EXPERIMENTAL DETERMINATION OF THE EFFECTIVE TAYLOR DISPERSIVITY IN A FRACTURE

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June 1984

Stanford Geothermal Program
INTERDISCIPLINARY RESEARCH
IN ENGINEERING AND EARTH SCIENCES
Stanford University, Stanford, California

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By

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Section 1: INTRODUCTION

Reinjection of waste hot water is commonly practiced in most geothermal fields, primarily as a means of disposal. Surface discharge of these waste waters is usually unacceptable due to the resulting thermal and chemical pollution.

Although reinjection can help to maintain reservoir pressure and fluid volume, in some cases a decrease in reservoir productivity has been observed (Horne, 1982). This is caused by rapid flow of the reinjected water through fractures connecting the injector and producers. As a result, the water is not sufficiently heated by the reservoir rock, and a reduction in enthalpy of the produced fluids is seen.

Tracer tests have proven to be valuable to reservoir engineers for the design of a successful reinjection program. By injecting a slug of tracer and studying the discharge of surrounding producing wells, an understanding of the fracture network within a reservoir can be provided.

In order to quantify the results of a tracer test, a model that accurately describes the mechanisms of tracer transport is necessary. One such mechanism, dispersion, is like a smearing out of a tracer concentration due to the velocity gradients over the cross section of flow. If a dispersion coefficient can be determined from tracer test data, the fracture width can be estimated.

The purpose of this project was to design and construct an apparatus to study the dispersion of a chemical tracer in flow through a fracture.

Section 2: LITERATURE SURVEY

The effects of water reinjection in geothermal systems worldwide is discussed in a paper by Horne(1983), which also includes a summary of tracer testing procedures and results.

In order to derive a model to accurately describe the transport of tracer through a fracture, the physics of dispersion must be understood. Taylor(1953) presented a classic study of dispersion in flow through a capillary tube. He showed that convective dispersion combines with transverse molecular diffusion in what we now know as "Taylor Dispersion" (Fig. 1). He showed that the tracer concentration is dispersed symmetrically about a plane that moves with the mean flow velocity. Taylor presents the equation governing the effective longitudinal dispersion:

$$\eta \frac{\partial^2 C}{\partial z^2} = \frac{\partial C}{\partial t} \quad (1)$$

where,

C = concentration

z = translated distance = x-ut

x = distance

t = time

u = mean velocity of flow

η = net longitudinal dispersivity (derived for pipe flow in Taylor's model)

The solutions to eqn (1) for different initial and boundary conditions can be found in Carslaw and Jaeger(1959). For a step input,

found in Carslaw and Jaeger(1959). For a step input,

$$C = C_0 + \frac{1}{2}(C_1 - C_0) \left[\operatorname{erfc} \left(\frac{x-ut}{2(\eta t)^{1/2}} \right) + \exp \frac{ux}{\eta} \operatorname{erfc} \left(\frac{x+ut}{2(\eta t)^{1/2}} \right) \right] \quad (2)$$

where,

C_0 = base concentration

C_1 = injected concentration

C = concentration at x

erfc = complimentary error function

Horne and Rodriguez(1983) used a method similar to Taylor's to derive an expression for the net longitudinal dispersivity, η , for flow in a fracture:

$$\eta = \frac{2}{105} \frac{b^2 u^2}{D} \quad (3)$$

where,

b = fracture half-width

u = mean flow velocity

D = coefficient of molecular diffusion

They also showed that, due to the effects of transverse molecular diffusion, any concentration gradients across the fracture would be equalized after a non-dimensional time, $t_D = 0.5$ (Fig. 2), where

$$t_D = \frac{D}{b^2} t \quad (4)$$

Fossum and Horne(1982) show how the subroutine VARPRO can be used to determine both linear and non-linear parameters from a set of experimental data. VARPRO uses a non-linear least squares method of curve fitting.¹² Fossum and Horne(1982) matched the calculated response to field data from tracer

convective dispersion

Taylor dispersion

(convective dispersion + transverse diffusion)

Fig. 1. Dispersion Schematics

Fig. 2. $\delta(t_d)$ vs. t_d (Horne & Rodriguez, 1983)

The present study set out to examine and confirm the applicability of Equation (3), which is only approximate, and to initiate broader investigations into dispersion in fractures. To these ends, an experimental program was undertaken.

The results of several experiments to study dispersion were found in the literature. Bear(1961) performed both one- and two-dimensional studies of dispersion through porous media and produced results which agreed with his theory. Hull and Koslow(1982) present the results of a study of dispersion in a network of channels.

Section 3: DESIGN

The design objectives were aimed at building an apparatus capable of studying dispersion through a fracture in both one- and two-dimensions. The possibility of testing both chemical and fluorescent tracers was another requirement.

1. HELE-SHAW CELL

The size of the model fracture, particularly its aperture, was constrained by the results of Horne and Rodriguez (1983). Any concentration gradient across the width of the fracture will be equalized after a non-dimensional time, $t_D = 0.5$ (Fig. 2). The real time it takes to become equalized is proportional to the square of the fracture half-width:

$$t = \frac{0.5 b^2}{D} \quad (5)$$

Using a diffusion coefficient for potassium iodide (KI) $\approx 2 \times 10^{-9} m^2/sec$ and a fracture half-width of 0.25 mm, the time required is about 16 sec. By using an aperture of 0.54 mm and flowrates of approximately 50 cc/min we could keep the apparatus small enough to fit on a lab bench! The cell is 6 ft long by 1 ft wide. Fig. 3 shows an overall view and Fig. 4 a detail of the design.

The lower plate is 1 in. thick cast aluminum alloy. It is hard anodized to prevent corrosion and provide a tough, non-conductive finish. The upper plate is 1/4 in. float glass and is separated from the aluminum by a gasket made up of three layers of plastic electrical tape. A series of aluminum clamps holds the cell together while four adjustable legs support it horizontally on the lab bench.

2. VALVES

The inlet and outlet ports were implemented by drilling holes 11 in. through the width of the plate. A 0.25 in. slit was then sawed through the surface (Fig. 4). Since the pressure drop across the length of the drilled hole is negligible com-

TOP VIEW

SIDE VIEW

FIG. 3 HELE-SHAW CELL

1 FT.

FIG. 4. DETAIL VIEW

FIG. 5. TRACER VALVE

pared with that across the slit, water will flow into the cell at uniform velocity over its width.

An on-off valve (Fig. 5) was designed to allow instantaneous injection of tracer. The valve is activated by hand and can be locked in either on or off position.

3. ELECTRODES

In order to continuously monitor the tracer concentration as it flows through the cell, an array of 96 electrodes was employed. For a KI tracer the conductivity of solution will increase linearly with the log of concentration. Thus we are able to measure the tracer concentration at any electrode location and any chosen time.

The coaxial electrodes were constructed using brass conductive elements and a teflon insulator (Fig. 6). The brass surfaces were electroplated with gold to prevent corrosion and polarization. Each electrode is press fit in to the aluminum plate and mounted flat to within .0015 in. The central electrode is connected to the data acquisition system, and the outer electrode is grounded to the plate.

An instantaneous current is flowed across the electrode while the resulting voltage is measured (requiring less than 0.1 sec.). Voltages can be measured once each second and are stored in a Compaq personal computer. The data is displayed on the screen so it is possible to "watch" the tracer as it flows through the cell. The curcuitry and electronics are described in Appendix A, and the computer scanning algorithms are described in Appendix B.

4. CONSTANT FLOWRATE SOURCE

Two constant pressure reservoirs, one for the base concentration and one for the tracer, were constructed (Fig. 7). The flowrate can be adjusted by

changing the height of the center tube.

Section 4: PROCEDURE

I. Solution Preparation

- (a) Prepare solutions of desired concentrations using distilled water and iodide standard. A base concentration of 175-200 ppm should be used. Voltage readings from lower concentrations tend to be too unstable for accurate analysis. By injecting ~300 ppm the electrode response remains in the linear portion of the voltage vs. log concentration curve, thus simplifying analysis. Using the 1000 ml volumetric flask, the concentration is:

$$C(\text{ppm}) = \frac{(x) \text{ liter of iodide standard} \cdot 12690 \text{ mg/l}}{1 \text{ liter}}$$

- (b) Clean reservoirs and fill with solution.

II. Assembly

- (a) Wipe clean the aluminum and glass plates with wet sponge and assemble; clamps should be finger tight.
- (b) Flush the Hele-Shaw cell with CO_2 (at $p < 2$ psi or glass may shatter).
- (c) Begin flowing water slowly, making sure to clear both the inlet and tracer valve of air, otherwise bubbles will become trapped in the cell. (distilled water was flowed until the cell was void of all air bubbles to save the prepared solutions for test experimental runs.) Pounding on the glass with the butt of your hand, or tapping the glass with the rubber mallet while flowing at high rate can prevent the water front from fingering and forming air pockets.
- (d) Now start flowing the solution of base concentration; allow 2 pore volumes (~ 500 cc) to flow before starting to SCAN (see next step).

III. Prepare computer for run

- (a) Plug in the multiplexor board power supply (Appendix A describes the design and operation of the multiplexor); turn on printer, then computer.
- (b) When "clock" appears (~ 60 sec) hit <F10>.
- (c) Type b:
- (d) Remove system diskette from the A drive; replace with a blank diskette.

IV. Ready to run

- (a) To begin recording, enter SCAN. Note time. SCAN will measure and store the voltage at each electrode, once per second, and will fill the C (internal) diskette after about 7 min. The voltages will be plotted against the location of each electrode on the screen (they will vary for each electrode since each has different sensitivity). SCAN will stop automatically after 7 min., or sooner if you hit <F1>.
- (b) Allow base concentration to flow for ~1 min. before injecting tracer. To inject, open gate valve, slide tracer valve into position and shut off inlet valve. Record time of injection.
- (c) When run is over, enter FIXUP. FIXUP processes the data (~15 min) and stores it in a new file named labfix.dat on the A diskette.
- (d) Enter PLOTFIX. This plots the voltage vs. time for each electrode, individually.
- (e) Take diskette out and label it; these are your results.
- (f) To start a new run, insert a blank disc in the A drive, shut off the tracer, and flow the base concentration (~500 cc). Repeat steps 7-11.
- (g) After runs are finished, disassemble and wash down thoroughly. Unplug the multiplexor board, shut off computer and printer.

Section 5: DATA PREPARATION

The processed data (labfix.dat) can be matched to the model given by Equation (2) to provide estimates of the mean speed of flow (u), and the effective dispersivity (η). A FORTRAN program, CURVEFIT, performs this operation, and simultaneously calibrates the measured voltages to concentrations. The program is run as follows:

- (a) Copy labfix.dat from drive A to drive C (C must be erased first).
- (b) Create a file PARAMS on C as follows:
 1. N = number of unknowns (12), [$=2$ (u and η), can be 3 if t_e is unknown as well].
 2. Initial estimates of u , η ; one per line (F10.4).
 3. Base concentration (C_0) and injected concentration (C_1); one per line (F10.4).
 4. Number of electrodes to be analyzed and electrode numbers (4012); e.g. 080104052122232932 will analyze 8 electrodes - 1,4,5,21,22,23,29 and 32.
- (c) Load a blank diskette in drive A.
- (d) Type a:
- (e) Type b:curvefit
- (f) Type c: params <CR>, con <CR>, c:labfix.dat <CR>, prn <CR>.
- (g) The program will output a summary of estimated parameters on the printer, and will create output files (1.dat through 32.dat) on A. These may be used for plotting. The program takes several hours to run.
- (h) On completion, type b:plot, this will plot the results on the screen for a specified range of electrodes.

Section 6: RESULTS

The results of seven one-dimensional experiments are presented in this section. The results from CURVEFIT are displayed in Tables 4-10, and are compared with the dispersivity predicted by Equation (3) in Figures 8-14. Graphs of the data collected from each electrode and the curve that fits it are also presented. Since it was a one-dimensional study, only the central row of 32 electrodes was used.

The actual fracture dimensions are shown in Table 1. Since the glass plate used for these runs was slightly curved, the aperture was measured at the centerline.

Table 1. FRACTURE DIMENSIONS (cm)		
Aperture	width	length
0.0515	24.46	179.1

Table 2 shows the concentrations of the base, (C_0), and injected (C_1) solutions for each run. The flowrate was measured by recording the time required to fill a 50 ml flask at the outlet. The time of injection, t_0 , was only recorded for runs 8, 9, and 10.

Table 2. MEASURED DATA				
Run #	C_0 (ppm)	C_1 (ppm)	q (cm ³ /sec)	t_0 (sec)
3	203	305	0.80	—
4	203	305	1.25	—
5	177	305	0.82	—
7	177	305	1.38	—
8	177	305	0.66	60
9	177	305	0.48	42
10	177	305	0.57	43

The velocities listed in Table 3 are values from CURVEFIT that best match the data (since the fracture cross-section did not have constant aperture, the

flow velocity was not constant over the width of the cell, and $u = q/A$ may not be accurate). These best-fit values are used in Equation (3) to provide an estimate of the dispersivity (η). The estimated times of injection, t_o , used in PARAMS are also listed in Table 3.

Table 3. ESTIMATED VALUES			
Run	u (cm/sec)	η (cm ² /sec)	t_o (sec)
3	0.542	0.210	73.0
4	0.824	0.486	56.6
5	0.539	0.208	79.5
7	0.890	0.567	73.4
8	0.451	0.146	60.0
9	0.375	0.101	42.5
10	0.416	0.124	42.5

The dispersivities predicted by Equation (3) are compared with those estimated by CURVEFIT in Figures 8-14. Only selected electrodes are plotted, as some are inconsistent due to faulty data collection or transmission. Electrodes 1 and 2 were usually inconsistent, probably because the tracer front had not yet become equalized.

Electrode	C_0 (ppm)	C_1 (ppm)	u (cm/sec)	η (cm ² /sec)	apparent t_0 (sec)
1	193.6091	305.1784	0.6992	1.0367	72.9238
2	201.2811	304.9338	0.6131	0.6220	72.6622
3	202.9373	305.0957	0.5908	0.4181	72.1451
4	200.0074	300.0581	0.5835	0.3585	72.6364
5	200.0959	300.0331	0.5846	0.2719	72.4763
6	202.9865	305.0680	0.5715	0.3113	72.3291
7	200.8080	449.3800	0.3034	2.8423	73.1011
8	199.9427	300.0960	0.5661	0.3005	73.4000
9	203.0065	305.0846	0.5711	0.2633	72.6024
10	203.0301	305.0725	0.5663	0.2930	72.8913
11	203.0107	305.1142	0.5647	0.3156	72.6204
12	200.0911	300.0483	0.5638	0.2449	73.4000
13	203.0176	305.1285	0.5593	0.2173	72.4066
14	203.0000	305.1503	0.5545	0.2358	73.1522
15	203.0166	305.1168	0.5543	0.2118	72.9312
16	203.0227	305.1229	0.5513	0.2623	72.9191
18	203.0147	305.1194	0.5498	0.2511	72.0139
19	203.0314	305.1045	0.5463	0.2341	72.6125
20	203.0683	305.1092	0.5455	0.2260	72.0204
21	203.0394	305.1175	0.5428	0.2326	72.6915
22	202.9910	305.1639	0.5442	0.2481	72.8799
23	202.9789	305.1957	0.5401	0.2652	72.8374
24	203.0310	305.1272	0.5410	0.2411	72.8148
25	202.9533	305.2214	0.5405	0.2708	72.2095
26	203.0517	305.1082	0.5406	0.1829	72.8609
27	203.0475	305.1186	0.5409	0.2078	72.5939
28	203.0491	305.1325	0.5416	0.1999	72.5580
29	203.0428	305.1451	0.5429	0.1909	72.8735
30	203.0459	305.1377	0.5435	0.1865	72.8586
31	203.0372	305.1767	0.5450	0.1787	72.3092
32	203.0167	305.2894	0.5471	0.2055	72.1761

Fig. 8. Comparison of estimated and calculated dispersivity. Run 3.

electrode	C_0 (ppm)	C_1 (ppm)	u (cm/sec)	η (cm^2/sec)	apparent t_0 (sec)
1	199.2807	300.1214	0.7571	0.9746	56.6000
2	196.9567	300.9046	0.6923	2.2386	56.6000
3	200.0594	300.0537	0.8099	0.5473	56.6000
4	200.2185	300.0896	0.8563	0.2747	56.6000
5	200.0233	300.0435	0.8318	0.4877	56.6000
6	200.0099	300.0408	0.8148	0.5452	56.6000
7	199.5614	299.8199	0.8558	0.3780	56.6000
8	200.0930	300.0335	0.8297	0.4787	56.6000
9	199.8557	300.0919	0.8154	0.7502	56.6000
10	199.8767	300.0824	0.8122	0.6532	56.6000
11	199.8754	300.0898	0.8163	0.7479	56.6000
12	199.9134	300.0852	0.8305	0.5706	56.6000
13	200.0458	300.0534	0.8392	0.5296	56.6000
14	199.3993	299.4178	0.8236	0.6332	56.6000
15	200.1344	300.0222	0.8343	0.4576	56.6000
16	200.0068	300.0574	0.8255	0.5355	56.6000
17	200.0289	300.0669	0.8352	0.5581	56.6000
18	200.0545	300.0575	0.8299	0.5820	56.6000
19	199.9969	300.0705	0.8276	0.5619	56.6000
20	200.0546	300.0801	0.8263	0.5019	56.6000
21	200.3710	300.0504	0.8425	0.3211	56.6000
22	200.0699	300.0592	0.8275	0.4723	56.6000
23	200.0753	300.0618	0.8253	0.4432	56.6000
24	200.1798	300.0337	0.8296	0.3779	56.6000
25	200.1206	300.0454	0.8245	0.4633	56.6000
26	200.0903	300.0514	0.8268	0.4436	56.6000
27	200.0851	300.0562	0.8275	0.4348	56.6000
28	200.1322	300.0412	0.8281	0.4114	56.6000
29	200.1982	300.0331	0.8323	0.3844	56.6000
30	200.0715	300.0772	0.8316	0.4726	56.6000
31	200.1124	300.0662	0.8383	0.4417	56.6000
32	200.1561	300.0574	0.8388	0.4319	56.6000

Fig. 9. Comparison of estimated and calculated dispersivity. Run 4.

Table 6. RUN 5 Results

Electrode	C_0 (ppm)	C_1 (ppm)	u (cm/sec)	η (cm ² /sec)	apparent t_0 (sec)
1	172.4794	305.4789	0.4567	0.4547	79.4469
2	172.6788	306.0025	0.4574	0.6216	79.3727
3	176.8712	305.1535	0.5017	0.2523	78.1311
4	177.0194	304.9398	0.5313	0.1454	79.7869
5	176.9580	305.1213	0.5216	0.2398	77.1715
6	176.9227	305.1701	0.5223	0.2561	78.5297
7	177.0075	305.1239	0.5370	0.2177	78.9163
8	176.9869	305.1637	0.5361	0.2446	79.3120
9	176.8473	305.2737	0.5305	0.3134	77.8365
10	176.9455	305.3065	0.5356	0.3051	79.2525
11	176.9516	305.2965	0.5380	0.3001	79.2787
12	176.9802	305.2385	0.5385	0.2701	78.9465
13	177.0278	305.1734	0.5407	0.2603	79.0606
14	176.9525	305.2849	0.5385	0.2628	79.0822
15	176.9865	305.2269	0.5396	0.2245	78.9550
16	176.9783	305.2457	0.5380	0.2272	79.0627
17	176.9859	305.2278	0.5433	0.2246	79.2794
18	176.9790	305.2643	0.5404	0.2555	78.9707
19	177.0272	305.2033	0.5403	0.2345	79.5479
20	177.0523	305.2374	0.5403	0.2236	79.1419
21	177.0293	305.2169	0.5406	0.2265	79.8696
22	177.0075	305.2479	0.5399	0.2226	79.1860
23	177.3219	305.0014	0.5539	0.1139	79.4677
25	177.2622	305.1656	0.5412	0.2284	79.9000
26	177.2921	305.1879	0.5403	0.2453	79.9000
27	177.8224	305.1204	0.5427	0.2068	79.9000
28	177.8538	305.1572	0.5426	0.2306	79.9000
29	177.5213	305.0650	0.5453	0.1940	79.9000
30	177.7206	305.1760	0.5471	0.1798	79.9000
31	177.8365	305.2394	0.5516	0.2109	79.9000
32	179.4189	305.0844	0.5596	0.2430	79.9000

Fig. 10. Comparison of estimated and calculated dispersivity. Run 5.

electrode	C_0 (ppm)	C_1 (ppm)	u (cm/sec)	η (cm ² /sec)	apparent t_0 (sec)
1	179.2666	305.6806	0.6717	0.4679	71.7837
2	178.6063	305.4210	0.6423	0.5171	72.7662
3	176.7776	305.0984	0.7263	0.3560	72.4584
4	176.9432	305.0747	0.7554	0.4441	71.7963
5	177.2003	305.0239	0.7962	0.3357	72.1182
6	176.8742	305.0944	0.8025	0.5089	72.0861
7	176.5317	304.7535	0.8446	0.3708	73.2562
8	177.0942	305.0427	0.8334	0.4695	72.5221
9	176.9618	305.1009	0.8464	0.5463	72.3593
10	178.5818	308.8341	0.7717	2.1440	69.0034
11	177.0820	305.0543	0.8744	0.4750	72.6505
12	176.9861	305.1233	0.8728	0.4913	72.6992
13	176.9890	305.1339	0.8793	0.4733	72.4796
14	176.9812	305.1491	0.8763	0.5617	72.4105
15	177.0046	305.1351	0.8821	0.4752	73.2172
16	176.9860	305.1614	0.8812	0.5194	73.3541
17	177.0174	305.1481	0.8890	0.4531	72.5010
18	177.0549	305.1027	0.8926	0.5390	73.2216
19	177.0296	305.1550	0.8914	0.5489	73.3826
20	177.0435	305.2248	0.8864	0.4589	72.9996
21	177.0824	305.1236	0.8990	0.4089	73.0323
22	176.9820	305.3146	0.8868	0.4852	72.6011
23	177.0107	305.2252	0.8908	0.4432	72.4894
24	177.0221	305.2593	0.8900	0.3673	72.6674
25	177.0153	305.1995	0.8953	0.4526	72.8346
26	177.0257	305.2189	0.8937	0.4460	72.8886
27	177.0623	305.1226	0.9012	0.4210	72.5135
28	177.0006	305.2429	0.9031	0.4885	73.2226
29	177.0204	305.2684	0.9005	0.4380	73.1053
30	176.9823	305.3311	0.9062	0.4202	72.5762
31	177.0160	305.2626	0.9069	0.4229	73.0941
32	177.0289	305.2506	0.9049	0.4531	73.0907

Fig. 11. Comparison of estimated and calculated dispersivity. Run 7.

Table 8. RUN 8 Results

Electrode	C_0 (ppm)	C_1 (ppm)	u (cm/sec)	η (cm ² /sec)	apparent t_0 (sec)
2	170.1399	304.8965	0.5513	0.9233	59.9313
3	179.8520	306.0940	0.4889	0.4890	57.0895
4	176.8666	305.3446	0.4808	0.2899	57.8292
5	176.7606	305.5164	0.4695	0.2779	58.7547
6	176.9526	306.1938	0.4655	0.2636	57.2856
7	176.9783	305.4040	0.4734	0.1685	59.8243
8	176.8539	305.5481	0.4669	0.2108	59.7648
9	176.8430	305.6109	0.4658	0.2067	58.6273
10	176.8034	305.7105	0.4613	0.2355	59.5570
11	176.8416	305.5755	0.4622	0.2043	59.7869
12	176.9483	305.3648	0.4629	0.1351	59.0176
13	176.9379	305.4082	0.4610	0.1314	59.4217
14	176.8700	305.4972	0.4588	0.1966	59.5951
15	176.9528	305.3391	0.4695	0.1341	59.7998
16	176.9221	305.3843	0.4569	0.1555	59.6897
17	177.1410	305.1577	0.4545	0.2325	59.5215
18	176.9500	305.3545	0.4557	0.1753	59.9117
19	176.9141	305.4277	0.4545	0.1594	59.1586
20	176.9196	305.5346	0.4527	0.1775	59.0275
21	177.0144	305.3112	0.4528	0.1384	59.8606
22	176.8555	305.6125	0.4489	0.1951	59.4112
23	176.9429	305.4110	0.4499	0.1390	59.7394
24	176.9425	305.4284	0.4502	0.1712	59.6374
25	176.9841	305.4207	0.4494	0.1376	59.7931
26	176.9982	305.4667	0.4490	0.1397	59.2542
27	176.9812	305.6110	0.4498	0.1285	59.4932
28	177.0255	305.6131	0.4507	0.1225	59.8619
29	176.9814	306.3142	0.4496	0.1238	59.7666
30	177.0031	306.6916	0.4511	0.1401	59.5834
31	176.9886	308.1960	0.4515	0.1342	59.6492
32	176.9780	310.5608	0.4541	0.1365	59.5192

Fig. 12. Comparison of estimated and calculated dispersivity. Run 8.

Table 9. RUN 9 Results

Electrode	C_0 (ppm)	C_1 (ppm)	u (cm/sec)	η (cm^2/sec)	apparent t_0 (sec)
1	92.4754	291.9013	0.8247	11.5627	42.0653
2	131.5554	304.1395	0.7286	4.8659	41.5202
3	176.0719	304.6890	0.6112	1.1524	42.2738
4	177.1845	305.0923	0.5164	0.5792	40.4316
5	177.1948	305.0734	0.4881	0.3830	40.2042
6	176.9417	305.2280	0.4528	0.4245	40.1679
7	177.1461	305.0948	0.4477	0.2400	39.9412
8	176.9773	305.1975	0.4303	0.2683	40.4746
9	177.0871	305.1323	0.4284	0.2139	42.1549
10	176.8942	305.2768	0.4158	0.2379	42.3783
11	177.0735	305.1511	0.4137	0.1834	42.4097
12	177.1774	305.1143	0.4067	0.1622	42.4030
13	177.2131	305.1068	0.4007	0.1483	42.3131
14	177.1551	305.1408	0.3955	0.1653	42.1147
15	177.1852	305.1264	0.3930	0.1385	41.9828
16	177.0934	305.1743	0.3876	0.1490	41.9846
17	177.1607	305.1516	0.3871	0.1339	41.6244
18	177.1774	305.1328	0.3851	0.1163	42.1718
19	177.4015	305.0996	0.3849	0.1121	41.8483
20	177.2561	305.2126	0.3806	0.0932	41.5267
21	177.1605	305.2447	0.3809	0.0827	42.4186
22	177.0856	305.2866	0.3759	0.1076	42.0458
23	177.0395	305.4663	0.3730	0.1034	42.0218
24	177.0251	305.6692	0.3728	0.0868	42.1476
25	177.0109	306.2703	0.3708	0.0879	41.6299
26	177.0091	306.7268	0.3721	0.0894	42.2254
27	177.0063	308.3059	0.3732	0.0773	41.6680
28	177.0743	314.0283	0.3721	0.0861	41.9085
29	177.4232	320.7780	0.3729	0.0551	42.1727
30	177.1694	210.5363	0.3841	0.0251	41.7788

Fig. 13. Comparison of estimated and calculated dispersivity. Run 9.

Table 10. RUN 10 Results					
Electrode	C_2 (ppm)	C_1 (ppm)	u (cm/sec)	η (cm^2/sec)	apparent t_0 (sec)
1	165.5770	301.8529	0.7688	1.4091	42.1981
2	165.8977	305.9257	0.6398	2.1460	41.6750
3	174.2000	304.6744	0.5717	0.7228	42.0110
4	176.9750	305.1616	0.5180	0.4317	40.5803
5	177.1250	305.1054	0.5005	0.3067	40.7347
6	176.9881	305.1995	0.4778	0.3406	40.2335
7	177.1116	305.1659	0.4784	0.2061	41.7093
8	177.1743	305.3780	0.4620	0.2459	40.7168
9	177.0269	305.1664	0.4610	0.2013	42.4894
10	176.9113	305.3792	0.4508	0.2708	41.7489
11	177.0229	305.1925	0.4496	0.1863	42.1240
12	176.9876	305.2614	0.4431	0.1885	41.6820
13	177.0255	305.2101	0.4389	0.1720	41.6887
14	177.0176	305.2232	0.4346	0.1776	42.3983
15	177.0756	305.1718	0.4322	0.1668	41.6534
16	177.0307	305.2253	0.4272	0.1629	41.5687
17	177.0804	305.2016	0.4290	0.1566	41.6011
18	177.0782	305.1882	0.4240	0.1542	42.1210
19	177.0610	305.1971	0.4258	0.1273	42.1639
20	177.0654	305.3203	0.4211	0.1299	42.4330
21	177.2330	305.1388	0.4212	0.1184	42.4194
22	177.0714	305.2319	0.4188	0.1301	40.7729
23	177.0949	305.1991	0.4164	0.1221	41.9701
24	177.1479	305.1808	0.4158	0.1174	42.3275
25	177.0338	305.4143	0.4142	0.1258	41.8614
26	177.0397	305.4812	0.4150	0.1262	42.2972
27	177.0394	305.5894	0.4147	0.1160	41.7694
28	177.0389	305.8245	0.4147	0.1092	41.5514
29	176.9966	306.9598	0.4128	0.0969	41.6143
30	177.0095	308.2523	0.4127	0.1110	42.1783
31	176.9899	314.8080	0.4107	0.0983	41.9902
32	177.0404	316.6703	0.4156	0.0831	42.4431

Fig. 14. Comparison of estimated and calculated dispersivity. Run 10.

Section 7: CONCLUSIONS

The approximation for Taylor Dispersion, Equation (3), is sufficiently accurate to be used in a fracture flow model for tracer test analysis.

The slight decline of dispersivity with distance suggests that further studies may find a stricter criterium for the non-dimensional time before tracer front equalization.

Section 8: REFERENCES

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APPENDIX A: MULTIPLEXER BOARD

The multiplexer board contains circuitry which switches through the electrode array, allowing instantaneous voltage measurement at each location. Its design is described by the flow diagram of Fig. A.1.1.

The circuitry is capable of testing 128 electrodes, divided into 8 groups of 16. The computer selects a bank of 16 electrodes and the voltage across each is measured individually using a resistance bridge. The signal is amplified, converted to digital logic, and stored in OUTPUT.DAT for later processing.

Fig. A.1.1. Multiplexer Board Design

APPENDIX B: COMPUTER SCANNING ALGORITHMS

Figure B.1.1 shows a flow diagram describing the computer scanning algorithms used for data acquisition, transmission, curve fitting and plotting.

Fig. B.1.1. Computer Scanning Algorithms

Appendix C: COMPUTER PROGRAMS AND SUBROUTINES

Directory of Lab Control Diskette

Volume in drive B has no label

Directory of B:\

SCAN	BAS	3153	5-10-84	10:26a
INIT	BAS	4779	5-10-84	10:15a
CALTABLE	DAT	128	5-29-84	11:01p
INIT	EXE	27611	5-10-84	10:18a
CURVEFIT	EXE	117428	6-05-84	3:10p
PLOTFIT	EXE	26362	5-30-84	5:02p
TEST	BAS	128	3-01-84	11:44a
PLOTLAB	EXE	24632	5-04-84	11:42a
CALVOLT	BAS	640	5-09-84	5:13p
INIT2	BAS	3200	5-09-84	1:36p
SCAN2	BAS	3200	5-09-84	5:36p
SCAN2	EXE	24889	5-09-84	5:39p
FIXUP	EXE	47668	5-11-84	2:36p
SCAN	EXE	24953	5-11-84	10:39a
DDTEST	BAS	128	3-02-84	3:50p
DACAL	BAS	512	3-06-84	2:20p
FIXUP	FOR	1926	5-11-84	2:34p
PARAMS		101	5-31-84	11:21a
PLOTFIX	BAS	2040	5-10-84	5:23p
PLOTFIT	BAS	3224	5-30-84	5:01p
PLOTFIX	EXE	24632	5-10-84	5:24p
GO		35	5-31-84	9:16a
CAL	DAT	3064	5-28-84	7:44p
23 File(s)			4096 bytes free	

SCAN

```

10 DEF FNRCONV(X)=X+6*INT(X/10)
20 HRS=0
30 MIN=0
40 SEC=0
50 OUT &H719,23
60 OUT &H718,3
70 OUT &H718,129
80 OUT &H719,1
90 OUT &H718,57
100 OUT &H718,31
110 OUT &H718,0
120 OUT &H718,FNRCONV(SEC)
130 OUT &H719,2
140 OUT &H718,57
150 OUT &H718,16
160 OUT &H718,FNRCONV(MIN)
170 OUT &H718,FNRCONV(HRS)
180 OUT &H719,71
190 OUT &H719,9
200 OUT &H718,0
210 OUT &H718,0
220 OUT &H719,10
230 OUT &H718,0
240 OUT &H718,0
250 OUT &H719,39
260 DIM ELEC VOLT(100),YFOLD(100),XFOLD(100)
270 FOR NUM=1 TO 100
280 XFOLD(NUM)=(NUM-1)*19+15: YFOLD(NUM)=130
290 NEXT NUM
300 SIGMAX=2!
310 ON KEY(1) GOSUB 1090
320 KEY OFF
330 OPEN "c:\output.dat" FOR OUTPUT AS #1
335 SCAN=0
340 SCREEN 2
350 CLS
360 PRINTCOL=1
370 TIME0#=TIME#
380 OLDTIME#=TIME#
390 THS=0
400 NUMCYCLE=0
410 ADHIGH=0
420 ADLOW=0
430 ADDRESS=&H710
440 OUT &H71F,&H80
450 OUT &H714,128
460 VOLTAGE=1

```

```

470 VOLTS=5
480 DECIMAL=204.7*VOLTS
490 DECIMAL=INT(DECIMAL): DAHIGH=INT(DECIMAL/256): DALOW=DECIMAL-256*DAHIGH
500 FOR CYCLE=1 TO 2
510 REM alternate voltage
520 REM IF CYCLE=1 THEN DAHIGH=&H7: DALOW=&HFF ELSE DAHIGH=&H8: DALOW=0
530 OUT &H711,DAHIGH
540 OUT &H710,DALOW
550 FOR CIRCUIT=1 TO 2
560 OUT &H71D,CIRCUIT
590 PRINTCOL=1+(CIRCUIT-1)*20
600 FOR ELECTRODE=0 TO 15
610 OUT &H715,ELECTRODE
620 OUT &H716,0
630 IF ELECTRODE=0 THEN GOTO 720
640 ADSIG=256*ADHIGH+ADLOW
650 IF ADSIG>32767 THEN ADSIG=ADSIG-65536!
660 ADVOLT=VOLTAGE*ADSIG/204.8
670 ELECNUM=(CIRCUIT-1)*16+ELECTRODE
680 ELEC VOLT(ELECNUM)=ADVOLT
690 PRINT #1, USING "&_ ###_ ###_ ###.####";OLDTIME$,THS,ELECNUM,ADVOLT
700 REM LOCATE ELECTRODE,PRINTCOL
710 REM PRINT USING "&_ ###_ ###_ ###.####";OLDTIME$,THS,(CIRCUIT-1)*16+ELECTRODE
;ADVOLT
720 IF INP(&H714)<128 THEN GOTO 720
730 ADLOW=INP(&H715)
740 ADHIGH=INP(&H716)
750 NEXT ELECTRODE
760 ADSIG=256*ADHIGH+ADLOW
770 IF ADSIG>32767 THEN ADSIG=ADSIG-65536!
780 ADVOLT=VOLTAGE*ADSIG/204.8
790 ELECNUM=(CIRCUIT-1)*16+16
800 ELEC VOLT(ELECNUM)=ADVOLT
810 PRINT #1, USING "&_ ###_ ###_ ###.####";OLDTIME$,THS,ELECNUM,ADVOLT
820 OLDTIME$=TIME$
830 OUT &H719,163
840 OUT &H719,17
850 THS=INP(&H718)
860 THS=THS-6*INT(THS/16)
870 NEXT CIRCUIT
875 OUT &H71D,0
880 NUMCYCLE=NUMCYCLE+1
890 OLDTIME$=TIME$
900 KEY(1) ON
910 MIN=1:MAX=32:YBASE=130
920 FOR NUM=1 TO 32
930 SIGNAL=ELEC VOLT(NUM)
940 YP=YBASE-60*SIGNAL/SIGMAX
950 XP=(NUM-MIN)*19+15
960 IF NUM<MAX THEN LINE (XPOLD(NUM),YPOLD(NUM))-(XPOLD(NUM+1),YPOLD(NUM+1)),0 :
LINE (XP,YP)-(XPOLD(NUM+1),YPOLD(NUM+1)),1
970 IF NUM>MIN THEN LINE (XPOLD(NUM),YPOLD(NUM))-(XPOLD(NUM-1),YPOLD(NUM-1)),0:
LINE (XP,YP)-(XPOLD(NUM-1),YPOLD(NUM-1)),1

```



```

980 XPOLD(NUM)=XP:YPOLD(NUM)=YP
990 REM IF NUM<17 THEN LOCATE NUM,1 ELSE LOCATE NUM-16,40
1000 REM PRINT NUM-1,SIGNAL
1010 NEXT NUM
1020 LINE (15,YBASE-60)-(624,YBASE-60)
1030 LINE (15,YBASE)-(624,YBASE)
1040 LINE (15,YBASE)-(15,YBASE-60)
1050 LINE (624,YBASE)-(624,YBASE-60)
1055 SCAN=SCAN+1 : IF SCAN>440 THEN GOTO 1090
1060 KEY(1) STOP
1070 NEXT CYCLE
1080 GOTO 500
1090 KEY(1) STOP
1100 CLOSE 1
1110 PRINT "start time",TIME0$,"finish time",OLDTIME$,"No. of cycles",NUMCYCLE
1120 STOP

```

FIXUP

```

COMMON /A/VOLT(32,500)
INTEGER ELEC,ELECN,HR,MIN,SEC,THS,RECNO
DIMENSION TIME(500),CTBL(20),VTBL(20,32)
CALBRT=0
IF (CALBRT.EQ.0) GO TO 200
C Next section skipped if no calibration desired
OPEN (3,FILE='cal.dat',STATUS='OLD',ACCESS='SEQUENTIAL')
READ (3,103) NPTS
103 FORMAT(I2)
DO 10 ELEC=1,32
DO 11 N=1,NPTS
READ(3,102) NDUMMY,VTBL(N,ELEC)
102 FORMAT (I4,11X,F6.4)
CTBL(N)=NDUMMY
11 CONTINUE
10 CONTINUE
END FILE 3
200 OPEN (1,FILE='c:output.dat',status='old',access='direct',
& recl=24,form='formatted')
READ(1,100,REC=1) HR,MIN,SEC,THS,ELEC,SIG
100 FORMAT (I2,1X,I2,1X,I2,2X,I2,2X,I2,2X,F6.4)

```

```

TZERO=3600.*HR+60.*MIN+SEC
DO 1 ELEC=1,32
RECNO=ELEC
I=1
3 READ (1,100,REC=RECNO,END=2) HR,MIN,SEC,THS,ELECN,SIG
IF (ELEC.EQ.1) TIME(I)=3600.*HR+60.*MIN+SEC-TZERO
IF (CALBRT.NE.0) CONCN=CONVRT(SIG,NPTS,ELEC,CTBL,VTBL(1,ELEC))
VOLT(ELEC,I)=SIG
C write (*,'(f7.1,1x,f10.4)') time(i),concn
I=I+1
RECNO=RECNO+32
GO TO 3
2 CONTINUE
WRITE (*,'(I4)') ELEC
1 CONTINUE
END FILE 1
OPEN (2,FILE='a:labfix.dat',status='new',access='sequential')
DO 4 ELEC=1,32
DO 5 II=1,I
WRITE (2,101) TIME(II),VOLT(ELEC,II)
101 FORMAT (F7.1,1X,F10.4)
5 CONTINUE
4 CONTINUE
END FILE 2
STOP
END
FUNCTION CONVRT(SIG,NPTS,ELEC,CTBL,V)
DIMENSION CTBL(20),V(20)
NPTS1=NPTS-1
IF (SIG.GT.V(1)) GO TO 1
DO 2 I=1,NPTS1
IF (SIG.GT.V(I+1)) GO TO 3
2 CONTINUE
SLOPE=1./(V(NPTS)-V(NPTS-1))/(CTBL(NPTS)-CTBL(NPTS-1))
CONVRT=CTBL(NPTS)+SLOPE*(V(NPTS)-SIG)
RETURN
3 SLOPE=(CTBL(I+1)-CTBL(I))/(V(I+1)-V(I))
CONVRT=CTBL(I+1)+SLOPE*(SIG-V(I+1))
RETURN
1 SLOPE=(CTBL(2)-CTBL(1))/(V(2)-V(1))
CONVRT=CTBL(1)+SLOPE*(SIG-V(1))
RETURN
END

```

PLOTFIT

```

10 DIM X(500),Y(500)
11 DIM FILE$(32)
12 DATA "1","2","3","4","5","6","7","8","9","10","11","12","13","14","15","16",
17,"18","19","20","21","22","23","24","25","26","27","28","29","30","31","32"
13 FOR ELEC%=1 TO 32
14 READ FILE$(ELEC%)
15 NEXT ELEC%
20 INPUT "Experiment number";NUMEXP$
30 INPUT "Range of electrodes";MINELEC%,MAXELEC%
40 FOR ELEC%=MINELEC% TO MAXELEC%
50 FILEN$=FILE$(ELEC%)+".dat"
60 TITLE$="Run - "+NUMEXP$+" ; Electrode "+FILE$(ELEC%)
70 OPEN FILEN$ FOR INPUT AS 1
80 INPUT #1,NUMPTS
90 XLOG%=0
100 YLOG%=0
110 PTX=0
120 FOR N=1 TO NUMPTS
130 IF EOF(1) THEN GOTO 180
140 INPUT #1,X(N),Y(N)
150 IF XLOG% THEN X(N)=LOG(X(N))/LOG(10!)
160 IF YLOG% THEN Y(N)=LOG(Y(N))/LOG(10!)
170 NEXT N
180 PRINT "number of data points is ",N
190 N1=N-1
200 YMIN=150!
210 YMAX=320!
220 XMAX=X(N-5)
230 XMIN=X(1)
240 IF XLOG% THEN XMAX=10!^XMAX: XMIN=10!^XMIN ELSE XMAX=XMAX: XMIN=XMIN
250 IF YLOG% THEN YMAX=10!^YMAX: YMIN=10!^YMIN ELSE YMAX=YMAX: YMIN=YMIN
260 PRINT "minimum & maximum x value",XMIN,XMAX
270 PRINT "minimum & maximum y value",YMIN,YMAX
280 XSCRA=9!
290 YSCRA=6!
300 XSCR=XSCRA-.5
310 YSCR=YSCRA-.5
320 PRINT "screen size is",XSCR,"wide, by",YSKR,"high"
330 XDIV=50!
340 YDIV=50!
350 IF XLOG% THEN XDIV=LOG(XDIV)/LOG(10!)
360 IF YLOG% THEN YDIV=LOG(YDIV)/LOG(10!)
370 DEF FNIX(P)=INT((P/XSCRA)*639!)
380 DEF FNIY(P)=199-INT((P/YSKRA)*199!)
390 XWID=XMAX-XMIN
400 YWID=YMAX-YMIN
410 SCREEN 2,0,0
420 KEY OFF
470 CLS
480 KEY OFF
490 KPASS=1
510 LINE (FNIX(1!),FNIY(1!))-(FNIX(1!),FNIY(YSKR))
520 LINE (FNIX(1!),FNIY(1!))-(FNIX(XSCR),FNIY(1!))

```

```

525 LOCATE 1,20;0 : PRINT TITLE$
530 NX=INT(XWID/XDIV)
540 NY=INT(YWID/YDIV)
550 FOR I=0 TO NX
560 PSET (FNIX(1!+I*(XSCR-1!)*XDIV/XWID),FNIY(1!))
570 LINE -STEP(0,199-FNIY(.125))
580 NEXT
590 FOR I=0 TO NY
600 PSET (FNIX(1!),FNIY(1!+I*(YSCR-1!)*YDIV/YWID))
610 LINE -STEP(-FNIX(.125),0)
620 NEXT
630 FOR I=0 TO NX
640 XVAL=XMIN+I*XDIV
650 XPOS=((XVAL-XMIN)/XWID)*(XSCR-1!)+.85
660 IX=INT(80!*XPOS/XSCRA)+1
670 IY=25-INT(12.5/YSCRA)
680 IF IX>80 OR IX<1 OR IY>25 OR IY<1 GOTO 720
690 LOCATE IY,IX
700 IF XLOG% THEN XVAL=10!^XVAL
710 PRINT XVAL
720 NEXT
730 FOR I=0 TO NY
740 YVAL=YMIN+I*YDIV
750 YPOS=((YVAL-YMIN)/YWID)*(YSCR-1!)+1!
760 IY=25-INT(25*YPOS/YSCRA)
770 IX=INT(.4*80!/XSCRA)
780 IF YMAX>1000 THEN IX=IX-1
790 IF YMAX>10000 THEN IX=IX-1
800 IF IX>80 OR IX<1 OR IY>25 OR IY<1 GOTO 840
810 LOCATE IY,IX
820 IF YLOG% THEN YVAL=10!^YVAL
830 PRINT YVAL
840 NEXT
850 LOCATE 1,1
860 IF KPASS=0 THEN KPASS=1: OUT &H3DD,&H20: GOTO 510
880 KPASS=0
890 XP=FNIX(1!+(XSCR-1!)*(X(1)-XMIN)/XWID): YP=FNIY(1!+(YSCR-1!)*(Y(1)-YMIN)/YWI
D)
900 IF PT% THEN CIRCLE (XP,YP),FNIX(.03): PAINT(XP,YP) ELSE PSET(XP,YP)
910 FOR I=2 TO N1
920 XP=FNIX(1!+(XSCR-1!)*(X(I)-XMIN)/XWID): YP=FNIY(1!+(YSCR-1!)*(Y(I)-YMIN)/YWI
D)
930 IF PT% THEN CIRCLE (XP,YP),FNIX(.03): PAINT (XP,YP) ELSE LINE -(XP,YP)
940 NEXT
950 IF KPASS>0 THEN GOTO 1060
960 KPASS=1
970 BEEP
980 PT%=1
1000 INPUT #1,NUMPTS
1010 FOR N=1 TO NUMPTS
1020 IF EOF(1) GOTO 890
1030 INPUT #1,X(N),Y(N)
1040 NEXT N
1050 GOTO 890
1060 BEEP
1070 CLOSE 1
1080 NEXT ELEC%
1090 SCREEN 0,0,0
1100 SYSTEM

```

PLOTFIX

```

20 DIM X(1000),Y(1000)
25 INPUT "ymin,ymax,ydiv";YMIN,YMAX,YDIV
30 CHOICE=0
35 OPEN "a:labfix.dat" FOR INPUT AS 1
40 I=0
45 XOLD=-1!
50 IF EOF(1) THEN GOTO 120
60 INPUT #1,X(I),Y(I)
90 IF X(I)<XOLD THEN GOTO 120
95 XOLD=X(I)
96 I=I+1
100 IF EOF(1) THEN GOTO 120
110 GOTO 50
120 N=I-1
125 CHOICE=CHOICE+1
126 PTX=0
140 XMAX=X(N)
150 XMIN=0
160 XSCRA=9!
170 YSCRA=6!
180 XSCR=XSCRA-.5
190 YSCR=YSCRA-.5
200 XDIV=60
220 IF XLOG% THEN XDIV=LOG(XDIV)/LOG(10!)
230 IF YLOG% THEN YDIV=LOG(YDIV)/LOG(10!)
240 DEF FNIX(P)=INT((P/XSCRA)*639!)
250 DEF FNIY(P)=199-INT((P/YS CRA)*199!)
260 XWID=XMAX-XMIN
270 YWID=YMAX-YMIN
280 SCREEN 2,0,0
290 KEY OFF
300 CLS
310 CLS
320 KEY OFF
325 LOCATE 1,40,0 : PRINT "electrode",CHOICE
330 KPASS=1
340 LINE (FNIX(1!),FNIY(1!))-(FNIX(1!),FNIY(YS CR))
350 LINE (FNIX(1!),FNIY(1!))-(FNIX(XSCR),FNIY(1!))
360 NX=INT(XWID/XDIV)
370 NY=INT(YWID/YDIV)
380 FOR I=0 TO NX
390 PSET (FNIX(1!+I*(XSCR-1!)*XDIV/XWID),FNIY(1!))
400 LINE -STEP(0,199-FNIY(.125))
410 NEXT I
420 FOR I=0 TO NY
430 PSET (FNIX(1!),FNIY(1!+I*(YS CR-1!)*YDIV/YWID))
440 LINE -STEP(-FNIX(.125),0)
450 NEXT I

```

```

460 FOR I=0 TO NX
470 XVAL=XMIN+I*XDIV
480 XPOS=((XVAL-XMIN)/XWID)*(XSCR-1!)+.85
490 IX=INT(80!*XPOS/XSCRA)+1
500 IY=25-INT(12.5/YSCRA)
510 IF IX>80 OR IX<1 OR IY>25 OR IY<1 GOTO 550
520 LOCATE IY,IX
540 PRINT XVAL
550 NEXT I
560 FOR I=0 TO NY
570 YVAL=YMIN+I*YDIV
580 YPOS=((YVAL-YMIN)/YWID)*(YSCR-1!)+1!
590 IY=25-INT(25*YPOS/YSCRA)
600 IX=INT(.6*80!/XSCRA)-2
610 IF IX>80 OR IX<1 OR IY>25 OR IY<1 GOTO 650
620 LOCATE IY,IX
640 PRINT YVAL
650 NEXT I
660 LOCATE 1,1
670 XP=FNIX(1!+(XSCR-1!)*(X(1)-XMIN)/XWID): YP=FNIIY(1!+(YSCR-1!)*(Y(1)-YMIN)/YWI
D)
680 IF PT% THEN CIRCLE (XP,YP),FNIX(.03): PAINT(XP,YP) ELSE PSET(XP,YP)
690 FOR I=2 TO N
700 XP=FNIX(1!+(XSCR-1!)*(X(I)-XMIN)/XWID): YP=FNIIY(1!+(YSCR-1!)*(Y(I)-YMIN)/YWI
D)
710 IF PT% THEN CIRCLE (XP,YP),FNIX(.03): PAINT (XP,YP) ELSE LINE -(XP,YP)
720 NEXT I
730 BEEP
735 IF CHOICE < 32 THEN GOTO 40
3000 CLOSE 1: STOP

```

CURVEFIT

```

C *****
C
C          PROGRAM BEGINS -Gilardi experiment analysis
C
C *****
C *****
C
C          IMPLICIT REAL*8(A-B,D-H,O-Z)
C
C          SET DIMENSIONS FOR VARPRO. BE CAREFUL WHEN SETTING THE
C          DIMENSIONS FOR THE INCIDENCE MATRIX INC. SEE NOTE.
C
C          DIMENSION Y(500),T(500),ALF(3),BETA(2),W(500),A(500,7),
*INC(12,8),NELTRY(32),NGO(32)
C          COMMON XPOS,TZERO
C          COMMON /DUMMY/C(500,6)
C          EXTERNAL ADA
C          CHARACTER*6 CFILE(32)
C          DATA CFILE/'1.dat','2.dat','3.dat','4.dat','5.dat',
@ '6.dat','7.dat','8.dat','9.dat',
1 '10.dat','11.dat','12.dat','13.dat','14.dat','15.dat',
@ '16.dat','17.dat','18.dat','19.dat',
2 '20.dat','21.dat','22.dat','23.dat','24.dat','25.dat',
@ '26.dat','27.dat','28.dat','29.dat',
3 '30.dat','31.dat','32.dat'/
C
C          SET PARAMETERS FOR VARPRO.
C
C          IPLOT=0
C          NMAX = 500
C          IPRINT=50
C
C          READ DATA SEQUENTIAL ORDERING AND
C          PROPER FORMATTING ARE IMPORTANT.
C
C          NL IS THE NUMBER OF NONLINEAR PARAMETERS
C
C          READ (5,311) NL
311  FORMAT (I1)
C          WRITE(6,12) NL
12   FORMAT (1H0,10X,'NUMBER OF NONLINEAR PARAMETERS'//(I3))
C
C          L IS THE NUMBER OF LINEAR PARAMETERS
C
C          L=2
C

```

ESTIMATES OF THE NONLINEAR PARAMETERS

```

C
C
C
C
      POS0=9.8425
      READ (5,310) VELOC,DIFFS,TZERO,DC0,DC1
310   FORMAT (F10.4)
      READ (5,312) NTRIES,(NELTRY(K),K=1,NTRIES)
312   FORMAT (33I2)
      DO 314 K=1,32
314   NGO(K)=1.
      DO 313 K=1,NTRIES
313   NGO(NELTRY(K))=0.
      WRITE (6,20) VELOC,DIFFS,TZERO
20    FORMAT (/, '0 Mean Velocity(cm/sec)   Diffusivity (cm2/sec)'
#      ' Time zero (sec)',/, (5X,F9.5,18X,F13.3,10X,F10.3))
C
C
      LPP2=L+NL+2
C
C
C
C
      N IS THE NUMBER OF OBSERVATIONS
      IV IS THE NUMBER OF INDEPENDENT VARIABLES T
C
C
      IV=1
C
C
C
C
      T IS THE INDEPENDENT VARIABLE
      Y IS THE N-VECTOR OF OBSERVATIONS
C
C
C
      WRITE (7,323)
323   FORMAT(2X,'Elec      Co      Cinj      Velocity ',
1     ' Disp      To',/,',
2     ' cm/sec      cm2/sec      sec') ppm      ppm      ',
      DO 200 NELEC=1,32
      N=0
      TOLD=0.
203   N=N+1
      READ (4,301,END=202) T(N),Y(N)
301   FORMAT (2F10.4)
      IF (TOLD.GT.T(N)) GO TO 202
      TOLD=T(N)
      GO TO 203
202   N=N-5
      IF (NGO(NELEC).GT.0) GO TO 200
      WRITE (6,320) NELEC
320   FORMAT (/,2X,'Electrode number ',I3,'*****')
      XPOS=POS0+5.08*(NELEC-1)
      ALF(1)=VELOC
      ALF(2)=DIFFS
      ALF(3)=TZERO

```


W(I) ARE THE WEIGHTING PARAMETERS

DO 1 I=1,N

W(I)=1.0

IMINF=IDINT(TZERO+XPOS/VELOC)-100

IMIN=IDINT(TZERO)

IF (IMIN.LT.IMINF) IMIN=IMINF

IF (IMIN.GT.N-200) IMIN=N-200

N=N-IMIN

IF (N.GT.200) N=200

DO 201 I=1,N

T(I)=T(IMIN+I)

Y(I)=Y(IMIN+I)

WRITE (6,*) T(I),Y(I)

CONTINUE

CALL VARPRO(L,NL,N,NMAX,LPP2,IV,T,Y,W,ADA,A,
*IPRINT,ALF,BETA,IERR,10)

V0=BETA(1)

V1=BETA(1)+2.*BETA(2)

BPAR=(V1-V0)/DLOG(DC0/DC1)

APAR=V1+BPAR*DLOG(DC1)

DO 211 I=1,N

Y(I)=DEXP((APAR-Y(I))/BPAR)

CALL VARPRO(L,NL,N,NMAX,LPP2,IV,T,Y,W,ADA,A,
*IPRINT,ALF,BETA,IERR,100)

LP1=L+1

CALL ADA (LP1,NL,N,NMAX,LPP2,IV,A,INC,T,ALF,1)

DO 8 I=1,N

C(I,LP1)=0.

DO 9 J=1,L

C(I,J)=BETA(J)*A(I,J)

C(I,LP1)=C(I,LP1)+C(I,J)

CONTINUE

OPEN (1,FILE=CFILE(NELEC),STATUS='NEW',ACCESS='SEQUENTIAL')

WRITE (1,305) N

DO 205 I=1,N

WRITE (1,14) T(I),Y(I)

WRITE (1,305) N

DO 206 I=1,N

WRITE (1,14) T(I),C(I,LP1)

FORMAT (1X,8F10.4)

FORMAT (I6)

END FILE 1

DCMID=BETA(1)+BETA(2)

DO 400 II=1,N

IF (C(II,LP1).GT.SNGL(DCMID)) GO TO 401

TMID=T(II)

CONTINUE

TOTALC=TMID-XPOS/ALF(1)

WRITE (7,302) NELEC,BETA(1),BETA(1)+2.*BETA(2),ALF(1),ALF(2)

,TOTALC

FORMAT (2X,I4,5F12.4)

CONTINUE

STOP

END


```

90      DO 81 I=1,N
        IF (ISEL.EQ.1) A(I,1)=1.0
        A(I,2)=0.
        TTI=T(I)-ALF(3)
        IF (TTI.LE.0.) GO TO 81
        TFUNC=DSQRT(ALF(2)*TTI)
        A(I,2)=DERFC((XPOS-ALF(1)*TTI)/(2.*TFUNC))
1      IF (XPOS*ALF(1)/ALF(2).GT.20.) A(I,2)=A(I,2)+DEXP(ALF
        (1)*XPOS/ALF(2))*DERFC((XPOS+ALF(1)*TTI)/(2.*TFUNC))

```

```

C
C 81      CONTINUE
C
C
C

```

```

C      IF (ISEL.EQ.2) GO TO 200
C
C
C
C
C
C
C

```

```

C 165      DO 170 I=1,N
C
        A(I,4)=0.
        A(I,5)=0.
        A(I,6)=0.
        TTI=T(I)-ALF(3)
        IF (TTI.LE.0.) GO TO 170
        E1=DEXP(-((XPOS-ALF(1)*TTI)**2)/(4.*ALF(2)*TTI))
        E2=DEXP(-((XPOS+ALF(1)*TTI)**2)/(4.*ALF(2)*TTI))
        E3=0.
        IF (ALF(1)*XPOS/ALF(2).LT.20.) E3=DEXP(ALF(1)*XPOS/ALF(2))
        ERF1=DERFC((XPOS+ALF(1)*TTI)/(2.*DSQRT(ALF(2)*TTI)))
        FUNC=2.*DSQRT(TTI*ALF(2)**3)
        A(I,4)=-TPI*E1*(-TTI/(2.*DSQRT(ALF(2)*TTI)))
        IF (E3.GT.0.) A(I,4)=A(I,4)+(XPOS/ALF
1      (2))*E3*ERF1+E3*(-TPI*E2)*TTI/(2.*DSQRT(ALF(2)*TTI))
        A(I,5)=TPI*E1*(XPOS-ALF(1)*TTI)/FUNC
        IF (E3.GT.0.) A(I,5)=A(I,5)-E3*ERF1*ALF(1)*XPOS/
1      ALF(2)**2+E3*TPI*E2*(XPOS+ALF(1)*TTI)/FUNC
        IF (NL.LT.3) GO TO 170
        A(I,6)=-TPI*E1*(XPOS/(2.*DSQRT(ALF(2)*TTI**3))+ALF(1)/
1      FUNC)
        IF (E3.GT.0.) A(I,6)=A(I,6)+E3*(-TPI*E2)
2      *(XPOS/(2.*DSQRT(ALF(1)*TTI**3))-ALF(1)/FUNC)

```

```

C
C 170      CONTINUE
C

```

```

FUNCTION DERFC(Y)
DOUBLE PRECISION      DERFC,Y
DIMENSION              P(5),Q(4),P1(9),Q1(8),P2(6),Q2(5)
DOUBLE PRECISION      P,Q,P1,Q1,P2,Q2,XMIN,XLARGE,SQRPI,X,
*                      RES,XSQ,XNUM,XDEN,XI,XBIG
INTEGER               ISW,I
DATA                  P(1)/113.8641541510502D0/,
*                      P(2)/377.4852376853020D0/,
*                      P(3)/3209.377589138469D0/,
*                      P(4)/.1857777061846032D0/,
*                      P(5)/3.161123743870566D0/
DATA                  Q(1)/244.0246379344442D0/,
*                      Q(2)/1282.616526077372D0/,
*                      Q(3)/2844.236833439171D0/,
*                      Q(4)/23.60129095234412D0/
DATA                  P1(1)/8.883149794388376D0/,
*                      P1(2)/66.11919063714163D0/,
*                      P1(3)/298.6351381974001D0/,
*                      P1(4)/881.9522212417691D0/,
*                      P1(5)/1712.047612634071D0/,
*                      P1(6)/2051.078377826071D0/,
*                      P1(7)/1230.339354797997D0/,
*                      P1(8)/2.153115354744038D-8/,
*                      P1(9)/.5641884969886701D0/
DATA                  Q1(1)/117.6939508913125D0/,
*                      Q1(2)/537.1811018620099D0/,
*                      Q1(3)/1621.389574566690D0/,
*                      Q1(4)/3290.799235733460D0/,
*                      Q1(5)/4362.619090143247D0/,
*                      Q1(6)/3439.367674143722D0/,
*                      Q1(7)/1230.339354803749D0/,
*                      Q1(8)/15.74492611070983D0/
DATA                  P2(1)/-3.603448999498044D-01/,
*                      P2(2)/-1.257817261112292D-01/,
*                      P2(3)/-1.608378514874228D-02/,
*                      P2(4)/-6.587491615298378D-04/,
*                      P2(5)/-1.631538713730210D-02/,
*                      P2(6)/-3.053266349612323D-01/
DATA                  Q2(1)/1.872952849923460D0/,
*                      Q2(2)/5.279051029514284D-01/,
*                      Q2(3)/6.051834131244132D-02/,
*                      Q2(4)/2.335204976268692D-03/,
*                      Q2(5)/2.568520192289822D0/
DATA                  XMIN/1.0D-10/,XLARGE/6.375D0/
DATA                  XBIG/13.3D0/
DATA                  SQRPI/.5641895835477563D0/

```

```

X = Y
ISW = 1
IF (X.GE.0.0D0) GO TO 5
ISW = -1
X = -X
5 IF (X.LT..477D0) GO TO 10
IF (X.LE.4.0D0) GO TO 30
IF (ISW .GT. 0) GO TO 40
IF (X.LT.XLARGE) GO TO 45
RES = 2.0D0
GO TO 70
10 IF (X.LT.XMIN) GO TO 20
XSQ = X*X
XNUM = P(4)*XSQ+P(5)
XDEN = XSQ+Q(4)
DO 15 I = 1,3
    XNUM = XNUM*XSQ+P(I)
    XDEN = XDEN*XSQ+Q(I)
15 CONTINUE
RES = X*XNUM/XDEN
GO TO 25
20 RES = X*P(3)/Q(3)
25 IF (ISW.EQ.-1) RES = -RES
RES = 1.0D0-RES
GO TO 70
30 XSQ = X*X
XNUM = P1(8)*X+P1(9)
XDEN = X+Q1(8)
DO 35 I=1,7
    XNUM = XNUM*X+P1(I)
    XDEN = XDEN*X+Q1(I)
35 CONTINUE
RES = XNUM/XDEN
GO TO 60
40 IF (X.GT.XBIG) GO TO 65
45 XSQ = X*X
XI = 1.0D0/XSQ
XNUM= P2(5)*XI+P2(6)
XDEN = XI+Q2(5)
DO 50 I = 1,4
    XNUM = XNUM*XI+P2(I)
    XDEN = XDEN*XI+Q2(I)
50 CONTINUE
RES = (SQRP1+XI*XNUM/XDEN)/X
60 RES = RES*DEXP(-XSQ)
IF (ISW.EQ.-1) RES = 2.0D0-RES
GO TO 70
65 RES = 0.0D0
70 DERFC = RES
RETURN
END

```

```

SUBROUTINE VARPRO (L, NL, N, NMAX, LPP2, IV, T, Y, W, ADA, A,
X IPRINT, ALF, BETA, IERR, ITMAX)
  DOUBLE PRECISION A(NMAX, LPP2), BETA(L), ALF(NL), T(NMAX, IV),
  2 W(N), Y(N), ACUM, EPS1, GNSTEP, NU, PRJRES, R, RNEW, XNORM,
  2 AS, BS, S
  INTEGER B1, OUTPUT
  LOGICAL SKIP
  EXTERNAL ADA
  DATA EPS1 /1.D-6/, OUTPUT /6/
  IERR = 1
  ITER = 0
  LP1 = L + 1
  B1 = L + 2
  LNL2 = L + NL + 2
  NLP1 = NL + 1
  SKIP = .FALSE.
  MODIT = IPRINT
  IF (IPRINT .LE. 0) MODIT = ITMAX + 2
  NU = 0.
  NU = 1.
10 CALL DPA (L, NL, N, NMAX, LPP2, IV, T, Y, W, ALF, ADA, IERR,
X IPRINT, A, BETA, A(1, LP1), R)
  GNSTEP = 1.0
  ITERIN = 0
  IF (ITER .GT. 0) GO TO 10
  IF (NL .EQ. 0) GO TO 90
  IF (IERR .NE. 1) GO TO 99
  IF (IPRINT .LE. 0) GO TO 10
  WRITE (OUTPUT, 207) ITERIN, R
  WRITE (OUTPUT, 200) NU
10 CALL ORFAC1(NLP1, NMAX, N, L, IPRINT, A(1, B1), PRJRES, IERR)
  IF (IERR .LT. 0) GO TO 99
  IERR = 2
  IF (NU .EQ. 0.) GO TO 30
25 CALL ORFAC2(NLP1, NMAX, NU, A(1, B1))
30 CALL BACSUB (NMAX, NL, A(1, B1), A(1, LNL2))
  DO 35 K = 1, NL
35   A(K, B1) = ALF(K) + A(K, LNL2)
40 CALL DPA (L, NL, N, NMAX, LPP2, IV, T, Y, W, A(1, B1), ADA,
X IERR, IPRINT, A, BETA, A(1, LP1), RNEW)
  IF (IERR .NE. 2) GO TO 99
  ITER = ITER + 1
  ITERIN = ITERIN + 1
  SKIP = MOD(ITER, MODIT) .NE. 0
  IF (SKIP) GO TO 45
  WRITE (OUTPUT, 203) ITER
  WRITE (OUTPUT, 216) (A(K, B1), K = 1, NL)
  WRITE (OUTPUT, 207) ITERIN, RNEW
45 IF (ITER .LT. ITMAX) GO TO 50
  IERR = -1
  CALL VARERR (IPRINT, IERR, 1)
  GO TO 95
50 IF (RNEW - R .LT. EPS1*(R + 1.D0)) GO TO 75
  IF (NU .NE. 0.) GO TO 60
  GNSTEP = 0.5*GNSTEP
  IF (GNSTEP .LT. EPS1) GO TO 95
  DO 55 K = 1, NL

```

```

55      A(K, B1) = ALF(K) + GNSTEP*A(K, LNL2)
      GO TO 40
60      NU = 1.5*NU
      IF (.NOT. SKIP) WRITE (OUTPUT, 206) NU
      IF (NU .LE. 100.) GO TO 65
      IERR = -2
      CALL VARERR (IPRINT, IERR, 1)
      GO TO 95
65      DO 70 K = 1, NL
      KSUB = LP1 + K
      DO 70 J = K, NLP1
      JSUB = LP1 + J
      ISUB = NLP1 + J
70      A(K, JSUB) = A(ISUB, KSUB)
      GO TO 25
75 R = RNEW
      DO 80 K = 1, NL
80      ALF(K) = A(K, B1)
      ACUM = GNSTEP*XNORM(NL, A(1, LNL2))/XNORM(NL, ALF)
      IF (ITERIN .EQ. 1) NU = 0.5*NU
      IF (SKIP) GO TO 85
      WRITE (OUTPUT, 200) NU
      WRITE (OUTPUT, 208) ACUM
85 IERR = 3
      IF (PRJRES .GT. EPS1*(R + 1.DO)) GO TO 5
90 IERR = ITER
95 IF (NL .GT. 0) CALL DPA(L, NL, N, NMAX, LPP2, IV, T, Y, W, ALF,
X ADA, 4, IPRINT, A, BETA, A(1, LP1), R)
      CALL POSTPR(L, NL, N, NMAX, LNL2, EPS1, R, IPRINT, ALF, W, A,
X A(1, LP1), BETA, IERR)
99 RETURN
200 FORMAT (9H      NU =, E15.7)
203 FORMAT (12H0  ITERATION, I4, 24H      NONLINEAR PARAMETERS)
206 FORMAT (25H      STEP RETRACTED, NU =, E15.7)
207 FORMAT (1H0, I5, 20H  NORM OF RESIDUAL =, E15.7)
208 FORMAT (34H      NORM(DELTA-ALF) / NORM(ALF) =, E12.3)
216 FORMAT (1H0, 7E15.7)
      END

```

C

```

SUBROUTINE ORFAC1(NLP1, NMAX, N, L, IPRINT, B, PRJRES, IERR)
DOUBLE PRECISION ACUM, ALPHA, B(NMAX, NLP1), BETA, DSIGN, PRJRES,
X U, XNORM, AS, BS, S

```

C

```

      NL = NLP1 - 1
      NL23 = 2*NL + 3
      LP1 = L + 1
      DO 30 K = 1, NL
      LPK = L + K
      ALPHA = DSIGN(XNORM(N+1-LPK, B(LPK, K)), B(LPK, K))
      U = B(LPK, K) + ALPHA
      B(LPK, K) = U
      BETA = ALPHA * U
      IF (ALPHA .NE. 0.0) GO TO 13
      IERR = -8
      CALL VARERR (IPRINT, IERR, LP1 + K)

```

```

      GO TO 99
13    KP1 = K + 1
      DO 25 J = KP1, NLP1
        ACUM = 0.0
        DO 20 I = LPK, N
          ACUM = ACUM + B(I, K) * B(I, J)
          ACUM = ACUM / BETA
          DO 25 I = LPK, N
            B(I, J) = B(I, J) - B(I, K) * ACUM
30    B(LPK, K) = -ALPHA
      PRJRES = XNORM(NL, B(LP1, NLP1))
      IF (IERR .EQ. 4) GO TO 99
      DO 50 K = 1, NL
        LPK = L + K
        DO 40 J = K, NLP1
          JSUB = NLP1 + J
          B(K, J) = B(LPK, J)
40    B(JSUB, K) = B(LPK, J)
50    B(NL23, K) = XNORM(K, B(LP1, K))
99  RETURN
      END

```

C

```

      SUBROUTINE ORFAC2(NLP1, NMAX, NU, B)
      DOUBLE PRECISION ACUM, ALPHA, B(NMAX, NLP1), BETA, DSIGN, NU, U,
X  XNORM, AS, BS, S
      NL = NLP1 - 1
      NL2 = 2*NL
      NL23 = NL2 + 3
      DO 30 K = 1, NL
        KP1 = K + 1
        NLPK = NL + K
        NLPKM1 = NLPK - 1
        B(NLPK, K) = NU * B(NL23, K)
        B(NL, K) = B(K, K)
        ALPHA = DSIGN(XNORM(K+1, B(NL, K)), B(K, K))
        U = B(K, K) + ALPHA
        BETA = ALPHA * U
        B(K, K) = -ALPHA
        DO 30 J = KP1, NLP1
          B(NLPK, J) = 0.
          ACUM = U * B(K, J)
          DO 20 I = NLP1, NLPKM1
            ACUM = ACUM + B(I, K) * B(I, J)
            ACUM = ACUM / BETA
            B(K, J) = B(K, J) - U * ACUM
          DO 30 I = NLP1, NLPK
            B(I, J) = B(I, J) - B(I, K) * ACUM
20    ACUM = ACUM + B(I, K) * B(I, J)
            ACUM = ACUM / BETA
            B(K, J) = B(K, J) - U * ACUM
          DO 30 I = NLP1, NLPK
            B(I, J) = B(I, J) - B(I, K) * ACUM
30    RETURN
      END

```

C

```

      SUBROUTINE DPA (L, NL, N, NMAX, LPP2, IV, T, Y, W, ALF, ADA, ISEL,
X  IPRINT, A, U, R, RNORM)
      DOUBLE PRECISION A(NMAX, LPP2), ALF(NL), T(NMAX, IV), W(N), Y(N),
X  ACUM, ALPHA, BETA, RNORM, DSIGN, DSQRT, SAVE, R(N), U(L), XNORM,
X  AS, BS, S
      INTEGER FIRSTC, FIRSTR, INC(12, 8)
      LOGICAL NOWATE, PHILP1
      EXTERNAL ADA

```



```

IF (ISEL .NE. 1) GO TO 3
  LP1 = L + 1
  LNL2 = L + 2 + NL
  LP2 = L + 2
  LPP1 = LPP2 - 1
  FIRSTC = 1
  LASTC = LPP1
  FIRSTR = LP1
  CALL INIT(L, NL, N, NMAX, LPP2, IV, T, W, ALF, ADA, ISEL,
X   IPRINT, A, INC, NCON, NCONP1, PHILP1, NOWATE)
  IF (ISEL .NE. 1) GO TO 99
  GO TO 30
3  CALL ADA (LP1, NL, N, NMAX, LPP2, IV, A, INC, T, ALF, MINO(ISEL, 3))
  IF (ISEL .EQ. 2) GO TO 6
  FIRSTC = LP2
  LASTC = LPP1
  FIRSTR = (4 - ISEL)*L + 1
  GO TO 50
6  FIRSTC = NCONP1
  LASTC = LP1
  IF (NCON .EQ. 0) GO TO 30
  IF (A(1, NCON) .EQ. SAVE) GO TO 30
  ISEL = -7
  CALL VARERR (IPRINT, ISEL, NCON)
  GO TO 99
30 IF (PHILP1) GO TO 40
  DO 35 I = 1, N
35   R(I) = Y(I)
  GO TO 50
40   DO 45 I = 1, N
45   R(I) = Y(I) - R(I)
50 IF (NOWATE) GO TO 58
  DO 55 I = 1, N
  ACUM = W(I)
  DO 55 J = FIRSTC, LASTC
55   A(I, J) = A(I, J) * ACUM
58 IF (L .EQ. 0) GO TO 75
  DO 70 K = 1, L
  KP1 = K + 1
  IF (ISEL .GE. 3 .OR. (ISEL .EQ. 2 .AND. K .LT. NCONP1)) GO TO 66
  ALPHA = DSIGN(XNORM(N+1-K, A(K, K)), A(K, K))
  U(K) = A(K, K) + ALPHA
  A(K, K) = -ALPHA
  FIRSTC = KP1
  IF (ALPHA .NE. 0.0) GO TO 66
  ISEL = -8
  CALL VARERR (IPRINT, ISEL, K)
  GO TO 99
66   BETA = -A(K, K) * U(K)
  DO 70 J = FIRSTC, LASTC
  ACUM = U(K)*A(K, J)
  DO 68 I = KP1, N
68   ACUM = ACUM + A(I, K)*A(I, J)
  ACUM = ACUM / BETA
  A(K, J) = A(K, J) - U(K)*ACUM
  DO 70 I = KP1, N

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```

70      A(I, J) = A(I, J) - A(I, K)*ACUM
75 IF (ISEL .GE. 3) GO TO 85
   RNORM = XNORM(N-L, R(LP1))
   IF (ISEL .EQ. 2) GO TO 99
   IF (NCON .GT. 0) SAVE = A(1, NCON)
85 IF (L .GT. 0) CALL BACSUB (NMAX, L, A, R)
   DO 95 I = FIRSTR, N
     IF (L .EQ. NCON) GO TO 95
     M = LP1
     DO 90 K = 1, NL
       ACUM = 0.
       DO 88 J = NCONP1, L
         IF (INC(K, J) .EQ. 0) GO TO 88
         M = M + 1
         ACUM = ACUM + A(I, M) * R(J)
88      CONTINUE
       KSUB = LP1 + K
       IF (INC(K, LP1) .EQ. 0) GO TO 90
       M = M + 1
       ACUM = ACUM + A(I, M)
90      A(I, KSUB) = ACUM
95      A(I, LNL2) = R(I)
99 RETURN
   END

```

C

```

SUBROUTINE INIT(L, NL, N, NMAX, LPP2, IV, T, W, ALF, ADA, ISEL,
X IPRINT, A, INC, NCON, NCONP1, PHILP1, NOWATE)
  DOUBLE PRECISION A(NMAX, LPP2), ALF(NL), T(NMAX, IV), W(N),
X DSQRT, AS, BS, S
  INTEGER OUTPUT, P, INC(12, 8)
  LOGICAL NOWATE, PHILP1
  EXTERNAL ADA
  DATA OUTPUT /6/
  LP1 = L + 1
  LNL2 = L + 2 + NL
  IF (L .GE. 0 .AND. NL .GE. 0 .AND. L+NL .LT. N .AND. LNL2 .LE.
X LPP2 .AND. 2*NL + 3 .LE. NMAX .AND. N .LE. NMAX .AND.
X IV .GT. 0 .AND. .NOT. (NL .EQ. 0 .AND. L .EQ. 0)) GO TO 1
  ISEL = -4
  CALL VARERR (IPRINT, ISEL, 1)
  GO TO 99
1 IF (L .EQ. 0 .OR. NL .EQ. 0) GO TO 3
  DO 2 J = 1, LP1
    DO 2 K = 1, NL
2      INC(K, J) = 0
3 CALL ADA (LP1, NL, N, NMAX, LPP2, IV, A, INC, T, ALF, ISEL)
  NOWATE = .TRUE.
  DO 9 I = 1, N
    NOWATE = NOWATE .AND. (W(I) .EQ. 1.0)
    IF (W(I) .GE. 0.) GO TO 9
    ISEL = -6
    CALL VARERR (IPRINT, ISEL, I)
    GO TO 99
9   W(I) = DSQRT(W(I))
  NCON = L
  NCONP1 = LP1
  PHILP1 = L .EQ. 0
  IF (PHILP1 .OR. NL .EQ. 0) GO TO 99

```

```

P = 0
DO 11 J = 1, LP1
  IF (P .EQ. 0) NCONP1 = J
  DO 11 K = 1, NL
    INCKJ = INC(K, J)
    IF (INCKJ .NE. 0 .AND. INCKJ .NE. 1) GO TO 15
    IF (INCKJ .EQ. 1) P = P + 1
11  CONTINUE
NCON = NCONP1 - 1
IF (IPRINT .GE. 0) WRITE (OUTPUT, 210) NCON
IF (L+P+2 .EQ. LPP2) GO TO 20
15 ISEL = -5
CALL VARERR (IPRINT, ISEL, 1)
GO TO 99
20 DO 25 K = 1, NL
25  IF (INC(K, LP1) .EQ. 1) PHILP1 = .TRUE.
99 RETURN
210 FORMAT (33H0  NUMBER OF CONSTANT FUNCTIONS =, I4 /)
END
SUBROUTINE BACSUB (NMAX, N, A, X)
DOUBLE PRECISION A(NMAX, N), X(N), ACUM, AS, BS, S
X(N) = X(N) / A(N, N)
IF (N .EQ. 1) GO TO 30
NP1 = N + 1
DO 20 IBACK = 2, N
  I = NP1 - IBACK
  IP1 = I + 1
  ACUM = X(I)
  DO 10 J = IP1, N
10  ACUM = ACUM - A(I, J)*X(J)
20  X(I) = ACUM / A(I, I)
30 RETURN
END
SUBROUTINE POSTPR(L, NL, N, NMAX, LNL2, EPS, RNORM, IPRINT, ALF,
X W, A, R, U, IERR)
DOUBLE PRECISION A(NMAX, LNL2), ALF(NL), R(N), U(L), W(N), ACUM,
X EPS, PRJRES, RNORM, SAVE, DABS, AS, BS, S
INTEGER OUTPUT
DATA OUTPUT /6/
LP1 = L + 1
LPNL = LNL2 - 2
LNL1 = LPNL + 1
DO 10 I = 1, N
10  W(I) = W(I)**2
  IF (L .EQ. 0) GO TO 30
  DO 25 KBACK = 1, L
    K = LP1 - KBACK
    KP1 = K + 1
    ACUM = 0.
    DO 20 I = KP1, N
20  ACUM = ACUM + A(I, K) * R(I)
    SAVE = R(K)
    R(K) = ACUM / A(K, K)
    ACUM = -ACUM / (U(K) * A(K, K))
    U(K) = SAVE
    DO 25 I = KP1, N

```

```

25      R(I) = R(I) - A(I, K)*ACUM
30 ACUM = 0.
   DO 35 I = 1, N
35      ACUM = ACUM + R(I)
   SAVE = ACUM / N
   IF (NL .EQ. 0) GO TO 45
   CALL ORFAC1(NL+1, NMAX, N, L, IPRINT, A(1, L+2), PRJRES, 4)
   DO 40 I = 1, N
       A(I, LNL2) = R(I)
       DO 40 K = LP1, LNL1
40      A(I, K) = A(I, K+1)
45 A(1, LNL2) = RNORM
   ACUM = RNORM*RNORM/(N - L - NL)
   A(2, LNL2) = ACUM
   CALL COV(NMAX, LPNL, ACUM, A)
   IF (IPRINT .LT. 0) GO TO 99
   WRITE (OUTPUT, 209)
   IF (L .GT. 0) WRITE (OUTPUT, 210) (U(J), J = 1, L)
   IF (NL .GT. 0) WRITE (OUTPUT, 211) (ALF(K), K = 1, NL)
   WRITE (OUTPUT, 214) RNORM, SAVE, ACUM
   IF (DABS(SAVE) .GT. EPS) WRITE (OUTPUT, 215)
   WRITE (OUTPUT, 209)
99 RETURN
209 FORMAT (1H0, 50(1H'))
210 FORMAT (20H0 LINEAR PARAMETERS // (7E15.7))
211 FORMAT (23H0 NONLINEAR PARAMETERS // (7E15.7))
214 FORMAT (21H0 NORM OF RESIDUAL =, E15.7, 33H EXPECTED ERROR OF O
      XERVATIONS =, E15.7, / 39H ESTIMATED VARIANCE OF OBSERVATIONS =
      X E15.7 )
215 FORMAT (95H WARNING -- EXPECTED ERROR OF OBSERVATIONS IS NOT ZE
      X. COVARIANCE MATRIX MAY BE MEANINGLESS. /)
      END
      SUBROUTINE COV(NMAX, N, SIGMA2, A)
      DOUBLE PRECISION A(NMAX, N), SUM, SIGMA2, AS, BS, S
C
      DO 10 J = 1, N
10      A(J, J) = 1./A(J, J)
      IF (N .EQ. 1) GO TO 70
      NM1 = N - 1
      DO 60 I = 1, NM1
          IP1 = I + 1
          DO 60 J = IP1, N
              JM1 = J - 1
              SUM = 0.
              DO 50 M = I, JM1
50          SUM = SUM + A(I, M) * A(M, J)
60          A(I, J) = -SUM * A(J, J)
70      DO 90 I = 1, N
          DO 90 J = I, N
              SUM = 0.
              DO 80 M = J, N
80          SUM = SUM + A(I, M) * A(J, M)
              SUM = SUM * SIGMA2
              A(I, J) = SUM
90          A(J, I) = SUM
      RETURN
      END

```

```

SUBROUTINE VARERR (IPRINT, IERR, K)
  DOUBLE PRECISION AS,BS,S
  INTEGER ERRNO, OUTPUT
  DATA OUTPUT /6/
  IF (IPRINT .LT. 0) GO TO 99
  ERRNO = IABS(IERR)
  GO TO (1, 2, 99, 4, 5, 6, 7, 8), ERRNO
1  WRITE (OUTPUT, 101)
  GO TO 99
2  WRITE (OUTPUT, 102)
  GO TO 99
4  WRITE (OUTPUT, 104)
  GO TO 99
5  WRITE (OUTPUT, 105)
  GO TO 99
6  WRITE (OUTPUT, 106) K
  GO TO 99
7  WRITE (OUTPUT, 107) K
  GO TO 99
8  WRITE (OUTPUT, 108) K
99 RETURN
101 FORMAT (46H0  PROBLEM TERMINATED FOR EXCESSIVE ITERATIONS //)
102 FORMAT (49H0  PROBLEM TERMINATED BECAUSE OF ILL-CONDITIONING //)
104 FORMAT (/ 50H INPUT ERROR IN PARAMETER L, NL, N, LPP2, OR NMAX. /)
105 FORMAT (68H0  ERROR -- INC MATRIX IMPROPERLY SPECIFIED, OR DISAGRE
  YES WITH LPP2. /)
106 FORMAT (19H0  ERROR -- WEIGHT(, I4, 14H) IS NEGATIVE. /)
107 FORMAT (28H0  ERROR -- CONSTANT COLUMN , I3, 37H MUST BE COMPUTED
  XONLY WHEN ISEL = 1. /)
108 FORMAT (33H0  CATASTROPHIC FAILURE -- COLUMN , I4, 28H IS ZERO, SE
  XE DOCUMENTATION. /)
  END
  DOUBLE PRECISION FUNCTION XNORM(N, X)
  DOUBLE PRECISION X(N), RMAX, SUM, TERM, DABS, DSQRT
C      FIND LARGEST (IN ABSOLUTE VALUE) ELEMENT
  RMAX = 0.
  DO 10 I = 1, N
    IF (DABS(X(I)) .GT. RMAX) RMAX = DABS(X(I))
10  CONTINUE
  SUM = 0.
  IF (RMAX .EQ. 0.) GO TO 30
  DO 20 I = 1, N
    TERM = 0.
    IF (RMAX + DABS(X(I)) .NE. RMAX) TERM = X(I)/RMAX
20  SUM = SUM + TERM*TERM
30 XNORM = RMAX*DSQRT(SUM)
99 RETURN
  END

```

APPENDIX E: ELECTRODE MOUNTING PROCEDURE

1. Drill .250 in. hole in plate.
2. Measure electrode O.D. with micrometer.
3. Using hand reamer with block to keep it aligned, ream hole so that the electrode can be press fit (if the fit is too tight, the electrode will deform and be wasted).
4. Clean electrode and hole with acetone.
5. Using the silver disc on the top of the electrode and the rubber hammer, tap the electrode part way into the plate.
6. Apply a bead of loctite cement around the electrode O.D.,
7. Tap electrode until it is flush with plate (+ .0015 in.). A dial indicator can be used to check this tolerance.
8. Wipe clean with acetone.

APPENDIX F: MATERIALS SUPPLIERS

The cast aluminum plate was purchased from Castle Metals, San Francisco, 321-2500.

The plate was anodized at Sanford Metal Processing, Menlo Park, 327-5172.

The float glass was purchased at Acme Glass, Palo Alto, 321-7781.

If a thicker, pyrex glass is desired, try Heussner Optics, Santa Clara, 988-4214.

The brass parts used to construct the electrodes were purchased at Don's Hobby shop, Menlo Park, 322-7176.

The electrodes were gold plated at Hansen Labs, on campus.