ce}^2/\omega_i^2 = 4.0$, while the lower

curve is for $\beta = 2.25$. Note that this form factor ratio decreases as $\omega_i \rightarrow \omega_{pe}$ [i.e., as $\alpha \rightarrow 1$] with the optimal choice being the smallest angular widths and higher magnetic field. It should be remembered, though, that the actual power reaching the detector increases significantly as $X \rightarrow 1$, as can be seen from the monotonic behavior in the respective form factors, Figs. 2 and 3.

Fig. 5 The dependence of the magnetic/density form factor on the scattering angle ϕ as a function of $\alpha = \omega_{pe}^2/\omega_i^2$ at fixed angular width Δ and B_0 . This form factor ratio decreases as $\omega_i \rightarrow \omega_{pe}$ [i.e., as $\alpha \rightarrow 1$] for fixed ϕ . Although forward scattering ($\phi = 0^\circ$) yields the largest form factor ratio, it does so at the expense of a large scattering volume V_{sc} and the loss of spatial resolution. This ratio is somewhat insensitive to the scattering angle if $\phi \leq 45^\circ$ over a large ratio of incident frequencies ω_i . For frequencies close to the O-mode cutoff layer, the form factor ratio is insensitive to ϕ .

Fig. 6 Ray tracing of an incident bundle of O-modes with beamwidth $\Delta = 5^\circ$, frequency $\omega_i/2\pi = 45$ GHz in TFTR, incident from the low field side. The O-modes approach the O-mode cutoff layer and are then refracted from the plasma. Three representative scattered X-mode bundles of angular width $\Delta = 5^\circ$ are considered for : (a) scattering angle $\phi = 0^\circ$, (b) scattering angle $\phi = 45^\circ$, and (c) scattering angle $\phi = 90^\circ$.

Fig. 7 Ray tracing of an incident bundle of O-modes with beamwidth $\Delta = 5^\circ$, frequency $\omega_i/2\pi = 45$ GHz in TFTR, incident from the high field side and oriented to pass through the plasma center. The O-modes approach the O-mode cutoff layer and are then refracted from the plasma. Three representative scattered X-mode bundles of angular width $\Delta = 5^\circ$ are

considered for : (a) scattering angle $\phi = 0^\circ$, (b) scattering angle $\phi = 45^\circ$, and (c) scattering angle $\phi = 90^\circ$.

Fig. 8 Ray tracing of an incident bundle of O-modes with beamwidth $\Delta = 5^\circ$, frequency $\omega_i/2\pi = 45$ GHz in TFTR, incident from below. The O-modes approach the O-mode cutoff layer and are then refracted from the plasma. Three representative scattered X-mode bundles of angular width $\Delta = 5^\circ$ are considered for : (a) scattering angle $\phi = 0^\circ$, (b) scattering angle $\phi = 45^\circ$, and (c) scattering angle $\phi = 90^\circ$.

Fig. 9 A plot of the relative form factors for $O \rightarrow X$ conversion from magnetic fluctuations to $O \rightarrow O$ scattering from density fluctuations. This curve is insensitive to $\beta = \omega_{ce}^2/\omega_i^2$. Only large angle scattering is considered, since for forward scattering it is relatively easy to choose parameters in an inhomogeneous plasma for the scattered O-mode to be reflected from its cutoff layer and out of the detector region.

Fig.10 The dependence of the magnetic/density form factor on the scattering angle ϕ as a function of $\alpha = \omega_{pe}^2/\omega_i^2$ for $X \rightarrow O$ mode conversion. For $\phi > 45^\circ$, these curves are very similar to those for $O \rightarrow X$ scattering, Fig. 5

Fig.11 Ray tracing of a bundle of X-modes with angular width $\Delta = 5^\circ$, frequency $\omega_i/2\pi = 45$ GHz, incident from below in a TFTR plasma. The incident X-mode is refracted as it passes through the cross section and the scattering volume is so chosen that (a) the scattered O-modes at $\phi = 90^\circ$ do not encounter the O-mode cutoff layer, but (b) all the scattered O-modes with $\phi < 50^\circ$ are reflected from the cutoff layer and refracted into a localized detector region.

(c) By lowering the location of the V_{sc} , the scattered O-mode with $\phi = 45^\circ$ does not encounter the cutoff layer and is refracted into a different region.

Fig.12 A plot of the relative form factors for $X \rightarrow O$ conversion from magnetic fluctuations to $X \rightarrow X$ scattering from density fluctuations for $\beta = 4.0$ [curves (a) $\phi = 0^\circ$, (b) $\phi = 45^\circ$, and (c) $\phi = 90^\circ$] and $\beta = 2.25$ [curves (a') $\phi = 0^\circ$, (b') $\phi = 45^\circ$, and (c') $\phi = 90^\circ$]. These polarization resolution curves should be compared with those for the incident O-modes, Fig. 9.

Fig. 1

Magnetic Form Factor Dependence on E_0

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6(a)

Fig. 6(b)

Fig. 6(c)

Fig. 7(a)

Fig. 7(b)

Fig. 7(c)

Fig. 8(a)

Fig. 8(b)

Fig. 8(c)

Polarization Resolution for Large Scattering Angles

Fig. 9

Fig. 10

Fig. 11(a)

Fig. 11(b)

Fig. 11(c)

Fig. 12

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