VARIABILITY OF SOLAR RADIATION
AT FOUR SELECTED STATIONS
IN THE UNITED STATES

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October 1979

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PREPARED FOR THE
U.S. DEPARTMENT OF ENERGY
OFFICE OF CONSERVATION AND SOLAR APPLICATIONS
UNDER CONTRACT EG-77-S-02-4521

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VARIABILITY OF SOLAR RADIATION AT FOUR SELECTED STATIONS IN THE UNITED STATES

ABSTRACT

Since many engineers use monthly averages of daily solar radiation as input when designing a solar system, this study addresses the variabilities of such data. The solar radiations considered are the total of direct and diffuse radiant energies received on a horizontal surface at ground level by a pyranometer during solar hours. Four stations, at Seattle-Tacoma, Washington; Madison, Wisconsin; Charleston, South Carolina; and Albuquerque, New Mexico were selected to represent different climatic conditions across the United States. The data were obtained from the U.S. National Climatic Center, Asheville, North Carolina.

We could not detect any significant trends in these data. The data can be fitted by either a normal or gamma distribution and, for simplicity, a normal distribution was used. The coefficients of variation for all stations over the years were relatively low, and in general, were lower for summer months than for winter months.

Of the four stations tested, Albuquerque had the lowest coefficients of variation and Seattle-Tacoma had the highest. We estimated that no more than 16 years of records are needed for any of these stations to estimate a mean value which is within 10 percent of the true mean. However, if one wishes to improve this estimate to within 5 percent of the true mean, we estimate that a maximum of 16, 25, 41 and 61 years of records are required, respectively, for Albuquerque, Charleston, Madison and Seattle-Tacoma. As a rule, more years of records are needed to determine reliable mean values for winter months than for summer months.

Both the continuous daylight hours of high solar radiation (over $1100 \text{ kJ/m}^2/\text{hr}$) and the accumulated hourly solar radiation for these durations were studied. Again, Albuquerque had the longest positive run length, the greatest accumulated daily solar radiation and the least coefficient of variation. We concluded that the available data should be sufficient for a general design at all four stations. However, if a large investment is involved, more detailed analysis is required.

I. INTRODUCTION

The amount of solar radiation reaching the earth's surface is affected by the atmospheric attenuation of the extraterrestrial solar radiation. Extraterrestrial solar radiation, by definition, is the amount of solar radiation received at the top of the atmosphere. The variation of the extraterrestrial radiation with time of the year is small and has been determined (see Thekaekara, 1965). Atmospheric attenuation is due to radiation scattering and absorption as it is transmitted through the atmosphere, and is a function of the meteorological conditions (see Moon, 1940; Fritz, 1958; Thekaekara, 1974; and Meinel and Meinel, 1977). Since meteorological conditions are highly variable, the amount of solar radiation received at ground level is subject to a certain degree of variation.

The two prevailing practices of using solar radiation data for design and planning purposes are (1) use of a monthly average value, and (2) run through (simulation) of all available data. Since the amount of available data is rather limited,

the immediate questions about the use of an average value are (1) how large a sample size (years of data) is needed to define a reasonable mean for different locations and different times of the year, and (2) is there a long-term trend (either periodic or non-periodic)? If there should be a significant medium- to long-term trend, use of available data may not be a reasonable approach because the past data may not be a good indication of future available solar radiation.

Thus the objective of this study is to answer the following questions:

1. Is it possible to detect a medium- or long-term trend from existing data?
2. What are the statistical characteristics of the solar radiation data?
3. How large a sample size (years of solar radiation data) is needed to estimate a mean value to within a certain degree of accuracy for the four different locations?
4. What are the probabilities of having continuous high solar radiation (over 1100 kJ/m²/hr) at a location? This item would be useful in the design of optimum storage capacity for a solar energy system.

Solar radiation data collected at four stations were analyzed. The four locations, chosen to represent different types of solar environment, are Madison, Wisconsin; Charleston, South Carolina; Seattle-Tacoma, Washington; and Albuquerque, New Mexico.

II. DESCRIPTION OF SOLAR RADIATION DATA

The solar radiation data used in this study were obtained from the U.S. National Climatic Center at Asheville, North Carolina. The data were stored on magnetic tapes with the format "SOLMET." The solar radiation considered in this study is the total of direct and diffuse radiant energies, measured hourly in kilojoules per square meter (kJ/m²/hr), received on a horizontal surface at ground level by a pyranometer during solar hours. The time periods covered at each station were: Madison, Wisconsin (1952-1974); Charleston, South Carolina (1952-1974); Albuquerque, New Mexico (1952-1974); and Seattle-Tacoma, Washington (1951-1966).

H, the monthly average of daily solar radiation, is calculated as the arithmetic mean of the total daily radiation measurements in a month, and is expressed as

$$H = \frac{\sum_{k=1}^m \sum_{\ell=1}^n H_{k\ell}}{m} \quad (1)$$

where $H_{k\ell}$ is the hourly solar radiation in the kth day of the month and ℓ th hour of the daylight, m is the total days in the month and n is the total daylight hours. The value of n for each day can be determined from the measurements.

The monthly average radiations H at ground level for each year were calculated and are shown in Tables I, II, III and IV for Madison, Seattle-Tacoma, Charleston and Albuquerque, respectively. These averages varied from year to year for each month. In these tables, missing data are indicated by a-1.

Table I

SUMMARY OF THE MONTHLY AVERAGE OF DAILY RADIATION: KJ/M²/DAY

MAUISON, WIS

YEAR	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
1952	-1	-1	-1	-1	-1	-1	23680	19237	16714	12526	6432	3656
1953	4865	8229	11488	13924	19533	23594	21406	20663	17992	11779	6817	4624
1954	5419	8050	12795	15204	20581	22443	21071	19330	13494	8483	4901	4216
1955	5470	8836	15143	17444	22006	21342	23344	21012	16627	9780	5758	4735
1956	6045	7734	13247	15904	18141	23644	20955	18413	15817	11899	4635	2753
1957	6077	7699	12873	15794	17045	22819	23746	18767	16040	10243	5194	5624
1958	6027	-1	-1	-1	-1	-1	-1	-1	14795	10022	5789	5475
1959	5442	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1960	-1	9883	16288	14606	16706	21509	24489	19904	15528	11321	6121	6337
1961	7473	8924	11034	14257	21572	26909	24485	20780	15092	10547	6246	4980
1962	6927	8513	14326	19373	18779	23762	20781	22497	12753	-1	-1	-1
1963	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1964	-1	-1	13327	16620	21740	22418	22243	28025	14754	12121	7269	4977
1965	5942	14268	12577	14577	-1	-1	-1	18525	11087	10376	6402	4293
1966	7505	9955	11737	14313	20605	22432	21454	19414	16491	10867	5754	5057
1967	6508	9543	12163	13256	20676	20297	22356	18776	15217	9229	4951	4376
1968	5930	10587	12714	15191	18406	21500	22674	21595	16797	11047	6455	4656
1969	6257	8781	15223	15789	18304	18744	20049	23208	18490	10626	6584	5761
1970	7493	10834	13997	18334	20342	22623	24201	21443	13864	11419	5551	5248
1971	7051	9590	13882	19200	20440	24526	23789	21709	16686	10844	7042	4549
1972	6259	10032	12984	15338	19867	23642	20490	17044	14636	10398	4642	4649
1973	5741	8593	9005	13703	16138	21039	22288	-1	-1	-1	3507	4819
1974	6176	9729	12879	14149	18499	20533	24121	19240	17540	11796	5973	4940

REMARKS...MISSING VALUE IS INDICATED BY -1

Table II

SUMMARY OF THE MONTHLY AVERAGE OF DAILY RADIATION, KJ/M²/DAY

SEATTLE-TACOMA, WA

YEAR	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
1951	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	2887
1952	3290	6321	10435	17947	22062	20196	25049	18894	15425	8946	5020	2585
1953	1806	5813	9436	14550	19445	20108	24020	19375	14030	8247	3533	2131
1954	2970	5310	13893	16674	22349	21770	20547	15415	13320	8548	3894	2627
1955	2409	5547	10691	16668	19315	21334	18202	19743	12531	6950	3738	2683
1956	3206	4988	9210	18419	21569	17160	23856	17930	13458	6663	4196	2609
1957	4347	5192	10100	14976	20388	20942	21331	19766	15552	8571	5363	2409
1958	3140	4961	11673	15690	23608	23143	25849	21628	13628	8786	3989	2213
1959	2946	4516	9836	13735	19532	21256	24560	19310	10058	7113	4843	2801
1960	3066	6241	10606	14368	17903	22111	25478	15118	11771	8286	4549	3256
1961	3662	3992	8813	12827	18520	23950	24420	21331	15205	6604	5521	2467
1962	3599	6810	9913	15907	16833	22580	23004	17228	15099	8083	3742	2990
1963	3347	-1	9966	12442	23067	19444	20212	18263	14435	8248	3958	2922
1964	2565	6850	8223	15147	20524	17991	21563	18767	13425	9589	3907	2853
1965	2628	4424	13430	14286	20300	23521	24431	18547	13912	9114	4248	3056
1966	2717	6031	9577	15716	22006	22986	21186	21542	14959	9035	4355	1898

REMARKS...MISSING VALUE IS INDICATED BY -1

Table III

SUMMARY OF THE MONTHLY AVERAGE OF DAILY NAVIGATION, KJ/M/M/DAY

CHARLESTON, S.C.

YEAR	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
1952	-1	-1	-1	-1	-1	-1	23452	19435	17297	14365	10701	8593
1953	10257	11764	15615	21520	25362	23388	22455	19018	16401	13927	10722	7633
1954	9019	13023	16444	19824	21063	25039	23704	23163	18681	16541	12000	10331
1955	9751	12007	17383	18896	22787	23087	22087	20408	12030	16357	12901	8878
1956	9886	12276	16675	20957	21729	22699	20971	20511	17782	12106	12735	10043
1957	8677	12149	14396	21054	19987	21308	21109	20801	16593	13496	10525	9923
1958	9296	13763	13644	18992	21742	21789	21151	19183	18472	13308	10736	7689
1959	9342	8718	14527	19229	21635	22012	18579	17212	14924	11705	9664	8592
1960	8962	12101	14018	19585	23710	20572	18294	21160	17687	14075	11742	10470
1961	9571	10858	16809	20062	21510	21363	22195	18800	17999	16239	10613	9755
1962	8521	12198	16831	23029	25253	20959	22990	21511	18792	17561	10968	9671
1963	10717	11270	18474	21281	15058	20235	22876	20991	17581	14830	12637	10478
1964	8990	14010	15781	20268	22873	21567	18923	20123	17962	14883	12269	7804
1965	10242	10732	19630	17028	23673	18288	23711	20381	16343	13912	11840	9831
1966	8536	11862	17174	21422	17556	22408	22085	20518	19548	15910	13661	9320
1967	9014	11979	18991	22966	25347	22453	22937	21359	17709	17656	13239	9430
1968	8945	13322	18072	20802	23025	21917	24302	22445	17037	14247	12387	9160
1969	9924	11184	18120	22143	21874	23225	22838	17697	14313	13981	11161	9209
1970	8289	13933	10069	18055	23985	22845	23000	18923	19905	15498	11475	10251
1971	8373	11708	15028	20918	21420	22021	18733	17669	16435	11595	10506	7835
1972	8481	9037	17328	18367	19604	22984	21321	18604	16874	14436	9780	6780
1973	7986	12172	17773	17879	21824	18474	21142	17785	17331	14842	11430	7879
1974	7488	11119	15796	19667	21085	19031	18584	15448	14334	14676	11064	7699

REMARKS...MISSING VALUE IS INDICATED BY -1

Table IV

SUMMARY OF THE MONTHLY AVERAGE OF DAILY RADIATION: KJ/M²/DAY

ALBUQUERQUE, N.M.

YEAR	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
1952	-1	-1	-1	-1	-1	-1	21842	25446	22081	14362	12623	11030
1953	13356	15554	21027	25101	28762	29977	27172	26952	24666	18619	13631	11254
1954	12103	17024	19588	26433	28131	31036	28829	25718	22629	18576	14581	11352
1955	11969	16709	21510	25420	28405	31917	27281	25361	24520	19349	13964	11405
1956	10737	16001	23282	26712	29857	28708	29181	26563	25426	18632	14490	11958
1957	10101	13928	18220	24388	28454	29391	26671	25238	23866	15374	11881	11091
1958	10874	14530	17239	23885	27145	30285	29729	26501	20148	15640	13342	11774
1959	13132	14337	21410	23004	27014	29087	28123	24578	23458	16928	13429	9396
1960	11030	14722	19556	23134	28441	27178	26486	28615	21711	16693	13330	10305
1961	11634	15504	18147	24771	28712	29942	28769	29770	20281	19215	11915	10660
1962	11323	14537	18823	23178	28633	29851	24965	25211	18957	17604	12755	10410
1963	11069	14369	16770	24385	25746	28668	24592	23651	20805	17971	12145	12427
1964	12314	14943	20690	24108	26917	29946	27270	25190	19934	16149	12771	8769
1965	10074	14105	17443	21839	27091	25939	25997	23503	21053	16094	11400	9188
1966	9856	13946	18572	23459	27218	28009	24447	17289	21833	16170	12672	10754
1967	11775	15505	18419	21289	28843	25849	28462	23693	20443	18573	11857	9429
1968	10744	13202	17515	22079	27142	29275	25353	23426	22243	16803	11321	9715
1969	10134	13061	18420	21111	25743	27139	25097	24022	19659	16513	12009	9868
1970	10462	13285	17719	26036	29820	30825	29801	26861	23377	17150	14287	11136
1971	12785	15896	22074	24934	29742	31029	29375	26071	22672	17231	13520	9225
1972	11793	15518	21815	25303	30302	27976	28230	25991	20993	15426	11392	10547
1973	9081	11912	17331	20839	23916	31224	25739	25060	22552	17975	13146	10335
1974	9046	14124	18997	23963	27261	26778	28493	22573	18023	15207	12124	10447

REMARKS...MISSING VALUE IS INDICATED BY -1

III. STATIONARITY OF SOLAR RADIATION

Due to the relative position of the sun and the earth, there are two obvious cycles, daily and annual, for the variation of solar radiation received on earth. In the daily cycle, solar radiation has a maximum value near noon; in the annual cycle, solar radiation normally reaches its maximum in May, June or July. The purpose of this study is to investigate the stationarity of average daily solar radiation for a particular calendar month over a period of time covering different calendar years. Otnes and Enochson's method (1972) will be used.

Suppose the averages for a specific month over N years of observation are denoted by

$$H_1, H_2, H_3, \dots, H_N$$

and, defining A_i as the number of reverse arrangements for a given value of index i with

$$H_j > H_i \text{ for all } i < j \leq N,$$

the statistics considered in this test of nonstationarity are expressed as follows

$$A = \sum_{i=1}^{N-1} A_i \quad (2)$$

It can be shown that for a random sequence of N observations, the expected value of A is

$$E[A] = \frac{N(N-1)}{4} \quad (3)$$

and the variance is

$$\text{Var}(A) = \frac{2N^3 + 3N^2 - 5N}{72} \quad (4)$$

For $N \geq 10$ it has been shown that the distribution of A may be closely approximated by the standard normal distribution of Z with

$$Z = \frac{A + 0.5 - E[A]}{\sqrt{\text{Var}[A]}} \quad (5)$$

For a nonstationary process of solar radiation with an increasing average for a specific month, one would expect A to be very large. This will result in a large positive value of Z . On the other hand, a nonstationary process with a decreasing monthly average of solar radiation will result in a very small value of A and a small negative value of Z . Since both a too large or a too small value for A tends to indicate a trend, a two-tailed test is used. A hypothesis of the existence of a trend is rejected if the calculated A is smaller than $Z_{\alpha/2}$ and larger than $Z_{-\alpha/2}$. This is equivalent to saying that a hypothesis of the existence of a trend is rejected if the tail probability corresponding to the calculated Z is smaller than half of the level of significance, α .

The investigational results of the stationarity of the solar radiation process for each month are shown in Table V for Madison, Seattle-Tacoma, Charleston, and Albuquerque stations. It can be seen that the monthly average solar radiation can be accepted as stationary for Madison and Seattle-Tacoma stations if the level of significance is taken at 0.05, while the solar radiation in January at Charleston, and in January, February, October and November at Albuquerque are rejected as stationary if the same level of significance is used. The calculated Z values for Albuquerque are all negative, except that calculated for May. This seems to indicate that the observed values decreased slightly over the observation period.

Conclusion: Since the absolute values of A for those months being rejected as stationary are rather small, it was concluded that no significant nonstationary trends could be detected in these four stations based on the available data.

IV. BASIC STATISTICS OF MONTHLY AVERAGE OF DAILY SOLAR RADIATIONS

Both normal and gamma distributions were used to fit the observed data. The probability density function for normal distribution has the form

$$p(H) = \frac{1}{S\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \frac{(H-H_m)^2}{S^2} \right\} \quad (6)$$

where H_m and S are mean and standard deviations, respectively. The probability density function for gamma distribution is expressed as follows

$$p(H) = \frac{\beta(\beta H)^{\alpha-1}}{\Gamma(\alpha)} \exp \left\{ -\beta H \right\} \quad (7)$$

where α and β are distribution parameters. The parameters in the distributions were estimated by the method of maximum likelihood. The agreement between the theoretical distributions and the data was investigated by chi-squared tests. Tables VI, VII, VIII and IX present the parameters and the tail probabilities for these data.

With a five percent significant level, both normal and gamma distributions are acceptable for all months at the four stations. Since the normal distribution is easier to interpret and can be adequately described by its mean and standard deviation, it was accepted for further analysis. Figures 1 through 48 show the probability density and cumulative functions for all stations. The basic statistics are given in Tables X, XI, XII and XIII.

The following trends may be concluded from these statistics.

A. The variation of mean monthly average of daily solar radiation (H_m) for different calendar months:

1. At all four stations the month of December has the lowest average daily solar radiation, with the month of January being slightly higher. The variation of daily solar radiation in December for all stations is large. The lowest value of 2637 kJ/m²/day occurred in Seattle-Tacoma and the greatest value of 10516 kJ/m²/day occurred in Albuquerque.

Table V

Test of Long-term Trend Based on Monthly Average of Daily Solar Radiation
according to Method Proposed by Otnes and Enochson, 1972.

Station Month	Madison		Seattle-Tacoma		Charleston		Albuquerque	
	Calculated z	Tail Probability	Calculated z	Tail Probability	Calculated z	Tail Probability	Calculated z	Tail Probability
January	1.539	0.062	-0.675	0.250	-2.933	0.002	-2.312	0.010
February	1.894	0.029	-0.594	0.276	-1.015	0.155	-2.538	0.006
March	-0.350	0.363	-1.306	0.096	0.364	0.286	-1.072	0.142
April	-0.490	0.312	-1.576	0.058	-0.508	0.306	-1.861	0.031
May	-0.833	0.202	-1.036	0.150	-0.169	0.433	0.395	0.347
June	-1.061	0.144	0.765	0.222	-1.410	0.080	-1.354	0.09
July	0.070	0.472	-0.495	0.310	-0.528	0.299	-1.321	0.093
August	0.770	0.221	0.396	0.346	-1.585	0.057	-1.796	0.036
September	-0.746	0.228	0.099	0.461	-0.317	0.376	-1.796	0.036
October	0.280	0.390	1.386	0.083	0.264	0.396	-2.166	0.015
November	-0.097	0.461	0.693	0.244	0.053	0.4789	-2.007	0.022
December	1.200	0.115	0.765	0.222	-1.268	0.103	-1.637	0.051

Table VI

Probability Distribution Parameters of Monthly Average of Daily Solar Radiation for Madison, WI.

Month	Normal Distribution			Gamma Distribution		
	Mean	Standard Deviation	Tail Probability	α	β	Tail Probability
January	6300	706	0.06	84	74	0.06
February	9322	900	0.15	112	83	0.34
March	13045	1568	0.33	72	182	0.33
April	15745	1809	0.53	83	189	0.48
May	19438	1739	0.57	129	151	0.57
June	22435	1838	0.72	159	141	0.49
July	22541	1414	0.33	267	84	0.33
August	20102	1593	0.31	168	119	0.43
September	15321	1662	0.17	83	184	0.47
October	10802	1025	0.41	113	95	0.41
November	5801	940	0.66	36	160	0.34
December	4756	744	0.29	38	123	0.34

Table VII
 i
 Probability Distribution Parameters of Monthly Average of Daily Solar
 Radiation for Seattle-Tacoma, WA.

Month	Normal Distribution			Gamma Distribution		
	Mean	Standard Deviation	Tail Probability	α	β	Tail Probability
January	3016	599	0.88	26	116	0.88
February	5478	858	0.89	43	127	0.70
March	10297	1553	0.25	51	201	0.64
April	15285	1657	0.88	90	168	0.88
May	20166	2297	0.88	78	257	0.64
June	21052	2060	0.88	107	196	0.88
July	22901	2256	0.26	106	216	0.28
August	18857	1944	0.89	97	194	0.89
September	13787	1498	0.39	83	166	0.39
October	8311	836	0.26	100	83	0.28
November	4324	613	0.37	56	77	0.37
December	2637	358	0.47	55	48	0.47

Table VIII

Probability Distribution Parameters of Monthly Average of Daily Solar Radiation for Charleston, SC.

Month	Normal Distribution			Gamma Distribution		
	Mean	Standard Deviation	Tail Probability	α	β	Tail Probability
January	9158	832	0.69	125	73	0.69
February	11985	1373	0.47	76	157	0.38
March	16165	1637	0.14	101	159	0.14
April	20234	1522	0.46	185	109	0.46
May	21895	2432	0.28	77	283	0.28
June	21720	1781	0.38	153	141	0.38
July	21622	1845	0.36	137	157	0.11
August	19704	1814	0.52	120	164	0.42
September	17044	1806	0.37	85	199	0.36
October	14608	1611	0.42	84	173	0.18
November	11514	1078	0.36	120	96	0.57
December	9045	1139	0.24	63	161	0.15

Table IX

Probability Distribution Parameters of Monthly Average of Daily Solar Radiation for Albuquerque, NM.

Month	Normal Distribution			Gamma Distribution		
	Mean	Standard Deviation	Tail Probability	α	β	Tail Probability
January	11236	1118	0.28	107	105	0.28
February	14722	1368	0.30	119	123	0.44
March	19298	1860	0.30	116	166	0.15
April	23987	1600	0.28	233	102	0.20
May	27571	1550	0.12	328	84	0.12
June	29124	1812	0.57	266	109	0.57
July	27155	1677	0.62	273	99	0.42
August	25039	2292	0.05	111	274	0.05
September	21806	1924	0.57	134	163	0.29
October	17373	1330	0.19	177	98	0.19
November	12799	1012	0.46	168	76	0.57
December	10516	969	0.57	122	85	0.71

Figure 1. Probability Distribution of Monthly Average of Daily Solar Radiation, January--Madison, Wisconsin

Figure 2. Probability Distribution of Monthly Average of Daily Solar Radiation, February--Madison, Wisconsin

Figure 3. Probability Distribution of Monthly Average of Daily Solar Radiation, March--Madison, Wisconsin

Figure 4. Probability Distribution of Monthly Average of Daily Solar Radiation, April--Madison, Wisconsin

Figure 5. Probability Distribution of Monthly Average of Daily Solar Radiation, May--Madison, Wisconsin

Figure 6. Probability Distribution of Monthly Average of Daily Solar Radiation, June--Madison, Wisconsin

Figure 7. Probability Distribution of Monthly Average of Daily Solar Radiation, July--Madison, Wisconsin

Figure 8. Probability Distribution of Monthly Average of Daily Solar Radiation, August--Madison, Wisconsin

Figure 9. Probability Distribution of Monthly Average of Daily Solar Radiation, September--Madison, Wisconsin

Figure 10. Probability Distribution of Monthly Average of Daily Solar Radiation, October--Madison, Wisconsin

Figure 11. Probability Distribution of Monthly Average of Daily Solar Radiation, November--Madison, Wisconsin

Figure 12. Probability Distribution of Monthly Average of Daily Solar Radiation, December--Madison, Wisconsin

Figure 13. Probability Distribution of Monthly Average of Daily Solar Radiation, January--Seattle-Tacoma, Washington

Figure 14. Probability Distribution of Monthly Average of Daily Solar Radiation, February--Seattle-Tacoma, Washington

Figure 15. Probability Distribution of Monthly Average of Daily Solar Radiation, March--Seattle-Tacoma, Washington

Figure 16. Probability Distribution of Monthly Average of Daily Solar Radiation, April--Seattle-Tacoma, Washington

Figure 17. Probability Distribution of Monthly Average of Daily Solar Radiation, May--Seattle-Tacoma, Washington

Figure 18. Probability Distribution of Monthly Average of Daily Solar Radiation, June--Seattle-Tacoma, Washington

Figure 19. Probability Distribution of Monthly Average of Daily Solar Radiation, July--Seattle-Tacoma, Washington

Figure 20. Probability Distribution of Monthly Average of Daily Solar Radiation, August--Seattle-Tacoma, Washington

Figure 21. Probability Distribution of Monthly Average of Daily Solar Radiation, September--Seattle-Tacoma, Washington

Figure 22. Probability Distribution of Monthly Average of Daily Solar Radiation, October--Seattle-Tacoma, Washington

Figure 23. Probability Distribution of Monthly Average of Daily Solar Radiation, November--Seattle-Tacoma, Washington

Figure 24. Probability Distribution of Monthly Average of Daily Solar Radiation, December--Seattle-Tacoma, Washington

Figure 25. Probability Distribution of Monthly Average of Daily Solar Radiation, January--Charleston, South Carolina

Figure 26. Probability Distribution of Monthly Average of Daily Solar Radiation, February--Charleston, South Carolina

Figure 27. Probability Distribution of Monthly Average of Daily Solar Radiation, March--Charleston, South Carolina

Figure 28. Probability Distribution of Monthly Average of Daily Solar Radiation, April--Charleston, South Carolina

Figure 29. Probability Distribution of Monthly Average of Daily Solar Radiation, May--Charleston, South Carolina

Figure 30. Probability Distribution of Monthly Average of Daily Solar Radiation, June--Charleston, South Carolina

Figure 31. Probability Distribution of Monthly Average of Daily Solar Radiation, July--Charleston, South Carolina

Figure 32. Probability Distribution of Monthly Average of Daily Solar Radiation, August--Charleston, South Carolina

Figure 33. Probability Distribution of Monthly Average of Daily Solar Radiation, September--Charleston, South Carolina

Figure 34. Probability Distribution of Monthly Average of Daily Solar Radiation, October--Charleston, South Carolina

Figure 35. Probability Distribution of Monthly Average of Daily Solar Radiation, November--Charleston, South Carolina

Figure 36. Probability Distribution of Monthly Average of Daily Solar Radiation, December--Charleston, South Carolina

Figure 37. Probability Distribution of Monthly Average of Daily Solar Radiation, January--Albuquerque, New Mexico

Figure 38. Probability Distribution of Monthly Average of Daily Solar Radiation, February--Albuquerque, New Mexico

Figure 39. Probability Distribution of Monthly Average of Daily Sola Radiation, March--Albuquerque

Figure 40. Probability Distribution of Monthly Average of Daily Solar Radiation, April--Albuquerque, New Mexico

Figure 41. Probability Distribution of Monthly Average of Daily Solar Radiation, May—Albuquerque, New Mexico

Figure 42. Probability Distribution of Monthly Average of Daily Solar Radiation, June—Albuquerque, New Mexico

Figure 43. Probability Distribution of Monthly Average of Daily Solar Radiation, July--Albuquerque, New Mexico

Figure 44. Probability Distribution of Monthly Average of Daily Solar Radiation, August--Albuquerque, New Mexico

Figure 45. Probability Distribution of Monthly Average of Daily Solar Radiation, September--Albuquerque, New Mexico

Figure 46. Probability Distribution of Monthly Average of Daily Solar Radiation, October--Albuquerque, New Mexico

Figure 47. Probability Distribution of Monthly Average of Daily Solar Radiation, November--Albuquerque, New Mexico

Figure 48. Probability Distribution of Monthly Average of Daily Solar Radiation, December--Albuquerque, New Mexico

Table X

BASIC STATISTICS OF THE MONTHLY AVERAGE OF DAILY RADIATION IN KJ/M²/DAY

MAUISUN, WIS

MONTH	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
YEARS	19	18	19	19	18	18	19	19	20	19	20	20
MEAN	6300.	9322.	13045.	15745.	19438.	22435.	22541.	20102.	15321.	10802.	5801.	4756.
S.D.	706.	900.	1568.	1809.	1739.	1838.	1414.	1593.	1662.	1025.	940.	744.
CV	.112	.097	.120	.115	.089	.082	.063	.079	.108	.095	.162	.156
SKEW	.298	-.129	.027	.679	-.335	.330	-.037	.197	-.725	-.380	-.606	-.546
KURT	2.672	1.994	3.138	2.489	2.130	3.582	1.515	2.301	3.413	2.774	2.955	4.577
AM2	.473E+06	.764E+06	.233E+07	.310E+07	.286E+07	.319E+07	.189E+07	.240E+07	.262E+07	.995E+06	.840E+06	.526E+06
AM3	.969E+08	-.862E+08	.957E+08	.371E+10	-.161E+10	.186E+10	-.970E+08	.733E+09	-.388E+10	-.378E+09	.466E+09	-.208E+09
AM4	.597E+12	.118E+13	.170E+14	.239E+14	.174E+14	.364E+14	.544E+13	.130E+14	.235E+14	.275E+13	.208E+13	.127E+13

NOTATIONS

YEARS = TOTAL YEARS OF OBSERVATION

MEAN, S.D., CV, SKEW AND KURT = MEAN, STANDARD DEVIATION, COEFFICIENT OF VARIATION, SKEWNESS AND KURTOSIS, RESPECTIVELY.

AM2, AM3 AND AM4 = THE SECOND, THIRD AND FOURTH MOMENTS, RESPECTIVELY

Table XI

BASIC STATISTICS OF THE MONTHLY AVERAGE OF DAILY RADIATION IN KJ/M²/DAY

SEATTLE-TALUMA, WA

MONTH	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
YEARS	16	15	16	16	16	16	15	15	15	15	15	16
MEAN	3416.	3478.	47297.	15485.	20166.	21052.	22981.	18857.	13767.	8311.	4324.	2637.
S.D.	599.	658.	1543.	1657.	2297.	2060.	2250.	1944.	1498.	830.	613.	358.
CV	.179	.197	.151	.108	.114	.098	.098	.103	.109	.101	.142	.136
SKEN	.147	.063	1.155	.195	-.472	-.447	-.524	-.413	-.997	-.728	.700	-.340
KURT	3.310	2.007	3.621	2.515	2.590	2.153	2.216	2.646	3.621	2.664	2.311	2.613
AM2	.137E+06	.086E+06	.220E+07	.257E+07	.444E+07	.378E+07	.475E+07	.353E+07	.210E+07	.653E+06	.350E+06	.120E+06
AM3	.240E+08	.359E+08	.393E+10	.607E+09	-.519E+10	-.355E+10	-.542E+10	-.273E+10	-.302E+10	-.384E+09	.146E+09	-.142E+08
AM4	.375E+12	.483E+12	.105E+14	.167E+14	.633E+14	.341E+14	.500E+14	.329E+14	.159E+14	.114E+13	.283E+12	.377E+11

NOTATIONS

YEARS = TOTAL YEARS OF OBSERVATION

MEAN, S.D., CV, SKEN AND KURT = MEAN, STANDARD DEVIATION, COEFFICIENT OF VARIATION, SKEWNESS AND KURTOSIS, RESPECTIVELY

AM2, AM3 AND AM4 = THE SECOND, THIRD AND FOURTH MOMENTS, RESPECTIVELY

Table XII

BASIC STATISTICS OF THE MONTHLY AVERAGE OF DAILY RADIATION IN KJ/M²/DAY

CHAMBERSBURG, Pa.

MONTH	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
YEARS	22	22	22	22	22	22	23	23	23	23	23	23
MEAN	9154.	11486.	18105.	20235.	21896.	23721.	21622.	19705.	17045.	14809.	11515.	9046.
S.D.	835.	1173.	1638.	1522.	1433.	1781.	1640.	1814.	1800.	1811.	1878.	1139.
CV	.091	.115	.101	.075	.111	.086	.085	.092	.100	.110	.099	.126
SKEW	-.008	-.032	.020	.110	-.064	-.103	-.029	-.098	-.060	-.047	.233	-.036
KURT	2.209	2.729	2.032	2.222	4.331	1.230	2.154	2.025	3.484	2.802	2.214	1.033
AM2	.0022+00	.0106+01	.0750+01	.2212+01	.5052+01	.3034+01	.3202+01	.3152+01	.3122+01	.2482+01	.1112+01	.1242+01
AM3	-.0002+00	-.0102+00	.0002+00	.0012+00	-.0242+01	-.0042+00	-.0302+00	-.0102+00	-.0322+00	-.0352+00	.2742+00	-.3272+00
AM4	.0072+01	.0442+01	.1322+01	.1092+01	.1302+01	.2902+01	.2292+01	.2802+01	.3082+01	.1802+01	.2702+01	.2982+01

NOTATIONS

YEARS = TOTAL YEARS OF OBSERVATION

MEAN, S.D., CV, SKEW AND KURT = MEAN, STANDARD DEVIATION, COEFFICIENT OF VARIATION, SKEWNESS AND KURTOSIS, RESPECTIVELY

AM2, AM3 AND AM4 = THE SECOND, THIRD AND FOURTH MOMENTS, RESPECTIVELY

Table XIII

BASIC STATISTICS OF THE MONTHLY AVERAGE OF DAILY RADIATION IN KJ/M²/DAY

ALBUQUERQUE, N.M.

MONTH	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
YEARS	22	22	22	22	22	22	23	23	23	23	23	23
MEAN	11236.	14722.	19298.	23987.	27571.	29124.	27155.	25039.	21806.	17373.	12799.	10516.
S.D.	1110.	1360.	1860.	1800.	1550.	1812.	1677.	2292.	1924.	1330.	1012.	969.
CV	.100	.093	.096	.067	.056	.062	.062	.092	.088	.077	.079	.092
SKW.	.252	-.038	.578	-.117	-.175	-.305	.004	-1.390	.067	-.093	.180	.041
KURT	2.009	3.572	2.160	2.342	2.781	2.200	1.815	7.246	2.287	1.772	1.922	2.193
AM2	.119E+07	.179E+07	.330E+07	.244E+07	.229E+07	.313E+07	.259E+07	.503E+07	.354E+07	.169E+07	.979E+06	.898E+06
AM3	.328E+09	-.919E+08	.347E+10	-.454E+09	-.807E+09	-.177E+10	.182E+08	-.150E+11	.449E+09	-.205E+09	.180E+09	.349E+08
AM4	.209E+13	.114E+14	.233E+14	.140E+14	.146E+14	.216E+14	.131E+14	.183E+15	.287E+14	.506E+13	.164E+13	.177E+13

NOTATIONS

YEARS = TOTAL YEARS OF OBSERVATION

MEAN, S.D., CV, SKW. AND KURT = MEAN, STANDARD DEVIATION, COEFFICIENT OF VARIATION, SKEWNESS AND KURTOSIS, RESPECTIVELY

AM2, AM3 AND AM4 = THE SECOND, THIRD AND FOURTH MOMENTS, RESPECTIVELY

2. The maximum average daily solar radiation in each year may occur in May, June, or July. The range of maxima was between 21896 (Charleston) and 29124 kJ/m²/day (Albuquerque). This range was much narrower than the range for the minimum values as stated in the previous item.

3. The ratios between the maximum average daily solar radiation for a calendar month to the minimum average daily solar radiation for another calendar month for the four stations are as follows:

Seattle-Tacoma, Washington	(July)/(December) = 8.68
Madison, Wisconsin	(July)/(December) = 4.74
Albuquerque, New Mexico	(June)/(December) = 2.77
Charleston, South Carolina	(May)/(December) = 2.42

B. The variation of monthly average values of solar radiation for different years and the coefficient of variation, which is the ratio between the standard deviation and the mean:

1. The variability of the daily solar radiation seems to be greater in the winter months (November, December, January, and February) than the summer months.

2. The coefficients of variation of daily solar radiation for both Albuquerque and Charleston are almost constant at a value near 10 percent for all months. This value indicates a low variability.

3. In summer months (May, June, July and August) the coefficients of variation for all four stations are low and at a constant value of about 10 percent.

4. Figures 49-52 show the variation of the mean monthly value for each calendar month and the 95 percent confidence intervals at each station.

C. The tendency of the variation of monthly average values for different years (the skewness). There is just about as much negative skewness as positive skewness. Since the skewness values varied more or less randomly, there appears no definite tendency for the variation of monthly average daily solar radiation for different years.

Conclusions: The monthly average daily solar radiation values can be fitted by either a normal distribution or a gamma distribution. For convenience, a normal distribution was used.

The maximum monthly average solar radiation values occurred in the summer months, and these values did not vary greatly at the four stations (maximum of 29,000 kJ/m²/day at Albuquerque and minimum of 22,000 kJ/m²/day at Charleston). The minimum monthly average solar radiation values occurred in the winter months and varied greatly, as indicated by data collected from these four stations (maximum of 10,500 kJ/m²/day at Albuquerque and minimum of 2,600 kJ/m²/day at Seattle-Tacoma).

The variations of monthly average values for each calendar month over different years were relatively small. The coefficients of variation were normally in the neighborhood of 10 percent. The greatest coefficient of variation occurring in January at Seattle-Tacoma, was about 20 percent. As a general rule, the coefficients of variation for summer months were lower than those for winter months. Of

Figure 49. Mean Monthly Average of Daily Solar Radiation for Madison, Wisconsin

Figure 50. Mean Monthly Average of Daily Solar Radiation for Seattle-Tacoma, Washington

Figure 51. Mean Monthly Average of Daily Solar Radiation for Charleston, South Carolina

Figure 52. Mean Monthly Average of Daily Solar Radiation for Albuquerque, New Mexico

the four stations tested, Albuquerque had the lowest coefficient of variation, ranging from 0.056 in May to 0.10 in January and Seattle-Tacoma had the greatest coefficient of variation, ranging from 0.098 in July to 0.199 in January.

V. SAMPLE SIZE NEEDED TO ESTABLISH A GOOD MEAN MONTHLY AVERAGE OF DAILY SOLAR RADIATION

Assuming that for each month at a given location the observed monthly averages of daily radiation are the physical realization of some normal distribution (see previous section), the true mean H_m can be estimated from the values observed in the past. The effect of the variation of the measured radiation on the estimation of true mean monthly average of daily solar radiation H_m can best be explained by the confidence interval of the estimates. A 100 (1- α) percent confidence interval for H_m is

$$(\bar{H} - t_{\alpha/2} \frac{S}{\sqrt{n}}, \bar{H} + t_{\alpha/2} \frac{S}{\sqrt{n}}) \quad (8)$$

In this equation, $\bar{H}$ is the estimated mean based on sample size, n , S is the estimated standard deviation, and $t_{\alpha/2}$ is the variate of a student's t -distribution with $n-1$ degrees of freedom, which corresponds to 0.5 α of the right tail probability. As shown by expression (8), the larger the sample size n , the narrower the confidence interval. In other words, the accuracy of the estimate increases with sample size, assuming that S remains constant. Thus, the record lengths should exceed a certain value in order to obtain an estimate with a certain degree of accuracy.

Again, assuming that the averages of daily solar radiation for each month at a particular location follow a normal distribution, the sample size N required to obtain an estimate of the mean value with (1- α) percent of confidence that the true value will be within a certain error of the estimated mean can be expressed as

$$N = 1 + \left(\frac{Z_{\alpha/2} S}{B_{1/2}} \right)^2 \quad (9)$$

In this above equation, $B_{1/2}$ is half of the tolerant error bound and $Z_{\alpha/2}$ is the variate of the standard normal distribution corresponding to 0.5 α of the right tail probability.

The required number of observations (in units of years) for various half-error bounds, namely 5, 10, 15, 20 and 25 percent of the estimated mean values (and centered at the estimated mean), are shown in Tables XIV, XV, XVI and XVII for the four stations. It is seen from these tables that there are already enough records available at all the stations to estimate the true means of all monthly average of daily solar radiation within 10 percent on either side of the estimated mean. Charleston and Albuquerque already have enough records to be within five percent of estimated mean, but Madison and Seattle-Tacoma require longer records for the winter months due to greater variations in the observed values. For instance, 61 years of record are needed to establish a mean value within ± 5 percent for January at Seattle-Tacoma.

Conclusion: Assuming that the monthly average values of daily solar radiation for each month at each location follow a normal distribution (which seems to be a

Table XIV

REQUIRED OBSERVATION YEARS FOR VARIOUS HALF-ERROR BOUNDS*

HALF-ERROR BOUND	MADISON, WIS				
	.05 MEAN	.10 MEAN	.15 MEAN	.20 MEAN	.25 MEAN
JANUARY	20	5	3	2	1
FEBRUARY	15	4	2	1	1
MARCH	23	6	3	2	1
APRIL	21	6	3	2	1
MAY	13	4	2	1	1
JUNE	11	3	2	1	1
JULY	7	2	1	1	1
AUGUST	10	3	2	1	1
SEPTEMBER	19	5	3	2	1
OCTOBER	14	4	2	1	1
NOVEMBER	41	11	5	3	2
DECEMBER	38	10	5	3	2

*REQUIRED OBSERVATION YEARS WERE CALCULATED WITH
95 PERCENT OF CONFIDENCE LIMITS

Table XV

REQUIRED OBSERVATION YEARS FOR VARIOUS HALF-ERROR BOUNDS*

HALF-ERROR BOUND	SEATTLE-TACOMA, WA				
	.05 MEAN	.10 MEAN	.15 MEAN	.20 MEAN	.25 MEAN
JANUARY	61	16	7	4	3
FEBRUARY	38	10	5	3	2
MARCH	35	9	4	3	2
APRIL	19	5	3	2	1
MAY	20	5	3	2	1
JUNE	15	4	2	1	1
JULY	15	4	2	1	1
AUGUST	17	5	2	2	1
SEPTEMBER	19	5	3	2	1
OCTOBER	16	4	2	1	1
NOVEMBER	31	8	4	2	2
DECEMBER	29	8	4	2	2

*REQUIRED OBSERVATION YEARS WERE CALCULATED WITH
95 PERCENT OF CONFIDENCE LIMITS

Table XVI

REQUIRED OBSERVATION YEARS FOR VARIOUS HALF-ERROR BOUNDS*

CHARLESTON, S.C.

HALF-ERROR BOUND	.05 MEAN	.10 MEAN	.15 MEAN	.20 MEAN	.25 MEAN
JANUARY	13	4	2	1	1
FEBRUARY	21	6	3	2	1
MARCH	16	4	2	1	1
APRIL	9	3	1	1	1
MAY	19	5	3	2	1
JUNE	11	3	2	1	1
JULY	12	3	2	1	1
AUGUST	14	4	2	1	1
SEPTEMBER	18	5	2	2	1
OCTOBER	19	5	3	2	1
NOVEMBER	14	4	2	1	1
DECEMBER	25	7	3	2	1

*REQUIRED OBSERVATION YEARS WERE CALCULATED WITH
95 PERCENT OF CONFIDENCE LIMITS

Table XVII

REQUIRED OBSERVATION YEARS FOR VARIOUS HALF-ERROR BOUNDS*

ALBUQUERQUE, N.M.

HALF-ERROR BOUND	.05 MEAN	.10 MEAN	.15 MEAN	.20 MEAN	.25 MEAN
JANUARY	16	4	2	1	1
FEBRUARY	14	4	2	1	1
MARCH	13	4	2	1	1
APRIL	7	2	1	1	1
MAY	5	2	1	1	1
JUNE	6	2	1	1	1
JULY	6	2	1	1	1
AUGUST	13	4	2	1	1
SEPTEMBER	12	3	2	1	1
OCTOBER	10	3	2	1	1
NOVEMBER	10	3	2	1	1
DECEMBER	14	4	2	1	1

*REQUIRED OBSERVATION YEARS WERE CALCULATED WITH
95 PERCENT OF CONFIDENCE LIMITS

reasonable assumption according to the previous section), there are more than enough data available to estimate a true mean within 10 percent of the estimated value for all months at all stations. Also, Charleston and Albuquerque already have sufficient record lengths to estimate the true mean value within five percent of the estimated value for each month. Only for the winter months at both Madison and Seattle-Tacoma are still more data needed to estimate the true mean values within five percent of the estimated value.

VI. PROBABILITY OF HAVING A CONTINUOUS HIGH LEVEL OF DAILY SOLAR RADIATION DURING DAYLIGHT HOURS

In order to design an optimum solar heating and cooling system, one should know the characteristics of solar radiation at a location as well as the heating and cooling demands. Since the solar radiation supply does not correlate with energy demand, units are usually provided to store converted solar energy. The statistics of continuous high levels of daily solar radiation during daylight hours can be useful for selecting an optimum solar heating and cooling system.

Not all solar radiation received at ground level is usable due to the amount of heat loss through the collector system. This cut-off level (minimum usable amount of solar radiation) is not actually a constant, but for simplicity, we took it to be $1100 \text{ kJ/m}^2/\text{hr}$. The characteristics of continuous hourly solar radiation (positive run-length) above this cut-off level were investigated for the months of January, April, July and October over the entire observation period for all four stations. These four months were selected to represent the four seasons.

The means, standard deviations, and coefficients of variation of the positive run-lengths are given in Table XVIII. For the same station, longer positive run-lengths occur more frequently in the summer months than in the winter. For the same month, stations in the southern United States have longer positive run-lengths than those in the north. Of all four stations, Albuquerque had the least coefficient of variation, with a value of about 0.30. Of course, if one changes the cut-off level, that statistics may change accordingly.

In addition to the knowledge of the duration of continuous high solar radiation, one may also be interested to learn the statistics of accumulated solar radiation above a certain cut-off level. Table XIX shows the mean, standard deviation and coefficients of variation for these accumulated solar radiations above a cut-off level of $1100 \text{ kJ/m}^2/\text{hr}$. Again, Albuquerque has the greatest accumulated radiation and the least coefficient of variation. In January, April and October, Charleston had the second highest accumulated solar radiation, but in July, Charleston had the least accumulated solar radiation.

Histograms of positive run-length and accumulated solar radiation above the cut-off level are shown in Figures 53-68. As these figures show, the histograms for Albuquerque are concentrated at the longer positive run-length and longer accumulated radiations. This would indicate the possibility of having continuous solar system operation over long periods. For both Seattle-Tacoma and Charleston, the histograms appeared to concentrate at both ends. This indicates perhaps a combination of both sunshine and rainy days. For Madison, the peaks appear to occur at random. It is obviously much easier to design an optimum solar system at Albuquerque than at the other locations, while the positive run-lengths at Madison are the most difficult to predict.

Table XVIII

Statistics of Positive Run-Length in Hours.

	January			April			July			October		
	Mean	Standard Deviation	Coeff. of Variation	Mean	Standard Deviation	Coeff. of Variation	Mean	Standard Deviation	Coeff. of Variation	Mean	Standard Deviation	Coeff. of Variation
Madison	3.36	1.34	0.40	6.15	3.13	0.51	7.09	3.36	0.47	4.99	2.20	0.44
Seattle- Tacoma	2.24	1.15	0.51	5.30	3.12	0.59	7.66	3.13	0.41	3.85	2.12	0.55
Charleston	4.39	1.93	0.44	7.36	2.86	0.39	6.71	3.24	0.48	5.94	2.39	0.40
Albuquerque	5.00	1.63	0.33	8.31	2.44	0.29	8.30	2.56	0.31	6.80	2.05	0.30

Table XIX

Statistics of Accumulated Solar Radiation above Cut-Off Level of $1100 \text{ kJ/m}^2/\text{hr}$

	January			April			July			October		
	Mean	Standard Deviation	Coeff. of Variation	Mean	Standard Deviation	Coeff. of Variation	Mean	Standard Deviation	Coeff. of Variation	Mean	Standard Deviation	Coeff. of Variation
Madison	4866	2175	0.45	13575	8361	0.62	16850	9326	0.55	9045	4575	0.51
Seattle-Tacoma	2927	1607	0.55	11034	7633	0.69	18237	8714	0.48	6495	4104	0.63
Charleston	7861	4016	0.51	17801	8263	0.46	15955	8919	0.56	12242	5746	0.47
Albuquerque	9075	3370	0.37	21323	7471	0.35	22890	7981	0.35	14954	5242	0.35

(a) POSITIVE RUN-LENGTH, HOURS

(b) ACCUMULATED RADIATION, KJ/M/M

Figure 53. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

Figure 54. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

(a) POSITIVE RUN-LENGTH, HOURS

(b) ACCUMULATED RADIATION, KJ/M/M

Figure 55. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

(a) POSITIVE RUN-LENGTH, HOURS

(b) ACCUMULATED RADIATION, KJ/M/M

Figure 56. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

(a) POSITIVE RUN-LENGTH, HOURS

(b) ACCUMULATED RADIATION, KJ/M/M

Figure 57. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

(a) POSITIVE RUN-LENGTH, HOURS

(b) ACCUMULATED RADIATION, KJ/M/M

Figure 58. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

Figure 59. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

(a) POSITIVE RUN-LENGTH, HOURS

(b) ACCUMULATED RADIATION, KJ/M/M

Figure 60. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

Figure 61. Histograms of Positive Run-Length and Accumulated Solar Radiation, above Cut-Off Level

Figure 62. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

(a) POSITIVE RUN-LENGTH, HOURS

(b) ACCUMULATED RADIATION, KJ/M/M

Figure 63. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

Figure 64. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

Figure 65. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

Figure 66. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

Figure 67. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

(a) POSITIVE RUN-LENGTH, HOURS

(b) ACCUMULATED RADIATION, KJ/M/M

Figure 68. Histograms of Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level

As shown by Figures 69-84, the mean positive run-lengths and mean accumulated radiations are highly linearly correlated.

Conclusions: The statistics of continuous high levels of daily solar radiation during daylight hours vary over different seasons at different locations. It appears that it is much easier to design an optimum solar system in Albuquerque than at any of the other three stations. The positive run-length peaks and the accumulated solar radiation peaks were the most difficult to predict.

VII. SUMMARY AND CONCLUSIONS

Since many engineers use the monthly average of daily solar radiation as input for the design of a solar system, this study addresses the variabilities of these monthly average of daily solar radiations. It was found that:

1. No significant long-term trends could be detected.
2. The data can be fitted either by a normal or a gamma distribution; for simplicity we have used a normal distribution.
3. The coefficients of variation for all stations were relatively low. In general, these values were lower for summer months than for winter months. Albuquerque had the lowest values, ranging between 0.06 in May and 0.10 in January, while the highest value of 0.2 occurred in January at Seattle-Tacoma.
4. The required sample size (equivalent to the total observation years) in order to obtain 95 percent confidence that the true mean is within an error bound of 5 percent, 10 percent, 15 percent, 20 percent and 25 percent of estimated mean was calculated for each month at each station. It is estimated that a maximum of 4, 7, 11 and 16 years of records are required, respectively, for Albuquerque, Charleston, Madison, and Seattle-Tacoma to determine average monthly values of daily solar radiation within 10 percent of their estimated means. It is also estimated that a maximum of 16, 25, 41 and 61 years of records are required, respectively, for Albuquerque, Charleston, Madison and Seattle-Tacoma to determine average monthly values of daily solar radiation within 5 percent of these estimated means. As a rule, a larger sample size is needed to determine reliable mean values for winter months than for summer months.
5. Both the daylight hours of continuous high solar radiation (above a cut-off level of $1100 \text{ kJ/m}^2/\text{hr}$) and the accumulated hourly solar radiation for those durations were studied. These statistics can be useful for deciding on optimal system sizes. Again, Albuquerque has the longest positive run-length (continuous high solar radiation), the greatest accumulated daily solar radiation, and the least coefficient of radiation. Thus it would be the easiest of the four locations for which to design an optimal solar system from the consideration of solar input. The positive run-lengths and the accumulated solar radiation are both greater in summer months than in winter months for all stations.

As a general conclusion, due to the low coefficients of variation of the monthly average of daily solar radiation over years, it appears that the use of monthly average values, as determined by the currently available data from these four station, is adequate for a general design. However, if a large investment is involved, such as that for a solar-powered generating station, depending on each location, the currently available data of 20-25 years may not be sufficient to

Figure 69. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, January--Madison, Wisconsin

Figure 70. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, April--Madison, Wisconsin

Figure 71. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, July—Madison, Wisconsin

Figure 72. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, October--Madison, Wisconsin

Figure 73. Relation between Positive Run-Length and Accumated Solar Radistion above Cut-Off Level, January--Seattle-Tacoma, Washington

Figure 74. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, April--Seattle-Tacoma, Washington

Figure 75. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, July--Seattle-Tacoma, Washington

Figure 76. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, October--Seattle-Tacoma, Washington

Figure 77. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, January—Charleston, South Carolina

Figure 78: Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, April--Charleston, South Carolina

Figure 79. Relation between Positive Run-Lengths and Accumulated Solar Radiation above Cut-Off Level

Figure 80. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, October--Charleston, South Carolina

Figure 81. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, January--Albuquerque, New Mexico

Figure 82. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, April--Albuquerque, New Mexico

Figure 83. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, July--Albuquerque, New Mexico

Figure 84. Relation between Positive Run-Length and Accumulated Solar Radiation above Cut-Off Level, October, Albuquerque, New Mexico

define an accurate mean value. In any case, there is an apparent need to investigate the number of years of data to determine an accurate mean for many more stations over the United States and to incorporate these and other relevant statistics into the design of optimal solar system sizes (including absorption and storage).

ACKNOWLEDGMENT

We wish to thank the Department of Energy for Sponsoring this study and also NCAR in Boulder for providing the use of their computers. Acknowledgement is also due to Dr. George Lof for his discussions.

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