SEISMIC DESIGN TECHNOLOGY FOR BREEDER REACTOR STRUCTURES

Volume 3: Special Topics in Reactor Structures

AGBABIAN ASSOCIATES
El Segundo, CA 90245

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SEISMIC DESIGN TECHNOLOGY FOR BREEDER REACTOR STRUCTURES

Volume 3:

Special Topics in
Reactor Structures

April 1983

Edited by:
D.P. Reddy

AGBABIAN ASSOCIATES

250 North Nash Street

P.O. Box 956

El Segundo, CA 90245-0956

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PREFACE

In 1974 Agbabian Associates embarked on a project to develop and update seismic design criteria and analysis methods as part of the national long-range program on Breeder Reactor Seismic Technology. This work was initiated by the Division of Reactor Development and Demonstration of the Energy Research and Development Administration. The work was continued under contract with the Department of Energy (DOE) and under contract with Union Carbide Corporation. The Technical Monitors for the project were C. Chester Bigelow for DOE and Claude Pugh and Richard Gwaltney for Union Carbide. The Project Managers at Agbabian Associates were Dr. George A. Young and Dr. Damoder P. Reddy.

The purpose of the study was to identify the problem areas associated with the application of seismic hazard assessment procedures and to establish guidelines for design and analysis of earthquake-resistant breeder reactor structures, equipment, and piping. The result of these research efforts was a series of thirty topical reports covering various topics related to the overall objective; these reports were produced over the span of years 1975-82.

The present volume is part of a subsequent shorter series that condenses the information from the original volumes into four volumes. There are three reasons for this condensed version. The first is that the original volumes were published on a limited distribution basis, and it is intended that the information published in the present volumes be available for a wide distribution and greater accessibility. Second, the information in the new, shorter form will be easier to use and therefore will find wider reception and greater utility in this way. Third, since much of the information contained in the topical reports has general application, the text of these four

volumes is prepared to serve as a useful reference on many general topics in the analysis and design of earthquake-resistant nuclear facilities.

The general title for the four volumes is Seismic Design Technology for Breeder Reactor Structures, and the volumes are entitled:

- Vol. 1. Special Topics in Earthquake Ground Motion
- Vol. 2. Special Topics in Soil/Structure Interaction
- Vol. 3. Special Topics in Reactor Structures
- Vol. 4. Special Topics in Piping and Equipment

The present volume, Volume 3, consists of six chapters which cover these respective topics pertaining to nuclear power plants:

<u>Chapter</u>	<u>Topic</u>
1	Seismic analysis techniques for structures
2	Equivalent viscous damping for seismic analyses
3	A reliability-based investigation of design factors
4	Seismic design verification of structures
5	Determination of equivalent response cycles for use in earthquake-induced fatigue analysis
6	The feasibility of seismic isolation of structures

These chapters are based on six topical reports SAN/1011-110, SAN/1011-103, SAN/1011-115, SAN/1011-117, SAN/1011-106, and SAN/1011-121.

Technical contributions to these six topical reports were made by M.S. Agbabian, A. Der-Kiureghian, L. Gebhardt, D.P. Reddy, H.S. Ts'ao, S.D. Werner, and G.A. Young. The four volumes were produced under the general direction of M.S. Agbabian; technical

editor was J.W. Athey. While preparing these volumes, Agbabian Associates received guidance from C. Chester Bigelow and Richard Gwaltney. It is hoped that these volumes will be a useful reference on topics that are essential for the safe design of nuclear facilities.

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CHAPTER 1

ANALYSIS TECHNIQUES

1.1 INTRODUCTION

1.1.1 SEISMIC CATEGORIES

Nuclear power plant structures, systems, and components must be classified for design and safety analysis into one of three seismic categories based on their criticality with regard to reactor operation and safety. The three seismic categories defined in Standard RDT F-9-2T are as follows:

Category I: Structures, systems, and components that are designed to remain functional for the Safe Shutdown Earthquake (SSE) vibratory ground motions. For reactor facilities this includes, but is not limited to, those structures, systems, and components which are necessary to assure the integrity of the reactor coolant pressure boundary; are necessary to provide the capability to shut down the reactor and maintain it in a safe shutdown condition; and are necessary to provide the capability to prevent or mitigate the consequences of accidents which could result in potential offsite radioactive exposures comparable to the guideline exposures in the Code of Federal Regulations, 10 CFR 100. (A more complete definition of Category I items is provided in Regulatory Guide 1.29).

Category II: Structures, systems, and components that are required to permit continued reactor operation, but are not included in Category I. This category may include items selected as requiring protection against reasonably expected earthquakes so as to protect plant investment and to assure continuance of priority programs in a facility.

Category III: Structures, systems, and components that are not included in Categories I or II, but are essential for maintaining support of normal plant operations.

1.1.2 ANALYSIS REQUIREMENTS

Federal regulatory codes, regulatory guides and Standard RDT-F-9-2T do not exclude or specify any method of seismic analysis for nuclear power plant structures, systems, and components. In fact, experimental verification can be used in lieu of analysis. Experimental procedures, however, are normally limited to small equipment items for practical reasons. When analytical methods are used to ensure that the required functions for Category I and II items are maintained, a dynamic analysis must be used except where it can be demonstrated that an equivalent static load method provides adequate conservatism. For large structures, dynamic analyses are nearly always required. Items falling in Category II can be designed in accordance with applicable national standards and codes, such as the Uniform Building Code and the ASME Boiler and Pressure Vessel Code which normally permit seismic analyses using equivalent static load techniques. The procedures presented here, therefore, are applicable to Category I and II structures for which dynamic analyses are selected.

1.1.2.1 Mathematical Models

Dynamic seismic analyses of nuclear power plant structures, systems, and components are usually based on several mathematical models. The primary model treats the mass, stiffness, and damping characteristics of the major Category I structures and, if essential, the soil-structure interaction effects. Masses of large

equipment items that cannot be justifiably uncoupled from the structure are also included. For complex structures, secondary models may be required to provide more detailed structure response. Separate mathematical models are also required to predict the dynamic response of equipment and piping. It should be noted that RDT F-9-2T (Sec. 6.2.1) refers to the primary model as the *supporting* model and the secondary models as the *supported* models. The Standard Review Plan (SRP), Section 3.7.2, refers to these as the *system* model and the *subsystem* models, respectively (USNRC, 1975). Procedures for decoupling primary and secondary models are given in RDT F-9-2T and the SRP. Only analyses based on primary models will be considered here.

Multiple-degree-of-freedom models are required to adequately represent the seismic response of nuclear power plant structures. Also rotation and translation about all principal axes must be considered at the foundation, or support level, when soil-structure interaction effects are considered. Damping characteristics of the structural and foundation materials must be included in the model analysis. Requirements of the mathematical models are given in RDT F-9-2T and related commentary is provided in the SRP.

Free-field earthquake ground motions are used as dynamic input to the primary model of the structures. Three orthogonal components of ground motion, two horizontal and one vertical, are considered. Each component of free-field ground motion is basically defined by response spectra (see RG 1.60), but time-motion records are also defined that give comparable spectra for each component. Seismic input for the primary model is discussed in Section 1.2.5.

1.1.2.2 Methods of Analysis

Dynamic seismic analyses may be performed using either time-motion or response spectrum methods. Although not required, it is general practice to provide both a response spectrum and a time-motion analysis using the primary model. The response spectrum analysis is usually done at an early stage to provide information for designing the structures. However, floor response spectra, which are required for input to the secondary equipment and piping models, can only be obtained from a time-history analysis. Therefore, at a later date, a time-history analysis is made using the primary model to provide dynamic input for the equipment and piping analysis.

Both discrete mass and finite element methods can be used to formulate the primary model (combinations of the two methods may also be used). In the former, the mass of the structure is given a realistic geometric distribution (i.e., location) by dividing the total mass into a series of discrete masses which are then connected by linear elastic elements (springs) having the stiffness characteristics of the structure. If soil-structure interaction effects are to be considered, the soil continuum is represented by equivalent elastic rotational and translational springs and by viscous dampers. The finite element method utilizes a system of discrete structural and soil continuum elements interconnected at their node points to represent the soil-structure system. The analysis technique satisfies time-dependent force equilibrium requirements at each node point and the stress-strain characteristics of each element.

1.2 FUNDAMENTAL RELATIONSHIPS

A dynamic seismic analysis can be defined as one that adequately accounts for the variation in magnitudes of forces and motions with time during the total duration of the postulated

ground motion, for the distribution of mass and stiffness properties of the system, and for the various forms of energy dissipation in the system. Basic to a dynamic analysis is the preparation of a mathematical model that represents the structure to be analyzed. Two different methods generally adopted in the preparation of the mathematical model are the discrete mass and the finite element methods. Formulation procedures applicable to either of these two methods are presented here.

1.2.1 EQUATIONS OF MOTION

Strictly speaking, all structures have an infinite number of dynamic degrees of freedom. However, the mathematical model that represents a given structure necessarily must consist of a finite number of degrees of freedom. Actually, a mathematical model with a finite number of node points is selected to represent the structure. Each node point may have up to six degrees of freedom, three translations and three rotations.

The selection of the nodes and the corresponding degrees of freedom greatly depends on the degree of complexity of the structure, the characteristics of ground motion, and the precision with which the response calculations are to be made. However, the judgment of the analyst plays the most important part in the selection of the nodes and the corresponding degrees of freedom.

If the mathematical model consisting of a total of n degrees of freedom is subjected to ground motion, the equations of motion for the system can be expressed in matrix form as

$$[M]\ddot{\{X\}} + [C]\dot{\{X\}} + [K]\{X\} = -[M][L]\ddot{S}(t) \quad (1-1)$$

where

$\ddot{\{X\}}$ $\dot{\{X\}}$ $\{X\}$ = Vectors of relative accelerations, velocities and displacements, respectively, at time t

$[M]$	= Mass matrix
$[C]$	= Damping matrix
$[K]$	= Stiffness matrix
$\{L\}$	= Vector governing the direction of excitation
$\ddot{S}(t)$	= Time history of ground or support motion

The square matrices $[M]$, $[C]$, and $[K]$ are of the order n by n .

Nuclear power plant facilities should be analyzed for three components (two horizontal and one vertical) of the ground motion. If all components of the ground motion are considered simultaneously, $\{L\}$ in Equation 1-1 consists of three columns each corresponding to one component of the support motion; in addition $\ddot{S}(t)$ is taken to be a row vector consisting of the three time histories of the support motion. For convenience, however, seismic analyses of nuclear facilities are frequently performed by considering one earthquake direction at a time, thus involving three separate analyses. The most adverse condition may result from the components acting alone, or from two or three components acting simultaneously. Since inelastic behavior is usually not considered in the analysis of nuclear power plant structures, the sum of the responses from the separate analyses should be equal, in principle, to the response due to input in three directions simultaneously. Usually, different mathematical models are chosen for each direction to take advantage of the fact that, for most cases, interaction of motions between the three directions is negligible. This leads to three simpler but different models. For a unidirectional analysis, $\{L\}$ is a column vector consisting of ones and zeros to properly select inertia forces in the direction being considered.

Upon solving Equation 1-1, the relative accelerations of the mass points are obtained. The absolute accelerations of these nodes, which may be necessary for the design of

certain equipment, can be obtained by the relation

$$\{\ddot{Z}\} = \{\ddot{X}\} + \ddot{S}(t) \quad (1-2)$$

where

$\{\ddot{Z}\}$ = Absolute acceleration vector at time t

1.2.2 MASS MATRIX

The mass matrix in Equation 1-1 represents the distribution of the inertia properties of the structural model. Two different methods are available for obtaining the mass matrix: the lumped mass and consistent mass formulations. Rotatory inertia has an important effect on the response of the structure. Formulation of the mass matrix for a finite element model of the structure is elaborated on in Section 1.3.5.

1.2.2.1 Diagonal Mass Matrix and Consistent Mass Matrix

If the lumped mass method is used to model a nuclear reactor structure, the mass matrix $[M]$ in Equation 1-1 is a diagonal matrix representing the mass of the facility to be concentrated at the node points. In the finite element method, the resulting mass matrix may contain off-diagonal terms. A diagonal mass matrix will result for the finite element method if the mass of an element is simply "lumped" at the corner nodes of the element. However, if the so-called consistent mass concept (Archer, 1963) is adopted, the resulting mass matrix will be nondiagonal, which indicates a coupling among the various mass points. In the consistent mass concept, the mass coefficients are made consistent with the stiffness coefficients by using the same element displacement functions for both the mass and stiffness matrices. Although the consistent mass approach has been shown to provide a better approximation of the inertia forces in the structure (Clough, 1969), it usually does not result in significant changes in the response. For example, Clough and Bathe (1972) computed the first twenty modes

of an arch dam by both consistent and lumped mass assumptions (Fig. 1-1). For this particular example, the consistent mass formulation gave higher frequencies although this trend cannot necessarily be generalized to other structural systems. Furthermore, the Clough-Bathe results did not indicate which frequencies are more accurate.

FIGURE 1-1. EFFECT OF MASS LUMPING ON DAM MODAL FREQUENCIES (Clough and Bathe, 1972)

The diagonal form of the mass matrix resulting from the lumped mass formulation offers significant savings in computational efforts over the consistent mass matrix. However, these savings may not be considerable if the response of the system is obtained through the use of the reduced mass and stiffness matrices, as discussed later in detail. In any case, it may be concluded that a slight improvement in the accuracy of calculations offered in some cases by the consistent mass approach does not justify a large increase in complexity of the calculations. Nuclear structures are massive and involve large masses concentrated at a few points, and any coupling among these masses should normally be insignificant. Thus the lumped mass formulation is recommended for analyses of nuclear facilities except for cases where it is difficult to devise a satisfactory physical lumping procedure.

1.2.2.2 Rotary Inertia Effects

It has been shown that if rotary inertia is neglected it can have a significant effect on the natural periods of a structure (Archer, 1963). Figure 1-2 illustrates the errors that were introduced for a nuclear structure which was idealized as a lumped mass cantilever system. Because of the large amount of mass distributed off the neutral axis, the inclusion of the translational mass only was not adequate. However, the current practice is to neglect the effects of rotary inertia on the response of all the systems in a nuclear facility. This results in a diagonal mass matrix with the diagonal terms corresponding to all the rotational degrees of freedom set to zero. Additional research is needed in this area to indicate if the above findings are generally applicable to all nuclear facilities.

FIGURE 1-2. EFFECT OF ROTARY INERTIA ON NATURAL PERIODS

1.2.3 STIFFNESS MATRIX

The computation of the stiffness matrix $[K]$ in Equation 1-1 is normally based on the

assumption that the mass points are connected by elastic elements with linear force-deflection properties. The elements of the stiffness matrix can be determined by several methods, e.g., beam theory, membrane shell theory, and finite element method. If a lumped mass model is used to represent a nuclear structure, the mass points normally representing floor levels are assumed to be connected by columns. The stiffness properties of these elements are computed from beam theory which may include both flexural and shear deformations. The stiffness coefficients thus calculated represent gross properties of the structure between floors or mass points. In the finite element method the structure is modeled by an assembly of structural elements such as beams, springs, plates, shell, and membranes. The element stiffness matrices for individual elements are used as building blocks to form the total structural stiffness matrix. In practice all the computations leading to the formulation of the stiffness matrix are routinely computed by the computer program used to perform the analysis.

The assumption of a linear force-deflection relationship for the structural elements used is of some concern. The evaluation of test data by Smith and Matthiesen (1971) indicates that the response of nuclear structures shows a significant deviation from linear response. Studies by Newmark (1971) show that inelastic response may be accounted for in the formulation of the input response spectrum. Hudson (1965) and Jennings (1968) show that inelastic one-degree-of-freedom systems can be simulated by equivalent linear systems by accounting for the hysteretic damping by viscous damping. Further research work is needed in this area.

A further discussion on the formulation of the stiffness matrix is provided in Section 1.3.5.

1.2.4 DAMPING MATRIX

The final step in mathematical modeling is the formulation of the damping matrix [C]. The actual damping in the system is a complex phenomenon that includes inelastic action, hysteresis damping, viscous damping, and frictional forces. There is not sufficient data available at present to quantitatively evaluate the parameters that affect damping. Although viscous damping is not realizable in structures, it is used as the only damping mechanism because it simplifies the differential equations of motion, Equation 1-1.

1.2.5 SEISMIC INPUT MOTION

The time history of ground motion, $\ddot{S}(t)$ on the right hand side of Equation 1-1, must be defined before the equations of motion can be solved. Procedures for defining the ground motion are given in 10 CFR 100 Appendix A, RG 1.60, and Section 3.7.1 of the SRP. Careful study of these documents should be made by the reader for a full understanding of this problem, which in some cases can be complex. Only a brief review of the general requirements is provided here.

Structures, systems, and components of nuclear power plants must be designed to withstand all postulated earthquake events that may be experienced during the lifetime of the plant. Two events that must be specifically considered are defined in 10 CFR 100, Appendix A as the Safe Shutdown Earthquake (SSE) and the Operating Basic Earthquake (OBE). The SSE is defined as the earthquake producing the maximum vibratory ground acceleration at the site. It further specifies that the characteristics of the SSE shall be defined by response spectra derived from more than one earthquake to assure that the maximum vibratory acceleration at the site throughout the frequency range of interest is included. Also,

it requires that the maximum vibratory accelerations of the SSE at each of the various foundation locations of the nuclear power plant structures be determined by taking into account the characteristics of the underlying soil material in transmitting the earthquake-induced motions. Smoothed response spectra (developed from a series of response spectra) are recommended for the criteria design spectra. Vibratory motion resulting from the OBE must also be defined for design by response spectra, and the maximum vibratory ground acceleration for the OBE must be at least one-half that defined for the SSE.

Regulatory Guide 1.60 defines response spectra acceptable for the SSE and OBE for rock or firm soil sites. These spectra are based on statistical comparisons by Newmark, Blume, and Kapur (1973) of response spectra from several past strong motion earthquakes. Figures 1-3 and 1-4 show response spectra defined in RG 1.60 for the horizontal and vertical components of ground motions, assuming a maximum horizontal vibratory ground acceleration of 1.0 g. For sites having different maximum horizontal vibratory ground accelerations, the design response spectra given in RG 1.60 (Figs. 1-3 and 1-4) should be linearly scaled in proportion to the maximum horizontal vibratory ground acceleration. RG 1.60 should be consulted for additional information on this procedure. It should be noted that RG 1.60 procedures do not apply to sites that are relatively close to the epicenter of an expected earthquake, or for sites that have physical characteristics that could significantly affect the spectral pattern of the input motion. Procedures for such site conditions are given in NUREG 75/072 (SW/AA, 1975) and are summarized in AA (1975b).

The SRP (Sec. 3.7.1) indicates that the SSE and OBE design spectra can be assumed to apply at either the finished grade (i.e., ground surface) or the foundation level of the nuclear

FIGURE 1-3. HORIZONTAL DESIGN RESPONSE SPECTRA — SCALED TO 1G HORIZONTAL GROUND ACCELERATION

FIGURE 1-4. VERTICAL DESIGN RESPONSE SPECTRA — SCALED TO 1G HORIZONTAL GROUND ACCELERATION

power plant structures. If the former is assumed and the structures are deeply embedded in soil or rock, then a separate free-field calculation is required to define the criteria ground motions at the base of the structures. If the latter is assumed and the structures are founded on rock having a shear wave velocity greater than 3500 fps (i.e., rock/structure interaction does not need to be considered), then the criteria spectra can be assumed to be the input to the base of the structures. However, if the structures for the latter case are founded on soil or rock having shear wave velocities less than 3500 fps, soil-structure interaction effects must be considered, and a separate analysis is again required to define the input to the structure. For simplicity in presenting the fundamental mathematical and mechanics relationships here, it will be assumed that the RG 1.60 criteria spectra represents the input to the base of the structures.

For analysis and design, criteria response spectra must be specified for three mutually orthogonal directions: two horizontal and one vertical. Since the two horizontal spectra are normally assumed to have an equal probability of occurrence in any horizontal direction, it is current practice to assume that the maximum accelerations in the two horizontal directions are equal, while the maximum vertical acceleration is normally assumed to be two-thirds of the maximum horizontal acceleration. Thus, the same horizontal spectra apply to the two horizontal components (Fig. 1-3), but separate spectra (Fig. 1-4) are required for the vertical component.

To obtain the dynamic response of nuclear power plant structures by integrating the equations of motion (Eq. 1-1) in the time domain, the input ground motions must be specified by a time history instead of by a response spectrum. However, the time history selected for the input must be compatible with

the design response spectrum. This compatibility is satisfied by requiring that the acceleration response spectrum derived from the time history envelop the design spectrum. Currently, there is no rigorous method for computing a time history to match a given response spectrum. Therefore, the required time history is obtained by modifying an existing earthquake record (Tsai, 1972), or by developing a synthetic time history whose spectrum envelops the design spectrum. Figure 1-5 shows a modified synthetic time history used for analyzing a nuclear plant structure (Tsai et al., 1971). Figure 1-6 shows a comparison of response spectra of the synthetic time history with RG 1.60 design spectra for 0.1 g maximum vibratory ground acceleration.

Tsai (1972) has presented an approximate method for producing time histories that match a given response spectrum. The procedure involves selecting any existing time history and modifying it through a trial and error approach to obtain the desired spectrum. The modification is achieved by spectrum suppression and spectrum raising techniques.

Spectrum suppression is achieved by passing the selected motion successively through certain frequency-suppressing filters. The filter used by Tsai consists of a two-degree-of-freedom mechanical system, and is described by three parameters, ω_0 , ξ , and F , that are related to the four physical quantities consisting of two damping constants, the stiffness coefficient, and the mass.

The spectrum-raising technique by Tsai can raise the input spectrum locally by any desired amount without significantly modifying the remaining spectrum. The principle is to superpose on the selected ground motion some sinusoidal motions of suitable amplitudes and phase angles. To raise the acceleration spectrum by an amount ΔS at the frequency ω_0 , the

but motion $\ddot{Z}(t)$ is modified as $\ddot{Z}(t) + A \sin(\omega_0 t + p)$.

In application, the spectrum suppression is usually done, when necessary, prior to the process of spectrum raising. This is because the amount of suppression cannot be precisely controlled, and it may take several trials before a satisfactory result can be obtained. Fortunately, the spectrum can be raised locally by a

FIGURE 1-5. A SAMPLE OF SYNTHETIC TIME HISTORY GENERATED TO ENVELOP THE DESIGN SPECTRA

desired amount whenever it is oversuppressed or originally below the design spectrum.

1.2.6 UNCERTAINTIES IN MODELING

Modeling of structures to perform dynamic analysis is not an exact science; it is somewhat an art. Two different engineers modeling a complex structure will inevitably produce two different mathematical models. However, if careful study and proper engineering judgment are used in modeling, the end results should be nearly the same in both cases. In modeling a complex structure, two sources of uncertainties can be noted: *subjective errors* and *physical errors*. Subjective errors are those that arise because of improper judgment, experience, and perception on the part of the engineer. Physical errors are those arising from variations in the material and geometrical properties of the

FIGURE 1-6. COMPARISON OF CRITERIA SPECTRA WITH SYNTHETIC TIME-HISTORY SPECTRA

structure, which affect the mass, stiffness, and damping properties of the model.

The errors in calculating the mass in a mathematical model may be due to the modeling errors and the errors in estimating the unit mass of structural materials. The unit weight of concrete, soil or other structure material generally can be estimated quite well, and the impact of errors in these estimates on the response of the structure usually can be neglected. Selection of mass points and decisions to include or exclude the masses of certain equipment, however, are subjective errors which affect the mass of the model and hence the natural frequencies.

Stiffness calculations of structural members, particularly of concrete sections, are subject to several sources of uncertainties, such as strength, quality control, aging, cracking pattern, and amount of reinforcement. Soil stiffness properties are governed by shear modulus, Poisson's ratio, and stress and strain levels. Stiffness estimates become more difficult when the structural materials are stressed beyond yield point values. Variabilities in damping are the least known quantities and the least studied. The impact of errors in damping is mainly on the amplitude of response and not on natural frequencies. Selection of damping is also dependent upon the method of analysis chosen.

The importance of correct modeling, and the errors that can occur when the model is inadequate are shown in studies reported by Plichon (1970) and Ibañez (1970). Both compared calculated results with data obtained from tests on nuclear facilities, and the original models gave poor results in both studies. However, these models were adjusted to give a good comparison with the test results. Evaluation of tests made on nuclear facilities similar to

those made by Matthiesen, Smith, and coworkers will aid in arriving at realistic models for nuclear systems (Matthiesen and Smith, 1966 and 1969; Ibañez et al., 1970; Smith and Matthiesen, 1969; Bleiweis, 1970; Matthiesen et al., 1970). Because of uncertainties in modeling techniques, it is prudent to perform different analyses with variations in the model to assess the impact of various parameters on the response of the structure.

1.3 SOLUTION TECHNIQUES

To obtain seismic response of a nuclear plant structure, the differential equations of motion as given in Equation 1-1 must be solved. There are three different approaches generally used to solve these equations: the modal superposition, the direct integration, and the Fourier analysis techniques. Along with these three approaches, a fourth method (Foss, 1958), which is not generally used for solving structural analysis problems, is presented because of its close resemblance to the modal superposition method.

1.3.1 MODAL ANALYSIS TECHNIQUES

The modal analysis techniques have been widely used in seismic analyses of power plant structures and are well documented in standard textbooks on structural dynamics (Biggs, 1964; Clough and Penzien, 1975). The basis for the modal superposition methods is the determination of the natural frequencies and mode shapes of the structure. For a system with n degrees of freedom, n frequencies and mode shapes can be defined. The mode shapes and frequencies in mathematical terminology are also referred to as eigenvectors and eigenvalues, respectively. To determine the eigenvalues and the associated eigenvectors, the free vibration properties of the structure are considered by setting the damping and external

terms in Equation 1-1 equal to zero to give

$$[M]\{\ddot{X}\} + [K]\{X\} = 0 \quad (1-3a)$$

A solution of the above equations is obtained by assuming

$$\{X\} = \{\phi_1\} \sin \omega_1 t \quad (1-3b)$$

Therefore, the equations of motion for free vibrations of the structure become

$$[K]\{\phi_1\} = \omega_1^2 [M]\{\phi_1\} \quad (1-3c)$$

where

$\{\phi_i\}$ = i th eigenvector or mode shape

ω_i^2 = i th eigenvalue or square of i th natural frequency

$[K]$ = Stiffness matrix

$[M]$ = Mass matrix

Equation 1-3 is the general form of eigenvalue equation whose solutions provide the mode shapes and frequencies of the structure.

Many different methods have been developed for the solution of eigenvalue problems in general (Wilkinson, 1965). However, for the solution of structural problems, the following procedures have been found to be most efficient:

- Householder-Inverse Iteration
- Generalized Jacobi Iteration
- Determinant Search
- Subspace Iteration

Clough and Bathe (1972) discuss in detail the principal features of the above methods, and their relative advantages and disadvantages, a summary of which is provided in Table 1-1. In general, no single method for extracting mode shapes and frequencies of nuclear structures is the most efficient for all problems. The optimum choice among the available methods

TABLE 1-1. COMPARISON OF METHODS FOR COMPUTING MODE SHAPES AND FREQUENCIES

No	Item for Comparison	Householder-inverse Iteration	Generalized Jacobi Iteration	Determinant Search	Subspace Iteration
1	Is the preliminary transformation to the standard form* ($[A]\{Y_1\} = \lambda_1\{Y_1\}$) required?	Yes	No	No	No
2	Major computational effort	Reduction of $[A]$ to tridiagonal form and iterations for computing eigenvectors	Jacobi rotation matrices used to transform $[K]$ and $[M]$ to diagonal form	Triangular factorization of $[K] - \omega^2[M]$ and inverse iterations	Solution for trial vectors and computations of reduced $[K]$ and $[M]$ and solutions of the reduced eigenvalue problems
3	Whether all or a few eigenvalues are computed in one operation	All eigenvalues and eigenvectors computed	All eigenvalues and eigenvectors computed	One eigenvalue and one eigenvector computed	A desired number of eigenvalues and eigenvectors computed in one operation
4	Whether the method takes advantages of the banded form of the stiffness matrix	No	Yes	Yes	Yes
5	Is the method suitable when the matrices have small bandwidth?	No	Yes	Yes	Yes
6	Is the method suitable when the matrices have large bandwidth?	Yes	No	No	Yes

* The transformation of Equation 1-3c can be accomplished in several ways as described by Clough (1969). For example, if the mass matrix is diagonal with nonzero positive elements the transformation is given by $[A] = [M]^{-1/2} [K] [M]^{-1/2}$, $\{Y_1\} = [M]^{-1/2} \{\phi_1\}$, $\lambda_1 = \omega_1^2$

depends upon the characteristics of the structure, number of mode shapes to be computed and the desired accuracy for the mode shapes. However, the subspace iteration method (Bathe, 1971) is usually the most effective method for all nuclear structure problems.

1.3.1.1 Decoupled Equations

The displacement vector $\{X\}$ can now be expressed in terms of the normalized mode shapes and the normal coordinate vector $\{q\}$ as

$$\{X\} = [\phi]\{q\} \quad (1-4)$$

in which $[\phi]$ is the matrix with the normalized mode shapes (vectors) as columns. The normalized modal matrix satisfies the following special relations:

$$\begin{aligned} [\phi]^T [M] [\phi] &= [I] \\ [\phi]^T [K] [\phi] &= [\omega^2] \end{aligned} \quad (1-5)$$

in which

$$\begin{aligned} [I] &= \text{Identity matrix} \\ [\omega^2] &= \text{Diagonal matrix of frequency squared} \end{aligned}$$

Equations 1-4 and 1-5 can be utilized to reduce the equations of motion as

$$\ddot{\{q\}} + [\bar{C}]\{\dot{q}\}[\omega^2]\{q\} = -\{\Gamma\} \ddot{S}(t) \quad (1-6)$$

where

$$\begin{aligned} [\bar{C}] &= [\phi]^T [C] [\phi] \\ \{\Gamma\} &= [\phi]^T [M] \{L\} = \text{Vector of modal participation factors} \end{aligned}$$

If the damping matrix $[C]$ is assumed to have the properties such that $[\bar{C}]$ is a diagonal matrix, Equation 1-6 can be written as a set of decoupled equations each of which resembles the equation of motion of a single-degree-of-freedom system:

$$\ddot{q} + 2[D][\omega]\{\dot{q}\} + [\omega^2]\{q\} = -\{\Gamma\} \ddot{S}(t) \quad (1-7)$$

where

$$\begin{aligned} [D] &= \text{Diagonal matrix of modal damping ratios} \\ [\omega] &= \text{Diagonal matrix of frequencies} \end{aligned}$$

This equation can also be written as

$$\begin{aligned} \ddot{q}_i + 2D_i \omega_i \dot{q}_i + \omega_i^2 q_i &= -\Gamma_i \ddot{S}(t) \\ i &= 1, 2, \dots, n \end{aligned} \quad (1-8)$$

An important advantage of the modal superposition methods arises from the fact that the essential seismic response of nuclear power plant structures is generally contained in the first few mode shapes. For example, although over a thousand degrees of freedom may be used to represent a complex structure, its seismic response is often determined with sufficient engineering accuracy by using no more than ten modes. Thus in practice, only the first few equations in Equation 1-8 are considered and the effect of the remaining equations on the response of the structure can be neglected.

To determine the seismic response of nuclear power plant structures, two different methods are employed to solve Equations 1-7 or 1-8: the time-history (modal) analysis method and the response spectrum analysis method.

1.3.1.2 Time-History (Modal) Analysis

In the time-history analysis, a time history of the support motion is used in Equation 1-7 to compute the response in each mode as a function of time. The total response is computed by summing the response of all significant modes as

$$\{X(t)\} = \sum_{i=1}^N \{\phi_i\} q_i(t) \quad (1-9)$$

where N is the number of modes considered in the analysis. In general, N is much smaller than n , the total degrees of freedom of the model. The velocity and acceleration responses (relative) are obtained using the equations similar to Equation 1-9. Finally, the absolute acceleration responses are obtained as

$$\ddot{Z}_1(t) = \ddot{X}(t) + \ddot{S}(t) \quad (1-10)$$

For nuclear power plant structures, the support motion is typically given in the form of response spectra (see Sec. 1.2.5). The current practice is to choose a time history whose response spectra essentially envelop the design spectra.

The solution of Equation 1-7 for each mode is obtained using a step-by-step numerical integration method such as the linear and constant acceleration methods (Biggs, 1964) and the Newmark Beta method (Newmark, 1959). For convenience in forming the summation shown in Equation 1-9, a constant integration time step is used for all modes. To ensure the stability of the numerical integration scheme, the time step should be smaller than one-fifth of the period of the highest mode considered in the analysis.

1.3.1.3 Response Spectrum Analysis

In the response spectrum method, the support motion is taken in the form of a response spectrum directly. Since the response spectrum is defined as a plot of maximum responses of single-degree-of-freedom systems to a given earthquake versus natural frequencies, the maximum response in each natural mode of vibration of the system can be written down from Equation 1-8 as

$$(q_i)_{\max} = \Gamma_i S_a(\omega_i)/\omega_i^2 \quad (1-11)$$

where $S_a(\omega_i)$ is the acceleration response spectral value corresponding to the frequency and damping of the structure in the i th mode. The maximum modal displacements, velocities, and accelerations in each mode are obtained as

$$\begin{aligned} \{X_i\}_{\max} &= \{\phi_i\}\{q_i\}_{\max} \\ \{\dot{X}_i\}_{\max} &= \omega_i \{X_i\}_{\max} \\ \{\ddot{X}_i\}_{\max} &= \omega_i^2 \{X_i\}_{\max} \end{aligned} \quad (1-12)$$

in which $\{X_i\}_{\max}$, $\{\dot{X}_i\}_{\max}$, and $\{\ddot{X}_i\}_{\max}$ are the maximum displacement, velocity, and acceleration vectors in the i th mode.

In the response spectrum method, only the maximum modal responses (Eq. 1-12) are known, and the phasing of modes cannot be determined as in the time-history analysis. The total response of the structure can, therefore, only be approximated. Two methods for combining the modal responses are currently used. A conservative estimate of the maximum structural response is obtained by summing the absolute values of the maximum modal responses, and is called the absolute sum method (ABS). Thus the maximum response, R , at a given point on the structure is computed as

$$R = \sum_{i=1}^N |\gamma_i| \quad (1-13)$$

in which γ_i denotes the maximum response at the point in the i th mode and N is the number of modes being considered. This method of combining the maximum modal responses is preferred when only a few modes are considered in the analysis.

The second method for estimating the maximum response is known as the square root of the sum of the squares or root-mean-square (SRSS) approach. This method generally provides a more realistic result than the ABS method, particularly if there is a considerable difference between the frequencies of successive modes. The maximum probable response of a point is computed from the SRSS method as

$$R = \sqrt{\sum_{i=1}^N \gamma_i^2} \quad (1-14)$$

The SRSS approach generally leads to results that are in close agreement with those obtained by the time-history analysis, provided the natural frequencies are well separated, i.e., statistically independent. For complex structures, the modal frequencies, especially in the higher modes, are not always well separated. In these cases, the SRSS method yields results which may be unrealistic. Singh et al. (1973) discuss this problem and recommend the

following method for combining the maximum modal response when certain modal frequencies are close.

$$R^2 = \sum_{i=1}^N \gamma_i^2 + 2 \left(\sum_{i=1}^N \sum_{j=i+1}^N \gamma_i \gamma_j \varepsilon_{ij} \right) \quad (1-15)$$

where

$$\varepsilon_{ij} = \left\{ 1 + \left(\frac{\omega_i' - \omega_j'}{D_i' \omega_i + D_j' \omega_j} \right)^2 \right\}^{-1}$$

in which

$$\omega_i' = \omega_i \sqrt{1 - (D_i')^2}$$

$$D_i' = D_i + \frac{2}{t_d \omega_i}$$

Here ω_i and D_i are the modal frequency and damping ratio in the i th mode, respectively, and t_d is the duration of the earthquake record. In forming the double sum in Equation 1-15, the terms with $\varepsilon_{ij} \leq 0.10$ can generally be neglected, for small values of ε_{ij} indicate that i th and j th modes are relatively independent.

When the higher mode responses are significant, the maximum responses in these modes can be combined algebraically (Stoykovich, 1973). This is because the higher modes correspond to a rigid structure which tends to follow the support motion, and for a relatively rigid structure the algebraic sum of the maximum modal responses becomes equal to the actual response.

Regulatory Guide 1.92 provides recommendations for combining the individual modal responses. Procedures are also discussed in more detail in AA (1975c). If the system modes are not closely spaced, the representative maximum value of a particular response of interest for design (e.g., stress, moment, or displacement) of a given element should be obtained by taking the SRSS (Eq. 1-14) of corresponding maximum values of the response

of the element attributed to individual modes. If certain system modes are closely spaced (i.e., two consecutive modes differ from each other by less than 10% of the lower frequency), then the closely spaced modes are divided into groups that include all modes having frequencies lying between the lowest frequency in the group and a frequency 10% higher. The representative maximum value of a particular response of interest for design should be computed using

$$R = \left[\sum_{k=1}^{N'} \gamma_k^2 + \sum_{q=1}^P \left\{ \sum_{m=1}^{n_q} \gamma_{mq} \right\}^2 \right]^{1/2} \quad (1-16)$$

where $N' = N - \sum_{q=1}^P n_q$ and where P is the number of groups of closely spaced modes, excluding individual separated modes; N' is the number of individual separated modes; n_q is the number of closely spaced modes in group number q ; and γ_{mq} is the peak value of response in the m th mode of group number q .

1.3.1.4 Comparison of Time-History and Response Spectrum Methods

The time-history method requires considerably more computational effort than the response spectrum method, because in the time-history analysis, complete time histories of stresses, displacement, and accelerations, etc., must be computed and searched for the maximum values which are necessary for design. The increased cost of the time-history analysis is the main disadvantage of this method. In fact, if it were not for this reason, an analyst would almost always choose the time-history method.

Because the design criteria for nuclear structures are normally specified in the form of response spectra (Figs. 1-3 and 1-4), the analyst is faced with a problem of selecting a time history, either an actual earthquake record

of a synthetic motion, whose response spectra agree with the design spectra. The matching of two spectra generally is difficult with an actual earthquake record. Therefore, an artificial record is commonly used in the analysis. The synthetic earthquake, being more broadband in nature than real earthquakes, excites higher and lower modes equally. The result is that the predicted response of the structure may significantly differ from the response under an actual earthquake of equal intensity. This problem may be corrected by analyzing the structure under a set of real earthquakes with narrow frequency bands that together envelop the design spectra. This, however, further increases the computational effort.

An important attraction of the response spectrum method is the economy with which the response calculation can be performed, for this method directly makes use of the design response spectra. The major criticism of the method is that any method of combining maximum modal responses is only approximate. Figure 1-7 provides a comparison of the response of a nuclear structure computed by the time-history and response spectrum methods (Bergstrom et al., 1971). The response by the

response spectrum method is computed using ABS and SRSS approaches. In this example, the time-history analysis and SRSS method give nearly the same results. Similar results may not be obtained with other cases.

The equipment and piping in a nuclear facility are designed separately from the floor response spectra generated from the building analysis. In the time-history method, the floor response spectra at a given location are computed in a routine fashion from the response time history of the location. There is no rational way of computing the floor response spectra when the response spectrum method is used. Kapur and Shao (1973) proposed a method for calculating the floor response spectra using the response spectrum method. Additional research work is necessary before this method can be adopted for the seismic design of equipment in nuclear power plant facilities.

A summary comparison of the time-history (modal) and response spectrum analyses is presented in Table 1-2.

1.3.2 DIRECT INTEGRATION METHOD

The modal superposition method is advantageous if the essential dynamic response of the structure is contained in the first few mode shapes. This will be the case if: (1) the seismic input can be approximated reasonably well by inertia force spatial distributions that are similar in shape to the eigenvectors of the first few modes; and (2) the frequency content of the input is primarily represented by the corresponding natural frequencies of the lowest modes. Otherwise, it will be necessary to include many modes of vibration in the analysis to obtain adequate accuracy by the mode superposition method. Also, for finite element analysis of soil/structure interaction problems, the mode shapes are not well defined and a large number of modes are required for accuracy. In such cases, a direct step-by-step

FIGURE 1-7. COMPARISON OF TIME HISTORY AND RESPONSE SPECTRUM METHODS

TABLE 1-2. COMPARISON OF NORMAL MODE METHODS

No	Item to be Compared	Response Spectrum Analysis	Time-History Analysis
1	Computational Economy	Most economical Cost is independent of the duration of the input record, since no earthquake record is actually used for the analysis	Computational effort increases with the duration of the input record
2	Accuracy of Solution	Approximations involved in considering a limited number of modes, and in the way the maximum modal response are combined	Depends on the number of modes considered in the analysis Desired level of accuracy can generally be obtained
3	Major Computational Effort	Computation of desired number of frequencies and mode shapes	Computation of desired number of frequencies and mode shapes
4	Input for Analysis	Design spectra can be directly used	A time history (real or artificial) to envelop the design spectra must be developed
5	Multidirectional Input	Responses from the individual analyses must be combined in some manner	Can be accounted for by simultaneously applying time histories of horizontal and vertical input motions
6	Linear and Nonlinear Analyses	Only linear analyses are possible	Only linear analyses are possible
7	Energy Absorbing Boundary	Not possible	Not possible
8	Form of Damping	Modal damping ratios can directly be used The damping matrix must be such that the equations are decoupled under appropriate transformation	Modal damping ratios can directly be used The damping matrix must be such that the equations are decoupled under appropriate transformation
9	Different Damping in Different Parts of the System	Cannot be handled directly An approximate procedure must be used to estimate effective damping ratios	Cannot be handled directly An approximate procedure must be used to estimate effective damping ratios
10	Nearly Equal Frequencies	Special procedure is required to combine the maximum modal response	Can be handled routinely
11	Relative Displacement Between Structures	Assumption must be made regarding phasing of adjacent building responses Out-of-phase assumption typically leads to overly conservative values	Can be estimated properly (if two buildings included in same model)
12	Floor Response Spectra for the Analysis of the Equipment	Special empirical methods must be used for estimation	Can be obtained routinely from computed time histories of motion at appropriate locations within the structure

integration procedure may be more efficient. Moreover, direct integration must be employed if the structural response is nonlinear due to either material properties or geometric effects. Current practice associated with nuclear plant structure analyses, however, is to consider only the nonlinear effects due to gaps that exist between parts of the structure such as those between vessel and barrel, and between adjacent buildings. Finally, when energy-absorbing boundaries are used to simulate the infinite extent of the soil, the direct integration methods must be used.

To obtain the dynamic response of a structure by a step-by-step method, the equations

of motion (Eq. 1-1) are integrated directly in incremental time steps and the response is obtained in the form of time histories. For linear systems, the matrices $[M]$, $[C]$, and $[K]$ remain constant, but for nonlinear systems the matrices $[C]$ and $[K]$ may change from step to step, but are assumed to remain constant during each time increment. If the system is strongly nonlinear (e.g., during the impact as the result of closing a gap), the time increment may have to be made very short, which increases the cost of the computations.

The integration technique affects the computer cost in two ways. First, the mathematical nature of the integration technique

determines the number of arithmetic operations that have to be taken as well as the amount of data that have to be maintained, processed, and transferred within the system. Secondly, the characteristic of the technique dictates the integration step size that has to be adopted for the stability and accuracy of the numerical solution. Since the length of the response history desired is predetermined, small time steps require large numbers of steps and a proportionate amount of computer time.

Many integration methods are currently in use but they can all be classified into two groups. One is the *implicit method*, in which equilibrium is expressed at nodal points by a system of coupled equations in which the unknown quantities are the generalized displacements, velocities, and accelerations. The solution of a coupled system of equations requires formulation of global stiffness, damping, and mass matrices and the inversion of a matrix whose order is the same as the number of equations. In contrast, a second group is the *explicit method*, in which equilibrium is expressed at nodal points by a system of uncoupled or near-coupled equations. It is possible to write these equations without formulating stiffness, mass or damping properties at the global level. For cost-effectiveness, the implicit methods are usually more stable and hence can tolerate larger step sizes. However, they require more computer core space because of the need to manipulate the global stiffness matrix, and they also require more time per step because the solution of a set of simultaneous equations is involved in each step. In contrast, the explicit methods usually require smaller time steps for numerical stability, but they require smaller core size and involve less calculations and hence less run time per step.

In the past few years, the implicit methods appear to be more widely used, even though the explicit methods are gaining popularity.

The wide use of the implicit schemes is apparently due to the large amount of past experience with linear response problems where conditions of stability are well established. For nonlinear problems, however, a typical implicit scheme requires the reformulation of the global stiffness matrix and the inversion of a matrix of the same order for each time step. For large scale systems, this can be prohibitive both in terms of computer time and storage.

1.3.2.1 Implicit Integration Method

The implicit integration scheme described here was developed by Wilson et al. (1973). This integration scheme is unconditionally stable and therefore the stability places no restriction on selection of the time step. However, the analyst must select and provide a time step that is based on the complexity of the mathematical model and the accuracy desired. The choice of the integration time step may also affect the level of the artificial damping introduced into the system.

In this method the accelerations are assumed to vary linearly within a time interval $\tau = \theta \Delta t$, where Δt is the time step and θ is a constant ≥ 1 which may take the values between 1.43 and 2.70 for unconditional stability.

Equation 1-1 at the time $t + \tau$ can be written as

$$[M]\{\ddot{X}\}_{t+\tau} + [C]\{\dot{X}\}_{t+\tau} + [K]\{X\}_{t+\tau} = \{P\}_{t+\tau} \quad (1-17)$$

where

$$\{P\}_{t+\tau} = -[M]\{L\} \ddot{S}(t)$$

From the linear acceleration assumption, the following expressions for $\{\ddot{X}\}_{t+\tau}$ and $\{\dot{X}\}_{t+\tau}$ in terms of the variables at time t and $\{X\}_{t+\tau}$ are established:

$$\begin{aligned} \{\ddot{X}\}_{t+\tau} &= \frac{6}{\tau^2} \{X\}_{t+\tau} - \{a\}_t \\ \{\dot{X}\}_{t+\tau} &= \frac{3}{\tau} \{X\}_{t+\tau} - \{b\}_t \end{aligned} \quad (1-18)$$

where $\{a\}_t$ and $\{b\}_t$ are computed from the known values of the state variables at time t

$$\{a\}_t = \frac{6}{\tau^2} \{X\}_t + \frac{6}{\tau} \{\dot{X}\}_t + 2 \{\ddot{X}\}_t$$

$$\{b\}_t = \frac{3}{\tau} \{X\}_t + 2 \{\dot{X}\}_t + \frac{\tau}{2} \{\ddot{X}\}_t$$

From Equations 1-17 and 1-18, $\{X\}_{t+\tau}$ can be evaluated in

$$[\bar{K}]\{X\}_{t+\tau} = \{\bar{P}\}_{t+\tau} \quad (1-19)$$

Here the effective stiffness matrix $[\bar{K}]$ is independent of time and the vector $\{\bar{P}\}$ is a function of input ground motion at time $t+\tau$ and of known values at time t and are given by

$$[\bar{K}] = [K] + \frac{3}{\tau} [C] + \frac{6}{\tau^2} [M]$$

$$\{\bar{P}\}_{t+\tau} = \{P\}_{t+\tau} + [M] \{a\}_t + [C] \{b\}_t$$

Since the effective force vector $\{\bar{P}\}_{t+\tau}$ can be computed, Equation 1-19 is a set of simultaneous equations in which $\{X\}_{t+\tau}$ are the unknowns. These equations are usually solved using the Choleski decomposition method that involves the triangularization of $[\bar{K}]$, the reduction of $\{\bar{P}\}_{t+\tau}$ and back substitution process involving the modified $[\bar{K}]$ and $\{\bar{P}\}_{t+\tau}$.

By substituting the $\{X\}_{t+\tau}$ into Equation 1-18, the values of the accelerations and velocities at this time are also known. Then the value of these variables at the desired time $t + \Delta t$ can be determined from the following equations:

$$\begin{aligned} \{\ddot{X}\}_{t+\Delta t} &= \left(1 - \frac{1}{\theta}\right) \{\ddot{X}\}_t + \frac{1}{\theta} \{\ddot{X}\}_{t+\tau} \\ \{\dot{X}\}_{t+\Delta t} &= \{\dot{X}\}_t + \frac{\Delta t}{2} \left(\{\ddot{X}\}_t + \{\ddot{X}\}_{t+\Delta t} \right) \\ \{X\}_{t+\Delta t} &= \{X\}_t + \Delta t \{\dot{X}\}_t + \frac{\Delta t^2}{6} \left(2 \{\ddot{X}\}_t + \{\ddot{X}\}_{t+\Delta t} \right) \end{aligned} \quad (1-20)$$

The procedure is repeated for the next time step until the solution advances to the specified point in time. For linear systems, the effective stiffness matrix need be formed only once. However, if the time step of integration is

altered, the effective stiffness matrix will have to be reformulated.

In the above derivations, the form of the damping matrix $[C]$ has been assumed to be arbitrary. For this case the calculations for obtaining $[\bar{K}]$ and $\{\bar{P}\}_{t+\tau}$ contain $[C]$. Even for linear problems, the vector $\{\bar{P}\}_{t+\tau}$ needs to be computed each time step, involving matrix manipulations with $[C]$. This process is not only time-consuming but also increases the required storage capacity of the computer. If it is assumed that the damping in the system is proportional to the mass and stiffness matrices, the integration procedure can be simplified considerably. Under this assumption, the damping matrix is given by

$$[C] = \alpha [M] + \beta [K] \quad (1-21)$$

For this case, Equation 1-19 simplifies to

$$[\bar{K}]\{\bar{X}\}_{t+\tau} = \{\bar{P}\}_{t+\tau} \quad (1-21)$$

where

$$\begin{aligned} [\bar{K}] &= [K] + c_4 [M] \\ \{\bar{X}\}_{t+\tau} &= c_1 \{\dot{X}\}_t + c_2 \{\ddot{X}\}_t + c_3 \{\ddot{X}\}_{t+\tau} \\ \{\bar{P}\}_{t+\tau} &= \{P\}_{t+\tau} + c_4 [M] \left[c_5 \{\dot{X}\}_t + c_6 \{\ddot{X}\}_t \right] \end{aligned}$$

and the C 's are constants:

$$\begin{aligned} c_1 &= \alpha + \tau \\ c_2 &= \frac{\tau^2}{3} + \frac{\alpha\tau}{2} \\ c_3 &= \frac{\tau^2}{6} + \frac{\alpha\tau}{2} \\ c_4 &= \frac{1 + \frac{\beta\tau}{2}}{c_3} \\ c_5 &= c_1 - \frac{\beta}{c_4} \\ c_6 &= c_2 - \frac{\beta\tau}{2c_4} \end{aligned}$$

With $\{\bar{X}\}_{t+\tau}$ determined from Equation 1-21, the acceleration vector at $t + \tau$ can be computed as

$$\{\ddot{\bar{X}}\}_{t+\tau} = \frac{1}{c_3} \left[\{\bar{X}\}_{t+\tau} - c_1 \{\dot{\bar{X}}\}_t - c_2 \{\ddot{\bar{X}}\}_t \right] \quad (1-22)$$

Now the displacement, velocity, and acceleration vectors at $t + \Delta t$ can be calculated from Equation 1-20.

Thus for proportional damping in the system, the expressions for $[\bar{K}]$ and $\{\bar{P}\}_{t+\tau}$ do not contain the damping matrix, which makes their evaluation simpler.

A flow chart showing the required steps in a typical finite element computer program that uses the implicit integration scheme is shown in Figure 1-8.

1.3.2.2 Explicit Integration Method

Explicit schemes have been successfully incorporated into major computer programs such as SLAM (Costantino et al., 1968); SAMSON (Belytschko and Chiapetta, 1973); FEAT (Leistner et al., 1969); and STARS (AA, 1981). Most of the computer programs that incorporate explicit methods also have nonlinear capabilities.

In integrating the basic equations of motion, the explicit scheme requires taking the following four basic steps for each time step:

Step 1 Use a suitable formula to predict the displacement of the structure at the end of the next time step.

Step 2 On the basis of the predicted displacement (deformed configuration of the structure) calculate the global internal load vector.

Step 3 Find the unbalanced force as the difference between the external and the internal load vector. Newton's Law is then used to calculate the nodal acceleration pertaining to the end of the time step.

FIGURE 1-8. FLOW CHART SHOWING IMPLICIT INTEGRATION SCHEME

Step 4 Use the newly found acceleration to calculate the nodal velocity at the end of the time step.

The accuracy of the predicted displacement in Step 1 may be checked in Step 4 by calculating the displacement in addition to the velocity,

and, if desired, the numerical computation may be repeated by using the calculated displacement as the revised, predicted displacement. However, since iteration is time consuming, one may choose to ignore this checking and simply keep on marching forward in time as is done in the SLAM code. Figure 1-9 shows a flow chart for such a computation scheme.

The prediction of the displacement, which is required to initiate the computation cycle, may be done using any formula that is based only on the response history up to the current time. Newmark's β -method (Newmark, 1959), as an example, affords such a formula when $\beta = 0$. This formula has the following form:

$$\{X\}_{t+\Delta t} = \{X\}_t + \Delta t \{\dot{X}\}_t + \frac{1}{2} (\Delta t)^2 \{\ddot{X}\}_t \quad (1-23)$$

Here X , $\dot{X}$, and $\ddot{X}$ represent the nodal displacement, velocity, and acceleration vectors of the system, while Δt is the step size for the next step. The $\beta = 0$ method is infinitely convergent, but for numerical stability, the time step size must be less than 0.318 of the shortest period of the finite element mesh. This may lead to a prohibitively small time step.

For the displacements at $t + \Delta t$, $\{X\}_{t+\Delta t}$, the unbalanced load vector, which is merely the difference between the external and internal load vectors, for the structure at $t + \Delta t$ can be computed. The external loads result from the application of the ground motions to the structure. The internal load vector is obtained by summing the internal loads in all the elements in the structural model. The internal load vector in an element is based on the state of stress in that element. The total stresses in turn may depend on the strain and strain rate levels. The stress/strain relations in an element are governed by the material properties that may be nonlinear, while the stress/strain rate relations depend on the damping properties of the element. Therefore, the case where the different

FIGURE 1-9. FLOW CHART SHOWING EXPLICIT INTEGRATION SCHEME USED IN SLAM CODE (Costantino, 1968)

damping values are present in the different regions of the structure may be handled routinely with this method. The nodal accelerations at $t + \Delta t$ then can be determined using Newton's Law:

$$[M]\{\ddot{X}\}_{t+\Delta t} = \{F^{unbal}\} = \{F^{ext}\} - \{F^{int}\} \quad (1-24)$$

re $\{F^{ext}\}$ and $\{F^{int}\}$ are respectively the external and internal load vectors at $t + \Delta t$, and $[M]$ is the mass matrix of the system. Since the computer programs in this category utilize the lumped mass approach in forming $[M]$, the above equations represent an uncoupled set that may be readily solved for the nodal accelerations. With the nodal accelerations at the end of the next step found, the nodal velocities at the same time instant may be determined from the following equation:

$$\{\dot{X}\}_{t+\Delta t} = \{\dot{X}\}_t + \frac{\Delta t}{2} \left[\{\ddot{X}\}_t + \{\ddot{X}\}_{t+\Delta t} \right] \quad (1-25)$$

This completes the calculation for one typical step. The procedure is repeated as many times as is necessary to obtain the desired response history of the system.

A summary of the comparison of the implicit and explicit integration methods is presented in Table 1-3.

1.3.3 FOURIER TRANSFORM TECHNIQUE

The Fourier transform technique has been gaining in usage for obtaining the seismic response of nuclear power plant structures. This method is directly applicable for solving linear systems. It solves the response of a system in the frequency domain by first transforming the input motion time histories into frequency domain and then uses inverse procedures to obtain the corresponding solution in the time domain. Once, the Fourier transform method was seldom used in the analyses of nuclear power plant structures because a large computational effort was required in finding Fourier transforms numerically. However, with the Fast Fourier Transform Algorithms developed by Cooley and Tukey (1965), this difficulty has been overcome. The Fast Fourier Algorithms significantly reduce the number of computations as compared with the direct method. Therefore, this method is now computationally very competitive with other methods.

TABLE 1-3. COMPARISON OF EXPLICIT AND IMPLICIT INTEGRATION METHODS

No	Item to be Compared	Explicit Method	Implicit Method
1	Form of Equations of Motion	Uncoupled equilibrium equations at nodal points	Coupled equilibrium equations at nodal points
2	Major Computational Effort	Reformulation of the internal load vector for each time step, in both linear and nonlinear solutions	Back substitution processes on the triangularized stiffness matrix. Reformulation and triangularization of the stiffness matrix for nonlinear problems
3	Cost Per Integration Time Step	Very low	Very high
4	Total Cost of Computation a Linear Problems b Nonlinear Problems	Since the time step must be very small, total cost may be slightly smaller or in some cases larger than the implicit method Large savings can be realized compared to the implicit method	For linear problems only one formulation and triangularization of the stiffness matrix required. The method can tolerate large time steps so that the total cost is comparable to the explicit method. The effective stiffness matrix needs to be reformulated and triangularized often. The total cost can be enormous
5	Accuracy and Stability of Solution	Less stable and accurate than the implicit method. May require a very small time step of integration	More stable and accurate than the explicit method. Can tolerate larger step sizes. Unconditionally stable for linear problems
6	Bandwidth of Stiffness Matrix	No bandwidth is considered	Bandwidth is an important factor in designing the structural model and for optimum computations and storage
7	Required Storage Capacity of Computer	Small	Large

In order to solve the equations of motion (Eq. 1-1), this method first assumes that the input motion is harmonic with the frequency ω

$$\ddot{S}(t) = e^{i\omega t} \quad (1-26)$$

If the system whose equations of motion are given by Equation 1-1 is linear, the response $\{X\}$ is also harmonic at the same frequency ω . The amplitude and the phase of the response depend on ω . This implies that the response due to harmonic input motion can be expressed as

$$\{X\} = \{H(\omega)\} e^{i\omega t} \quad (1-27)$$

in which $\{H(\omega)\}$ is the complex frequency response function. It is noted that the complex frequency response and the impulse response functions are Fourier transforms of each other. The substitution of Equations 1-26 and 1-27 into Equation 1-1 gives $\{H(\omega)\}$ for the system

$$\left[[K] + i\omega [C] - \omega^2 [M] \right] \{H\} = - [M]\{L\} \quad (1-28)$$

where $i = \sqrt{-1}$. The above equations form a set of linear simultaneous equations and can generally be solved using the usual methods such as the Gaussian elimination method. If the damping matrix is identically zero and if ω is a natural frequency of the system, the above set becomes singular. However, structural systems are always analyzed with some damping so that, in general, the damping matrix $[C]$ never becomes zero.

After the frequency response, $\{H(\omega)\}$, of the system due to harmonic input is computed, the response in the frequency domain due to an arbitrary input motion $\ddot{S}(t)$ can be obtained as

$$\{\bar{X}(\omega)\} = \bar{S}(\omega) \{H(\omega)\} \quad (1-29)$$

where $\bar{S}(\omega)$ is the Fourier transform of $\ddot{S}(t)$ given by

$$\bar{S}(\omega) = \int_{-\infty}^{\infty} \ddot{S}(t) e^{-i\omega t} dt$$

The response $\{X(t)\}$ in the time domain due to $\ddot{S}(t)$ can be written as the inverse Fourier integral

$$\begin{aligned} \{X(t)\} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \{\bar{X}(\omega)\} e^{i\omega t} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{S}(\omega) \{H(\omega)\} e^{i\omega t} d\omega \end{aligned} \quad (1-30)$$

The three important steps in the analysis by this method are indicated in Equations 1-28, 1-29, and 1-30. Since the set of simultaneous equations, Equation 1-28, cannot in general be solved in closed form, the set is solved by numerical procedures for each value of ω starting from 0 to $\omega_{\max}$ in increments of $\Delta\omega$. As discussed later, $\omega_{\max}$ and $\Delta\omega$ depend on the duration and the interval of digitization of the input motion. After the frequency response function for a range of values of ω is computed, the Fourier transform and the inverse Fourier transform are computed by the Fast Fourier Transform method.

The numerical evaluation of the Fourier transform and the inverse Fourier transform are performed by expressing the integrals in discrete form. If the input earthquake record is digitized with N points at the time interval Δt , the discrete Fourier transform of the record can be expressed as

$$\bar{S}_j = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} \ddot{S}_k e^{-i\omega_j k \Delta t}, & \text{for } j = 0, j = \frac{N}{2} \\ \frac{2}{N} \sum_{k=0}^{N-1} \ddot{S}_k e^{-i\omega_j k \Delta t}, & \text{for } 1 \leq j < \frac{N}{2} \end{cases} \quad (1-31)$$

where

$$\omega_j = \frac{2\pi j}{N\Delta t}, \quad j = 0, 1, \dots, \frac{N}{2}$$

$$\ddot{S}_k = \ddot{S}(k\Delta t)$$

$$\bar{S}_j = \bar{S}(\omega_j)$$

Similarly, the inverse Fourier transform is given by

$$\ddot{S}(t) = \text{Re} \left\{ \sum_{k=0}^{N/2} \bar{S}_k e^{i\omega_k t} \right\} \quad (1-32)$$

The algorithm of the superfast Fast Fourier Transform technique evaluates Equation 1-31 or 1-32 in a computational time proportional to $N \log N$ as opposed to N^2 by the straightforward calculations.

One advantage of the Fourier transform method is that viscous damping in continuum finite elements (representing soils, etc.) can be introduced simply by using complex moduli in the formation of the stiffness matrix $[K]$. It can be shown that the use of the complex moduli

$$G^* = G \left(1 - 2\beta^2 + 2i\beta \sqrt{1 - \beta^2} \right) \quad (1-33)$$

$$E^* = E \left(1 - 2\beta^2 + 2i\beta \sqrt{1 - \beta^2} \right)$$

will lead to the same response as a modal analysis with the uniform ratio of critical damping β (Lysmer et al., 1974). In Equation 1-33, G^* and E^* are equivalent complex shear and Young's moduli and G and E are shear and Young's moduli, respectively. This method is also suitable when the damping is not uniform. In such a case, different values of G , E , and β are used in each element.

In practice, the seismic motions input to a structure are given as a digitized record with a total of N points at the time interval of Δt . The total duration of the record is then given by $N\Delta t$. In using the Fast Fourier Transform method to solve the system, the complex frequency response function $H(\omega_k)$ is computed for each ω_k (Lysmer et al., 1974)

$$\omega_k = \frac{2\pi k}{N\Delta t}, \quad k = 0, 1, \dots, \frac{N}{2} \quad (1-34)$$

Therefore, the maximum value of ω_k considered in the analysis $\omega_{\max}$ is equal to $\pi/\Delta t$, and $\Delta\omega$, the increment in ω_k , is given by $2\pi/N\Delta t$.

The Fourier Transform method makes extensive use of superposition and is therefore,

strictly speaking, applicable only to linear systems. However, some computer programs (LUSH, SHAKE, etc.) based on this method include nonlinear properties of materials through an "equivalent" linear method. This method takes into account, in an approximate manner, nonlinear strain or strain-rate dependent material properties through several linear analyses. Initially, a set of material properties is estimated for each element of the model based on estimated levels of strain or strain-rate values in the element. The system is analyzed using these properties, and strain and strain-rate time histories are obtained for each element. Based on these time histories, a revised set of material properties for each element is estimated and the system is analyzed again with the revised material properties. This procedure is repeated several times until convergence has occurred. The response from the last analysis is taken as the nonlinear response of the system.

There are two basic disadvantages to the equivalent linear method. The first is that for nonlinear problems, the principle of superposition, which is the basis for the Fourier transform method, is not valid. Also, the number of iterations in some cases may be large, leading to too large a computational effort to justify this method. The second disadvantage is in estimating a set of strain or strain-rate time history from the previous analysis. Since the material properties change with time, a single set of properties is used in the analysis to represent some kind of average. This may in general lead to significant errors.

A summary of the comparison of the modal analysis, direct integration, and Fourier analysis methods is presented in Table 1-4.

1.3.4 FOSS METHOD

Another approach for obtaining the response of a system whose governing equations

TABLE 1-4. COMPARISON OF DIFFERENT METHODS OF ANALYSIS FOR NUCLEAR PLANT STRUCTURES

No	Item to be compared	Modal Analysis Methods ¹	Direct Integration Methods ²	Fourier Transform Method
1	Major Computational Effort Involved	Extract mode shapes and frequencies	Triangularization and back substitution for IIM and formulation of the load vector for EIM	Frequency response functions and forward and inverse Fourier transforms
2	Total Cost of Computations	Most economical Cost is essentially independent of the duration of the input record	Most expensive Cost is directly proportional to the duration of the input record	Intermediate Cost is directly proportional to the duration of the input record
3	Input for Analysis	Design spectrum for RSM A time history to envelop design spectrum for THM	A time history to envelop design spectrum	A time history to envelop design spectrum
4	Multidirectional Input	In RSM, responses from the individual analyses must be combined in some manner Can be accounted for in one operation for THM	Can be accounted for in one operation by simultaneously applying time histories for horizontal and vertical input motions	Responses due to each input motion must be combined in the frequency or time domain to obtain the total responses
5	Types of Analyses	Only linear analyses are possible	Linear and nonlinear analyses are possible	Nonlinear analyses through equivalent linear analyses are possible
6	Energy Absorbing Boundary Using Dashpots	Not possible	Can be used effectively to simulate infinite extent of soil media	Theoretically possible However, the existing computer programs do not seem to include this capability
7	Form of Damping	Modal damping ratios can directly be used The damping matrix must be such that the equations are decoupled under appropriate transformation	Damping matrix must be constructed from the given modal damping ratios using an available method Also the form of damping matrix is not restricted Rayleigh type damping, when used, conserves computer storage	Damping matrix must be constructed from the given modal damping ratios using an available method Also the form of the damping matrix is not restricted Rayleigh type damping, when used, conserves computer storage
8	Different Damping In Different Parts of the System	Cannot be handled directly An approximate procedure must be used to estimate effective damping ratios	Can be handled directly if a damping matrix can be provided for each part	Can be handled directly if a damping matrix can be provided for each part
9	Accuracy of Solution	Depends on the number of modes considered For RSM, the maximum modal responses are combined in some approximate way	Desired accuracy can be obtained, in general, by adjusting the integration time step	Desired accuracy can, in general, be obtained by adjusting the number and range of frequencies considered

¹Response Spectrum Method (RSM), Time History Method (THM)²Implicit Integration Method (IIM), Explicit Integration Method (EIM)

of motion are given by Equation 1-1 was proposed originally by Foss (1958). The Foss method has been applied to aerospace type problems (Hasselman, 1972) and soil/structure interaction problems (Jennings and Bielak, 1973), but seldom, if ever, to nuclear power plant analyses. This method very closely resembles the normal mode method, which is probably the most common method used for structural analysis problems. A description of

this method may also be found in Hurty and Rubenstein (1964)

1.3.5 FINITE ELEMENT METHODS

The finite element method has been used extensively in recent years to solve nuclear power plant structural analysis problems. The main reason for the great popularity of this method is that it can be used to solve complex structural and continuum mechanics problems

with a general procedure that is suitable for present-day high-speed digital computers. This method can be routinely programmed to accommodate such complex and difficult problems as nonhomogeneous materials, nonlinear stress/strain relations, and complicated boundary conditions. In addition, the method is equally applicable to static and dynamic problems in structural analysis. There are now in use more than a dozen large general-purpose computer programs using the finite element method. These programs are capable of solving almost any type of structural analysis problem, cost and computer capacity being the only limitations.

1.3.5.1 Summary of Basic Concepts

The finite element method has now become recognized as a general method of wide applicability to structural problems. This method is discussed in several textbooks, e.g., Zienkiewicz (1971), Desai and Abel (1972), Oden (1972), and Przemieniecki (1968), and these can be consulted for background information.

The finite element method is essentially a process through which a structure or continuum system with infinite degrees of freedom (continuous distribution of mass and stiffness) can be approximated by an assemblage of subregions, or elements each with a finite number of unknowns. Instead of analyzing the entire system in one operation, the solutions are formulated for each element and combined to obtain the solution for the entire structure.

Problems in the field of structural or continuum analysis can usually be solved by one of three approaches: the displacement method, the force or equilibrium method, and the hybrid or mixed method. Displacements (including rotations) are treated as primary unknown quantities in the displacement method; stresses

(including moments) are assumed as primary unknown quantities in the second method; and some displacements and some stresses are treated as unknown quantities in the hybrid method. However, the displacement method is the only approach that is used for solving nuclear power plant structures because it is more amenable to computer analyses. Therefore, the discussion here is limited to the displacement method only. The following are the important steps necessary in the analysis of a structural system by the finite element method.

- a. Discretization of the system by appropriate finite elements
- b. Identification of the degrees of freedom necessary
- c. Selection of displacement (shape) functions for each type of finite element
- d. Formulation of strain/displacement and stress/strain matrices for each finite element
- e. Formation of element stiffness matrices
- f. Formation of global stiffness matrix
- g. Formation of mass matrix
- h. Solution of equations of equilibrium

The details describing these steps may be found in the standard texts listed above.

1.3.5.2 Material Representations

Various material models are currently available for analyzing nuclear power plant structures. These models are applicable to all finite element computer programs with capabilities to solve linear two- or three-dimensional problems. Two different approaches to nonlinear material models are discussed here: Equivalent linear models as incorporated in the LUSH code (Lysmer et al., 1974), and inelastic

models based on theory of plasticity that are available in the FEDRC code (AA, 1975d). Although other types of models are available for nonlinear materials, e.g., concrete models, these two models are used more often for nuclear power plant analyses.

Linear Materials. For linear, isotropic, and elastic materials, the stress/strain matrix [D] for a three-dimensional finite element is given by the generalized Hooke's law (Desai and Abel, 1972):

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & \frac{1-2\nu}{2} & 0 & 0 \\ \text{Symmetrical} & & & & \frac{1-2\nu}{2} & 0 \\ & & & & & \frac{1-2\nu}{2} \end{bmatrix} \quad (1-35)$$

It is seen that the stress/strain matrix [D] for linear isotropic materials is completely given by two elastic constants: Young's modulus E and Poisson's ratio ν . Since E and ν are related to the bulk modulus K and the shear modulus G, the stress/strain matrix above can also be expressed completely in terms of K and G.

For two-dimensional problems certain strain and stress components vanish so that the matrix [D] can be simplified considerably. For a plane strain finite element, the stress/strain relation simplifies to (Desai and Abel, 1972):

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{pmatrix} = \frac{E}{(1-\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix} \quad (1-36)$$

For a plane stress finite element, the stress/strain relation becomes (Desai and Abel, 1972):

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{pmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix} \quad (1-37)$$

Similarly, the stress/strain relation for axisymmetric solid finite element is (Desai and Abel, 1972):

$$\begin{pmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{\theta\theta} \\ \tau_{rz} \end{pmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{pmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{\theta\theta} \\ \gamma_{rz} \end{pmatrix} \quad (1-38)$$

in which σ_{rr} , σ_{zz} , and $\sigma_{\theta\theta}$ are the normal components of stress; τ_{rz} is the shear stress; ϵ_{rr} , ϵ_{zz} , and $\epsilon_{\theta\theta}$ are the normal components of strain; and γ_{rz} is the shear strain, all referred to a cylindrical coordinate system.

Equivalent Linear Models. When soil media are subjected to earthquake motions, they exhibit nonlinear properties, which are primarily due to the strain-dependence of their stiffness and energy-dissipation characteristics. Therefore, to take the nonlinear effects into consideration in the seismic analyses of nuclear power plants, several finite element computer programs have included capabilities to consider the nonlinear properties of certain finite elements. These elements in general are two-dimensional and are intended to model soil media.

An approximate method for considering strain-dependent soil properties in the seismic analyses of nuclear power plants is provided in the LUSH code (Lysmer et al., 1974). This code approximates a nonlinear solution by a linear analysis in which the stiffness and damping values used in the analysis are compatible with the effective shear strain amplitudes at all points of the system. This procedure is an equivalent linear method.

The nonlinear stress/strain relations for soils used in the LUSH code are based on the published data on strain compatible properties for typical clays and sand by Seed and

Idriss (1970). According to this data, the nonlinear response of soils results from strain-dependent shear moduli and hysteretic characteristics of soils. Changes in Poisson's ratio are small and hence it is considered a constant for each soil. Since the damping factor is proportional to the area inside the hysteresis loop, the nonlinear characteristics of the soil are described by the shear modulus and damping factor of the soil.

Since the LUSH code is based on the method of complex response, the damping in each element can be introduced by the use of the complex shear modulus G^* and Young's modulus E^* from Equation 1-33, to ensure that the element will have a uniform critical damping ratio. The stress/strain matrix $[D]$ then can be formed by replacing E in Equation 1-35 by this E^* .

Inelastic Models Based on Theory of Plasticity. A second class of nonlinear material models used with finite element techniques utilizes variable modulus and theory of plasticity. Unlike the equivalent linear model used in the LUSH code, these models exhibit material laws that appropriately change with time. However, the computer time required to perform a dynamic analysis of a system using these models is greatly increased.

Although in recent years a number of earthquake analyses of structures have been carried out using inelastic finite element codes with plasticity-based models, e.g., Dibaj and Penzien (1969), the models described here are primarily based on those used in the FEDRC code (AA, 1975d). FEDRC is a dynamic, inelastic two-dimensional finite element computer program and incorporates a library of sophisticated nonlinear material models. The program has been used in several earthquake-related applications requiring special nonlinear material models (Reddy and Ts'ao, 1975; Werner and VanDillen, 1977).

For materials obeying the theory of plasticity, the stress/strain relations before yielding (initial elastic constants) can be based on constant values or variable modulus models. Several different variable modulus models are provided in FEDRC. The user can select a model to fit the experimental data for the material. For example, for rock type material, the bulk modulus B and the shear modulus G are assumed to depend on the current value of elastic volumetric strain, μ , and its previous maximum value $\mu_{\max}$ (AA, 1975d).

1.3.5.3 Energy-Absorbing Boundaries

For the structural system of a nuclear facility, the foundation medium extends indefinitely in one or more directions, but a finite element model of the system must necessarily be of finite dimensions. This truncation of space creates an artificial boundary in the finite element model; therefore, reflections from the fictitious boundary intermingle with the real response of the structure and introduce undefined errors in the solution. A possible remedy is to move the artificial boundaries far away so that reflections do not reach the points of interest during the time of interest. In general, this procedure results in a model with a large number of finite elements and is therefore impractical, especially if the time of interest is long. To suppress artificial reflections from a boundary, several approaches have been developed, which are described briefly here.

Transmitting Boundary. Ang and Newmark (1971) developed the concept of a transmitting boundary for use in finite difference formulations. Although this approach can be extended to finite element problems, it is not directly applicable to such problems.

The transmitting boundary is based theoretically on the concept of the step-wise transmission of D'Alembert forces. This approach

simulates exactly the effects of an infinite space if the wave propagation velocity in the medium is known. Therefore, for linear one-dimensional elastic propagation problems, the transmitting boundary is exact, in the sense that no errors, other than those associated with the basic finite difference approximations, are introduced at the artificial boundary. This concept can be extended approximately to problems in two and three dimensions and to inelastic media.

Ang and Newmark applied this approach in solving several wave propagation problems in infinite media. These problems include one-dimensional, two-dimensional plane strain and axisymmetrical propagation problems in elastic media, and an axisymmetric propagation problem in a layered elastoplastic medium. The results from these problems in general were satisfactory and agreed well with the closed-form solutions where available.

Quiet Boundary. The concept of a quiet boundary was developed for use in a two-dimensional finite element computer program called DEPS (AA, 1972a). The DEPS code is capable of solving dynamic, elastic, two-dimensional, and axisymmetric problems. The quiet boundary procedure in the DEPS code attempts to prevent unwanted reflections from boundaries of a finite element mesh. This procedure when applied to a boundary anticipates a reflected signal from the boundary and cancels it by superimposing a signal of equal magnitude and of opposite sign. Therefore, reflections at a boundary are significantly reduced.

Energy-Absorbing Boundary. Lysmer and Kuhlemeyer (1969) developed a general method consisting of a special viscous boundary (or an energy-absorbing boundary) which absorbs the energy of incoming waves so that little or no

reflections occur at the boundary. The boundary is subjected to special conditions which ensures that all energy arriving at the boundary is absorbed. For this purpose Lysmer and Kuhlemeyer proposed the following conditions at the boundary:

$$\begin{aligned}\sigma &= a \rho c_p \dot{w} \\ \tau &= b \rho c_s \dot{u}\end{aligned}\tag{1-39}$$

Here σ and τ are the normal and shear boundary stresses, respectively; $\dot{w}$ and $\dot{u}$ are the normal and tangential boundary velocities, respectively; ρ is the mass density; c_p and c_s are the velocities of P-waves and S-waves, respectively; a and b are dimensionless parameters whose values are chosen to minimize the energy reflected by the boundary. The above boundary conditions correspond to a situation in which dashpots oriented normal and tangential to the boundary are attached.

If a harmonic P-wave is incident at the boundary of an elastic medium as shown in Figure 1-10, the ratio of the incident energy E_i and the reflected energy E_r depends only on the incident angle θ and Poisson's ratio ν (Lysmer and Kuhlemeyer, 1969). The variation of the energy ratio with θ for a Poisson's ratio of 0.25 is shown in Figure 1-11. Other values of Poisson's ratio give similar results. It is noted that a unit ratio corresponds to perfect reflection while a zero energy ratio corresponds to complete absorption. It is seen from Figure 1-11 that the maximum absorption is given when $a = b = 1$. A more accurate measure of the overall ability of the boundary to absorb the incident energy is the average of the product $(E_r/E_i) \sin \theta$. This quantity is shown in Figure 1-12 for $a = b = 1$. The area under the curve represents the average of reflected energy. From this figure, it is seen that the viscous boundary is 98.5% effective in absorbing P-waves (Lysmer and Kuhlemeyer, 1969).

FIGURE 1-10. INCIDENT AND REFLECTED ENERGIES - P-WAVE INCIDENCE

FIGURE 1-11. ENERGY RATIO FOR INCIDENT P-WAVE

FIGURE 1-12. EFFECTIVE ENERGY RATIO OF INCIDENT P-WAVE

Similar results are expected for incident S-waves so that the maximum absorption of the incident energy is obtained when $a = b = 1$. The boundary is about 95% effective for incident S-waves. Therefore, the viscous boundary with conditions defined by Equation 1-39 with $a = b = 1$ is a nearly perfect absorber of P- and S-waves. For other types of waves, e.g., Rayleigh waves, the viscous boundary is not as effective.

The energy-absorbing boundary by Lysmer and Kuhlemeyer (1969) is exact for one-dimensional elastic wave propagation problems. In two- and three-dimensional problems, the method is approximate and the efficiency in transmitting the energy depends on the angle of incident waves with respect to the boundary. This technique has been incorporated in several linear and nonlinear, two- and three-dimensional

programs, e.g., TRI/SAC (AA, 1976) and FEDRC (AA, 1975d). The usage of this boundary in several applications appears to provide satisfactory solution for most problems.

Consistent Boundary. Lysmer and Waas (1972) developed a technique for modeling a semi-infinite domain by a finite region using a consistent boundary separating the finite and infinite regions. This technique is essentially based on the substructuring method of Rubenstein (1966). Lysmer and Waas developed this technique for a special case in which Love waves propagated in a two-dimensional medium; however, the concept is applicable to more general cases.

Basic to the consistent boundary technique by Lysmer and Waas is the division of a semi-infinite region as shown in Figure 1-13. In Figure 1-13, the irregular zone in the middle is

FIGURE 1-13. SUBDIVISION OF A LAYERED SEMI-INFINITE REGION

region of interest. This zone is flanked on either side by a semi-infinite layered medium. All the zones were underlain by a rigid half-space. Lysmer and Waas considered only Love waves propagating in the medium shown in Figure 1-13 in which no motions occur in the plane of the paper.

If the irregular core region is removed and substituted by equivalent distributed forces corresponding to the internal stresses, the dynamic equilibrium of the far field will be preserved. The boundary displacements resulting from these boundary forces were determined using a semianalytic technique leading to the dynamic stiffness of the far-field regions. The dynamic stiffness was then used to analyze accurately the core region alone to include the effects of the far-field regions. The analysis of the core region was then performed using the finite element method.

For this case, the displacements in the semi-infinite layered regions can be expanded in terms of eigenfunctions corresponding to natural modes of wave propagation in each stratum. For each mode, stresses are determined at the consistent boundaries. The stresses are actually expressed in terms of the nodal displacements using modal participation factors. The resulting relationship between boundary stresses and displacements defined the dynamic stiffness of the far-field layered regions. The dynamic stiffnesses of the semi-infinite layered regions are then used to determine the response of the irregular region of interest. Lysmer and Waas demonstrated that excellent results can be obtained with their technique even for the case when the consistent boundary is very close to the source of excitation. However, the procedure is limited to ideal horizontal layers.

Kausel, Roesset, and Waas (1975) extended the Lysmer-Waas approach to problems with axisymmetric geometry and arbitrary loading.

This axisymmetric system was idealized by a finite region surrounded by a semi-infinite far field as shown in Figure 1-13. All the regions were underlain by a rigid half-space. The irregular region is discretized using toroidal finite elements having three degrees of freedom per nodal ring. The far field consisting of horizontal layers is represented by a semi-analytic consistent boundary.

The solution technique used by Kausel, Roesset, and Waas consists of a representation of the loads and displacement in a Fourier series similar to that used by Ghosh and Wilson (1969). For displacements, this representation takes the form

$$\begin{aligned}\mu &= \sum_n \bar{u} \begin{pmatrix} \cos n\theta \\ \sin n\theta \end{pmatrix} \\ \omega &= \sum_n \bar{w} \begin{pmatrix} \cos n\theta \\ \sin n\theta \end{pmatrix} \\ v &= \sum_n \bar{v} \begin{pmatrix} -\sin n\theta \\ \cos n\theta \end{pmatrix}\end{aligned}\tag{1-40}$$

where u , w , and v are the components of radial, vertical, and tangential displacement, respectively. The terms of $\cos n\theta$ for u and w and $-\sin n\theta$ for v are used to represent symmetric components of displacement, while the remaining terms correspond to antisymmetric displacement components.

The above displacement expansions are used to develop a stiffness matrix for the finite element region that relates the in-plane and out-of-plane degrees of freedom to the corresponding external forces. This stiffness matrix is a function of the Fourier term number, n . Then, in representing the far-field region, the finite element region is considered as being removed and replaced by equivalent distributed forces corresponding to the internal stresses. The dynamic stiffness matrix for the far-field

region relating the boundary forces and displacements is then computed using a procedure that combines analytic techniques with the finite element method. This can be superposed onto the stiffness matrix representing the finite element region; this resultant stiffness matrix is used in the final solution to the total problem.

1.3.5.4 Size of Elements

An important decision in the discretization of a structural system by finite element procedures is to specify the maximum sizes of the finite elements to be used in the discretization. The sizes of the elements affect the quality of the results and the cost of computations. In general, the smaller the finite elements, the more accurate are the results. In fact, the basic assumption in the usage of the finite element method is that the solution of a problem by this method converges to the exact solution if the finite elements used are made infinitesimally small. Thus it appears that it is always advisable to use very small elements. However, the use of very small elements results in a large number of elements, which may result in excessive cost and numerical inaccuracies in the results.

For static analyses, the factors to be considered in the design of finite element models include the type of output desired, the presence of regions with high stress concentrations and whether a nonlinear or linear analysis is required. If the output desired consists of displacements only, larger size elements may be permissible. However, if the stresses are to be calculated, a finer mesh consisting of smaller sized elements is desirable as the stresses are calculated from the derivatives of displacements. In regions of high stress concentration, the elements chosen should be small enough to describe accurately the high stress gradients. Away from these regions, where the stress distribution is nearly uniform, the elements can

be very large in size. In problems in which there are regions in which the stress is expected to exceed the elastic range and nonlinear behavior is to be considered, the elements in this region may have to be smaller in size than elsewhere as the nonlinear properties are stress or strain dependent and cause local inhomogeneities.

For dynamic analyses, the design of a finite element model is not only influenced by the distribution of stiffness properties as in static analyses, but also by the distribution of inertia properties of the structural systems. Therefore, the discretization process should properly consider variations in the distributions of stiffness and mass properties. For uniform and homogeneous regions such as free-field soil media, the basic element size should be selected such that the element size is small compared to the wave length of the slowest waves propagating through the medium at the highest frequency of interest. Experience with the dynamic analyses using the finite element method has shown that the maximum dimension of an element $h_{\max}$ should satisfy the following inequality:

$$h_{\max} \leq \frac{V}{b f_{\max}} \quad (1-41)$$

where V is the velocity of propagation of the waves, $f_{\max}$ is the highest frequency of interest and b is a constant that usually has a value between 4 and 5. If the wave propagation phenomenon is complex, it is not possible to identify the type of wave that is significant. In such cases, V is generally taken as equal to the velocity of the shear waves in the medium, although other waves, such as Rayleigh waves, may travel somewhat slower than the shear waves. If the maximum element size satisfies the above equation, the finite element mesh should not distort all waves of interest due to the discretization process. Otherwise the waves

will be artificially distorted and the response will be in error.

The selection of the basic element sizes for modeling structures is based primarily on the capability of the elements to respond to the desired frequencies. Specifically, the fundamental frequency of each element should be higher than the highest frequency of interest. The fundamental frequency of an element may be estimated based on analytical considerations. The fundamental frequencies of elements considered should correspond to the primary deformation modes of interest. For example, only the fundamental bending frequency of a beam element is to be considered if the element is expected to resist loads through bending only. Thus even if the frequencies corresponding to axial and torsional modes are lower, they are ignored for the purpose of determining the maximum element sizes. The highest frequency

of interest for a given problem may be affected by the frequency content of the input seismic motions. For example, if these motions are digitized, then the highest frequency that is significant is related to the time interval at which the motions are digitized. If the input is digitized at the time interval of Δt , the highest frequency represented by the data is given by

$$f_{\max} = \frac{1}{2\Delta t} \quad (1-42)$$

Another characteristic of the discretization that affects a static or dynamic analysis is the aspect ratios of the elements. The aspect ratio indicates the general shape of the element. For a two-dimensional rectangular element, it is defined as the ratio of the length to the width of the element. Experience has shown that this ratio should be kept below 5; otherwise errors in excess of 10% may result.

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CHAPTER 2

EQUIVALENT DAMPING VALUES

2.1 INTRODUCTION

The current technology for representing energy losses in nuclear power plants as equivalent viscous damping involves a broad spectrum of research programs for analytical methods, testing and data processing procedures, and techniques for defining damping for design applications. Applications of these procedures to conventional structures and to nuclear plant structures, equipment, and piping are included here, along with general trends indicated by the results of these applications. A specific investigation of material and radiation damping in soil materials is covered in Werner (1976).

2.2 FORMULATION OF DAMPING MATRICES

For a mathematical model of a nuclear plant subjected to earthquake ground motions and consisting of n degrees of freedom, the equations of motion can be expressed in matrix form as:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = -[M][L]\{\ddot{S}(t)\} \quad (2-1)$$

where

$\{\ddot{X}\}$, $\{\dot{X}\}$, $\{X\}$ = Vectors of relative accelerations, velocities, and displacements, respectively, at time t

$[M]$ = Mass matrix

$[C]$ = Damping matrix

$[K]$ = Stiffness matrix

$[L]$ = Matrix governing the direction of excitation

$\{\ddot{S}(t)\}$ = Vector of input ground motion time histories

This system of equations can be solved numerically, through either step-by-step integration procedures or normal mode methods.

Normal mode procedures, which are most widely used in current practice for a linear system, require diagonalization of the system stiffness and damping matrices. Diagonalization of the stiffness matrix is a straightforward result of the eigenvalue extraction process; however, diagonalization of the damping matrix requires certain assumptions pertaining to the form of this matrix. The assumption of Rayleigh damping (i.e., the damping matrix expressed as a weighted sum of the stiffness and mass matrices) is most commonly employed in current practice; however, other more elaborate assumptions (e.g., Caughey, 1960; Wilson and Penzien, 1972; Nielsen, 1964) have been used and are summarized by Werner and Reddy (1975).

2.3 TECHNIQUES FOR DEFINING EQUIVALENT VISCOUS DAMPING FACTORS

Techniques for defining equivalent viscous damping ratios for nuclear plants fall into two classes: analytical methods, which include techniques for representing certain known energy loss mechanisms, such as hysteresis effects, Coulomb damping, etc., as equivalent viscous damping factors; and experimental methods, which include approaches for defining equivalent viscous damping factors from response measurements without particular regard toward which energy loss mechanisms actually exist in the elements being tested.

2.3.1 ANALYTICAL METHODS

A number of techniques have been devised for estimating equivalent viscous damping that represents actual energy losses in structures or structural elements. Perhaps the first approach of this type was developed by Jacobsen (1930)

in his classic study of the response of single-degree-of-freedom systems under the influence of various types of damping and subjected to sinusoidally varying external forces. For a system with Coulomb damping or with a damping force proportional to the n th power of the velocity, Jacobsen was able to develop an equivalent viscous damping coefficient by equating the work done by the actual system during a single cycle of loading to the work done by the equivalent viscous damped system (Fig. 2-1a).

Other analytical methods for estimating modal damping ratios include the work of Jacobsen (1960), Hudson (1965), and Jennings (1968). Jacobsen obtained an equivalent viscous damping factor as the ratio of the hysteresis loop area (i.e., half the area of the complete hysteresis loop) to the work area under the skeleton curve (which is a measure of the strain energy in the system) (Fig. 2-1b). Hudson established viscous damping ratios for hysteretic structures by varying the damping in the linear oscillator until its peak response amplitude matched that of the structure (Figs. 2-1c,d). Jennings assessed alternative methods for computing equivalent damping ratios for an elastoplastic single-degree-of-freedom system subjected to harmonic base excitations (Fig. 2-2). Another pertinent analytical study was made by Mayes and Mowbray (1975), who showed that Coulomb damping representation using equivalent viscous damping is not fully satisfactory.

In summary, the various analytical methods have shown that, under many circumstances, only limited success can ever be obtained by describing energy losses as viscous damping. Among the variables that greatly affect this equivalent viscous damping prediction are:

- The type of nonlinear energy losses being represented (i.e., Coulomb

damping, velocity-squared damping, hysteresis effects, etc.)

- The nature of the input forcing function; i.e., harmonic support motions or random type earthquake motions
- The dynamic characteristics of the system being represented
- The degree of inelastic deformation (i.e., ductility ratio) anticipated in the nonlinear system
- The particular method used to estimate the equivalent viscous damping

2.3.2 EXPERIMENTAL METHODS

Experimental methods used to estimate damping ratios are:

- Free-vibration tests, which use the logarithmic decrement to estimate damping ratios (Fig. 2-3)
- Steady-state vibration tests, which measure the steady-state response of the structure to harmonic forces and estimate damping from properties of the resonant frequency peak of the measured response or from structure response characteristics based on eccentric mass excitation (Nielsen, 1964; Hoerner and Jennings, 1969; Jennings et al., 1972) (Fig. 2-4).
- Random vibration measurements, which arise from ambient vibrations (e.g., Stubbs and McLamore, 1973; Trifunac, 1972), from blast loads (e.g., Bleiweis et al., 1973; Blume, 1969) and from earthquakes (Hart and Vasudevan, 1975). The basic approaches used to estimate damping from these measurements involve (1) obtaining structure transfer

(a) Frequency dependence of equivalent viscous damping force for different values of velocity - damping power, n and harmonic loading (Jacobsen, 1930)

(b) Definition of equivalent viscous damping based on maximum strain energy considerations (Jacobsen, 1960)

(c) Effect of shape of hysteretic force-deflection curve and ductility ratio on equivalent viscous damping ratio under harmonic loading (Hudson, 1965)

(d) Earthquake-induced spectral amplitudes for hysteretic structures with ductility ratios ranging from 2 to 4 (Hudson, 1965)

FIGURE 2-1. RESULTS FROM ANALYTICAL METHODS FOR DEFINING EQUIVALENT VISCOUS DAMPING

(a) Damping ratio vs. ductility ratio for different methods of estimating damping

	Name	Description
Amplitude Method	1 Resonant Amplitude	Linear structure selected to have same mass, stiffness and natural period (at small oscillations), and resonant response amplitudes as elastoplastic structure. Damping factor of linear structure chosen so that linear structure and elastoplastic structure at resonance dissipate equal energies per cycle.
Dynamic Methods	2 Dynamic Stiffness	Linear structure has same mass as elastoplastic oscillator, and spring constant of linear oscillator chosen so that resonant frequencies of two structures are identical. Resonant amplitudes are matched, and damping ratio for linear structure chosen so that energy dissipated per cycle is same as for elastoplastic structure.
	3 Dynamic Mass	Stiffness of linear structure set equal to initial stiffness of elastoplastic structure, and mass of linear structure varied to match resonant frequency of elastoplastic structure. Resonant amplitudes and energy dissipated per cycle set equal for two structures.
	4 Constant Critical Damping	Product of stiffness and mass of linear structure set equal to amplitude-dependent product for elastoplastic structure. In addition, ratio of stiffness and mass of linear structure set equal to amplitude-dependent frequency of elastoplastic structure.
Geometrical Methods	5 Geometrical Stiffness	Stiffness of linear structure set equal to slope of lines joining ends of hysteresis loop of elastoplastic structure, in addition, masses of two structures also set equal. Resonant amplitudes of linear structure and elastoplastic structure as well as energies dissipated per cycle at resonance are equated.
	6 Geometrical Energy	Geometry of skeleton curve and hysteresis loop defines equivalent viscous damping, other properties of equivalent linear structure not considered. Corresponds to approach devised by Jacobsen (1960).

(b) Description of methods for computing damping ratios

FIGURE 2-2. COMPARISON OF METHODS FOR COMPUTING EQUIVALENT DAMPING RATIOS FOR HYSTERETIC SIMPLE STRUCTURE AND HARMONIC INPUT (Jennings, 1968)

(a) Definition of logarithmic decrement

(b) Variation of damping with logarithmic decrement

FIGURE 2-3. MEASUREMENT OF DAMPING USING FREE VIBRATION TESTS (Thomson, 1965)

NOTE: HALF-POWER METHOD: $\delta_n = \frac{\Delta\omega}{2\omega_n}$

$$Q \text{ METHOD: } Q = \frac{1}{2\delta_n(1 - \delta_n^2)^{1/2}} \approx \frac{1}{2\delta_n}$$

(a) Half-power method and Q method

NOTE: $\frac{A_1}{A_2} = \frac{(1 + 8\delta_n^2)^{1/2}}{8\delta_n} \approx \frac{1}{8\delta_n}$

(b) Measurement of damping from tests using eccentric rotating mass oscillators (Hudson, 1961)

(c) Determination of damping from out-of-phase component of acceleration for system with closely spaced modes (Jennings, et al., 1972)

FIGURE 2-4. MEASUREMENT OF DAMPING USING STEADY-STATE RESPONSE TESTS

functions from ratios of Fourier spectra of the structure and its base, (2) using curve-fitting techniques in the frequency domain (Fletcher and Powell, 1963) or time domain (Holtz, 1973), or (3) finding the poles in the complex plane of the response transfer function, where each mode of the system will have an associated pole (Otnes, 1974)

These experimental methods are described in Werner and Reddy (1975), and the reader is also referred to standard textbooks on vibrations and dynamics (e g , Biggs, 1964, Thomson, 1965, Richart et al , 1970)

2 4 DESIGN ESTIMATES AND MEASUREMENTS OF VISCOUS DAMPING RATIOS

To define damping for the design of nuclear plant elements, two options can be pursued by the engineer First, he can directly utilize the damping ratios set forth in Regulatory Guide 1 61 Alternately, appropriate tests can be used to define the damping characteristics of various plant elements

2 4 1 REGULATORY GUIDE 1 61

The current (1973) practice regarding the specification of equivalent viscous damping in nuclear plant structural elements and in supports and foundations of the equipment is provided in Regulatory Guide 1 61 (USAEC, 1973) This guide delineates damping values acceptable to the Regulatory staff for use in the elastic modal dynamic seismic analysis of Category I structures, systems, and components According to Regulatory Guide 1 61, the specified damping ratios should be used for all modes considered in the dynamic analysis, higher damping values can be used only if supported by documented test data The specified damping values are provided in Table 2-1

The selection of these damping values appears to be based primarily on the judgment of experienced engineers, although there are no tests or analysis programs that have been cited as backup information for these specified damping ratios The recommendations contained in Regulatory Guide 1 61 were first published by Newmark, Blume, and Kapur (1973), similar

TABLE 2-1. REGULATORY GUIDE 1.61 DAMPING VALUES¹

Structure or Component	Percent of Critical Damping	
	Operating Basis Earthquake or 1/2 Safe Shutdown Earthquake ²	Safe Shutdown Earthquake
Equipment and large-diameter piping system ³ , pipe diameter greater than 12 in	2	3
Small-diameter piping systems, diameter equal to or less than 12 in	1	2
Welded steel structures	2	4
Bolted steel structures	4	7
Prestressed concrete structures	2	5
Reinforced concrete structures	4	7

¹Based on recommendations by Newmark, Blume, and Kapur (1973)

²In the dynamic analysis of active components as defined in Regulatory Guide 1 48 (USAEC 1973b), these values should also be used for SSE

³Includes both material and structural damping If the piping system consists of only one or two spans with little structural damping, use values for small-diameter piping

Recommendations in a slightly different form were provided in earlier publications (Newmark, 1969, 1972; Newmark and Hall, 1969). None of these references provide any quantitative background information, although some qualitative discussion which describes the judgment factors behind the selection of these damping values was provided by Newmark (1969).

2.4.2 DAMPING MEASUREMENTS FROM VIBRATION TESTS OF STRUCTURES, EQUIPMENT, AND PIPING

Because it is permissible to utilize damping values higher than those provided in Regulatory Guide 1.61 if the damping values are supported by adequate tests, it is appropriate to review some of the past test programs associated with nuclear plant structures, equipment, and piping. Some tests have also been carried out that are not related solely to nuclear plants but whose results can be utilized to help define damping in nuclear plant elements.

2.4.2.1 UCLA Tests

The UCLA tests of nuclear plant structures, internals, and piping are documented in numerous reports and summarized in Hart and Ibanez (1973) and Hart et al. (1973). These tests were conducted on numerous nuclear reactor facilities including the experimental gas-cooled reactor (EGCR) at Oak Ridge, Tennessee, (Smith and Matthiesen, 1969); the UCLA research reactor (Matthiesen and Smith, 1966); the Carolinas-Virginia tube reactor (Hornbuckle et al., 1969); the Enrico Fermi I sodium cooled reactor (Bleiweis, 1970); and the San Onofre Nuclear Generating Station, Unit I in Southern California (Smith et al., 1971). In addition, a one-fifth scale model of a reactor pressure vessel was tested at the UCLA Structures Laboratory (Hart et al., 1973).

The response and damping characteristics of these various reactor facilities were investigated using sinusoidal steady-state vibrators,

free vibrations induced by snapback loads, and blast-induced ground vibrations. The steady-state vibrators were originally developed at Caltech (Hudson, 1961) and consist of eccentric masses rotating about a horizontal axis. For this case, the force level is proportional to the eccentric mass, the amount of eccentricity, and the square of the rotational frequency. The free vibration snapback tests were conducted by applying static loads using a hydraulic ram and then suddenly releasing the load. The blast tests were performed by detonating high explosives in the soil at varying distances from the plant (from 25 to 1000 ft away) and at varying depths in the soil (from 0 to 60 ft deep). The type and quantities of explosives (from 10 to 2000 lb) was also varied in these tests. A summary of the results of these studies is provided in Figure 2-5. These results show that the full-scale tests were conducted at vibration levels well below those that might be anticipated as resulting from the OBE or SSE for the site. However, the model tests were conducted at amplitude levels more nearly representative of strong earthquake response.

2.4.2.2 Indian Point Primary Cooling Loop Tests

Tests of primary loop components at the Indian Point Unit 2 reactor were conducted by Olsen, Singleton, and Bohm (1973). Items tested in this program included the steam generator, main coolant pump, and primary coolant piping. Low amplitude, simple harmonic excitations were applied in a frequency sweep covering the range of interest, and the response was recorded by suitably located accelerometers. For closer examination of the system modes, and to obtain damping values at higher excitation levels, complementary forced vibration tests were conducted at frequencies near the system resonances. These complementary tests consisted of a single wave forcing

(a) One-fifth model tests

(b) Full scale tests

FIGURE 2-5. DEPENDENCE OF DAMPING ON RESPONSE AMPLITUDE — UCLA TEST RESULTS (Hart et al., 1973)

function of increasing amplitude that is cut off at its peak amplitude in the first "beat"; damping was obtained from the free vibrations after termination of the excitation. Results from these tests are summarized in Figure 2-6. These tests indicate damping levels that: (1) are of the same order as the UCLA steam generator test results; (2) increase with increasing amplitude of excitation; and (3) vary from mode to mode. The damping in the steam generator was shown to arise primarily from motions of the internals, and, when compared to the damping of the pump, exhibited larger values for the same deflection and frequency. The apparent damping in the crossover leg was shown to also include energy dissipated in the steam generator and pump.

2.4.2.3 Nuclear Plant Equipment Tests

General recommendations for testing to define the seismic resistance of Class I electrical equipment in nuclear plants are provided in a standard published by the Institute of Electrical and Electronic Engineers (IEEE 344-1973) and in a subsequent review of this standard by Skreiner et al. (1974). Two experimental methods for defining damping in equipment are recommended, namely the implementation of free-vibration measurements after removal of a driving force and the use of the half-power method. The Q-method, which is based on the amplitude of the resonance peak, was judged to be unsuitable for equipment damping measurements. Illustrations of equipment tests, which

LEGEND:

- △ 2.9 Hz
  - 6.5 Hz
  - 17.8 Hz
  - 2.48 Hz
  - 3.15 Hz
  - 2.4-3.25 Hz
  - 2.4-3.25 Hz
- SHAKER TESTS (150°F)
- SHAKER TESTS (480°F)
- PREOPERATIONAL TESTS (270-530°F) (3σ)
- PREOPERATIONAL TESTS (270-530°F) (1σ)

(a) Steam generator - radial modes

LEGEND:

- △ 2.6 Hz
  - 6.9 Hz
  - 2.43 Hz
  - ▽ 3.15 Hz
  - 2.4-3.25 Hz
  - 2.4-3.25 Hz
- SHAKER TESTS (150°F)
- SHAKER TESTS (480°F)
- PREOPERATIONAL TESTS (270-530°F) (3σ)
- PREOPERATIONAL TESTS (270-530°F) (1σ)

(b) Steam generator - tangential modes

LEGEND:

- △ 4.5 Hz TANGENTIAL
  - 5.3 Hz RADIAL
  - 4.0-4.9 Hz
  - 4.0-4.9 Hz
- SHAKER TESTS (150°F)
- PREOPERATIONAL TESTS (270-530°F) (3σ)
- PREOPERATIONAL TESTS (270-530°F) (1σ)

(c) Coolant pump

LEGEND:

- 35.1 Hz
  - 41.5 Hz
  - △ 56.6 Hz
- SHAKER TESTS (150°F)

(d) Crossover leg

FIGURE 2-6. DAMPING RATIOS FOR INDIAN POINT, UNIT 2 PRIMARY LOOP
(Olsen, Singleton and Bohm, 1973)

indicate the nonlinear response characteristics of equipment enclosures, are provided in Table 2-2.

2.4.2.4 Damping Investigations for General Structures

In addition to tests that deal specifically with nuclear reactor structures, piping, and equipment, other experimental investigations that may provide important insights into basic energy dissipation mechanisms in building structures have been conducted. The investigations include (1) dynamic tests of reinforced concrete frame structures constructed at the Nevada Test site and subjected to underground blast loads (Blume et al., 1971); (2) steady-state forced vibration tests of a 22-story steel frame building (Jennings et al., 1972); and (3) tests of 8-story and 9-story reinforced concrete structures subjected to excitations

from an eccentric mass oscillator (Jennings and Kuroiwa, 1968; Englekirk and Matthiesen, 1967). All of these results illustrate the nonlinear nature of the building response (e.g., Fig. 2-7), and the latter set of investigations provide comparisons of buildup damping estimates using different estimation procedures (Table 2-3).

Other building damping estimates have been obtained directly from earthquake motion measurements. Studies along these lines by Hart et al. (1973) and by Hart and Vesudevan (1975) illustrate how the building damping ratio increases with an increasing level of excitation (Fig. 2-8). Iemura and Jennings (1974) use earthquake response measurements from a 9-story reinforced concrete structure to illustrate time dependent variations in building energy-dissipation characteristics (Fig. 2-9).

TABLE 2-2. DAMPING MEASUREMENT IN ELECTRICAL EQUIPMENT

Vibration Direction	Peak Acceleration, g	Peak Displacement, in	Natural Frequency, Hz	Damping Ratio, Percent
Side-to-Side	0.91	0.8	3.35	8.6
	0.3	0.18	4.07	5.2
	0.1	0.04	4.6	4.0
Front-to-Back	1.8	0.76	4.8	4.2
	1.0	0.42	4.86	3.7
	0.42	0.16	5.0	2.3
	0.24	0.09	5.1	1.9
	0.10	0.03	5.3	1.6

(a) Results from free vibration tests of 1600 lb electrical equipment enclosure (Prause et al., 1973)

Equipment Base Acceleration, g*	Equipment Top Acceleration, g*	Magnification Factor	Damping, Percent Critical
0.11	1.2	11.0	4.5
0.25	2.1	8.4	6.0
0.39	2.6	6.7	7.5
0.67	2.9	4.3	11.6

*Constant-amplitude horizontal sinusoidal vibration at 8 Hz natural frequency

(b) Results from harmonic load tests of "Typical" equipment cabinet (Prause and Ahlbeck, 1972)

FIGURE 2-7. DAMPING FACTORS MEASURED IN REINFORCED CONCRETE FRAME STRUCTURES AT NEVADA TEST SITE (Blume, Sharpe and Kost, 1971)

TABLE 2-3. COMPARISONS OF DAMPING VALUES COMPUTED BY DIFFERENT METHODS -- BUILDING VIBRATION TESTS¹

Structure Description and Reference	Mode	Displacement Amplitude Ratio or (Applied Force, kips) ²	Damping in Percentage of Critical		
			Half Power ³	Hudson (1961)	Nielsen (1964)
Nine-story Reinforced Concrete Shear Wall Structure (Jennings and Kuroiwa, 1968)	North-South Fundamental Mode	1 00	1 16	1 44	1 24
		1 62	1 45	Insuff data	1 35
		1 97	1 48	1 51	1 39
		2 45	1 75	1 88	1 49
		3 02	1 64	1 93	1 60
		0 75	1 54	1 74	1 47
	East-West Fundamental Mode	1 00	0 69	0 97	0 80
		1 71	1 06	0 96	0 89
		2 74	1 12	1 17	0 97
		3 44	1 12	1 15	1 01
		4 38	1 21	Insuff data	1 16
		5 08	1 48	1 50	1 20
	Fundamental Mode in Torsion	1 00	0 93	1 51	0 98
		1 70	0 96	1 40	0 99
		2 04	1 05	1 65	1 65
Eight-Story Reinforced Concrete Building with Lateral Resistance Provided by Shear Walls in East-West Direction and Space Frame in North-South Direction (Englekirk and Matthieson, 1967)	East-West Fundamental Mode	(1 2)	2 5	3 3	2 0
		(0 65)	2 5	3 3	1 9
	North-South Fundamental Mode	(0 4)	2 75	3 1	3 5

Notes ¹For both buildings, tests conducted using eccentric mass oscillator located on top floor to apply load to building

²Numbers in parentheses refer to applied force in kips while those not in parentheses correspond to displacement amplitude ratio

³See discussion and Eq 15, where $\delta_n = \Delta\omega/2\omega_n$, in Werner and Reddy (1975)

(a) Damping vs. resonant frequency amplitude of basement Fourier modulus (Hart, Lew and Di Julio, 1973)

(b) Damping vs. amplitude of spectral velocity (Hart and Vesudevan, 1975)

FIGURE 2-8. DAMPING RATIOS DERIVED FROM BUILDING RESPONSE MEASUREMENTS DURING SAN FERNANDO EARTHQUAKE

(a) Hysteretic response of single-degree-of-freedom oscillator

(b) Variation of damping ratio with time

FIGURE 2-9. INVESTIGATION OF ENERGY LOSSES IN MILLIKAN LIBRARY DURING SAN FERNANDO EARTHQUAKE (Iemura and Jennings, 1974)

2.5 ENGINEERING REPRESENTATION OF DAMPING FOR NUCLEAR PLANT SEISMIC ANALYSES

For the representation of damping for nuclear plant dynamic analyses, two basic approaches can be used: first, the complete damping matrix for the system can be formed and appropriate numerical procedures can be used to obtain the solution for the complete damped system, either in the time or frequency domain; second, a more common approach for nuclear plant applications is to use approximate techniques to define an equivalent modal damping for the system, and from this the dynamic response of the system can be computed using the normal mode method.

2.5.1 APPROACHES USING COMPLETE DAMPING MATRIX

The complete damping matrix for the system can be formed in two ways. One approach is to utilize the mass matrix, stiffness matrix, mode shapes, and frequencies of the system to define the damping matrix according to one of the idealized forms described in Section 2 of Werner and Reddy (1975). These forms permit a decoupling of the original equations of motion of the system so that the normal mode methods can be utilized. A second approach is to develop a more general damping matrix by first defining the damping characteristics for individual elements within the system; these estimates can be based on AEC design guidelines, such as Regulatory Guide 1.61 or from tests and data processing procedures of the type previously described here. Then using the geometric relationships and mutual connectivity of these various elements, the damping matrix for the entire system can be constructed. This damping matrix will in general not have an idealized form. There are various alternative techniques for obtaining the dynamic response of the system for this case; these include the use of direct integration

methods in the time domain, Fourier transform procedures to obtain the solution in the frequency domain, and nonproportional damping procedures.

2.5.1.1 Direct Integration Procedures

To obtain the dynamic response of a system using a step-by-step method, the equations of motion of the system (Eq. 2-1) are integrated directly in incremental time steps. The resulting response is obtained in the form of time histories corresponding to each dynamic degree of freedom in the system. For linear systems, the damping and stiffness matrices, $[C]$ and $[K]$, remain constant, while for nonlinear systems these matrices change from step to step but are assumed to remain constant during each time increment. The time step used in the direct integration procedure is selected by the analyst and is based on the highest frequency of interest in the calculations, the degree of nonlinearity in the system, and the characteristics of the particular integration procedure used in the calculations. The cost of a computerized dynamic analysis using direct integration procedures is affected by the number of time steps required, the size of the problem being analyzed, and the mathematical nature of the integration technique. Either implicit or explicit integration techniques can be used (Chuang and Isenberg, 1973).

2.5.1.2 Fourier Transform Techniques

A second approach, for use with linear or equivalent-linear systems, is to carry out the system response analysis in the frequency domain and then to use inverse procedures to obtain the corresponding solution in the time domain. This involves transforming the input time history to the frequency domain $\ddot{X}_g(i\omega)$ and determination of a complex frequency response function for each degree of freedom of interest within the system being analyzed, $H_j(i\omega)$. The

CHAPTER 3

PROBABILISTIC DESIGN FACTORS

3.1 INTRODUCTION

The basic objective of structural design is to assure the safety and serviceability of the proposed facilities. A nuclear power plant, for example, should safely withstand any set of seismic loads that may reasonably be expected to occur during its service life and should be capable of economically performing its operational functions under all normal load conditions. Ideally, for safe design, the designer must evaluate the effect of the most severe combination of potential loads, including loads due to accidents or natural hazards, and select structure, component, and equipment proportions so that these loads can be safely resisted.* Also, to maintain serviceability, the designer must evaluate the effect of loads that occur frequently and make certain that a satisfactory operating performance is possible during the occurrence of these loads.

Future loads on a structure, however, are uncertain and act in unpredictable combinations. Also, structural material properties are random and structural member behaviors are not fully understood. In addition, there are always errors in estimation of design variables, structural analysis, and construction techniques. Under these conditions of uncertainty, the *absolute* assurance of safety and serviceability, i.e., zero probability of failure, for most structures becomes impossible. Specifically, the absolute assurance of safety of a structure,

which would require consideration of extremely rare phenomena such as loads due to catastrophic events, would result in such a huge and expensive design that its construction would be economically prohibitive.

For this reason, the safety and serviceability of a structure may only be assured within acceptable levels of risk, i.e., small probabilities of structural failure or unserviceability are tolerated. The *acceptable level of risk* for a proposed structure should ideally be obtained from a cost-optimization analysis which considers the initial cost of the design as well as the expected socioeconomic losses due to its potential failure or unserviceability. This type of analysis indicates that structures whose functions are of high socioeconomic value, as well as structures whose failures may be associated with large socioeconomic loss, should have higher levels of reliability (i.e., lower levels of acceptable risk) than ordinary structures. For example, the acceptable risk for a hospital building, which has a critical function for an entire community, or a nuclear power plant, whose failure may have catastrophic consequences for large populations, should be smaller than the risk accepted for a residential building.

The uncertainties that are present in structural design, require that a rational structural design procedure be based on concepts of probability theory. This has long been recognized by many investigators who have studied various aspects of probabilistic structural analysis and design (e.g., Freudenthal, 1974; Bolotin, 1969). The probabilistic design of a structure requires (1) probabilistic assessment of loads, structural properties, and other

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*The material in this chapter has equal applicability to the structural design of nuclear power plant structures, components, and systems. However, for simplicity, reference will hereafter be made only to a structure.

design variables, and (2) probabilistic assessment of structural response to applied loads. The latter involves the evaluation of risks associated with unfavorable structural response or failure. Hence, a design is accepted if the calculated risk is smaller than the tolerable value of risk. This type of rigorous probabilistic design procedure, however, has seldom been practically applied.

The structural design procedures currently applied are based on deterministic principles. These procedures do not recognize the randomness inherent in future loads, structural properties, or other design variables. Also, they do not attempt to measure structural safety or serviceability in terms of the involved risks. Instead, these procedures assume *nominal* values of loads, material properties, etc., and on the basis of these values attempt to evaluate structural safety or serviceability in terms of arbitrary criteria that are not necessarily realistic. In this regard, it is often incorrectly implied that a structure designed in accordance to these procedures has zero risk of failure or unserviceability. In view of the condition of uncertainty prevailing for most design parameters, this implication is erroneous. In fact, the major disadvantage of the deterministic design method is that the true reliability of a structure designed with these procedures remains unknown and it may be different from the desired reliability. For example, the structure may be too conservative (i.e., from an economic point of view) or it may be unconservative (i.e., associated with undesirably high levels of risk). This disadvantage is of particular concern in designing costly structures that require very high levels of reliability, such as nuclear power plants.

There are several reasons why probabilistic design is not yet practically applicable: (1) probabilistic design procedures are increasingly complicated relative to the deterministic

procedures; for instance in many cases closed-form solutions are not possible and extensive additional computations are necessary, (2) the required statistical data, which must be the basis for probabilistic assessment of design variables, are not adequately available, and (3) the engineering community in general is not familiar with probability theory and has often rejected its use in structural design.

In recent years, interest in probabilistic methods in engineering has greatly increased, and considerable research in the areas of structural reliability and probabilistic design has been reported. Due to the complexity and impracticality of accurate analysis as well as the lack of complete probabilistic information, various approximate methods have been studied. Among these, the probabilistic structural analysis and design based on second moment theories has become the most promising (e.g., Cornell, 1969; Lind, 1971; Ang and Cornell, 1974). In this method only means, variances, and covariances (i.e., first and second moments) of variables, rather than their full probability distributions, are used. The failure probabilities are then evaluated on the basis of reasonable distributions assigned to specified performance functions. In this method, uncertainties due to various errors or approximations may also be included in the analysis. Clearly, the failure probability evaluated with such an approximate procedure will not be accurate. In particular, for low probabilities of failure, which are of most practical interest, the evaluated probabilities are found to be sensitive to the distribution assigned to the performance function (Ang, 1973). As a result, failure probabilities evaluated on the basis of second moment theories may be regarded as only *relative* measures of the true risks. However, such relative measures of the true risk are all that are necessary for most practical applications.

A particular advantage of the second moment probabilistic design procedure is that it can be developed in formats which are similar and comparable to deterministic design formats currently in use. This similarity will undoubtedly make the adoption of such a probabilistic design format into building codes much more feasible. In addition, this similarity in format allows direct comparison of probabilistic and deterministic formats, thus providing a theoretical basis for critical investigation of design criteria currently used in practice.

3.2 BACKGROUND INFORMATION

Current design philosophies include (1) deterministic design methods and their desirability from the viewpoint of a rational structural design procedure, and (2) basic criteria for safety and serviceability as applied in a probabilistic procedure and the concepts of system reliability versus element reliability and their relevance in the second moment probabilistic design.

3.2.1 DETERMINISTIC DESIGN PROCEDURES

Two design methods currently in use in the United States are the working stress design (WSD) and the load factor design (LFD) methods. Both methods are basically deterministic in the sense that they do not consistently include the randomness and uncertainty associated with loads, material properties, structural behavior, etc., or the probabilities that are associated with the occurrence of various loads or their possible combinations. Moreover, safety and serviceability criteria in these methods are not connected to any rationally defined risk level.

The WSD method requires that under nominal (service) load conditions, stresses in a structure remain below an "allowable stress" level as defined by codes (usually about 50 to

60% of yield or ultimate strength of the material used). Thus safety against unusual overloads or unpredictable behavior is provided by maintaining reserve strength, in terms of maximum stress value, in structural components. The safety margin or safety factor is expressed in terms of the ratio of ultimate or yield stress to the allowable stress. The design equation for any combination of loads in this method may be expressed as:

$$R' \geq \gamma \sum f_i X_i' \quad (3-1)$$

where R' = the nominal value of structural strength at a critical point in terms of yield or ultimate stress, X_i' = the nominal value of a load (e.g., dead, live, or earthquake load), f_i = the influence coefficient defining the stress at the point due to a unit value of the applied load, and γ = the safety factor, i.e., the ratio of R' to the allowable stress, as specified by the code. The summation is over all loads included in the combination.

In order to assure safety against rare overloads or unpredictable structural behavior, the LFD method attempts to directly include these effects into the design process. Thus, it requires that the "dependable" strength of each structural component, including the adverse effect of variables in material properties and behavior, be greater than the largest load effect resulting from the most critical combination of loads, including the effect of rare overloads. In order to arrive at an estimate of possible overloads, each nominal load value is multiplied by a *load factor* (usually greater than unity) which depends on the variability of the load. Thus a highly unpredictable load (e.g., earthquake load) has a larger load factor than a load which has small variability (e.g., dead load). The evaluation of the strength of structural components is based on approximate analysis techniques and nominal values of design variables, e.g., material properties or

member dimensions. Thus the estimated strength represents a nominal value. To include the adverse effect of uncertainties in these factors, the nominal strength is multiplied by a *strength* factor which is smaller than unity. The design equation for any combination of loads in this method may be expressed as:

$$\phi R' \geq \sum \delta_i f_i X_i' \quad (3-2)$$

where R' = the nominal value of strength of a particular component, X_i' = the nominal value of the load, f_i = the influence coefficient defining the load effect (bending moment, shear, etc.) at the component due to a unit value of the applied load, δ_i = the load factor (usually greater than or equal to unity), and ϕ = the strength factor (smaller than or equal to unity). The summation is over all loads included in a combination.

It must be realized that although load and strength factors are to include the effects of uncertainties in loads and resistances, they have not been evaluated on a probabilistic basis. Their values, as specified by codes, commonly remain invariant irrespective of the type of structure, its function, or its location, and regardless of the planned useful life of a structure. The determinations of these values have mainly been subjective and based on the calibration with other methods of design (e.g., the WSD method).

In designing for a specific load combination, both methods simply add the effects of various loads. No modifications in the safety or load and strength factors are provided, with the exception of combinations involving rare loads where usually a 25% reduction in safety or load factors is permitted. These modifications reflect the low probabilities that are associated with the occurrence of, for example, a major earthquake at the same time as the occurrence of extreme wind loads. Although these modifications are reasonable, they are arbitrary

because they are not related to the rate of occurrence of earthquakes or extreme winds.

From the viewpoint of an ideal design approach, the WSD and LFD methods have several shortcomings:

- a. Design loads and resistances are not clearly and consistently defined. Since these are uncertain, a statistical definition, such as the number of standard deviations above or below mean values, is more desirable.
- b. In the WSD method, the safety criterion which is defined in terms of stress is not a realistic criterion.
- c. In the WSD method, a single safety factor, which remains constant for all types of structures and site locations and for all load combinations, cannot consistently account for the combined effect of uncertainties in these variables.
- d. In the LFD method the variabilities in loads and resistances are roughly included in the design through the load and strength factors. However, these factors are not directly related to the variabilities of loads and resistances. In particular, a constant set of factors for all structures and site locations is not adequate.
- e. Neither method is able to consistently account for the uncertainties that may arise from approximations in structural analysis and errors in construction.
- f. In both methods, various loads are combined with no regard to the probabilities and correlations associated with their simultaneous occurrence.

- g. The safety margins as defined in the WSD and LFD methods are not consistent measures of structural reliability. As a result, structures designed on the basis of either method would generally not have a consistent level of safety. Moreover, if a structure with an unprecedented level of reliability is to be designed, there is no rational basis for specifying appropriate safety or load and strength factors that must be used.

The latter point is particularly significant in developing design criteria for nuclear power plants. Due to the high economic value of these structures and the catastrophic consequences that may result from their failures, exceptionally high levels of reliability are demanded. As a result, in addition to regular loads, loads due to extremely rare events (e.g., extreme intensity ground motions, tornado wind velocities, aircraft crash, and potential accidents during a natural hazard), which are commonly neglected in structural design, become significant and must be included in the design process. Since no theoretical basis exists for the evaluation of load factors, it is not clear what the appropriate factors should be for combinations involving these loads. In the absence of more appropriate load factors, the current practice (see USNRC Standard Review Plan, Sec. 3.7.1) is to use values that are determined on a subjective basis, or are arbitrarily derived from load factors used in conventional design. As a result, the true reliability of a nuclear power plant designed according to current load factors may not be consistent with the desired levels of safety. A conclusive solution to this problem is only possible if a theoretical basis is developed for the specification of design factors. As demonstrated later, such a theoretical basis can be

developed using the second moment probability theory.

3.2.2 CRITERIA FOR SAFETY AND SERVICEABILITY

Because structural design is invariably performed under conditions of uncertainty that are associated with applied loads and structural resistances, the probability of an adverse event, such as excessive deformations or collapse of structural components or an entire structure, still exists, however small it may be. This probability, known as the "risk" or "probability of failure," can be the basis for the specification of criteria for safety and serviceability.

The safety and integrity of a structure are of concern during the entire time of its operation. For this reason, the *useful life* of a structure is of primary interest in its safety analysis. In particular, the evaluation of the risk of structural damage or collapse should consider the following two modes:

- a. Structural damage or collapse under lifetime maximum load
- b. Structural damage or collapse under repeated (operational) loads which may result from fatigue phenomenon

For the first Case, the lifetime maximum load would be a function of the structure's life. For Case b the number of applications of the load also depends on this lifetime.

Besides being safe, a structure must also be able to perform its intended function economically. Excessive deformations, large amplitude vibrations, wide cracks, or frequent repairable damages (due to local yielding) could interfere with structural performance. Therefore, the risk of such events must be confined to acceptably small values, such that their

occurrences are infrequent and therefore tolerable. In this concern, it is the percentage of time that a structure fails to operate, not its useful life, that is important. Therefore, a rational criterion for serviceability may be specified in terms of an acceptable level of risk during a unit period of time.

It is important to realize that the value of acceptable risk for structural components is generally very small (i.e., of the order of 10^{-3} or smaller). Since such small values of probabilities may be sensitive to assumed distributions or various approximations in an analysis, the calculated value of risk must be regarded only as a consistent relative measure of the underlying risk. Thus, for example, a component designed for a risk of 10^{-6} is regarded to be two orders of magnitude safer than a component that is designed (with the same procedure) for a risk of 10^{-4} , although each of these values may not correctly represent the absolute values of the risks that are involved.

3.3 SECOND-MOMENT PROBABILISTIC DESIGN

In the development of second-moment probabilistic design concepts presented here, a single combination of loads is considered; the case of multiple load combinations as well as the stochastic nature of loads is discussed later.

3.3.1 EVALUATION OF RISK

Consider a component of a structure that is affected by a specific combination of loads. It is desired to evaluate the probability of failure of the component in a specific mode. Let S denote the combined load effect in a critical section of the component and R denote the corresponding resistance. As an example, in evaluating the probability of failure of a beam in flexural mode, S would correspond to the applied moment in the critical section whereas R would describe the flexural capacity

of the section. Due to the uncertainties in loads and resistances, S and R are random variables. Let $f_S(s)$ and $f_R(r)$ describe the probability density functions (PDF) of S and R respectively. A typical illustration of these functions is shown in Figure 3-1.

FIGURE 3-1. PROBABILITY DENSITY FUNCTIONS FOR LOAD AND RESISTANCE

The failure of the component is defined as the event $\{R < S\}$, i.e., the event that the applied load effect exceeds the capacity of the component. The corresponding probability of failure, denoted by p_f , is then obtained as,

$$p_f = P(R < S) \\ = \int_{-\infty}^{\infty} P(R < S | S = s) f_S(s) ds$$

which assuming statistically independent R and S becomes

$$p_f = \int_{-\infty}^{\infty} P(R < s) f_S(s) ds \\ = \int_{-\infty}^{\infty} F_R(s) f_S(s) ds \quad (3-3)$$

where $F_R(s) = \int_0^s f_R(r) dr$ is the cumulative distribution function (CDF) of R evaluated at $r = s$. The statistical independence of R and S is a reasonable assumption for most practical cases where the variabilities in load effects are mainly due to variabilities in loads and the resistances are independent of applied loads.

In this formulation of p_f , two separate distributions must be prescribed, i.e., one for R

and one for S. This, however, is not necessary if the failure event is defined in terms of a performance function of R and S, in which case only one distribution for the performance function must be prescribed. Two performance functions that give useful results for practical purposes are considered here.

First consider the performance function $Z_1 = R - S$, where failure is defined by the event $\{Z_1 < 0\}$. Therefore the probability of failure may be obtained as:

$$p_f = P(Z_1 < 0) = F_{Z_1}(0) \quad (3-4)$$

where $F_{Z_1}(0)$ is the CDF of Z_1 evaluated at $z_1 = 0$. The distribution of Z_1 is all that is necessary to evaluate p_f . A second performance function of interest may be defined as $Z_2 = R/S$. In this case, failure is defined by the event $\{Z_2 < 1.0\}$, and the probability of failure may be obtained as:

$$p_f = P(Z_2 < 1.0) = F_{Z_2}(1.0) \quad (3-5)$$

where $F_{Z_2}(1.0)$ is the CDF of Z_2 evaluated at $z_2 = 1.0$. Again, the distribution of Z_2 must be prescribed in order to evaluate p_f .

In prescribing distributions for Z_1 and Z_2 the following must be considered:

- The distributions must be physically consistent, i.e., since Z_1 may have both positive and negative values $f_{Z_1}(z_1)$ must range over both positive and negative values of z_1 ; also, since Z_2 is always positive, $f_{Z_2}(z_2)$ must exist over only positive values of z_2 .
- The assumed distributions must be consistent with the supporting statistical data of R and S. If the available data suggest normal distributions for both R and S, the function $R - S$ would also be normally distributed, and hence a normal distribution for Z_1 would be appropriate. Also, if

the available data suggest lognormal distributions for both R and S, the function R/S would also be lognormally distributed and therefore a lognormal distribution for Z_2 would be consistent with the data.

- For practical purposes it is desirable that the assumed distributions give simple analytical results. A normal distribution for Z_1 and a lognormal distribution for Z_2 give simple formulations for the probability of failure, as shown below.

Normally Distributed Z_1 . Let μ_{Z_1} , μ_R , and μ_S be the means and σ_{Z_1} , σ_R , and σ_S be the standard deviations of Z_1 , R, and S, respectively. Then for $Z_1 = R - S$,

$$\mu_{Z_1} = \mu_R - \mu_S \quad \text{and} \quad \sigma_{Z_1} = \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3-6)$$

Assuming the normal distribution for Z_1 yields

$$p_f = 1 - \phi\left(\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right) \quad (3-7)$$

where $\phi(\ )$ is the standard normal probability (Ang and Tang, 1975).

Lognormally Distributed Z_2 . A lognormal distribution for Z_2 would imply a normal distribution for $\ln Z_2 = \ln R/S$. The mean and standard deviation of the latter may be obtained from first-order approximations as:

$$\mu_{\ln Z_2} \cong \ln \frac{\mu_R}{\mu_S} \quad \text{and} \quad \sigma_{\ln Z_2} \cong \sqrt{\Omega_R^2 + \Omega_S^2} \quad (3-8)$$

where Ω_R and Ω_S are the coefficients of variation of R and S respectively. The probability of failure may then be evaluated as

$$p_f = P(\ln Z_2 < 0) \cong 1 - \phi\left(\frac{\ln \frac{\mu_R}{\mu_S}}{\sqrt{\Omega_R^2 + \Omega_S^2}}\right) \quad (3-9)$$

where $\phi(\ )$ is as defined above.

For other distributions of Z_1 or Z_2 , the Equations 3-4 or 3-5, respectively, must be directly evaluated. In such cases, p_f is not a simple function of the first and second moments. In most cases the available data is not adequate to justify the numerical evaluation of Equations 3-4 and 3-5.

For a general case, the values of p_f as evaluated from Equations 3-7 and 3-9 are approximate. This is especially true for small values of the probability, i.e., $p_f \leq 10^{-3}$, for which the far tails of the distributions are important. Thus, such calculated values of p_f may be regarded only as relative measures of the true risk. It is noted that in most practical design applications, a relative measure of the safety or serviceability is all that is necessary.

3.3.2 PROBABILISTIC DESIGN FORMATS

A probabilistic design format would seek to proportion structural members such that a prescribed level of reliability is assured, i.e., such that the failure probability is less than the specified acceptable level. Suppose that the acceptable risk p_f is specified for a particular component. In solving for the mean resistance μ_R in either of Equations 3-7 and 3-9, the minimum required mean strength can be determined. Thus, on the basis of Equation 3-7, a design format may be derived as:

$$\mu_R \geq \mu_S + \phi^{-1}(1 - p_f) \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3-10)$$

and on the basis of Equation 3-9 an alternative format may be derived as:

$$\mu_R \geq \mu_S \exp \left[\phi^{-1}(1 - p_f) \sqrt{\Omega_R^2 + \Omega_S^2} \right] \quad (3-11)$$

where $\phi^{-1}(1 - p_f)$ is the value of the standard normal variate at the probability level $1 - p_f$ (i.e., the required reliability). As in deterministic design procedures, the actual proportioning of components requires iterative calculations; that is, first a set of dimensions should

be assumed leading to an estimate of the right-hand side of the design inequality (note that in general μ_S , σ_S , and Ω_S depend on the assumed dimensions); if this inequality is not satisfied, a new set of dimensions must be assumed and the process must be repeated until an acceptable design is obtained.

The above formats were based on normal distributions for $R - S$ and $\ln(R/S)$, respectively. For any other viable distribution, the above formats may be generalized as:

$$\mu_R \geq \mu_S + \beta_1 \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3-12)$$

$$\mu_R \geq \mu_S \exp \left(\beta_2 \sqrt{\Omega_R^2 + \Omega_S^2} \right) \quad (3-13)$$

where β_1 and β_2 now depend on the assumed distributions for $R - S$ and $\ln(R/S)$, respectively, and on the required level of reliability. Ang and Ellingwood (1971) have shown that β_1 and β_2 become sensitive to the assumed distributions when $p_f \leq 10^{-5}$. Since in design of important structures, such as nuclear power plants, risk levels smaller than 10^{-5} may be required, the prescribed values of β_1 or β_2 may be regarded only as *relative* measures of the required reliability.

Cornell (1969), Lind (1971), and Rosenblueth and Esteva (1972) suggested that β_1 or β_2 be used as a safety index with no explicit reference to the failure probability. This idea is of interest in developing building codes where a direct reference to the risk is not desirable. For the purpose of illustration, the values of β_1 and β_2 for normal Z_1 and lognormal Z_2 , respectively, are listed in Table 3-1 for various levels of p_f . Note that for the assumed distributions $\beta_1 = \beta_2 = \phi^{-1}(1 - p_f)$.

3.3.3 EVALUATION OF STATISTICS OF S AND R

In the above derivation, S was defined as the combined load effect in a critical section and R was defined as the resistance of the

TABLE 3-1. VALUES OF β_1 AND β_2 ASSUMING
NORMAL Z_1 AND LOGNORMAL Z_2

P_f	β_1 or β_2
10^{-1}	1.28
10^{-2}	2.33
10^{-3}	3.09
10^{-4}	3.72
10^{-5}	4.25
10^{-6}	4.75
10^{-7}	5.20
10^{-8}	5.60

component. Both of these random variables are generally functions of other variables. For example, S is a function of the applied loads, whereas R is a function of material property constants and member dimensions. Usually the statistics of S and R are not directly available, and one has to obtain them on the basis of the statistics of the related function variables.

For the purpose of generality, consider the random variable Z as a function of random variables $X_1, \dots, X_n$,

$$Z = g(X_1, X_2, \dots, X_n) \quad (3-14)$$

where Z may represent R or S . Let the random variables X_i have means μ_{X_i} , standard deviations σ_{X_i} , and coefficients of variation (COV) $\Omega_{X_i} = \sigma_{X_i}/\mu_{X_i}$. On the basis of first-order approximations (Benjamin and Cornell, 1970; Ang and Tang, 1975), the mean and variance of Z approximately are:

$$\mu_Z \cong g(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n}) \quad (3-15)$$

$$\sigma_Z^2 \cong \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial g}{\partial X_i} \right)_0 \left(\frac{\partial g}{\partial X_j} \right)_0 \rho_{ij} \sigma_{X_i} \sigma_{X_j} \quad (3-16)$$

in which $(\partial g/\partial X_i)_0$ indicates the derivatives evaluated at the mean values, and ρ_{ij} indicates the coefficient of correlation between X_i and X_j . For uncorrelated X_i and X_j the above equation simplifies to:

$$\sigma_Z^2 \cong \sum_{i=1}^n \left(\frac{\partial g}{\partial X_i} \right)_0^2 \sigma_{X_i}^2 \quad (3-17)$$

With mean and variance of Z known, its COV value may be obtained from:

$$\Omega_Z = \frac{\sigma_Z}{\mu_Z} \quad (3-18)$$

The above derivation assumes that the mean and variance of X_i are available and that the function g correctly describes the relation of Z to X_i . In practice, the statistics of X_i usually are based on incomplete data, and the function g is often an incomplete mathematical idealization. In order to include these additional uncertainties, Ang (1973) introduced the extended reliability model. According to this procedure, each variate X_i is modeled as:

$$X_i = N_{X_i} \hat{X}_i \quad (3-19)$$

where $\hat{X}_i$ is an estimator of X_i with the mean $\bar{x}_i$ and COV δ_{X_i} , which are estimated from the available data which may be incomplete, and N_{X_i} is a random correction factor with mean $\bar{N}_{X_i}$ and COV Δ_{X_i} , representing the bias and the error in the estimation, respectively. For an unbiased estimator, $\bar{N}_{X_i}$ is equal to one; also as a first approximation Δ_{X_i} is taken equal to the error in the estimated mean $\bar{x}_i$ (Ang, 1973). Thus, the mean and COV of X_i are evaluated as:

$$\mu_i \cong \bar{x}_i \quad (3-20)$$

and

$$\Omega_{X_i} \cong \sqrt{\delta_{X_i}^2 + \Delta_{X_i}^2} \quad (3-21)$$

where

$$\sigma_{X_i} = \Omega_{X_i} \mu_{X_i} \cong \Omega_{X_i} \bar{x}_i$$

In a similar manner, the true function g is modeled as:

$$g = N_g \hat{g}(X_1, X_2, \dots, X_n) \quad (3-22)$$

where $\hat{g}$ is the assumed function which may be imperfect, whereas N_g is a random correction factor representing the error in the assumed

function. Here, two kinds of variabilities should be considered. First, consider the variability in function $\hat{g}$ about its mean value, for a given set of data (for known X_i); this is represented by the COV value δ_g . Second, consider the error due to limited amount of data which is represented by the COV of N_g denoted by Δ_g . Thus the total uncertainty of the function g is obtained as:

$$\Omega_g \cong \sqrt{\delta_g^2 + \Delta_g^2} \quad (3-23)$$

Finally, assuming that $\hat{g}$ is an unbiased estimator of g , the mean of Z is obtained from Equations 3-15, 3-20, and 3-22 as:

$$\mu_z \cong \bar{z} \cong \hat{g}(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) \quad (3-24)$$

and the total COV of Z is obtained from Equations 3-16, 3-21, and 3-23 as:

$$\Omega_z \cong \sqrt{\Omega_g^2 + \frac{1}{\bar{z}^2} \left[\sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial g}{\partial X_i} \right)_0 \left(\frac{\partial g}{\partial X_j} \right)_0 \rho_{ij} \sigma_{X_i} \sigma_{X_j} \right]} \quad (3-25)$$

which for uncorrelated X_i becomes

$$\Omega_z \cong \sqrt{\Omega_g^2 + \frac{1}{\bar{z}^2} \left[\sum_{i=1}^n \left(\frac{\partial g}{\partial X_i} \right)_0^2 \sigma_{X_i}^2 \right]} \quad (3-26)$$

The total standard deviation of Z is then obtained as:

$$\sigma_z = \bar{z} \Omega_z \quad (3-27)$$

3.3.3.1 Statistical Evaluation of Load Effects

The critical design load effect, S , in general results from a combination of several loads. Let X_i describe the random intensity of the i th load. On the basis of the principle of superposition, the load effect S is obtained as:

$$S = f_1 X_1 + f_2 X_2 + \dots + f_n X_n = \sum_{i=1}^n f_i X_i \quad (3-28)$$

where f_i are influence coefficients which depend on the load effect under consideration as well as the loading pattern. Assuming the influence coefficients are deterministic, the mean and COV of the load effect S for uncorrelated loads are:

$$\mu_s \cong \bar{s} = \sum_{i=1}^n f_i \bar{x}_i \quad (3-29)$$

$$\Omega_s = \frac{1}{\bar{s}} \sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2} \quad (3-30)$$

where the COV of each load X_i is obtained according to Equations 3-20 and 3-21. As an example, consider the evaluation of the statistics for the earthquake-induced forces. The seismic risk analysis required for such an evaluation is usually based on incomplete historical data of earthquakes and certain imperfect idealizations (Der Kiureghian and Ang, 1975). Accordingly, these errors may be incorporated through Δ_{X_i} , while δ_{X_i} would describe the variability in ground motion intensities as estimated on the basis of the available data and idealized models.

In design for safety, limit states of structural behavior are of interest where the response is no longer linear. As a result, the principle of superposition does not hold and the linear relation assumed in Equation 3-28 would not be perfect. For such cases an additional uncertainty term may be included and Equation 3-30 then becomes:

$$\Omega_s = \sqrt{\Omega_g^2 + \frac{1}{\bar{s}^2} \sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2} \quad (3-31)$$

where Ω_g represents the error due to the superposition assumption. The value of Ω_g may be assigned subjectively on the basis of the expected variability of the true structural response from the assumed linear response.

A further refinement may be made by observing that the influence coefficients are in fact random variables. This results from approximations such as the replacement of complicated loading patterns with simple idealizations, the replacement of dynamic loads with "equivalent" static loads, and the use of idealized dimensions. The effect of these errors could directly be included in Ω_{X_i} of

Equation 3-31; however, if an explicit formulation is required, this equation may be written as:

$$\Omega_s = \sqrt{\Omega_g^2 + \frac{1}{S^2} \sum_{i=1}^n \bar{f}_i^2 \bar{x}_i^2 (\Omega_{f_i}^2 + \Omega_{X_i}^2)} \quad (3-32)$$

where $\bar{f}_i$ is the mean value of the influence coefficient and Ω_{f_i} is its estimated variability.

3.3.3.2 Statistical Evaluation of Resistance

The resistance R of a structural component for a particular load effect, such as the flexural capacity of a beam against applied bending moments, may be described in a functional form as:

$$R = g(X_1, X_2, \dots, X_n) \quad (3-33)$$

where X_i are random variates describing material properties, member dimensions, etc.

As an example, consider the flexural capacity of a reinforced concrete beam with width b , effective depth d , tension steel area A_s , steel yield strength f_y , and concrete strength f_c . Assuming the reinforcement ratio is well below the balanced ratio (to ensure zero probability of brittle failure), the flexural capacity may be obtained as:

$$R = A_s f_y d \left(1 - \eta \frac{A_s}{bd} \frac{f_y}{f_c} \right) \quad (3-34)$$

where η is the concrete stress block parameter (ACI, 1971). On the basis of Equations 3-24 and 3-26, the mean and COV of R for uncorrelated parameters are:

$$\mu_R \cong \bar{R} = \bar{A}_s \bar{f}_y \bar{d} \left(1 - \eta \frac{\bar{A}_s}{\bar{b}\bar{d}} \frac{\bar{f}_y}{\bar{f}_c} \right) \quad (3-35)$$

and

$$\begin{aligned} \Omega_R^2 \cong \Omega_g^2 &+ \left(\frac{1 - 2\bar{\eta}\bar{q}}{1 - \bar{\eta}\bar{q}} \right)^2 \left(\Omega_{f_y}^2 + \Omega_{A_s}^2 \right) \\ &+ \left(\frac{\bar{\eta}\bar{q}}{1 - \bar{\eta}\bar{q}} \right)^2 \left(\Omega_{f_c}^2 + \Omega_b^2 + \Omega_d^2 \right) \\ &+ \left(\frac{1}{1 - \bar{\eta}\bar{q}} \right)^2 \Omega_d^2 \end{aligned} \quad (3-36)$$

where $\bar{q} = (\bar{f}_y/\bar{f}_c) \bar{A}_s/(\bar{b}\bar{d})$ and a bar over a symbol connotes the mean value. The term Ω_η reflects the error in the idealization of the nonlinear concrete stress block by a simple rectangular (or triangular) stress block. The term Ω_g reflects the error in the assumed function for R ; this may result as the effect of tension stresses in uncracked concrete, the effect of shear stresses, etc., which are not included in the derivation of Equation 3-34. Each of the COV terms $\Omega_g, \dots, \Omega_d$ on the right-hand side is composed of a basic variability term and an error term. For example, the term Ω_d is obtained as $\Omega_d = \sqrt{\delta_d^2 + \Delta_d^2}$, where δ_d is the coefficient of variation of the effective depth, as observed from data on actual construction practices, whereas Δ_d is the error in estimation arising from insufficiency of such data. This resistance function, as well as other resistance functions for reinforced concrete members, is studied in detail by Ellingwood and Ang (1972, 1974). Similar studies for metal structures may be found in ASCE (1972).

3.3.4 PRACTICAL DESIGN FORMATS

For practical purposes it is desirable to develop probabilistic design formats which are similar and comparable to the deterministic formats currently in application. Such a format would be more readily received by the engineering community and could be used as a theoretical basis for a critical examination of the current design criteria.

In order to develop formats that are comparable to the deterministic formats of the WSD and LFD methods (i.e., Eqs. 3-1 and 3-2), it is necessary that (1) the terms corresponding to the resistance and the load-effect be separated, as they are in the WSD and LFD formats, (2) relationships between nominal and statistical values of the resistance and loads be

specified, and (3) factors comparable to safety or load and strength factors be derived.

First consider the probabilistic format of Equation 3-12. With the means of R and S obtained from (unbiased) statistical data, this equation becomes:

$$\bar{R} \geq \bar{S} + \beta_1 \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3-37)$$

In order to separate the terms corresponding to the resistance and load effect, the relation

$$\sqrt{\sigma_R^2 + \sigma_S^2} = \alpha_R (\sigma_R + \sigma_S) \quad (3-38)$$

is assumed where α_R is a decoupling factor equal to $\sqrt{\sigma_R^2 + \sigma_S^2} / (\sigma_R + \sigma_S)$. It can be shown that α_R is relatively insensitive to variations in σ_R and σ_S . For most cases where σ_R and σ_S are of the same magnitude, $\alpha_R = 0.75$ can be used. It is only when one of the two standard deviations is much smaller than the other, say less than 20%, that α_R becomes sensitive. In that case, the variable with small standard deviation can be treated deterministically and $\alpha_R = 1.0$ can be used.

Substitution of Equation 3-38 in Equation 3-37 and rearranging gives

$$\bar{R} - \beta_1 \alpha_R \sigma_R \geq \bar{S} + \beta_1 \alpha_R \sigma_S$$

or

$$(1 - \beta_1 \alpha_R \Omega_R) \bar{R} \geq (1 + \beta_1 \alpha_R \Omega_S) \bar{S} \quad (3-39)$$

The above probabilistic design format is seen to be similar to the WSD format of Equation 3-1, where the safety factor with respect to the mean resistance and mean load effect is

$$\bar{\gamma} = \frac{1 + \beta_1 \alpha_R \Omega_S}{1 - \beta_1 \alpha_R \Omega_R} \quad (3-40)$$

This is the *central safety factor*, and its relationship with the conventional safety factor, i.e., γ in Equation 3-1, will be explored later.

The coefficient of variation Ω_S (from Equation 3-31) involves the influence coefficients, f_i . This means that Ω_S , and therefore $\bar{\gamma}$, depend on the component or the location of the

critical section considered. Therefore, a risk-consistent design would require a different safety factor at each critical section. This may be explained by noting that since different sections are affected differently under a given load, and since various loads may have different degrees of variability, it follows that a section which is highly affected by loads with large variabilities must have a larger safety factor than a section which is not very much affected by such loads.

A change in safety factor from section to section may not be desirable for practical purposes. One solution to this problem would be to separate the terms for each load, which leads to a format similar to that of the LFD method. For this purpose, the value of Ω_S from Equation 3-31 is first substituted into Equation 3-39, resulting in:

$$(1 - \beta_1 \alpha_R \Omega_R) \bar{R} \geq \left(1 + \beta_1 \alpha_R \sqrt{\Omega_g^2 + \frac{1}{\bar{S}^2} \sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2} \right) \bar{S} \quad (3-41)$$

In order to separate the terms on the right-hand side, decoupling factors

$$\alpha_g = \frac{\sqrt{\Omega_g^2 + \frac{1}{\bar{S}^2} \sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}}{\Omega_g + \frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}} \quad (3-42)$$

and

$$\alpha_s = \frac{\sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}}{\sum_{i=1}^n f_i \bar{x}_i \Omega_{X_i}} \quad (3-43)$$

are assumed and Equation 3-41 is rearranged to read:

$$(1 - \beta_1 \alpha_R \Omega_R) \bar{R} \geq \sum_{i=1}^n \left[1 + \beta_1 \alpha_R \alpha_g (\Omega_g + \alpha_s \Omega_{X_i}) \right] f_i \bar{x}_i \quad (3-44)$$

where the equality $\bar{S} = \sum_{i=1}^n f_i \bar{x}_i$ is used.

The above probabilistic format is similar to the LFD format of Equation 3-2, where relative to the mean resistance, $\bar{R}$, the central strength factor is:

$$\bar{\phi} = 1 - \beta_1 \alpha_R \Omega_R \quad (3-45)$$

and relative to the mean value of the i th load, the central load factor is

$$\bar{\delta}_i = 1 + \beta_1 \alpha_R \alpha_g \left(\Omega_g + \alpha_S \Omega_{X_i} \right) \quad (3-46)$$

The relationships between the central factors and the conventional factors, i.e., ϕ and δ_i of Equation 3-2, are discussed later.

The decoupling factor α_g as defined in Equation 3-42 is of the same type as α_R ; therefore, it may be assumed to be 0.75 or 1.0 depending on the relative values of Ω_g and

$$\frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}$$

The factor α_S of Equation 3-43 is used to separate the effects of various loads. It is seen in this equation that α_S depends on the relative variability of the various load effects. This factor, like α_R discussed earlier, remains fairly constant for various load combinations. For practical use, values of α_S in Table 3-2 are recommended. These values are appropriate when the standard deviations of various load effects are more or less of the same order, the smallest ones not being less than 20 or 30% of the largest ones.

TABLE 3-2. ASSUMED VALUES OF THE DECOUPLING FACTOR α_S

Number of Loads, n	α_S
1	1.00
2	0.75
3	0.62
4	0.55
5	0.50
6	0.45

Considering the second probabilistic design format given in Equation 3-13, for R and S obtained from (unbiased) statistical data, the design equation becomes:

$$\bar{R} \geq \bar{S} \exp \left(\beta_2 \sqrt{\Omega_R^2 + \Omega_S^2} \right) \quad (3-47)$$

This format is similar to the WSD format of Equation 3-1; thus the central safety factor is

$$\bar{Y} = \exp \left(\beta_2 \sqrt{\Omega_R^2 + \Omega_S^2} \right) \quad (3-48)$$

Since Ω_S is a function of the influence coefficients, the central factor is again seen to depend on the location of the section or the component in the structure under consideration.

In order to obtain a format similar to the LFD format, it is necessary that the strength and load terms be separated. Using the decoupling factor α_R of Equation 3-38, Equation 3-47 may be written as:

$$\bar{R} \geq \bar{S} \exp \left[\beta_2 \alpha_R (\Omega_R + \Omega_S) \right] \quad (3-49)$$

or

$$\exp (-\beta_2 \alpha_R \Omega_R) \bar{R} \geq \exp (\beta_2 \alpha_R \Omega_S) \bar{S} \quad (3-50)$$

Substituting Ω_S from Equation 3-31 and α_g from Equation 3-42, the above equation can be written as:

$$\begin{aligned} & \exp (-\beta_2 \alpha_R \Omega_R) \bar{R} \\ & \geq \exp (\beta_2 \alpha_R \alpha_g \Omega_g) \\ & \times \exp \left(\beta_2 \alpha_R \alpha_g \frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2} \right) \bar{S} \end{aligned} \quad (3-51)$$

Now introduce a new decoupling factor α_S such that:

$$\begin{aligned} & \exp \left(\beta_2 \alpha_R \alpha_g \frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2} \right) \bar{S} \\ & = \sum_{i=1}^n \exp \left(\beta_2 \alpha_R \alpha_g \alpha_S \Omega_{X_i} \right) f_i \bar{x}_i \end{aligned} \quad (3-52)$$

Substituting this equation in Equation 3-51 yields:

$$\exp(-\beta_2 \alpha_R \Omega_R) \bar{R} \geq \sum_{i=1}^n \exp \left[\beta_2 \alpha_R \alpha_g \left(\Omega_g + \alpha_S \Omega_{X_i} \right) \right] f_i \bar{x}_i \quad (3-53)$$

The central strength and load factors in this case are

$$\bar{\phi} = \exp(-\beta_2 \alpha_R \Omega_R) \quad (3-54)$$

and

$$\bar{\delta}_i = \exp \left[\beta_2 \alpha_R \alpha_g \left(\Omega_g + \alpha_S \Omega_{X_i} \right) \right] \quad (3-55)$$

respectively.

An examination of the decoupling factor α_S , defined by Equation 3-52, for a number of selected design situations has revealed that this factor remains quite close to the factor defined by Equation 3-41 and, therefore, the values recommended in Table 3-2 can be used.

These safety, load, and strength factors correspond to the mean values of loads and resistances. For this reason they are known as *central* factors. The relations of these factors with the factors used in practice are discussed later.

3.4 LOADS AND THEIR COMBINATIONS

In an actual situation, several different combinations of loads -- not just a specific set of loads -- may act on the structure during its useful life. Therefore, it is necessary to investigate all such combinations of loads and to determine those combinations that may be critical to the safety of the structure. In addition, the loads in each particular combination may vary with time. Under such conditions, adding the peak values of loads would not be reasonable since the peaks in general would not occur at the same time. In such cases it is the peak value of the combined load effect that is critical and should be determined.

3.4.1 STOCHASTIC EVALUATION OF LOAD EFFECTS

In evaluating load effects due to an applied load on a structure, three types of uncertainties are encountered: (1) temporal and spatial randomness in load intensity and distribution, (2) uncertainties in influence coefficients resulting from variabilities in structural dimensions and strengths, and (3) errors in methods of analysis and prediction. It is expected that uncertainties resulting from the temporal and spatial randomness in load intensities have the dominant effect.

Some aspects of this problem have been looked upon by a number of investigators (e.g., Peir and Cornell, 1973, and McGuire and Cornell, 1974, for treatments of live loads; and Davenport, 1967, for analysis of wind loads).

Consider a case where a load is applied on the Area A of a structure. Let $w(x,y,t)$ denote the random intensity of the load (i.e., load per unit area) at the location described by the coordinates x and y and at time t . For a particular load effect under consideration let $I(x,y)$ denote the influence coefficients, i.e., the load effect due to a unit load at x, y . The required load effect is

$$S(t) = \int_A w(x,y,t) I(x,y) dA \quad (3-56)$$

The evaluation of the statistics of $S(t)$ from those of $w(x,y,t)$ in this equation is very complicated. In order to simplify the problem, it is often reasonable to assume that the unknown shape of the load distribution remains unchanged with time. With this assumption, the load intensity can be described as

$$w(x,y,t) = w_0(t) \psi(x,y) \quad (3-57)$$

where $\psi(x,y)$, a random process, is the load distribution shape function such that

$$\int_A \psi(x,y) dA = A \quad (3-58)$$

and $w_0(t)$ is a stochastic process denoting the load intensity when it is uniformly distributed.

Using Equation 3-57 in Equation 3-56 one obtains

$$\begin{aligned} S(t) &= \int_A w_o(t) \psi(x,y) I(x,y) dA \\ &= w_o(t) \int_A \psi(x,y) I(x,y) dA \end{aligned} \quad (3-59)$$

In this equation the stochastic nature of load effects appears through $w_o(t)$, whereas uncertainties due to the spatial randomness of the load and to variabilities in influence coefficients are reflected through $\psi(x,y)$ and $I(x,y)$, respectively.

For practical analysis and design purposes it is convenient to assume simple deterministic shape functions and use influence coefficients based on mean structural strengths and dimensions. Let $\psi^*(x,y)$ and $I^*(x,y)$ denote such values. To include the uncertainties from these sources, a random correction factor H may be included and Equation 3-59 may be written as

$$S(t) = w_o(t) H \int_A \psi^*(x,y) I^*(x,y) dA \quad (3-60)$$

where H is such that

$$H \int_A \psi^*(x,y) I^*(x,y) dA = \int_A \psi(x,y) I(x,y) dA \quad (3-61)$$

The variabilities of H for various types of structures and loads may be estimated through measurements and observations in existing structures as well as through analytical techniques. A number of structural types for each type of loading will have to be considered to cover most practical situations. Note that separate analysis must be made for each typical load effect. With the descriptions of $w_o(t)$ and H known, the moments of the load effect may be estimated from Equation 3-60.

The load-load effect relationship has been described here (see Eq. 3-28) as

$$S = fX \quad (3-62)$$

where f is a deterministic influence coefficient and X is the random value of load intensity. From the discussion above,

$$X = w_o(t)H \quad (3-63)$$

and

$$f = \int_A \psi^*(x,y) I^*(x,y) dA \quad (3-64)$$

With f a deterministic value, the stochastic characteristics of S are obtainable from those of X from Equation 3-62.

3.4.2 CATEGORIZATION OF LOADS

With respect to the nature of their occurrence in time, loads affecting a structure can be grouped in five categories:

1. Time-independent loads
2. Time-dependent sustained loads
3. Time-dependent transient loads
4. Occasional short-duration loads
5. Consequential loads

The first category includes loads which do not vary with time, e.g., dead load. A plot of such a load as a function of time would appear as a horizontal line with a random amplitude, as shown in Figure 3-2a. A single probability density function, as shown in Figure 3-2b, would fully describe the probabilistic character of the load.

The second category includes loads that act at all times and vary with time, such as sustained live loads. A typical sample function of such a load appears as in Figure 3-3a. For safety analysis, two values of a load in this category are of interest: the value of the load at any instant of time (the instantaneous load), and the peak value of the load during a period of T years (the T -year extreme load). For the sample function in Figure 3-3a, the probability density functions of the instantaneous and T -year extreme values are shown schematically in Figure 3-3b.

The third category includes loads that vary with time and are active during certain intervals of time only. Seasonal loads, such as

snow loads, are included in this group. A sample function for such a load appears as in Figure 3-4a. For this category of loads, a parameter of interest is the duration of the load at each occurrence. This parameter is a random variable denoted by τ . For this group of loads, we again define instantaneous and T-year extreme values, with the probability density functions as shown schematically in Figure 3-4b.

The fourth category includes loads that act occasionally but for very short periods of time. The durations of these loads are so short that their joint occurrences are unlikely. Therefore, in combining loads, only one load at a time from this category should be included in any combination. Examples for this category may include loads resulting from certain natural hazards, such as tornados and short duration ground shocks, and loads due to certain accidents, such as an aircraft crash or explosions. A typical sample function for such a load is shown in Figure 3-5a, where each vertical bar indicates an occurrence, with the height of the bar showing the random magnitude of the load. Again, two different distributions may be identified, one corresponding to the load at any occurrence and the other corresponding to the peak load during a period of years.

The last category includes loads that may be triggered as a consequence of other loads,

especially occasional severe loads. Examples for this category include loads that are due to an explosion or the failure of certain structural or nonstructural components and that happen as a result of the occurrence of a major earthquake or tornado. A key parameter to consider for this group of loads is the probability of occurrence of the event given a major earthquake or tornado, which is designated P_e . This category of loads has sample functions which are similar to those portrayed in Figure 3-5 for Category 4 loads. The number of occurrences of these loads in unit time, however, would depend on the probability P_e .

3.4.3 EVALUATION OF STATISTICS OF EXTREME LOADS

The two values of a stochastic load of interest in structural reliability are: the instantaneous value at an arbitrary point in time, and the peak or extreme value over a period of time, say T years. The statistics of the instantaneous load may be obtained from data collected through measurements at arbitrary points in time. Usually, measurements at various typical buildings or locations may be combined in order to increase the sample size. The evaluation of the statistics of extreme loads, on the other hand, is more difficult since it requires collection of data over long periods of time. Another approach is to obtain the

FIGURE 3-2. TIME-INDEPENDENT LOAD, CATEGORY 1

(a) Sample function

(b) Probability density functions for instantaneous and extreme loads

FIGURE 3-3. TIME-DEPENDENT CONTINUOUS LOAD, CATEGORY 2

(a) Sample function

(b) Probability density functions for instantaneous and extreme loads

FIGURE 3-4. TIME-DEPENDENT PIECEWISE CONTINUOUS LOAD, CATEGORY 3

(a) Sample function

(b) Probability density functions for any occurrence, annual peak, and extreme loads

FIGURE 3-5. OCCASIONAL SHORT-DURATION LOAD, CATEGORY 4

statistics of the T-year extreme load on the basis of theoretical models and statistical data that are obtained for a relatively short period of time. A number of studies on this topic are available. For example, according to Gumbel (1958), if various occurrences of a load are statistically independent and identically distributed, then the distribution of the extreme value after a number of occurrences will asymptotically approach to one of three standard distributions, known as Type 1, Type 2, and Type 3 Extreme Value Distributions. The selection of one of these distributions depends on the shape of the far tail of the distribution of the instantaneous value.

For use in the second-moment reliability approach, only the mean and the variance of extreme loads, rather than entire distributions, are required. Most investigators, on the other hand, have been concerned with deriving the distribution of extreme loads (e.g., Bosshard, 1975; Grigoriu, 1975; Wen, 1977a, 1977b). By proper integrations the mean and the variance can be evaluated from these distributions, but this approach has several disadvantages. First, in order to derive distributions, one is bound to make simplifying assumptions and idealizations, which may not be realistic. Second, the solutions that are obtained are often very complicated, and considerable additional computations are required in order to evaluate the mean and the variance. Finally, since only the mean and the variance are utilized, the additional information that is contained in the distribution remains unused.

In view of this, a simple method was developed for obtaining the mean and the variance of extremes (Der Kiureghian, 1977). In this approach the mean and the variance of the extreme load are evaluated as a function of the mean and the variance of the instantaneous load and a set of parameters describing the stochastic variations of the load. This approach has a

number of advantages, including (1) there is less need for idealizations and simplifying assumptions; (2) the mean and variance of extreme loads can be obtained with straightforward calculations using simple design charts, (3) the method can be further developed using analytical as well as Monte-Carlo simulation techniques, and (4) it can be extended to the case of multiple loads.

The above approach can be applied to cases involving one load or a combination of loads. The discussion here is limited to stationary and statistically independent loads.

3.4.3.1 One-Load Case

Let S denote the instantaneous value of a stationary stochastic load effect and S_T denote its extreme value over a period of T years, i.e.,

$$S_T = \max_T(S) \quad (3-65)$$

$\bar{s}$ and σ_S are the mean and the standard deviation of S ; these values are assumed to be known. Also, $\bar{s}_T$ and σ_{S_T} are the mean and standard deviation of S_T .

One would expect that the value of $\bar{s}_T$ increases with increasing values of $\bar{s}$, σ_S , and T . Also, it is expected that σ_{S_T} be closely related to σ_S . With this in mind, the following formats are assumed for the evaluation of $\bar{s}_T$ and σ_{S_T} :

$$\bar{s}_T = \bar{s} + p\sigma_S \quad (3-66)$$

$$\sigma_{S_T} = q\sigma_S \quad (3-67)$$

where p and q are yet unknown functions of T and other parameters which describe the stochastic variations of S . It is noted that the parameter p must necessarily be a monotonically increasing function of T . These formats can be applied to several processes useful in modeling stochastic loads.

Stationary Gaussian Process: The stationary Gaussian process (Fig. 3-6) has often been used to model continuous loads of stochastic nature. This process can be used to model certain loads of Categories 2 and 3.

FIGURE 3-6. SAMPLE OF STATIONARY GAUSSIAN PROCESS

It can be shown that if the upcrossings of a stationary Gaussian process from a level $\bar{s}$ are assumed to constitute a Poisson process, then, the functions p and q can be evaluated from

$$p = \int_{-\infty}^{\infty} u f_U(u) du \quad (3-68)$$

and

$$q = \left[\int_{-\infty}^{\infty} u^2 f_U(u) du - p^2 \right]^{1/2} \quad (3-69)$$

where $f_U(u)$ is density function of the normalized variate $U = (S_T - \bar{s})/\sigma_S$ and is shown to be

$$f_U(u) = \left[\frac{1}{\sqrt{2\pi}} + vTu \Phi(u) \right] \times \exp \left[-\frac{u^2}{2} - vT \exp \left(-\frac{u^2}{2} \right) \right] \quad (3-70)$$

where $\Phi(u)$ is the standard normal probability and v is the mean rate of upcrossings of level $\bar{s}$ (i.e., mean number of upcrossings per unit time) of the process. The parameter v describes the stochastic fluctuations of the process; a large v would indicate rapid variations with time whereas a small v would mean slow variations. This parameter is obtained from the statistical data. It can be observed in

Equations 3-68 to 3-70 that p and q are functions of vT only. This product describes the mean number of upcrossing of level $\bar{s}$ over the time period T . From Equations 3-68 and 3-69, the values of p and q are evaluated numerically and are shown in Figure 3-7.

FIGURE 3-7. p AND q FUNCTIONS FOR A GAUSSIAN PROCESS

Poisson Square Wave Process: This process has been used to model sustained loads (Bosshard, 1975; Wen, 1977a). It is assumed in this model that the load intensity changes at random points in time following a simple Poisson process. It is also assumed that the intensities within intervals are independent and identically distributed random variables. A sample function of this process is shown in Figure 3-8.

If v denotes the mean rate of load changes and $F_S(s)$ describes the distribution of load intensities, the occurrence of $\{S > s\}$ following load changes is a Poisson process with the mean occurrence rate $v[1 - F_S(s)]$. Therefore, the cumulative distribution of the extreme value may be obtained as

$$F_{S_T}(s) = P(S < s)_{t=0} \times P(S \leq s \text{ following load changes})_{0 \rightarrow T} = F_S(s) \exp \{-vT [1 - F_S(s)]\} \quad (3-71)$$

which upon differentiating with respect to s gives the density function

$$f_{S_T}(s) = f_S(s) [1 + vT F_S(s)] \times \exp \{-vT [1 - F_S(s)]\} \quad (3-72)$$

where $f_S(s)$ is the probability density function of S . For any arbitrary distribution of S , this density function may be used to evaluate p and q as functions of vT from the relations

$$p = \frac{\int_{-\infty}^{\infty} s f_{S_T}(s) ds - \bar{s}}{\sigma_S} \quad (3-73)$$

and

$$q = \frac{\left[\int_{-\infty}^{\infty} s^2 f_{S_T}(s) ds - (\bar{s} + p\sigma_S)^2 \right]^{1/2}}{\sigma_S} \quad (3-74)$$

Filtered Poisson Process: This process is often used to describe transient loads defined as Category 3 loads (Bosshard, 1975; Grigoriu, 1975; Wen, 1977a, 1977b). It is assumed for this process that each occurrence has a random intensity as well as a random duration, and that occurrence times follow a simple Poisson process. It is assumed that at each occurrence the intensity and the duration are independent of one another. Also, intensities and durations at various occurrences are assumed to be independent and identically distributed. A sample function of this process is shown in Figure 3-9.

It can be shown that with v denoting the mean rate of occurrences and $F_S(s)$ denoting the distribution of intensity at an arbitrary occurrence, the cumulative distribution of S_T is

$$F_{S_T}(s) = \exp \{-vT [1 - F_S(s)]\} + [H(s) - 1] \exp(-vT) \quad (3-75)$$

where $H(s)$ is the unit step function, i.e.,

$$H(s) = \begin{cases} 1.0 & \text{for } s > 0 \\ 0.0 & \text{for } s < 0 \end{cases}$$

FIGURE 3-8. SAMPLE OF POISSON SQUARE WAVE PROCESS

FIGURE 3-9. SAMPLE OF FILTERED POISSON PROCESS

Upon differentiating, the density function is obtained as

$$f_{S_T}(s) = vT f_S(s) \exp \{-vT [1 - F_S(s)]\} + \delta(s) \exp(-vT) \quad (3-76)$$

where $\delta(s)$ is the Dirac delta function representing a spike of unit area at $s = 0$. With the density function known, p and q are then evaluated from Equations 3-73 and 3-74.

Figures 3-10 to 3-12 show the values of p and q as functions of vT for the Poisson square wave and the filtered Poisson processes and for a number of distributions of S . Note that the parameter vT is defined differently for the two processes. Specifically, it denotes the mean number of intensity changes for the Poisson square wave process as opposed to the mean number of occurrences for the filtered Poisson process. These should be compared with the definition of vT for the stationary Gaussian

(a) p functions

(b) q functions

FIGURE 3-10. p AND q FUNCTIONS FOR POISSON SQUARE WAVE AND FILTERED POISSON PROCESSES WITH NORMAL DISTRIBUTION

(a) p functions

(b) q functions

FIGURE 3-11. p AND q FUNCTIONS FOR POISSON SQUARE WAVE AND FILTERED POISSON PROCESSES WITH LOGNORMAL DISTRIBUTION

process as the mean number of upcrossings of the mean level $\bar{S}$.

It is generally observed that the values of p and q for the two processes are identical for large values of vT , i.e., greater than 10. This is due to the fact that for large vT the density functions in Equations 3-72 and 3-76 both approach to

$$vT f_S(s) \exp \{-vT [1 - F_S(s)]\}$$

On the other hand, for small vT the two processes possess markedly different behaviors. It is observed that for the Poisson square wave process, as with the stationary Gaussian process, the values of p and q respectively approach to 0.0 and 1.0, as vT approaches to zero. This indicates that with diminishing mean number of intensity changes, i.e., when intensity is likely to remain constant over the time

(a) p functions

(b) q functions

FIGURE 3-12. p and q functions for POISSON SQUARE WAVE AND FILTERED POISSON PROCESSES WITH VARIOUS DISTRIBUTIONS

T, the extreme value S_T approaches to the instantaneous value S. For the filtered Poisson process, the functions p and q at small vT are highly dependent on the coefficient of variation. It can be shown that as vT approaches to zero,

i.e., as the possibility of no occurrence increases, the values of p and q approach to

$$(vT - 1)/\Omega_S \text{ and } \sqrt{vT \left[1 + (1 - vT) \frac{1}{\Omega_S^2} \right]}$$

respectively. These predict $\bar{s}_T = \sigma_{S_T} = 0.0$ for vT approaching to 0.0, which is consistent with the expected behavior.

Similar results can be obtained for other stationary load processes. In this respect, Monte-Carlo simulation techniques could be used to generate the p and q functions for any type of model that is too complicated to be solved analytically. More important, however, is that such results could be obtained from the statistical data of a load, if such data were available. For this purpose once the occurrence rate v has specifically been defined, the values of p and q for any T could be evaluated using the mean and standard deviation of the instantaneous and the peak values of the load, obtained respectively by sampling at random points and at peaks over period T. Thus, without a need for idealizations and assumptions, which are often unrealistic, one could develop the above useful functions to relate statistics of the instantaneous and extreme values of a load.

3.4.3.2 Multiple-Load Case

Consider a combination of n stationary and stochastically independent loads, giving load effects $S_1, S_2, \dots, S_n$ at a particular component. If this combination includes transient loads of Categories 3 and 4, the concurrent action of these load effects would only occur during a portion of time period T. We define S as the combined load effect due to this particular combination of loads such that

$$S = \begin{cases} S_1 + S_2 + \dots + S_n & \text{when all } S_i \text{ are acting} \\ 0.0 & \text{otherwise} \end{cases} \quad (3-77)$$

With this definition, S has a zero value when any of the loads in the combination are not

acting. A sample of such a combination for three loads is schematically illustrated in Figure 3-13.

FIGURE 3-13. SAMPLE COMBINATION OF THREE LOAD EFFECTS

The intention here is to determine the peak value of S over the time T as defined by

$$S_T = \max_T(S) \quad (3-78)$$

According to the above definition, S_T is the peak load effect under the specific combination $S_1 + S_2 + \dots + S_n$. It is not necessarily the peak load effect resulting from the acting loads, since the largest peak may occur under another combination. All possible combinations will, therefore, have to be analyzed in order to account for the largest peak.

The mean and standard deviation of S at an arbitrary occurrence are obtained from Equation 3-77 as

$$\bar{s} = \sum_i \bar{s}_i \quad (3-79)$$

and

$$\sigma_S = \sqrt{\sum_i \sigma_{S_i}^2} \quad (3-80)$$

where $\bar{s}_i$ and σ_{S_i} are the mean and standard deviation of S_i . From these values and the formats in Equations 3-66 and 3-67, the mean and standard deviation of S_T can be obtained as

$$\bar{s}_T = \sum_i \bar{s}_i + p \sqrt{\sum_i \sigma_{S_i}^2} \quad (3-81)$$

and

$$\sigma_{S_T} = q \sqrt{\sum_i \sigma_{S_i}^2} \quad (3-82)$$

where p and q are yet unknown functions of T for the process S .

On the basis of such limited information, exact evaluations of p and q will generally be impossible, and approximate relationships will have to be developed. For this purpose, suppose it is possible to obtain for each load effect S_i a set of parameters p_i and q_i defining its peak S_{iT} during the time of combination. It can be shown (Der Kiureghian, 1978) that a set of good approximations for p and q are

$$p = \frac{\sum_i p_i \sigma_{S_i}^2}{\sum_i \sigma_{S_i}^2} \quad (3-83)$$

and

$$q = \frac{\sum_i q_i \sigma_{S_i}^2}{\sum_i \sigma_{S_i}^2} \quad (3-84)$$

Other formats may also give good results. In selecting such formats, the convenience in application and the accuracy as checked with correct analytical or simulation results should be the determining factors.

In order to demonstrate the applicability of the above procedures and to examine the accuracy of results, a number of combinations are studied below.

Sum of Stationary Gaussian Processes: In this case for Category 2 loads, it is possible to obtain exact values of p and q in terms of the

first two moments of the individual loads. It is well known that the sum of Gaussian processes is itself a Gaussian process. Based on this, it can be shown that for independent processes, the mean rate of upcrossings of the combined process above its mean level is

$$v = \sqrt{\frac{\sum_i v_i^2 \sigma_{S_i}^2}{\sum_i \sigma_{S_i}^2}} \quad (3-85)$$

where v_i is the mean upcrossing rate of S_i from its mean level. With the value of v known, p and q can be determined from Figure 3-7 for the combined load.

Sum of Poisson Square Wave Processes:

Consider n Category 2 load effects modeled as Poisson square wave processes. Let v_i denote the mean rate of intensity for S_i . Depending on the type of distribution of the load intensity (as inferred from statistical data of the load or as prescribed subjectively on the basis of experience and judgment), p_i and q_i for each load effect S_i are obtained from Figures 3-10, 3-11, or 3-12. These values are read against the mean number of intensity changes, $v_i T$, and for the instantaneous coefficient of variation of the load. Equations 3-83 and 3-84 are then used to evaluate p and q .

To check the method, computed mean and standard deviation values for combinations of two loads were compared with results obtained from Monte Carlo simulations (see Fig. 3-14). The distributions of intensities for the two loads were assumed to be normal and lognormal.

Figure 3-14a compares computed and simulated results for $v_1 = v_2$, representing cases where mean rates of intensity changes are comparable. Results for four sets of standard deviations are given. A remarkable agreement is observed in this figure over a wide range of the mean rate of intensity changes.

To examine combinations of loads where mean rates of intensity changes are different, Figure 3-14b compares computed and simulated results for $v_2 T = 1000$ with $v_1 T$ varying from 10 to 1000. Although the agreement seems to deteriorate with increasing difference between the mean rates of intensity changes, a fair approximation is obtained even for v_2/v_1 approaching 100.

When the ratio v_2/v_1 is very large, a better approximation is possible by first finding the peak of S_2 during the time $1/v_1$, and then combining this peak with S_1 , using the mean rate of intensity change v_1 . For this purpose, it is necessary to use an appropriate extreme value distribution for the peak of S_2 during the time $1/v_1$. The dashed curves in Figure 3-14b were obtained following this procedure.

Sum of Filtered Poisson Processes: Consider first the combination of two independent filtered Poisson load effects S_1 and S_2 . Let v_1 and v_2 denote the mean occurrence rates and τ_1 and τ_2 denote the random durations. Neglect the effect of self-overlapping of the respective processes. As indicated by Wen (1977a), the combined load effect $S = S_1 + S_2$ would also have Poisson occurrences. The mean joint occurrence rate, denoted by v_{12} , is approximately obtained as

$$v_{12} = v_1 v_2 (\bar{\tau}_1 + \bar{\tau}_2) \quad (3-86)$$

where $\bar{\tau}_1$ and $\bar{\tau}_2$ are the mean durations. The above mean rate of occurrence of the combination is used to obtain p_1, q_1 and p_2, q_2 for the two loads from Figures 3-10, 3-11, or 3-12, depending on the distribution of intensities. With these values known, p and q are obtained from Equations 3-83 and 3-84.

It can be shown (Wen, 1977a) that the mean duration of the joint occurrence of two loads is

(a) Equal occurrence rates

(b) Varied occurrence rates

FIGURE 3-14. COMPUTED AND SIMULATED RESULTS FOR COMBINATION OF POISSON SQUARE WAVE LOADS: S_1 - NORMAL, S_2 - LOGNORMAL

$$\bar{\tau}_{12} = \frac{\bar{\tau}_1 \bar{\tau}_2}{\bar{\tau}_1 + \bar{\tau}_2} \quad (3-87)$$

This result, together with Equation 3-86, can be used to obtain mean joint occurrence rates for combinations of more than two filtered

Poisson processes. For example, in the case of three loads

$$\begin{aligned} v_{123} &= v_{12}v_3 (\bar{\tau}_{12} + \bar{\tau}_3) \\ &= v_1v_2v_3 (\bar{\tau}_1\bar{\tau}_2 + \bar{\tau}_2\bar{\tau}_3 + \bar{\tau}_3\bar{\tau}_1) \end{aligned} \quad (3-88)$$

Also

$$\begin{aligned}\bar{\tau}_{123} &= \frac{\bar{\tau}_{12}\bar{\tau}_3}{\bar{\tau}_{12} + \bar{\tau}_3} \\ &= \frac{\bar{\tau}_1\bar{\tau}_2\bar{\tau}_3}{\bar{\tau}_1\bar{\tau}_2 + \bar{\tau}_2\bar{\tau}_3 + \bar{\tau}_3\bar{\tau}_1}\end{aligned}\quad (3-89)$$

To examine the method, computed and simulated results for combinations of two filtered Poisson processes were compared (see Fig. 3-15). Durations of the two loads were assumed to be uniformly distributed with mean values of 0.01 years and coefficients of variation of 0.30. Normal and lognormal distributions were assumed for intensities of the two loads.

Figure 3-15a shows computed and simulated results for $v_1 = v_2$. The mean number of joint occurrences, $v_{12}T$, varies from 1 to 400. Very good agreement is obtained, except for $v_{12}T = 1$. The error in computed values at that point is attributed to the sharp sensitivity of the p and q functions to the instantaneous coefficient of variation and to the product vT at small values of this product; see Figures 3-10 to 3-12. This, however, is not alarming since such combinations (i.e., with very small mean number of occurrences) would probably not be critical in most practical applications.

Figure 3-15b shows computed and simulated results for $v_2T = 1000$, with v_1T varying from 50 to 1000. The mean number of joint occurrences, $v_{12}T$, varies from 20 to 400. A very good agreement with simulation results is again observed.

Sum of Poisson Square Wave and Filtered Poisson Processes: Consider first the combination of a Poisson square wave process S_1 and a filtered Poisson process S_2 . Let v_1 denote the mean rate of intensity changes of S_1 and v_2 denote the mean rate of occurrences of S_2 . Assume that the duration of occurrence of S_2 ,

denoted by τ_2 , is much smaller than the time between intensity changes of S_1 . Consider the following two cases:

1. $v_1 \ll v_2$: In this case one would expect several occurrences of S_2 between consecutive changes of S_1 . The mean number of intensity changes of S_1 in the combination, therefore, is v_1T . The values of p_1, q_1 and p_2, q_2 for the combination should, therefore, be obtained on the basis of v_1T and v_2T , respectively, using the appropriate curves in Figures 3-10, 3-11 and 3-12.
2. $v_1 \gg v_2$: In this case multiple occurrences of S_2 between consecutive changes of S_1 would be unlikely. The mean number of intensity changes of S_1 would, therefore, be approximately equal to v_2T . The values of p_1, q_1 as well as p_2, q_2 for the combination should, therefore, be obtained on the basis of v_2T , using the appropriate curves in Figures 3-10, 3-11 and 3-12.

When v_1 and v_2 are comparable, the above procedures could still be used. Reasonably accurate results are expected due to the relatively slow variations of p and q with vT for the Poisson square wave process. With the values of p_1, q_1 and p_2, q_2 known, p and q for the combination are then obtained from Equations 3-83 and 3-84.

Figure 3-16 shows computed and simulated results for combinations of a Poisson square wave process and a filtered Poisson process. For simulation analysis, the duration of occurrence τ_2 was assumed to be uniformly distributed with mean of 0.01 years and coefficient of variation 0.30. The distributions of intensities for the two processes were respectively assumed to be normal and lognormal. Results are shown for $v_1T = 50$ with v_2T varying from 5 to 500.

(a) Equal occurrence rates

(b) Varied occurrence rates

FIGURE 3-15. COMPUTED AND SIMULATED RESULTS FOR COMBINATION OF FILTERED POISSON LOADS: S_1 — NORMAL, S_2 — LOGNORMAL

wave and filtered Poisson processes. For this purpose one should first determine the joint occurrence rate of the filtered Poisson processes. This occurrence rate is used to determine the p_i and q_i values for each filtered Poisson process. The values of p_i and q_i for each Poisson square wave process are then obtained from either the procedure 1 or 2, depending on the magnitude of its intensity change rate relative to the above occurrence rate.

3.5 DESIGN FOR MULTIPLE LOAD COMBINATIONS

Structures are usually affected by loads resulting from several independent sources. For the safety analysis and design of a structure under these conditions, it is necessary to evaluate the risk of failure within the life of structures under all possible combinations of loads. Then reliability-based design factors corresponding to various combinations of loads are formulated on the basis of concepts discussed previously.

3.5.1 EVALUATION OF RISK UNDER MULTIPLE LOAD COMBINATIONS

Assume that a structure may be affected by a number of loads that may act in m different combinations during the time period T . For a specific component of the structure, we define the events C_i , $i = 1, 2, \dots, m$, where C_i = the component survives under the i th combination of loads during period T , and $\bar{C}_i$ = the complementary event. Consider the following example: assume that a structure may be affected by dead, live, wind, and earthquake loads, denoted by S_D , S_L , S_W , and S_E , respectively. Also assume that only three combinations, namely $S_D + S_L$, $S_D + S_L + S_W$, and $S_D + S_L + S_E$, denoted by the numbers 1, 2, and 3, respectively, are considered to be critical for this structure. Now imagine three separate experiments in which we expose the structure

FIGURE 3-16. COMPUTED AND SIMULATED RESULTS FOR COMBINATION OF POISSON SQUARE WAVE AND FILTERED POISSON LOADS: S_1 - NORMAL, S_2 - LOGNORMAL

A very good agreement between the two sets of results is observed in this figure.

The above procedure can be extended to combinations involving several Poisson square

each of the load combinations separately. For example, in the first experiment for Combination 1 we isolate the structure from wind and earthquake loads and let the dead and live loads act for a period of T years. In the second experiment for Combination 2 we isolate the structure from any combination other than dead, live, and wind loads for the period of T years; that is, we isolate the structure from earthquake loads and from dead and live loads when no wind loads are acting. Finally, in the third experiment we isolate the structure from all loads other than the concurrent action of dead, live, and earthquake loads. According to the above definition, the possible outcomes of these experiments, respectively, are $\{C_1, \bar{C}_1\}$, $\{C_2, \bar{C}_2\}$, and $\{C_3, \bar{C}_3\}$. That is, for example, if the component survived during the second experiment we say the event C_2 has occurred. If the component failed during the third experiment we conclude that the event $\bar{C}_3$ has occurred.

In order for the component to survive during the time period T , it must survive under each of the m load combinations. In other words, it will fail if it fails under any combination, i.e.,

$$P(\text{Failure}) = P(\bar{C}_1 \cup \bar{C}_2 \cup \dots \cup \bar{C}_m) \quad (3-90)$$

where $\cup$ indicates union of events. Considering that C_i are generally positively correlated, the above probability is bounded by

$$\max_i P(\bar{C}_i) \leq P(\text{Failure}) \leq \sum_{i=1}^m P(\bar{C}_i) \quad (3-91)$$

which indicates that the actual failure probability is greater than the failure probability under the most critical combination and is smaller than the sum of the failure probabilities under all combinations.

In reliability-based design, an acceptable risk for component failure, p_f , is specified.

Under multiple load combinations it is necessary that this acceptable risk be allocated to the various load combinations that may be critical. The allocation can be done in any manner as long as it is assured that the actual probability of failure of the design remains smaller than p_f . Using the upper bound in Equation 3-91, this can be done by requiring that the sum of acceptable risks for all possible combinations be equal to or smaller than p_f .

For economical reasons it may be desirable to have various load combinations require more or less equal resistances. For this purpose one would have to allocate larger portions of p_f to combinations that are more critical (i.e., combinations which require larger resistances for the same risk level). On the other hand, if for any reason it is desired to maintain a high level of reliability under a specific combination, then one would have to allocate a smaller acceptable risk to that combination, which may result in that combination effectively determining the required resistances.

3.5.2 DESIGN FACTORS UNDER MULTIPLE LOAD COMBINATIONS

Consider the general case of a combination including time-independent (dead) load effects D_j and stochastic load effects S_i . Let p_f^* denote the acceptable risk allocated to the combination.

The peak combined load effect within the life of structure T is defined as

$$S_T = \sum_j D_j + \max_T \left(\sum_i S_i \right) \quad (3-92)$$

The mean and standard deviation of S_T are

$$\bar{s}_T = \sum_j \bar{d}_j + \sum_i \bar{s}_i + p \sqrt{\sum_i \sigma_{S_i}^2} \quad (3-93)$$

$$\sigma_{S_T} = \sqrt{\sum_j \sigma_{D_j}^2 + q^2 \sum_i \sigma_{S_i}^2} \quad (3-94)$$

p and q in the above equations are specific to the combination and are evaluated as described previously (see Eqs. 3-83 and 3-84).

With the coefficient of variation obtained as $\Omega_{S_T} = \sigma_{S_T} / \bar{s}_T$, the reliability-based central safety factor corresponding to the formulation on $Z_1 = R - S_T$ is obtained as (see Eq 3-40)

$$\bar{Y} = \frac{1 + \beta_1 \alpha_R \Omega_{S_T}}{1 - \beta_1 \alpha_R \Omega_R} \quad (3-95)$$

The central safety factor corresponding to the formulation on $Z_2 = R/S_T$ is obtained as (see Eq 3-48)

$$\bar{Y} = \exp \left(\beta_2 \sqrt{\Omega_R^2 + \Omega_{S_T}^2} \right) \quad (3-96)$$

β_1 and β_2 in these expressions are the corresponding reliability indices obtained in terms of p_f^* . These factors are to be applied on the mean value of S_T as obtained from Equation 3-93

To obtain reliability-based load factors, it is necessary to separate the effects of loads in the combination. For this purpose, Equations 3-93 and 3-94 are written as

$$\bar{s}_T = \sum_J \bar{d}_J + \sum_1 \bar{s}_1 + \alpha_{s2}^p \sum_1 \sigma_{S_1} \quad (3-97)$$

and

$$\sigma_{S_T} = \alpha_{s1} \left(\sum_J \sigma_{D_J} + q \sum_1 \sigma_{S_1} \right) \quad (3-98)$$

where

$$\alpha_{s2} = \frac{\sqrt{\sum_1 \sigma_{S_1}^2}}{\sum_1 \sigma_{S_1}}$$

$$\alpha_{s1} = \frac{\sqrt{\sum_J \sigma_{D_J}^2 + q^2 \sum_1 \sigma_{S_1}^2}}{\sum_J \sigma_{D_J} + q \sum_1 \sigma_{S_1}}$$

Using these equations together with the method developed previously, it is easily verified that for the formulation based on $Z_1 = R - S_T$, the central load factors for D_J and S_1 , respectively, are

$$\bar{\delta}_{D_J} = 1 + \beta_1 \alpha_R \alpha_{s1} \Omega_{D_J} \quad (3-99)$$

and

$$\bar{\delta}_{S_1} = 1 + \beta_1 \alpha_R \alpha_{s1} q \Omega_{S_1} + \alpha_{s2}^p \Omega_{S_1} \quad (3-100)$$

These expressions do not include the effect of additional uncertainty Ω_g that is due to errors in formulation of loads. The inclusion of this effect gives

$$\bar{\delta}_{D_J} = 1 + \beta_1 \alpha_R \alpha_g \left(\Omega_g + \alpha_{s1} \Omega_{D_J} \right) \quad (3-101)$$

and

$$\bar{\delta}_{S_1} = 1 + \beta_1 \alpha_R \alpha_g \left(\Omega_g + \alpha_{s1} q \Omega_{S_1} \right) + \alpha_{s2}^p \Omega_{S_1} \quad (3-102)$$

The above factors are to be applied on the instantaneous mean values $\bar{d}_1$ and $\bar{s}_1$

For the formulation based on $Z_2 = R/S_T$, the central load factors for D_J and S_1 may be obtained as

$$\bar{\delta}_{D_J} = \exp \left[\beta_2 \alpha_R \alpha_g \left(\Omega_g + \alpha_{s1} \Omega_{D_J} \right) \right] \quad (3-103)$$

and

$$\bar{\delta}_{S_1} = \exp \left[\beta_2 \alpha_R \alpha_g \left(\frac{\Omega_g + \alpha_{s1} q \Omega_{S_1}}{1 + \alpha_{s2}^p \Omega_{S_1}} \right) \right] \left(1 + \alpha_{s2}^p \Omega_{S_1} \right) \quad (3-104)$$

where α_R , α_g , and α_{s2} are as before, but α_{s1} must be obtained from a transcendental equation similar to Equation 3-52. It is felt, however, that Equation 3-98 could still be used to obtain good approximations of α_{s1} . Note that Table 3-2 may be used to obtain approximate values of α_{s1} and α_{s2} depending on the number of loads in a combination.

The strength factors corresponding to the two formulations remain as given in Equations 3-45 and 3-54.

3 5 3 VARIABLES AFFECTING RELIABILITY-BASED DESIGN FACTORS

Reliability-based design factors derived in this study were seen to be functions of various design variables. Some of these variables are

significant, whereas others may be insignificant. A brief qualitative discussion of variables affecting design factors is presented below.

Central Safety Factor: From Equations 3-95 and 3-96, variables affecting the central safety factor are seen to be the acceptable risk p_f^* allocated to the combination (this is through the reliability index β_1 or β_2), and the coefficients of variation Ω_R and Ω_{ST} of the resistance R and of the peak combined load effect S_T , respectively. The parameter α_R in Equation 3-95 is effectively a constant.

It is observed in Equations 3-95 and 3-96 together with Table 3-1 that the central load factor $\bar{\gamma}$ is larger for a larger β_1 or β_2 (i.e., a higher required reliability) or for a smaller p_f^* (i.e., a lower acceptable risk). This indicates a more conservative design when higher reliability is required.

While the coefficient of variation Ω_R remains unchanged from one combination to another, the coefficient of variation Ω_{ST} of the peak combined load effect, obtained from Equations 3-93 and 3-94 as

$$\Omega_{ST} = \frac{\sqrt{\sum_j \sigma_{D_j}^2 + q^2 \sum_i \sigma_{S_i}^2}}{\sum_j \bar{d}_j + \sum_i \bar{s}_i + p \sqrt{\sum_i \sigma_{S_i}^2}} \quad (3-105)$$

is clearly dependent on the combination under consideration. This is not only due to the dependence of Ω_{ST} on the (instantaneous) means and standard deviations of loads in combination, but also due to its dependence on the second-moment descriptors p and q . As was shown before, p and q depend on the stochastic behavior of each load and on the useful life, T , assumed for the structure. It is recalled that the mean and standard deviation values of load effects are related to the mean and standard deviation values of loads through the influence

coefficients. This means that for a given combination of loads and for a specific T , the value of Ω_{ST} and therefore the safety factor $\bar{\gamma}$, would change from one component to another component in the structure.

Central Strength and Load Factors: The central strength factor, $\bar{\phi}$, given in Equations 3-45 and 3-54 is seen to be a function of the acceptable risk p_f^* through the reliability index β_1 or β_2 , and the coefficient of variation Ω_R . For both formulations $\bar{\phi}$ is seen to decrease with increasing β_1 or β_2 or with decreasing p_f^* . The strength factor is also seen to decrease with increasing values of Ω_R . These indicate a more conservative estimate of strength when a high reliability is required or when a large uncertainty is associated with the strength evaluation.

Variables affecting central load factors for dead loads, $\bar{\delta}_{D_j}$, are seen in Equations 3-101 and 3-103 to be the reliability index β_1 or β_2 (or the corresponding acceptable risk p_f^*), the coefficient of variation Ω_{D_j} , and the decoupling factor α_{s1} . The factors α_R and α_g may be regarded as constants. The coefficient of variation Ω_g is the same for all loads and combinations.

It is observed in Equations 3-101 and 3-103 that the central dead load factor increases with increasing β_1 or β_2 or with decreasing p_f^* . It is also seen to increase with increasing values of the coefficient of variation of the load. The parameter α_{s1} , as given in Equation 3-98, is primarily a function of the number of loads in a combination, as suggested in Table 3-2. It is concluded, therefore, that the dead load factor is dependent on the number of loads in combination; it decreases with increasing number of loads. The dependence of the factor on individual load characteristics and the values of p and q as well as the influence coefficients are expected to be insignificant. The dead load

factors are, therefore, effectively independent of the location of a component in a structure.

Variables affecting central load factors for stochastic loads, $\bar{\delta}_{S_i}$, are seen in Equations 3-102 and 3-104 to be the reliability index β_1 or β_2 (or the corresponding acceptable risk p_f^*); the instantaneous coefficient of variation of load, Ω_{S_i} ; the second-moment descriptors p and q ; and the decoupling factors α_{s1} and α_{s2} for each combination. The factors α_R and α_g , and the coefficient of variation Ω_g are the same for all loads and combinations.

It is observed in Equations 3-102 and 3-104 that the central load factor for a stochastic load increases with increasing β_1 or β_2 or with decreasing p_f^* . It is also observed to increase with increasing value of the instantaneous coefficient of variation Ω_{S_i} . The relation of the load factor to the stochastic behavior of loads appears through the p and q values for each combination. Since these values are dependent on the standard deviation of individual load effects and, therefore, on the influence coefficients, it is concluded that the load factor for a stochastic load in a specific combination would depend on the location of the component in the structure. The significance of this dependence is not clear at this time.

The load factor $\bar{\delta}_{S_i}$ is also dependent on the useful life of the structure, T , through the second-moment descriptors p and q . Based on the studied behaviors of p and q with respect to T , it is observed that the load factor increases with increasing value of T .

The decoupling factors α_{s1} and α_{s2} generally depend on the characteristics of loads in the combination. As mentioned before, however, these factors can effectively be determined in terms of the number of loads in the combination (see Table 3-2). Based on such an approximation, the load factor $\bar{\delta}_{S_i}$ is

found to decrease with increasing number of loads in a combination.

3.5.4 ILLUSTRATIVE EXAMPLE

To demonstrate the application of the theory developed here, a hypothetical example is provided below. This problem is presented in greater detail in Section 5.4 of AA (1978).

Consider a structural component affected by a dead load effect D , a sustained live load effect L_s , extraordinary transient live load effects L_e , a sustained temperature load effect T_o , and earthquake load effects E . The assumed models for these loads are described in Table 3-3.

On the basis of stiffnesses assumed in a preliminary design, the random-point-in-time mean and standard deviation values of load effects in various components can be evaluated in terms of the influence coefficients and the random-point-in-time mean and standard deviation of the corresponding loads. For the component under consideration, assume these values are evaluated as shown in Table 3-3. The assumed values of the parameters v_i for each load (i.e., the mean intensity change rate of L_s , the mean occurrence rates of L_e and E , and the mean upcrossing rate of T_o) and the mean durations, $\bar{t}_i$, for the transient loads L_e and E , are also given in this table.

To investigate the effect of the structure lifetime T , the values of $T = 25$ and 50 years are assumed. The lifetime peak load effects of individual loads for the two values of T , obtained using Equations 3-66 and 3-67 and Figures 3-7 and 3-12, are given in the lower portion of Table 3-3.

Four combinations of the loads are considered in Table 3-4. These include combination 1 involving the sustained loads D , L_s , and T_o ; combinations 2 and 3 involving the

TABLE 3-3. ASSUMED LOAD PARAMETERS FOR ILLUSTRATIVE EXAMPLE

Load	Load Model	Random-Point-in-Time Load Effect			v_1 1/year	Mean Duration $\bar{t}_1$, Years
		Mean	St Dev	COV		
D	Random Variable	100	20	0 20	--	--
L_s	Poisson Square Wave with Gamma Dist	50	15	0 30	0 5	--
T_o	Stationary Gaussian	50	10	0 20	20 0	--
L_e	Filtered Poisson with Gamma Dist	60	24	0 40	10 0	10^{-2}
E	Filtered Poisson with Exponential Dist	50	30	0 60	0 5	10^{-6}

Load	Load Model	Lifetime Peak Load Effect					
		T = 25 years			T = 50 years		
		Mean	St Dev	COV	Mean	St Dev	COV
D	Random Variable	100 0	20 0	0 20	100 0	20 0	0 20
L_s	Poisson Square Wave with Gamma Dist	77 0	11 8	0 15	83 6	11 1	0 13
T_o	Stationary Gaussian	86 8	3 3	0 04	88 5	3 2	0 04
L_e	Filtered Poisson with Gamma Dist	150 7	17 8	0 12	160 3	17 2	0 11
E	Filtered Poisson with Exponential Dist	113 0	38 4	0 34	134 0	38 4	0 29

TABLE 3-4. COMPUTED RESULTS FOR COMBINATIONS OF LOADS

Comb No	Load Combination	T = 25 Years					T = 50 Years				
		p	q	Lifetime Peak Combined Load Effect			p	q	Lifetime Peak Combined Load Effect		
				Mean	St Dev	COV			Mean	St Dev	COV
1	D + L_s + T_o	2 38	0 65	242 9	23 2	0 10	2 74	0 61	249 4	22 8	0 09
2	D + L_s + T_o + L_e	3 18	0 71	355 5	29 2	0 08	3 59	0 68	367 6	28 7	0 08
3	D + L_s + T_o + E	2 01	1 13	320 4	45 0	0 14	2 63	1 13	342 0	45 0	0 13
4	D + L_s + T_o + L_e + E	0 11	1 28	314 7	57 9	0 18	0 67	1 24	338 4	56 0	0 17

sustained loads plus the transient loads L_e and E, respectively; and combination 4 involving the sustained loads plus both of the transient loads. The last combination thus considers the possibility of coincidence of the two transient loads.

The second moment descriptors p and q for each combination are given in Table 3-4 for

T = 25 and 50 years. The lifetime peak combined load effects for each combination are also given in this table. From these values, it is then possible to obtain the central safety and load factors for each combination. For this example it is assumed that the acceptable risk is allocated equally to the four combinations. To investigate the effect of the acceptable risk

level, the cases $\beta_1 = \beta_2 = 3.72$ (i.e., $p_f^* \cong 10^{-4}$; see Table 3-1), and $\beta_1 = \beta_2 = 4.25$ (i.e., $p_f^* \cong 10^{-5}$) are considered. Assuming $\Omega_R = 0.20$, $\Omega_g = 0.0$, $\alpha_R = 0.75$, and $\alpha_g = 1.0$, the central safety, strength, and load factors corresponding to the formulations based on $Z_1 = R - S_T$ and $Z_2 = R/S_T$, for each combination can be calculated as shown in AA (1978).

3.5.4.1 Factors Comparable with Current Procedures

In order to make a proper comparison of safety and load factors it is necessary that they apply to a consistent set of loads and resistances. Values of loads and resistances used in current procedures, however, are seldom well-defined with respect to their distributions, and accurate comparison is difficult. It is assumed here that design resistances and loads used in current procedures correspond to the mean values of resistance and lifetime peak loads, respectively. When these are different from the mean values, an appropriate adjustment must be made.

As indicated earlier, the safety factor formulated in Equation 3-95 or 3-96 refers to the mean value of lifetime peak combined load effect, $\bar{s}_T$. In current procedures this factor is applied on the sum of design loads, i.e., $\sum_j \bar{d}_j + \sum_i \bar{s}_{iT}$, where $\bar{s}_{iT}$ is the mean lifetime peak of S_i . The safety factor comparable with current procedures, denoted by $\bar{\gamma}^*$, may therefore be obtained as

$$\bar{\gamma}^* = \bar{\gamma} \frac{\bar{s}_T}{\sum_j \bar{d}_j + \sum_i \bar{s}_{iT}} \quad (3-106)$$

Values of $\bar{\gamma}^*$ evaluated for the example under consideration are tabulated in Table 3-5 for formulations based on Z_1 and Z_2 and for $T = 50$ years.

It was also indicated before that load factors formulated in Equations 3-101 to 3-104

refer to the mean instantaneous load values. Load factors comparable with current procedures, denoted by $\bar{\delta}_i^*$, may therefore be obtained from

$$\bar{\delta}_i^* = \bar{\delta}_i \frac{\bar{s}_i}{\bar{s}_{iT}} \quad (3-107)$$

Results for this set of load factors for $T = 50$ years are shown in Table 3-6.

Shown in Table 3-6 is also a total load factor for each combination. This factor, which has not been previously discussed, relates to the total load in each combination. Consistent with the factors introduced earlier, the central total load factor $\bar{\xi}$ is used in the design equation:

$$\bar{\phi} \bar{R} \geq \bar{\xi} \bar{s}_T \quad (3-108)$$

It is observed that

$$\bar{\xi} = \bar{\phi} \bar{\gamma} \quad (3-109)$$

In particular, for comparison with current procedures, we obtain

$$\bar{\xi}^* = \bar{\phi} \bar{\gamma}^* \quad (3-110)$$

The values of $\bar{\xi}^*$ as evaluated for $\beta_1 = \beta_2 = 3.72$ and $T = 50$ years are shown in Table 3-6.

3.5.4.2 Discussion of Numerical Results

Significance of Distribution. It is observed that the results for the two formulations based on Z_1 and Z_2 are generally different. This indicates that for the levels of risk considered (i.e., 10^{-4} and 10^{-5}), the reliability-based safety, load, or strength factors are not insensitive to the distributions of R and S_T . In particular, the results based on Z_1 would be accurate if the underlying distributions were normal, whereas the results based on Z_2 would be accurate if they were lognormal. It is generally believed, however, that the lognormal distribution is more realistic for loads and resistances (Ang and Cornell, 1974). Based on this, the values corresponding to the formulation on Z_2 are considered to be more realistic.

TABLE 3-5. RELIABILITY-BASED SAFETY FACTORS $\bar{\gamma}^*$ COMPARABLE WITH CURRENT PROCEDURES

Load Combination	T = 50 Years			
	$Z_1 = R - S_T$		$Z_2 = R/S_T$	
	$\beta_1 = 3.72$	$\beta_1 = 4.27$	$\beta_2 = 3.72$	$\beta_2 = 4.27$
$D + L_s + T_o$	2.60	3.30	2.08	2.35
$D + L_s + T_o + L_e$	2.35	2.96	1.89	2.13
$D + L_s + T_o + E$	2.60	3.33	2.05	2.34
$D + L_s + T_o + L_e + E$	1.98	2.55	1.57	1.81

Note These safety factors are relative to the sum of mean lifetime peak load effects and the mean resistance

TABLE 3-6. RELIABILITY-BASED STRENGTH AND LOAD FACTORS COMPARABLE WITH CURRENT PROCEDURES

(a) $Z_1 = R - S_T$

Load Combination	T = 50 Years													
	$\beta_1 = 3.72$							$\beta_1 = 4.27$						
	$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}^*$	$\bar{\delta}_{T_o}^*$	$\bar{\delta}_{L_e}^*$	$\bar{\delta}_E^*$	$\bar{\zeta}^*$	$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}^*$	$\bar{\delta}_{T_o}^*$	$\bar{\delta}_{L_e}^*$	$\bar{\delta}_E^*$	$\bar{\zeta}^*$
$D + L_s + T_o$	0.44	1.36	1.15	0.91	--	--	1.14	0.36	1.41	1.18	0.93	--	--	1.19
$D + L_s + T_o + L_e$	0.44	1.30	1.18	0.93	0.86	--	1.03	0.36	1.34	1.20	0.94	0.88	--	1.07
$D + L_s + T_o + E$	0.44	1.30	1.20	0.94	--	1.13	1.14	0.36	1.35	1.25	0.97	--	1.18	1.20
$D + L_s + T_o + L_e + E$	0.44	1.27	0.96	0.79	0.67	0.82	0.87	0.36	1.31	1.00	0.82	0.71	0.88	0.92

(b) $Z_2 = R/S_T$

Load Combination	T = 50 Years													
	$\beta_1 = 3.72$							$\beta_1 = 4.27$						
	$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}^*$	$\bar{\delta}_{T_o}^*$	$\bar{\delta}_{L_e}^*$	$\bar{\delta}_E^*$	$\bar{\zeta}^*$	$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}^*$	$\bar{\delta}_{T_o}^*$	$\bar{\delta}_{L_e}^*$	$\bar{\delta}_E^*$	$\bar{\zeta}^*$
$D + L_s + T_o$	0.57	1.43	1.17	0.92	--	--	1.19	0.53	1.51	1.21	0.94	--	--	1.25
$D + L_s + T_o + L_e$	0.57	1.35	1.19	0.94	0.87	--	1.08	0.53	1.41	1.23	0.96	0.90	--	1.13
$D + L_s + T_o + E$	0.57	1.32	1.23	0.95	--	1.19	1.17	0.53	1.41	1.32	1.00	--	1.33	1.24
$D + L_s + T_o + L_e + E$	0.57	1.29	1.02	0.81	0.74	0.99	0.89	0.53	1.34	1.08	0.85	0.80	1.11	0.96

- Note
- 1 Load factors are relative to mean lifetime peak loads
  - 2 Strength factor is relative to mean resistance
  - 3 The total load factor $\bar{\zeta}^*$ is relative to the sum of mean lifetime peak loads

It is important to realize that regardless of the sensitivity to distribution, either of these formulations can be useful.

Acceptable Risk. The significance of the acceptable risk level can be observed by comparing the results for risk levels of 10^{-4} and 10^{-5} . It is observed that an order of magnitude reduction in the risk level results in larger safety and load factors and in smaller strength factors. These variations due to risk are found to be quite significant. As an example, an increase of about 14% in the safety factors based on Z_2 result from the said reduction in the acceptable risk (see Table 3-5).

Structure Life. The significance of structure lifetime can be observed by comparing the load factors for $T = 25$ and 50 years. It appears that the structure lifetime has an appreciable influence on the factors for stochastic loads, whereas it has practically no significance on dead load factors. The increase in stochastic load factors for going from $T = 25$ to 50 years is estimated to be about 3% to 4% for the formulation based on Z_2 .

Combination of Loads. It is observed that, in general, safety and load factors for various combinations are different. These variations are related to the number of loads that are combined, and to the stochastic nature of loads and their combinations. These variations are of particular interest for the factors shown in Tables 3-5 and 3-6, which are comparable with current procedures. It is apparent from these tables that variations in safety or load factors from one combination to another can be significant, particularly when transient loads are combined. As an item of special interest, observe the reduction in safety and load factors in going from the first three combinations to the last combination, which includes the extraordinary live load, L_e , as well as the earthquake load, E . With respect to the first or the

third combination, the reduction in the safety or the total load factor is about 23%, whereas with respect to the second combination this reduction is about 15%. It is recalled that current procedures would normally allow a fixed 25% reduction in these factors.

The reduction in individual load factors varies from one load to another. This variation is due to the characteristics of each load as well as the assumed procedure for separating the effects of the loads through the factors α_s . Reduction in factors of stochastic loads are observed to range from 10% to 27%. A significant observation here is that the dead load factor exhibits only a reduction of less than 5%. This reduction can only be attributed to the increased number of loads, since the dead load factor as given in Equation 3-101 or 3-103 is virtually independent of p and q . These observations contradict the current procedure which permits a fixed reduction of 25% in factors of all loads including the dead load.

Comparison with Current Factors. The results shown in Tables 3-5 and 3-6 appear to be significantly different from factors used in current procedures. To avoid possible misinterpretation of the results, it is necessary to emphasize that these factors refer to the mean values of the resistance and the lifetime peak load. It is believed that values of resistance estimated in current design are generally conservative and well below the mean. The values of design loads may also be different from the mean values. A proper adjustment of these factors can yield results which are in the neighborhood of current factors.

It should also be realized again that these results apply only to the specific example with the assumed value of $\Omega_R = 0.20$ and the acceptable risk levels of 10^{-4} and 10^{-5} . These assumptions, as well as those regarding the characteristics of individual loads, influence the

derived values of safety, strength, and load factors.

3.6 DEVELOPMENT OF CURRENT DESIGN PROCEDURES

These formulations of reliability-based safety, load, and strength factors may not give absolutely correct values of the design factors, especially at very low risk levels. In particular, the results obtained for these factors may significantly depend on the assumed form of the performance function. On a relative basis, however, these formulations are believed to give consistent results. On such grounds, therefore, they can be used as a theoretical basis to critically examine the current procedures, derive design criteria for unconventional structures, and develop probabilistic design procedures.

3.6.1 EXAMINATION OF CURRENT DESIGN FACTORS

A comparison of design factors recommended by current methods to those formulated here reveals inconsistencies or deficiencies in several areas of the existing procedures.

3.6.1.1 Risk Level

While various types of engineering structures require different levels of reliability, currently recommended values of the design factors (i.e., the safety or the load and strength factors) do not include any measure of the required reliability or the acceptable risk. Consider, for example, the design of a hospital versus a residential building. From a socioeconomic viewpoint, clearly the former would demand a higher level of reliability than the latter; yet, the current practice recommends using the same set of factors for the two structures. This would obviously result in either the hospital not being reliable enough (from a

social viewpoint) or the residential building being overconservative (from an economic viewpoint).

In the formulations developed here, the required reliability was included through the reliability index β_1 or β_2 . It was found that for a higher reliability the safety and load factors should be increased while the strength factor should be decreased.

Any future study to improve the current procedures may:

- a. Quantify relative measures of required reliabilities associated with various categories of structures.
- b. Use the formulations developed here to evaluate the variations in reliability-based design factors corresponding to the above variations in the risk level.
- c. If the variations in these factors are significant, implement corresponding variations (on a relative basis) in the existing design factors for the various categories of structures.

3.6.1.2 Structure Lifetime

It has been observed that because of the stochastic nature of loads acting on a structure, the lifetime T influences the reliability-based safety and load factors. It was shown that the variations in these factors are such that they require a larger resistance for the same reliability but for a longer lifetime.

Engineering structures are designed for various anticipated lifetimes. While a lifetime of 30 to 50 years is common, there are structures of temporary nature with lifetimes as short as a few years (e.g., military structures) and structures with intended lifetimes of hundreds of years (e.g., bridges).

Current deterministic design procedures, on the other hand, do not explicitly include the significance of the lifetime. In most cases it is entirely neglected, and loads as well as design factors are specified regardless of its anticipated value. For certain structures (e.g., dams) the lifetime is included in the evaluation of loads or other environmental effects. The design factors, however, are virtually always kept constant, which introduces another element of inconsistency in the current procedures: in the example of the previous section, it was shown that even when the safety and load factors are applied on the lifetime peak loads (which are calculated with due consideration of the lifetime), these factors would still be affected by variations in the lifetime, a variation which results from the stochastic nature of loads and their combinations.

Any future study to improve the current procedures may:

- a. Determine anticipated lifetimes associated with various groups of structures.
- b. Using appropriate models of stochastic loads and the design factor functionals developed here, evaluate the variations in the reliability-based factors corresponding to the above variations in the structure lifetime.
- c. If variations in these factors are significant, implement corresponding variations (on a relative basis) in the existing design factors for the various groups of structures.

3.6.1.3 Variabilities in Loads and Resistances

Although the significance of variabilities in loads and resistances is recognized, these variabilities are included in existing deterministic procedures in only a crude and subjective manner. Both the WSD and the LFD methods

attempt to account for these variabilities through purposely overestimated loads and underestimated resistances. In addition, a reserved resistance is provided through applying a safety factor (greater than one) in the WSD method and strength and load factors (the former less than one and the latter greater than one) in the LFD method. However, these factors have been developed with no consistent consideration of the underlying actual variabilities in loads and resistances, and are essentially subjective.

For this item, the WSD method is believed to be less desirable than the LFD method, since it provides no means for accounting for differences in uncertainties of various loads. By using larger load factors for less predictable loads, the LFD method subjectively accounts for such differences. One should note, however, that the same results could be achieved in the WSD method by using more conservative values for less predictable loads.

In order to improve the existing design procedures on this vital item, it is necessary that variabilities in loads and resistances be explicitly included in the design process. For convenience, the coefficients of variation of loads and resistances may be used as measures of the corresponding variabilities. The formulations developed here can form the framework for establishing appropriate relations of the design factors to the coefficients of variation of loads and resistances.

3.6.1.4 Nominal Loads and Resistances

In the second-moment approach presented here, the mean values of loads and resistances were used to specify the design equation. The values of design loads and resistances used in current procedures, on the other hand, are not consistently defined. Generally, nominal values are used which are either specified by codes or

obtained following code specified procedures. The basis for these values, or their relation with respect to the underlying distributions of actual loads and resistances, in most instances, remains indeterminated.

While it is recognized that any appropriate set of design loads and resistances may be used (i.e., by appropriately adjusting the corresponding safety, strength, or load factors), it is essential that the assumed procedure remain consistent for all loads and resistances under all circumstances. Such a consistent procedure may be obtained by specifying nominal loads and resistances in terms of their mean values. Expressions that have often been proposed are (e.g., Ang and Cornell, 1974)

$$X' = \bar{X} (1 + k_X \Omega_X) \quad (3-111)$$

and

$$R' = \bar{R} (1 - k_R \Omega_R) \quad (3-112)$$

where X' and R' are the nominal load and resistance and $\bar{X}$, $\bar{R}$, and Ω_X , Ω_R are the corresponding mean and coefficient of variation values. The constants k_X and k_R denote the number of standard deviations between the nominal and the mean values. Through these constants it is possible to specify consistent measures of nominal loads and resistances.

Consistent with the above definition of nominal loads and resistances, a set of safety, load, and strength factors may be introduced. These factors may be obtained in terms of the corresponding central factors and the constants k_R and k_X . For example, the strength and load factors for the nominal resistance R' and the nominal load X' can be written as

$$\phi = \bar{\phi} \frac{\bar{R}}{R'} = \frac{\bar{\phi}}{1 - k_R \Omega_R} \quad (3-113)$$

and

$$\delta = \bar{\delta} \frac{\bar{X}}{X'} = \frac{\bar{\delta}}{1 + k_X \Omega_X} \quad (3-114)$$

The selection of k_X and k_R should have no influence on the outcome of a design. Their selection should, therefore, be based on convenience only. In particular, a convenient choice might be $k_X = k_R = 0$; i.e., nominal loads and resistances equal to the mean values.

A direct comparison (i.e., on an absolute basis) of the existing factors with those obtained here could be achieved if the nominal loads and resistances in the current procedures are specified as shown in Equations 3-111 and 3-112.

3.6.1.5 Combination of Loads

Variations of reliability-based safety and load factors from one combination to another, as observed in the example problem, are attributed to the number of loads in a combination and to the stochastic fluctuations of loads. It was observed that with increasing number of loads in a combination, the individual load factors tend to decrease. This was apparent through the decoupling factors α_s which decreased with increasing number of loads (see Table 3-2). This behavior results from the fact that with increasing number of loads the probability of all loads having unusually large values is smaller. This effect is entirely absent in the existing procedures, which use the same load factors regardless of the number of loads combined. It is believed that this problem would be especially critical in design of nuclear power plants, where many independent loads are combined to design for the safety of containment structures.

The stochastic variabilities of loads and the resulting effects on their combinations pose a very complex problem. In the formulation developed here, these effects were included through the second-moment descriptor functions p and q . These include the most significant parameters that may define stochastic loads,

i.e., the occurrence rate, the duration, and the form of the instantaneous distribution.

It was observed in the example problem that the stochastic variabilities of loads have highly significant effects on the load factors in various combinations. In particular, when two transient loads were included in a combination, the load factors for the same set of (stochastic) loads were substantially smaller than in combinations involving only one or no transient loads. Meanwhile, the dead load factor remained unaffected (except for a small reduction due to the increase in number of loads). These reductions in load factors are related to the above mentioned parameters pertaining to the stochastic loads in the combination and, therefore, would generally vary from one situation to another.

Perhaps current procedures are most inadequate when it comes to the combination of stochastic loads. The current methods exclude certain combinations that may have large combined effects but are thought to be very unlikely, i.e., the probability of occurrence is small. One example is the combination of earthquake and wind loads, which is almost never considered. Also, standard reduction of load factors (or increase of allowable stresses, in WSD) for combinations involving one or more transient loads is permitted. An example is the combination of live loads with earthquake or wind loads, wherein a reduction of 25% is specified.

This practice, however, has been established on the basis of experience and engineering judgment only, for no theoretical studies have been conducted to show, for example, that the combination of earthquake and wind loads is insignificant. Although this may be suitable for certain groups of structures, for others that demand very high levels of reliability (e.g., nuclear power plants) it may be unsuitable. A

proper investigation is required before such procedures are justified.

The presently specified reduction in load factors is also subjective. While it is expected that the proper reductions depend on the various parameters of the individual loads in a combination, a standard reduction is currently specified with no consideration of the respective load occurrence rates or the corresponding durations. The adequacy of this procedure should also be investigated, especially with regard to design of critical structures such as nuclear power plants.

The theoretical development presented here can form the framework for the above investigations. It may be used to justify the current specifications for a certain class of structures or load combinations; or, it may be used to develop improved procedures as necessary. In any case, thorough studies are required to resolve such vital problems.

An improvement on the load factor reduction procedure that can be recommended at this time, follows. It was observed in the example problem that the dead load factor is essentially independent of combination, except for the number of loads involved. This indicates that the reduction in load factors should not be applied to the dead load factor. Current procedures specify a blanket reduction of load factors, including that of the dead load. It is felt that this inconsistency can be avoided by specifying that load factor reductions do not apply to the dead load. A somewhat smaller adjustment of the dead load factor based on the number of loads in a combination may be specified.

3.6.2 DESIGN FACTORS FOR UNCONVENTIONAL STRUCTURES

There is always a need to design structures markedly different from the ordinary ones, i.e., unconventional structures. These

CHAPTER 4

DESIGN VERIFICATIONS

4.1 INTRODUCTION

Seismic design verification of nuclear power plant structures has two important objectives. First, it must verify that the predicted response of the structures at equipment and piping support points has been reliably estimated, as the structure response at these points is used as input to the seismic design analysis of the equipment and piping systems. Second, seismic design verification of the structure is needed to assure satisfactory and reliable behavior of the structure in strong earthquake ground motion environments. A review of seismic design verification requirements for nuclear power plant structures and an assessment of current procedures are presented here. Seismic design verification of piping and equipment is not included. An important objective here is to determine if improved verification procedures for future Liquid Metal Fast Breeder Reactor (LMFBR) structures are needed.

4.2 ASSESSMENT OF SEISMIC DESIGN VERIFICATION PROCEDURES

4.2.1 INTRODUCTION

Federal requirements pertaining to seismic design verification are set forth in Code of Federal Regulations 10 CFR 50, Appendix B, which provides quality assurance requirements for nuclear power plants and fuel reprocessing plants. Guidance in interpretation of the requirements is provided in Regulatory Guide (RG) 1.64 by the U.S. Nuclear Regulatory Commission (USNRC, 1975a). RG 1.64 by reference incorporates the American Standards Institute Standard N45.2.11 (ANSI, 1974), with some exceptions. The requirements of the U.S.

Energy Research and Development Administration are provided in RDT Standards RDT-F-2-2 and RDT-F-2-4T (ERDA, 1973, 1974). These documents provide a complete background in regulatory requirements.

4.2.2 REGULATORY REQUIREMENTS

Appendix B of 10 CFR 50 requires that every applicant for a construction permit include in the Preliminary Safety Analysis Report (PSAR) a description of the quality assurance program to be applied to the design, fabrication, construction, and testing of the structures, systems, and components of the facility. Information pertaining to the managerial and administrative controls to assure safe plant operation must also be provided in the Final Safety Analysis Report (FSAR). Thus, Appendix B establishes quality assurance requirements not only for design, but for construction and operation of the facility as well. Also, the requirements apply to all activities affecting safety-related functions, and include designing, purchasing, fabricating, handling, shipping, storing, cleaning, erecting, installing, inspecting, testing, operating, maintaining, repairing, refueling, and modifying. Before proceeding further, definitions of the terms *quality assurance*, *quality control*, *design* and *design verification* are needed.

Quality assurance is defined in 10 CFR 50, Appendix B, as "all those planned and systematic actions necessary to provide adequate confidence that a structure, system, or component will perform satisfactorily in service." *Quality assurance* is further defined to include *quality control* which is defined as "those quality assurance actions related to the physical

characteristics of a material, structure, component, or system which provides a means to control the quality...to predetermined requirements."

Design control procedures set forth in 10 CFR 50, Appendix B provide that measures be established to assure that applicable regulatory requirements and the design basis for the structures, systems, and components be correctly translated into specifications, drawings, procedures, and instructions, and that procedures be provided for verifying, *or checking*, the adequacy of the design. Three procedures for verifying the adequacy of the design are enumerated: (1) performance of design reviews, (2) use of alternate or simplified calculational methods, or (3) performance of a suitable testing program. It is important to note that the above provisions are defined as design *control* procedures and that the conjunction *or* is used rather than *and*. This could be interpreted to imply that any one of the control procedures may be used, and that all three procedures are not required for a given facility. However, a slightly different interpretation is given in ANSI N45.2.11.

Design is defined in ANSI N45.2.11, as the "technical and management processes which commence with identification of design input and which leads to and includes the issuance of design output documents." *Design input* is defined as, "those criteria, parameters, bases, or other design requirements upon which detailed final design is based." *Design output* is defined as, "documents such as drawings, specifications, and other documents defining technical requirements of structures, systems, and components." Finally, *design verification* is defined as, "the process of reviewing, confirming, or substantiating the design by *one or more* methods to provide assurance that the design meets the specified design inputs."

Here, it should be noted that *one or more* methods is indicated rather than any *one* method.

The methods of providing design verification described in ANSI 45.2.11 are the same three design control methods defined in 10 CFR 50, Appendix B (i.e., design reviews, alternate calculations, and qualification testing). The extent of the design verification required is indicated in ANSI 45.2.11 to be "a function of the importance to safety of the item under consideration, the complexity of the design, the degree of standardization, the state of the art, and the similarity with previously proven designs. However, the applicability of standardized or previously proven designs, with respect to meeting pertinent design inputs, including environmental conditions, shall be verified for each application." With this as background, description of the three methods of providing design verification are quoted below from ANSI 45.2.11.

4.2.2.1 Verification By Design Reviews

Design reviews are critical reviews to provide assurance that design documents such as drawings, calculations, analyses or specifications are correct and satisfactory. Design reviews can range from multi-organization reviews to single-person reviews. The depth of review can range from a detailed check of the complete design to a limited check of such things as the design approach and the results obtained. The results of the review shall be documented and measures taken to ensure that the findings are implemented. Whether the review is conducted by one individual or a multi-organization there are a number of basic questions that shall be addressed such as:

1. Were the inputs correctly selected and incorporated into design?
2. Are assumptions necessary to perform the design activity adequately described and reasonable? Where necessary, are the assumptions identified for subsequent re-verifications when the detailed design activities are completed?

3. Are the appropriate quality and quality assurance requirements specified?

4. Are the applicable codes, standards and regulatory requirements including issue and addenda properly identified and are their requirements for design met?

5. Have applicable construction and operating experience been considered?

6. Have the design interface requirements been satisfied?

7. Was an appropriate design method used?

8. Is the output reasonable compared to inputs?

9. Are the specified parts, equipment, and processes suitable for the required application?

10. Are the specified materials compatible with each other and the design environmental conditions to which the material will be exposed?

11. Have adequate maintenance features and requirements been specified?

12. Are accessibility and other design provisions adequate for performance of needed maintenance and repair?

13. Has adequate accessibility been provided to perform the in-service inspection expected to be required during the plant life?

14. Has the design properly considered radiation exposure to the public and plant personnel?

15. Are the acceptance criteria incorporated in the design documents sufficient to allow verification that design requirements have been satisfactorily accomplished?

16. Have adequate pre-operational and subsequent periodic test requirements been appropriately specified?

17. Are adequate handling, storage, cleaning and shipping requirements specified?

18. Are adequate identification requirements specified?

19. Are requirements for record preparation review, approval, retention, etc., adequately specified?

4.2.2.2 Verification By Alternate Calculations

Verification of some types of calculations or analyses may be achieved by comparison with alternate methods of

calculation or analyses. This shall be performed by a person or persons other than those who performed the original calculation. Where alternate calculations are performed to verify the correctness of the original calculation, a review shall be performed to address the appropriateness of assumptions, input data, and the code or other calculation method used.

The alternate method used for comparison may be a more simplified approach or less rigorous, such as when a hand calculation is used to check the computer code output. Although the simplified or less rigorous method may not exactly check the original calculation or analysis, it must provide results consistent with the original calculation or analyses.

4.2.2.3 Verification By Qualification Testing

Design verification for some designs or specific design features can be achieved by suitable qualification testing of a prototype or initial production unit.

In those cases where the adequacy of a design is to be verified by a qualification test, the testing shall be identified and documented. Testing shall demonstrate adequacy of performance under the most adverse design conditions. All pertinent operating modes shall be considered in determining these design conditions where it is intended that the test program confirm the adequacy of the overall design. Where the test is only intended to verify a specific design feature, the other features of the design shall be verified by other means. For example, it may be most effective to verify that an instrumentation cabinet is designed to withstand the maximum earthquake-caused vibratory motions by actually subjecting the cabinet and its associated components to shaker tests which correspond to these vibratory motions. The shaker tests will not, however, verify that the circuitry is designed correctly or that the component in the cabinet will perform its intended function. Other tests or verification means are required to confirm that remaining design functions are adequately performed by the instrumentation and that those components perform the intended functions for the varying design conditions to which they are subjected.

Qualification testing shall be performed in accordance with written test procedures which incorporate or reference the

requirements and acceptance limits contained in applicable design documents. The test procedures shall include provisions for assuring that prerequisites for the given test have been met, that adequate instrumentation of that required range and accuracy is available and used, and that necessary monitoring is performed. Prerequisites include such items as calibrated instrumentation, appropriate equipment, trained personnel, condition of test equipment and the item to be tested, suitable environmental conditions and provisions for data acquisition. Test results shall be documented and evaluated by the responsible designer to assure that test requirements have been satisfied.

If testing indicates that modifications to the item are necessary to obtain acceptable performance, the modification shall be documented and the item modified and retested or otherwise verified to assure satisfactory performance. When tests are being performed on models or mock-ups, scaling laws shall be established and verified. The test configuration shall be clearly defined and documented. The results of model test work shall be subject to error analysis, where applicable, prior to use in final design work.

4.2.3 CURRENT SEISMIC DESIGN VERIFICATION PROCEDURES FOR STRUCTURES

The qualification test procedure described above requires that testing shall demonstrate adequacy of performance under the most adverse design conditions, which would require seismic design verification of nuclear power plant structures at both the Operating Basis Earthquake (OBE) and Safe Shutdown Earthquake (SSE) levels. The latter would require testing under very intensive and, in some cases, almost destructive ground-shaking environments. Since nuclear power plant structures are massive and complex, and their construction must be integrated with the installation of equally complex piping and equipment component systems, the qualification testing of complete nuclear power plant structures is not currently used: First, there are no practical

means of producing either the OBE or SSE ground-shaking environments for such massive structures without damaging other structures; second, the plant could not be placed in service after testing without extensive refurbishing, because of potential damage to Category II and III items (i.e., Category II items are required to survive the OBE but not the SSE environments). Qualification testing at the most adverse conditions is therefore not feasible. As a result, seismic design verification of nuclear power plant structures is currently provided by using both design reviews and alternate calculations supported by numerous quality control tests of materials, but not by qualification testing of the completed structure.

4.2.4 ASSESSMENT OF CURRENT SEISMIC DESIGN VERIFICATION PROCEDURES FOR STRUCTURES

There is an important difference in verification based on design reviews and alternate calculations, and in verification based on qualification testing, keeping in mind that quality control tests of materials are common to both procedures. The former provides a verification of the *design* as defined in ANSI N45.2.11, which is the "technical and management processes which commence with identification of design input and which leads to and includes the issuance of design output documents." This procedure is, therefore, actually a verification of the correctness of documents. In contrast, the latter would demonstrate the adequacy of a complete prototype structural system in the most adverse seismic design environment. In order to evaluate current seismic design verification procedures, it will be assumed at this point that qualification testing of nuclear power plant structures is a sizable alternative verification procedure. Based on this assumption, two questions need to be considered: Is one procedure more reliable

than the other? and If so, what does the more reliable procedure provide that is not provided by the other?

Qualification testing provides proof through testing that a completely integrated structural system is, or is not, adequate when subjected to the most adverse seismic design environment. This is true regardless of whether the design procedures are correct or incorrect. For example, the design procedures could be grossly incorrect, yet a very conservative structure could result that would be more than adequate for the seismic design environment considered. The test in this case would verify the adequacy of the structural system, but it would not verify the correctness of the design documentation, nor the correctness of the design assumptions.* However, if special tests are adequately planned and conducted, the results could be used to verify the adequacy of both the design assumptions and the completed structural system. Therefore, qualification testing, if properly planned and conducted, could provide additional important information that may not be provided by the design review and alternate calculation procedure which would be a verification of design assumptions.

It follows that only when the design assumptions are known to be completely correct, can it be assumed that the design review and alternate calculation procedure will lead to a complete seismic design verification for nuclear power plant structures. When this is not known to be true, the qualification test procedure becomes the more reliable procedure, if it

can be provided. This leads to two additional questions on current seismic design verification procedures: What is the confidence level in the assumptions used in current seismic design procedures, i.e., are there design assumptions that need to be verified? and If there are, can this be accomplished by other procedures (such as testing in a less severe seismic environment than the OBE and SSE) than by qualification testing?

4.2.4.1 Confidence Level In Seismic Design Assumptions

Confidence level is used here in a qualitative sense. On this basis, the confidence level in the assumptions used in the seismic design of a nuclear power plant structure is low, for example, when compared to the confidence level in the assumptions used to design a cylindrical steel vessel for an internal pressure, for several reasons. Basically, the pressure vessel loading is simple and well understood, the structural configuration is simple and easy to analyze, the material behavior can be easily defined, and the engineering profession has a long experience record of successfully designing and testing such items under service load and overload conditions. In contrast, the structural configuration of a nuclear power plant and its seismic loading are both complex, and the material behavior under strong seismic loading may not be well represented in the analytical models. Also, there have been no planned tests of such structures in OBE and SSE environments.

The above comparison is somewhat unfair since a pressure vessel is usually only one item in a much more complex mechanical system, and only one load has been considered. When the confidence level in the assumptions for the seismic design of the nuclear power plant structure is compared to the confidence level in the assumptions used for the design of a complete

* *Design assumption* is used here as an inclusive term that includes all assumptions pertaining to analysis, design, construction, and operation that could affect the integrity of the completed structure in a seismic environment.

mechanical system for all loads, including seismic, the two levels of confidence will be found to be more comparable. This is particularly true for the seismic design since the input to the mechanical system is filtered through the structure. Therefore, it is evident that either a complete system test should be provided, or the uncertainties in the design assumptions for the overall system should be isolated and special verification procedures developed. Uncertainties in the seismic design assumptions for nuclear power plant structures pertain to seismic input, material properties, dynamic analyses, and design details.

Seismic Input. Seismic engineering is a developing technology and it is generally recognized that there is limited information available on which to base predictions of the level of vibratory ground motion that should be assigned to the OBE and SSE for most nuclear power plant sites. This uncertainty is considered by requiring the use of conservative criteria. At the present time, the basic seismic input can be verified only by the design review which makes certain that conservative regulatory requirements have been correctly interpreted and applied. Once established and verified, it becomes basic input to both the seismic design and the seismic design verification. In other words, the seismic environment defined for the design is the same seismic environment that must be produced if qualification testing is provided. Uncertainties in assumptions relative to the definition of the criteria ground motions, therefore, do not impact the determination of the adequacy of the seismic design verification procedure. However, this applies only to the basic site input and does not apply to free-field calculations based on the seismic input criteria.

Material Properties. A great number of field and laboratory tests are required to provide design information on the physical

properties of the soil and rock supporting the nuclear power plant structures. Laboratory tests and quality control tests are also required to determine and control the physical properties of the materials incorporated into the structure. Information on the unit weights as well as on the strength and stress/strain properties under both static and dynamic loads are required for both the subsurface and structural materials. The testing procedures are well defined, and reasonable estimates of the properties of the test samples can be made. However, there is some uncertainty in converting test sample properties to in situ soil and rock properties which are needed as basic input to the mathematical models used to provide seismic analyses of the free field and of the soil/structure system. The uncertainty in converting sample properties of concrete to in-place concrete structural properties is considerably less than for the soil and rock, but greater uncertainty exists for concrete than for steel.

The uncertainty in converting soil and rock sample properties to in situ properties results primarily from three factors. First, in many soils, considerable sample disturbance results during the drilling and sample extraction process which may alter the material properties. Second, tests of small core samples may not properly represent the mass, or in situ properties. Core samples in rock, for example, may not have an adequate representation of joints, weathered zones, etc. Also the in situ, state of stress, which should be duplicated in testing of the samples, may not be known. Third, soils and some rock have nonlinear stress/strain properties that are nearly always strain dependent. These properties are difficult to determine experimentally, and even more difficult to approximate by an equivalent elastic assumption, which may be used in free-field and soil/structure interaction mathematical model analyses.

Procedures used to determine the soil and rock properties are discussed in several documents (e.g., SW/AA, 1972). Uncertainties in design are usually handled by assuming that critical properties, such as the shear moduli, may vary by as much as $\pm 50\%$. Parametric analyses are made, therefore, using a range in material properties. The density of soil or rock can be closely estimated (within 2 to 3%) and presents no problem. Experimental verification of the in situ properties using large or full-scale tests, if physically possible, would of course reduce the uncertainty in the assigned material properties. The uncertainty associated with the assumptions used to express the nonlinear soil and rock behavior as equivalent linear behavior in mathematical models for dynamic analyses is a different problem.

Modulus, density, and stress/strain properties of steel and concrete construction materials can, as noted above, be determined with greater certainty than soil and rock properties as these materials can be produced under controlled conditions. Variations in concrete modulus may have to be assumed in analyses to test the effect of modest variations in this property, but this is not required with steel. The nonlinear stress/strain characteristic of concrete, and the yield behavior of steel are well understood and can be defined by control tests. Uncertainties do not arise in this area until attempts are made to model this behavior for elements of the structure for the dynamic analysis.

Dynamic Analyses. It has been indicated that the uncertainties in defining the seismic input are currently considered by using conservative criteria, and that basic material properties can be reasonably well defined by field and laboratory testing, with the uncertainties in properties being treated conservatively by making parametric analyses that consider the

predicted range in material properties. However, assumptions of material behavior in the formulation of mathematical models for dynamic analysis also introduce uncertainties. The analytical procedures consist of both free-field and structure analyses.

Free-Field Analyses. Acceptable procedures for defining vibratory ground motion are described in the Standard Review Plan (SRP) of the U.S. Nuclear Regulatory Commission (USNRC, 1975b). Section 2.5.2 of this document requires that the wave transmission characteristics of the site materials be described as a function of the significant frequencies, and that the type of seismic waves producing the maximum ground motion be determined. This section indicates that the SHAKE computer program (Schnabel et al., 1972) can be used for this frequency-dependent analysis when the maximum ground motion results from vertically propagating shear waves. It also indicates that the shear wave model is currently accepted by the NRC for site conditions where the maximum ground motion may result from compressional and surface waves, although reference is made to the work of Trifunac and Udawadia (1974) and Drake (1972) for these conditions.

Section 3.7.1 of the SRP describes acceptable procedures for defining design response spectra and time histories to be used as input in seismic analyses of nuclear power plant structures. This section indicates that the design response spectra should be defined for the free field or rock conditions (i.e., with no structures present) and that it should ordinarily be considered applicable at the finished grade level (ground surface) of the site. If the proposed structures are to be deeply embedded, then the free-field ground motion at the elevation of the base of the structures will be different than that defined for the ground surface. Acceptable procedures for computing the ground motion input at the base level of the

proposed structures are also set forth in Section 3.7.1 which requires time histories to be input *only* at the base of the model. A method approved by the SRP for obtaining these time histories is through deconvolution analysis using the SHAKE and LUSH (Lysmer et al., 1974) computer codes. Even though other equivalent computer codes and analysis techniques are acceptable, the combined application of the SHAKE/LUSH computer codes for computing the base input and the soil/structure response have become standard procedure, even for deeply embedded structures in rock. This has apparently (and unintentionally) resulted from the emphasis given in the SRP to only these two codes and to the requirement for only base input which is a requirement unique to these two codes. Because of the nearly universal use of the SHAKE code to define the ground motion at the base level of the proposed structures, a review of important assumptions in this modeling procedure is essential. To avoid confusion, it should be noted that the deconvolution routine in the LUSH and FLUSH (Lysmer, 1975) codes (which are used for the soil/structure interaction models) is comparable to the SHAKE code.

In order to demonstrate the uncertainties associated with the assumptions currently used in free-field analyses, Figure 4-1 provides ground surface (El 815) and foundation level (El 721) response for a soil site based on a 1-D SHAKE code analysis and a 2-D TRI/SAC code (AA, 1976) analysis. If the applicant uses the 1-D code for predicting the seismic motion at the foundation level, an attenuation of 70% will be indicated at a frequency of 2 Hz and present NRC procedures would permit a reduction of up to 40% for this case. However, if the 2-D code is used to make the prediction, no attenuation would be indicated, or permitted, at 2 Hz. Since the material property assumptions are approximately the same in both models, the

discrepancy is probably due to the overly simplified characteristics of the 1-D model. Unfortunately, there is no experimental verification for either the 1-D or 2-D model, and neither model can be verified by design reviews and conventional alternate calculations.

Three alternatives are available. First, no attenuation in ground motion with depth might be assumed for the site, which could be a very conservative assumption. Second, expensive calculations could be made using 3-D elastic and 2-D nonlinear mathematical models. However, significant questions might still be raised concerning the adequacy of the modeling assumptions used, in the absence of any experimental verification of the results. The third alternative would be to provide experimental verification of the computed attenuation in seismic motion with depth, assuming that this is possible. This would be desirable regardless of whether 1-D or 2-D mathematical free-field models are used. Therefore, it must be concluded that there are modeling assumptions used in free-field analyses in which the confidence level would be greatly enhanced, if experimental verification could be provided.

Structure Analyses. Dynamic seismic analysis of Category I nuclear power plant structures, systems, and components are usually based on several mathematical models. The primary model treats the mass, stiffness, and damping characteristics of the structures, including the soil/structure interaction effects. Masses of large equipment items that cannot be justifiably uncoupled from the structure are also included. For complex structures, secondary models may be required to provide more detailed structure response. Separate mathematical models are also required to predict the dynamic response of equipment and piping. Free-field earthquake ground motions are used as dynamic input to the primary soil/structure interaction

FIGURE 4-1. COMPARISON OF COMPUTED FREE-FIELD SPECTRA FOR SOIL SITE
USING SHAKE AND TRI/SAC COMPUTER CODES

models and are developed by the procedures discussed above. The floor response computed from the primary model is then used as dynamic input to the mathematical models of equipment and piping. The soil/structure interaction analysis performed with the primary model is, therefore, one of the fundamental steps in the overall seismic dynamic analysis.

Soil/structure interaction analysis procedures fall in two broad categories, depending upon the type of model used. One category uses discrete mass models of the structures which are supported by a series of linear elastic springs that introduce the equivalent stiffness of the supporting soil, or rock. Radiant energy losses and the hysteretic damping characteristics of the soil are approximated by equivalent viscous dampers.

The second general model category uses a system of discrete structural and soil continuum elements interconnected at their node points to represent the soil/structure system, and is referred to as a finite element model. Although the present analytical state of the art includes 3-D finite element capability, few if any authentic 3-D dynamic finite element soil/structure interaction analyses have been provided for nuclear power plant structures. This limitation, similar to the limitation in free-field analyses, has apparently been imposed by computer costs, and for complex structures, by computer program capacity. Therefore, the present state of the art consists of either 3-D discrete mass, or 2-D finite element modeling procedures. Each type of model has certain advantages and disadvantages.

The advantages of the discrete mass model when compared to the 2-D finite element model are that the former can treat 3-D response and the analysis costs are less in computer-time charges. Generally, models for both procedures require about the same man-hours to

prepare. The primary disadvantages of the discrete mass models are nearly all related to the simplified manner in which soil/structure interaction is treated. In order to select the soil springs and damping elements, the structure is assumed to have a rigid base, the foundation shape is assumed to be rectangular or circular, and the depth of embedment of the structure is usually neglected. Based on these assumptions, and assuming that the soil behaves elastically, soil spring constants are conveniently computed from equations based on static loads and elastic half-space theory. These equations were initially developed to treat foundation vibration problems induced by heavy rotation, or reciprocating, equipment (i.e., steady-state vibrations of equipment supported by relatively rigid foundations). Actually, nuclear power plant structures are not rigid, the foundation shapes are usually irregular, the complex may consist of several closely spaced structures, the structures may be deeply embedded, and the supporting soil behavior may be nonlinear. Also, the soil spring constants are frequency dependent which is usually neglected. Radiation dampers* are selected from equations also based on elastic half-space theory and are frequency dependent (which is usually neglected), and there may be interaction between two or more structures, which is also neglected. Thus, the greatest uncertainties associated with this type of model are in the interaction representation assumptions.

The advantages of a finite element model over a discrete mass model result primarily from the fact that the finite element model provides

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*Loss of energy from the model due to reflected stress waves that radiate away from the base of the structure is modeled through the use of viscous dampers. For embedded structures, radiant damping may reach 60 to 120% of critical damping.

an excellent distribution of the mass and stiffness of the structure and supporting rock or soil. For this reason, this model can more realistically treat the flexibility of the structure, the effects of embedment, layered supporting media, and the presence of adjacent structures. A proper finite element model automatically introduces radiant energy losses, and can respond in an almost unlimited number of (planar) modes in contrast to the limited number of interaction modes possible with discrete mass models. Nonlinear behavior also can be considered with nonlinear finite element models, but few analyses have been provided for nuclear power plant structures. Also, it should be noted that the FLUSH code is nearly universally used as the finite element model. However, this code has all of the problems noted above for the SHAKE code. For example, the horizontal and vertical components of motion are uncoupled; simple, vertically propagating 1-D wave motion is used as input; and energy cannot radiate away from the structure nor out of the model.

It should be apparent that some of the assumptions associated with the soil/structure interaction representations for discrete mass models (also with the FLUSH code models) could be verified by alternate analyses using 2-D nonlinear and 3-D linear finite element models. However, as noted in the discussion of free-field analyses, in the absence of experimental verification some uncertainty would still exist.

It should also be noted that within a typical engineering organization there frequently exists only one opinion on how best to analyze certain difficult problems, such as soil/structure interaction, which may inhibit its members from viewing a problem from a different perspective. In the absence of experimental data, an entire industry, or a significant school of thought within the industry, can be affected. This condition may well exist today

in the nuclear power plant industry relative to methods of providing soil/structure interaction analyses, and methods of verification. There is therefore a strong need for some form of experimental verification of dynamic seismic free field and soil/structure interaction analyses to advance the confidence level in the analysis assumptions.

Because structure models currently used simplify the true structure response, good engineering judgment must be exercised to make certain that the simplified response is a conservative estimate of the true response. However, engineering judgment and intuition do not provide design verification. There is also the problem that simplified conservative analyses may result in unnecessarily expensive designs. Damping in structural materials, for example, is difficult to estimate, and, as a result, is invariably conservatively underestimated in the analysis assumptions. Therefore, there is no question that verification of the design assumptions associated with the dynamic analyses of nuclear power plant structures by some form of experimental testing would greatly enhance the confidence level in the assumptions, and perhaps lead to less conservative design.

Design Details. In nearly every strong earthquake in which severe damage has resulted to engineering structures, some of the damage has invariably been traced to faulty design details. Design codes are, therefore, altered after each major earthquake to provide more stringent detailing requirements. Qualification tests of military structures subjected to blast and ground shock effects, and qualification testing of military equipment have also frequently revealed omitted details in the design, or overlooked modes of response, that are strikingly evident from the test results. Because of constant improvements in the design codes, and because of the special care that is exercised in the design of nuclear power plant structures,

the likelihood of critical omissions in predicted nuclear power plant structural behavior is significantly reduced, but in the absence of some form of experimental verification of response, it is not possible to assure that such omissions have been completely eliminated. Some form of experimental verification of response is therefore justified, solely for the purpose of detecting unforeseen modes of response and errors in design details.

4.2.4.2 Verification of Design Assumptions

Design verification of nuclear power plant structures is currently provided only by design reviews and alternate calculations. Qualification testing is not used because of the expense and the possible damage to other nearby structures, and because of the possible damage to Category II and III items of the nuclear power plant. It has been shown that design reviews and alternate analyses do not provide as reliable a verification procedure as qualification testing (assuming it could be provided), unless the design assumptions are known to be correct (i.e., have been verified). Design assumptions need additional verification in order to enhance the confidence in the design review-alternate calculation design verification procedure. In some cases this could be provided by more sophisticated analyses, but it is evident that some form of test verification is also needed to enhance the confidence level in the design assumptions.

To illustrate that test verification of design assumptions is not a simple exercise, assume that the tests will be performed on a prototype structure. Two characteristics of the tests are immediately evident. First, because of the damage that might otherwise result to Category II and III items, and also because of the practical limits on the energy that can be introduced in the input, the seismic test environment will need to be significantly below the

OBE and SSE environments. Second, the dynamic response of a nuclear power plant structure (including the soil) may be nearly linearly elastic at the test level, but will usually become nonlinear in the stronger SSE environment. Therefore, extrapolation of response into the nonlinear range from the measured response in the linear elastic range will be difficult.

There are certain benefits that would result from lower level testing. For example, unique modes of structural response and some errors in design details should be detectable. However, it will be difficult to verify assumptions associated with free field and soil/structure interaction analyses, and associated with the damping assumptions assigned to the soil and structural materials. To some extent, this will also apply to the stress-strain behavior of the structural materials in the regions where yield behavior is approached in the SSE environment. Proper verification of design assumptions may, therefore, require both experimental tests and more sophisticated analyses.

4.3 COMPARISON OF DYNAMIC TESTING METHODS

4.3.1 DYNAMIC TEST METHODS

Methods of dynamic testing of large structures as presented by Hudson (1970, 1976), include free vibration, forced vibration, and vibration table tests with the following characteristics:

Free Vibration Tests

- Initial Displacement -- pull-back and quick release

- Initial Velocity -- impacts, rockets

Forced Vibration Tests

Transient Excitations

- Wind
- Microtremors
- Explosions
- Natural Earthquakes

Steady-State Resonance Excitation

Mechanical Oscillators
Man-excited

Variable Frequency Excitation

Sweep Frequency -- rundown
Pulse Sequences

Vibration Table Tests

Dynamic testing of structures requires excitation, or loading, instrumentation to measure response, and data processing to aid in the interpretation of the results. Since the primary concern here is in the feasibility of different methods of loading, instrumentation and data processing will not be discussed. However, these are important aspects of any test program and must be carefully considered when planning the tests. Also, the discussion here applies largely to the testing of nuclear power plant structures.

4.3.1.1 Free Vibration Tests

Initial displacement tests, which are also known as snap-back, pull-back, or "twang" tests, are provided by giving the structure an initial displacement by means of an applied force which is suddenly released allowing the structure to vibrate freely. The free, transient vibration of the structure is then recorded and analyzed to obtain frequencies, mode shapes, and damping values. One of the difficulties associated with the use of this method for testing nuclear power plant structures is that the very large forces needed to provide adequate displacements require strong anchor points as well as points on the structure that can withstand large loads. Also, as a practical matter, the most that can be achieved with this method is excitation of the horizontal modes by applying loads to the above ground more flexible portions of the structure. Such tests have been performed on water tanks (Carder, 1936) and steel stacks (Cloud, 1963). Even for simple structures the results are not always

entirely satisfactory. If the loads are not carefully applied, or if the structures are not symmetrical, response occurs about both principal axes, which produces records that exhibit beating and make the data hard to interpret. Clearly, simple vertical cantilever type structures are more amenable to this method of testing than a nuclear power plant because of their greater flexibility and fewer significant modes.

Another method of exciting test structures is to provide an impulse or initial velocity. This can be accomplished by means of rockets, explosive cartridges, pendulums, or falling weights, whose time of load application is short compared to the period of the structure. Rockets appear to be the most promising loading concept and have been used to test a concrete chimney (Hudson, 1970). However, since only a short pulse load is applied, the method is again much more applicable to simple vertical cantilever structures than to massive complex nuclear power plant structures. A more sophisticated extension of this method will be discussed later.

4.3.1.2 Forced Vibration Tests

Transient Excitation. Two general types of transient excitations used to excite structure response for dynamic tests are: relatively low level ambient vibrations that result from wind, operating machinery, street traffic and microseismic activity; and stronger input resulting from underground explosions and from natural earthquakes.

Ambient vibrations from wind, operating machinery, street traffic, and microseismic activity have been used for more than four decades to measure the dynamic response of structures. The U.S. Coast and Geodetic Survey (USCGS, 1936) was one of the first to use the measured wind induced response of

buildings to calculate building frequencies. In this method, highly sensitive seismometers are placed at various points on the structure, depending upon the mode shapes of interest. The response measured by the seismometers are amplified and recorded on magnetic tape. The analog data is then digitized, filtered, and processed using random vibration theory and spectral analyses to obtain estimates of the modal frequencies, vectors, and damping.

Advancements in instrumentation and data processing during the past two decades have refined the method to where it is now possible to obtain a relatively complete set of data including higher modal frequencies, vectors, and damping values (see Crawford and Ward, 1964; Ward and Crawford, 1966; and Cherry and Brady, 1965). The method has been used to determine dynamic characteristics of a variety of structures including high rise buildings (Trifunac, 1970), suspension bridges (McLamore et al., 1971), the concrete intake tower of a dam (Keightley et al., 1961), and a nuclear power plant (ES, 1968).

The responses of structures to ground motions induced by underground explosions have also been measured in sufficient tests to indicate that this is an acceptable loading procedure. Both underground chemical and nuclear explosions have been used. This is one of the few procedures in which the structure is excited by base motion due to stress waves propagating through the soil and rock. The response, therefore, includes soil/structure interaction effects. The method has great promise but has two strong disadvantages. First, it is quite expensive, and second, because of other cultural developments that may be near many nuclear power plant sites, the liability for damage to these developments during planned tests could be quite high.

An early example of using the ground motion generated by an underground explosion as a dynamic test environment for a structure was reported by Hudson et al. (1954). In this example 370,000 lb of explosives was detonated in a rock quarry approximately 400 yd from the structure. The test was repeated six years later in conjunction with a second blast in which 1,347,000 lb of explosives were detonated with two time delays of 17 msec each. Studies performed by the U.S. Coast and Geodetic Survey (Carder and Cloud, 1959) have established relationships between charge size, distance from point of detonation, and peak ground acceleration. Underground military structures have also been tested with chemical explosives. Test procedures for military structures are summarized in a recent Corps of Engineers guideline manual on Hardness Verification (COE, 1977).

The ground motion produced by underground nuclear explosions have also been used as a means of excitation of various structures (AEWES, 1972; Medearis, 1975). For example, responses of a microwave tower, an athletic gymnasium, and an earthdam to the detonation of three 30 kt nuclear devices located from 4 to 40 mi from the facilities and more than 1 mi below ground were measured. The ground motion resulting from an underground nuclear explosion more nearly resembles that of an earthquake than any other type of input, because the depth of the detonation is deep and the distance to the point of detonation is great enough to permit the stress waves to propagate by multiple paths. This produces an input having a broad band response spectrum similar to an earthquake. However, because of the restrictions on underground nuclear testing, and the broad land area affected by the detonation, tests of this type can be carried out only in limited areas and under limited conditions.

In Los Angeles and various other California cities, high-rise buildings have been instrumented with strong motion recorders at the base and at two upper levels. As a result, the response of some of the structures to strong earthquake ground motion has been recorded (NOAA, 1973) under the most realistic test conditions possible. However, long periods of waiting are required to collect response information with this type of testing, and the instrumentation usually is not adequate to permit optimum interpretation of results. Nevertheless, on a long term basis it represents a test procedure for obtaining reliable information, if the instrumentation is adequately planned.

Steady-State Resonance Excitation. Steady-state resonance excitation of test structures using a sinusoidally varying, unidirectional force is probably the most common type of dynamic testing of structures currently used. In this test, the frequency of the applied force is first held constant at one value while measurements of the strong response are recorded. The frequency of the applied force is then changed, and the response measurements repeated. The end product of such a test is a set of frequency response curves for various points on the structure from which natural frequencies, mode shapes, and damping can be extracted. Nonlinearities in the test system can also be investigated by varying the amplitude of the applied load at the different test frequencies and noting the change in resonant frequency.

Acceleration of mass is the method normally used to generate the forces applied to the structure with this method of testing, as there is usually no adequate means of reacting the large mechanical forces necessary to drive a massive structure. Among the devices developed for this purpose are ones that utilize

rotating eccentric masses, rectilinear reciprocating masses, and electromagnetic drives (Hudson, 1976). The type that has been used most often employs two counterrotating eccentric masses of equal moment, arranged so that the reaction force on the structure is rectilinear. Devices employing this principle have been used for many years. For a detailed description of a modern force generator, see Hudson (1962).

Hudson (1976) also denotes tests in which steady-state response has been excited by a man moving his center of mass at the fundamental frequency of the structure. His example indicates that a 150 lb man moving his body with a double amplitude of 6 in. at a frequency of 1 Hz will exert an inertia force of 46 lb. When compared to the effective forces listed in Table 1, it is apparent that the excitation level is too low to be of interest in testing nuclear power plant structures.

Variable Frequency Excitation. Variable frequency excitation is common in the testing of equipment with low damping. Here there is great danger that testing at a resonant frequency will quickly lead to equipment damage because of the force buildup. Thus, the frequency of excitation is varied gradually through a range of frequencies to identify beat frequencies from which the mode frequencies can be extracted. This can also be accomplished by operating the oscillator at a high speed and then cutting all power, allowing it to coast or "run down" through the mode frequencies of the test item. While this method of excitation simplifies the speed control and instrumentation problems, the energy input is too low to represent a viable test procedure for nuclear power plant structures.

An innovative system of loading a structure with a series of pulses geometrically located and systematically timed to cause the structure to respond in a specified manner has

been proposed by Safford and Masri (Masri and Safford, 1976, and Safford et al., 1977). In this procedure, pulses consisting of cold gas, hot gas, or solid fuel propellants, depending on the impulse needed, are placed at optimum points on the structure. Feasibility studies have been conducted to determine the size, quantity, and location of pulses required for a 25-story building. An optimization algorithm operates with the building system functions (impedance or transfer function) to compute the pulse train required to produce the specified response. Pulses are varied in amplitude, duration, and initiation time. This system could be used to determine mode response characteristics, or to cause the structure to develop a specified response. This system has great potential and is in the process of being developed.

4.3.1.3 Vibration Table Tests

At the present time, a limited number of shake tables are available to test scale models of relatively simple structures. However, shake table as well as field deployed model tests do not appear to be an attractive means of providing experimental verification of the dynamic response of nuclear power plant structures. For example, the large shake tables could not accommodate even a one-tenth scale model of a structural complex such as the Clinch River Breeder Reactor Plant. Model construction is also expensive, and scaling problems become very difficult, if not impossible, to treat. Soils are nonlinear, and the properties of cohesionless materials, for example, are a function of the depth of the overlying soil. Dynamic similarity would have to be maintained (the relative frequency of the structure and input wave motions would have to be scaled) as well as the site and structure geometry, and the soil properties. This is beyond present-day

modeling techniques and is not therefore recommended.

4.3.2 COMPARISON OF TEST RESULTS

Only four of the methods described to dynamically load structures justify additional discussion relative to their applicability to testing of nuclear power plant structures: loads induced by ambient transient vibrations and steady-state mechanical oscillators, and loads induced by explosions and natural earthquakes. The first two represent relatively low energy level loads while the last two represent loads that approximate those developed in an OBE or SSE environment.

Examples of test results for each method and comparison of these results are provided here. An important consideration in evaluating test procedures is the ability to verify test results by either theoretical calculations or by comparison with results from other test procedures. Fortunately, there has been sufficient duplication in analytical studies and test methods that a meaningful comparison of test results can be provided.

4.3.2.1 Comparison of Ambient Transient Test Results with Theoretical Analyses

There are numerous examples in the technical literature of the use of ambient transient wind loads as the source of excitation of buildings in tests designed to determine the fundamental periods, modal vectors, and damping values. The representative examples cited here also have published values for the computed theoretical fundamental periods.

Table 4-1 provides a comparison of fundamental periods determined by theoretical calculations and ambient wind vibration measurements for five buildings. The first four buildings, in Ottawa and Montreal, vary from 10 to 44 stories in height, and both steel and reinforced

TABLE 4-1. COMPARISON OF FUNDAMENTAL PERIODS OF BUILDINGS DETERMINED FROM THEORETICAL COMPUTATIONS AND FROM AMBIENT TRANSIENT WIND LOADS

Building	Description	Direction	Fundamental Period, sec		
			Theoretical	Ambient Measured	Ratio T/A
Health and Welfare Building Ottawa, Canada ¹	17 stories, 140 ft x 88 ft plan, 235 ft high, reinforced concrete core with steel frame	Long	1.4	1.3	1.1
		Transverse	1.4	1.0	1.4
		Torsional	-	0.9	-
Imperial Bank Montreal, Canada ²	44 stories, 140 ft x 100 ft plan, 603 ft high, steel frame	Long	3.3	4.7	0.7
		Transverse	3.9	4.7	0.8
		Torsional	-	3.9	-
CIL House Montreal, Canada ²	34 stories, 168 ft x 112 ft plan 430 ft high, steel frame	Long	3.0	3.9	0.8
		Transverse	2.6	4.5	0.6
		Torsional	-	3.4	-
Post Office Ottawa, Canada ²	10 stories, 266 ft x 74 ft plan, 148 ft high, reinforced concrete	Long	0.9	0.6	1.5
		Transverse	0.8	0.7	1.1
		Torsional	-	0.3	-
Holiday Inn 8244 Orion St Los Angeles, CA ³	7 stories, 160 ft x 62 ft plan 86 ft high (est)	Long	0.8	0.5	1.6
		Transverse	0.9	0.5	1.8
		Torsional	-	-	1.7

¹Crawford and Ward, 1964

²Ward and Crawford, 1966

³NOAA, 1973

concrete types of construction are represented. The fifth structure is a reinforced concrete seven-story motel located in the San Fernando Valley of California that is of interest for later comparisons because it was subjected to strong earthquake ground motion during the 1971 earthquake. The analytical models used to compute the fundamental periods in most cases were crude compared to present-day capability. One-dimensional discrete mass models were used, and the torsional modes were not computed. Soil/structure interaction effects, which should have little influence on ambient transient wind vibration results, were also neglected in the calculations.

Two points should be noted from the data in Table 4-1, first the errors in the computed periods vary both above and below the measured periods by an average of 35%, with the greatest errors of 60 and 70% occurring for the Holiday Inn building, which is the most recent

structure. Other comparisons will confirm that differences between computed and measured periods of buildings will frequently reach 50 to 100%, and the average value of 35% deviation is a typical mean value. The second point is the close spacing of the torsional and flexural mode periods for all buildings where the torsional mode period was measured. When the modes are closely spaced, coupling between the torsional and flexural modes should be anticipated, a type of behavior that has been measured on other structures. It implies that the torsional mode should not be overlooked in design.

Damping values were also estimated for three of the Canadian buildings (Ward and Crawford, 1966), which will be used later. By power spectral analyses, damping of from 1.0 to 1.8% was measured for the fundamental modes of response for the three buildings. Values of 1.6 and 1.8 were measured for the respective fundamental modes of the two buildings by auto-correlation analysis.

4.3.2.2 Comparison of Ambient Transient and Steady-State Vibration Test Results

Two multistory buildings that have been subjected to both ambient transient and steady-state vibratory tests are: the Millikan Library (Jennings et al., 1972; Jennings and Kuroiwa, 1968) and the San Diego Gas and Electric building (Trifunac, 1972). For both buildings the fundamental periods determined by ambient transient and steady-state vibratory tests compare very well. The mode shapes derived by the two methods for the San Diego Gas and Electric building show excellent agreement. Damping values for the Millikan Library are in close agreement for both test methods. However, for the San Diego Gas and Electric building the damping values from the ambient transient test are not considered reliable.

Comparative ambient transient and steady-state vibratory tests were also conducted for the Carolinas-Virginia Tube Reactor (CVTR) building. The ambient transient vibratory measurements were performed by Earth Sciences (ES, 1968) and the steady-state vibratory tests were performed by UCLA (Matthiesen and Smith, 1969). An evaluation and comparison of the results of the two studies was prepared by the Idaho Nuclear Corporation (Schmitt, 1970). A comparison of the fundamental frequencies of the CVTR building showed reasonable agreement between the two test methods. The steady-state vibratory tests indicated high damping, whereas the ambient transient tests indicated lower damping values.

It should be noted that ambient transient vibration tests are usually conducted at a level of about two orders of magnitude below the steady-state vibratory tests. As a consequence, the nonlinear effects are generally manifested only in the steady-state vibratory tests.

4.3.2.3 Comparison of Testing with Buried Explosives and Steady-State Vibratory Test Results

Two interesting examples of a ground motion environment induced by underground chemical explosives have been reported by Hudson et al. (1954, 1961). In both cases a large rock quarry blast was used to obtain ground motion and structural response measurements.

In the 1952 test, 370,000 lb of explosives (Nitramon) were detonated at a distance of about 1100 ft from the base of a steel mill building. In the 1958 test, 1,347,000 lb of explosives (Nitramite and Nilite) were detonated at a distance of about 1200 to 1300 ft from the building. The 1952 test had a maximum recorded horizontal acceleration of 0.10 g and a maximum vertical acceleration of 0.12 g, while the 1958 test produced maximum horizontal and vertical accelerations of 0.23 g at the mill building. A comparison of the response spectra of the horizontal motion with Regulatory Guide 1.60 (USAEC, 1973) spectra indicated that underground detonations of 200 to 500 tons of explosives can produce a strong motion environment at distances of 1000 to 2000 ft that are quite similar to that of a magnitude 5 to 6 earthquake (i.e., comparable to the Golden Gate Park 1957 record, $M = 5.7$).

A comparison can be made of steady-state vibratory test results with the results of relatively low yield explosives from tests that have been performed on the Experimental Gas Cooled Reactor (EGCR) located at Oak Ridge, Tenn. In the steady-state tests, two mechanical oscillators were placed on the fourth floor, and the response was measured at several points to obtain mode shapes and frequencies (Smith and Matthiesen, 1969). Table 4-2 shows a comparison of frequencies from the explosion tests. Computer theoretical frequencies from the initial

TABLE 4-2. COMPARISON OF EGCR CONTAINMENT STRUCTURE FUNDAMENTAL FREQUENCIES DETERMINED FROM STEADY-STATE VIBRATIONS, AND UNDERGROUND EXPLOSIONS WITH THEORETICAL VALUES

Direction	Fundamental Frequencies, Hz		
	Theoretical	Steady State	Explosion
EW	6 to 12	4.65	4.4 to 4.5
NS	6 to 12	4.2	3.8 to 4.0
Torsion	--	8.2	8.0 to 8.6

design analysis are also listed. All the acceleration measurements at resonance varied from 0.0006 g to 0.0036 g; thus all measurements were at least two orders of magnitude below those that would result in a strong earthquake environment. No significant frequency shift was noted when the loads increased by a factor of three.

For the explosive tests, charge weights of principal tests were 100, 600, and 2000 lb. These were detonated at a depth of 40 to 50 ft and at distances from the building of only 260 to 290 ft. As a result of the close proximity of the blasts, very intense high-frequency accelerations were measured at the engulfment of the structure by the blast wave (0.8 g at 60 Hz for 2000-lb charge). The primary measurements, however, were the rocking mode response after engulfment. For these responses, accelerations of about 0.02 to 0.03 g were recorded on the fourth floor of the containment building in the EW direction for 600- to 2000-lb charges. Thus, a response was obtained that was about one order of magnitude higher than obtained for the steady-state vibratory tests. Table 4-2 indicates that only a slight drop in frequency resulted for the order of magnitude increase in response. Since this response is an order of magnitude lower than would be experienced in a strong earthquake environment, the frequency shift that would take place under strong motion is uncertain.

The fundamental mode damping estimated from the steady-state tests varied between about 2 and 3.5%. For the flexural modes, computed damping values varied from about 2.5 to 4.5% for the explosive tests. Both sets of data appear reasonable, considering the higher level of response for the latter. The rock-like foundation and the shallow depth of embedment contributed to the success of the tests. However, larger explosive charge weights placed a greater distance from the structure would help eliminate the high-frequency accelerations and would give much longer response times and higher accelerations, which would be desirable.

4.3.2.4 Comparison of Measured Earthquake Building Response with Ambient Transient and Steady-State Vibratory Test Results

Strong motion recorders have been installed in multistory buildings in Los Angeles and other large California cities to collect information on the response of buildings to vibratory earthquake ground motion. In general, the recorders have been installed in the basements, near the midheights, and on the roofs of the buildings. This is an excellent test procedure; however, it also has major disadvantages. First, many years may elapse before useful information can be collected. Second, it is expensive to monitor all multistory buildings in order to make certain that a few will experience strong motion. Third, to keep costs within reasonable bounds, only a minimum of instrumentation is provided. As a result, it is usually difficult to determine whether significant torsional response occurred, and in no case has there been sufficient instrumentation to evaluate the influence of soil/structure interaction on the response measurements. Therefore, a complete interpretation of structure response from records that have been

obtained is generally not possible. Nevertheless, this is an important method of collecting test information. The practice of installing strong motion recorders in all nuclear reactor structures should eventually yield useful information on the response of these more massive and more deeply embedded structures. A review of some of the results of strong motion measurements in conventional structures follows.

The San Fernando earthquake of February 9, 1971 is one of the first earthquakes to provide strong motion records for a significant number of conventional multistory buildings. Records were obtained for about 70 buildings that varied from 7 to 42 stories in height (NOAA, 1973). Table 4-3 provides a summary of data from five buildings (varying from 7 to 20 stories in height) that is characteristic of the general behavior of the multistory buildings on soil foundations during the 1971 earthquake.

Postearthquake analyses based on the acceleration records of these and other structures are provided in NOAA (1973).

The Millikan Library and the Kajima building sites have the firmest soil conditions of the five sites. Spread footing foundations were used for these two buildings. The Kajima site has only 30 ft of sand overlying siltstone, and the footings were carried down about 15 ft below grade, the equivalent of one story. The Millikan Library is also carried down a depth of 14 ft, but this site has a greater depth of soil overlying bedrock. The 1901 Avenue of Stars Building site has a relatively deep alluvial soil condition, but four sub-basement stories were excavated below grade, and the tower is supported on 72-ft-long steel H piles that are capped in clusters of 3 to 10 piles. All pile caps are tied laterally with reinforced concrete tie beams. This structure, therefore, is deeply anchored into the soil.

TABLE 4-3. COMPARISON OF FUNDAMENTAL BUILDING PERIODS BEFORE, DURING, AND AFTER THE SAN FERNANDO EARTHQUAKE OF FEBRUARY 9, 1971 (NOAA, 1973)

Building	Structural Type ¹	Total Stories Above/Below Grade, Stories	Length and Width, ft	Distance to Epicenter, mi	Peak Ground Accel, g	Direction	Fundamental Period ²		
							Before, sec	During, sec	After, sec
Holiday Inn 8244 Orion Los Angeles	RC (FP)	7/0	63 150	13	0.18 0.24	NS EW	0.5 0.5	1.6 1.2	0.5 0.7
Bank of California 15250 Ventura Los Angeles	RC (MR)	12/0	161 60	17	0.23 0.14	S9°W S81°E	- -	2.2 3.0	1.7 1.6
Kajima Bldg 250 E First Los Angeles	Steel	15/1	96 66	26	0.09 0.13	N36°E N54°W	1.8 1.9	2.9 2.8	2.1 2.2
1901 Avenue of the Stars Los Angeles	Steel	20/4	112 242	24	0.12 0.17	N46°W S44°W	2.6 2.5	3.5 3.6	2.8 2.7
Millikan Library Caltech Pasadena	RC	9/1	75 69	23	0.18 0.22	EW NS	0.7 0.5	1.0 0.6	0.8 0.5

Notes ¹Fundamental periods before and after earthquake determined from ambient transient vibrations, fundamental periods during earthquake determined from acceleration records

²RC = Reinforced concrete
FP = Flat plate
MR = Moment-resisting frame

The Holiday Inn and the Bank of California building are located on deep alluvial soil sites. At the Holiday Inn the soil is at least 800 ft deep. The soils at the Bank of California building are shallower, but of unknown depth. Both structures were built on cast-in-place piles that were capped at grade. No basement was provided for either structure. The piles are 40 to 50 ft in length.

The most important information resulting from the analyses of the acceleration records for these structures was the very significant shift in the fundamental mode periods under strong earthquake input as compared to the periods determined from ambient transient and steady-state vibratory loads. Starting first with the Millikan Library, Table 4-3 indicates ambient and steady-state test values of 0.7 sec in the EW direction and 0.5 sec in the NS direction for this building. During the 1971 earthquake, the period in the EW direction increased 40% and in the NS direction 20%.

The data in Table 4-3 indicate that the Kajima and 1901 Avenue of the Stars buildings also had a 40 to 50% increase in fundamental period during this earthquake, as compared to the period determined by ambient transit vibration tests. The Kajima site had a very firm foundation. While the 1901 Avenue of the Stars site is a deeper soil site, this building was embedded to a depth of 45 ft. This structure was also founded on piles. The intensity of ground motion at the 1901 Avenue of the Stars building was comparable to that experienced at the Millikan Library, but the ground motion recorded at the Kajima building was slightly lower. Thus, it is reasonable to conclude that the general response of these three buildings was quite comparable. However, Hart was the only investigator of the responses of these buildings to indicate that the shift in fundamental period was probably significantly

affected by soil/structure interaction (NOAA, 1973).

The data for the Bank of California and the Holiday Inn buildings provide an interesting comparison with the data for the buildings discussed above. It will be noted that the input at the Bank of California site was comparable to that recorded at the Millikan Library and at the 1901 Avenue of the Stars building, but the increase in fundamental period in the transverse (east-west) direction of the building was nearly 90% as compared to 40% for the other two buildings. The shift in the longitudinal direction was only 30%. These comparisons, however, are with ambient transient vibration measurements made after the earthquake rather than before. Those made after the earthquake for the other buildings were equal to or slightly higher than measurements made before the earthquake. The shift in fundamental period was therefore higher than observed for the three previous structures. While it is not possible to primarily associate this increase to soil/structure interaction effects, the foundation and structural arrangement were such that greater effects should be anticipated. This becomes more evident when the response of the Holiday Inn is examined.

The input to the Holiday Inn was stronger than to the Bank of California building. However, the shift in fundamental period was more than would be indicated by a linear extrapolation of input from the Bank of California site. For the Holiday Inn, the increase in fundamental model period was 200% in the transverse direction (north-south) of the building and 140% in the longitudinal direction, or an average of 170%. In this case, the site is a deep, saturated alluvial soil, and the structure has negligible embedment. Again, it is not possible to determine how much of the shift in period was due to nonlinear soil/structure interaction and

how much as due to yielding of structural members. However, conditions at this site are quite favorable to accentuated soil/structure interaction, and it was not possible in the postearthquake analysis to satisfactorily model the shift in period by changes in structural material properties alone. Two conclusions can be drawn. First, fundamental periods of multi-story buildings determined by ambient transient and steady-state vibratory test procedures have been found to be in error by 50 to 200% when compared with the measured response of the structures in a moderately strong earthquake ground motion environment. (Damping in the earthquake environment has also been demonstrated to be 5 to 10% of critical as compared to 2 to 3% at steady-state vibration test levels.) Second, additional strong motion instrumentation is needed to permit determination of torsional response and the effect of soil/structure interaction on the structure response in strong earthquake ground motion environments.

Steady-state vibratory tests have been conducted at the San Onofre nuclear power plant, and response to the 1971 San Fernando earthquake was also recorded (Ibanez et al., 1970; Matthiesen et al., 1970; Smith et al., 1971). However, the steady-state vibratory tests were planned primarily for the determination of equipment response, and the input from the San Fernando earthquake was too low (0.01 g) to provide meaningful response. A comparison of results from these investigations does not, therefore, contribute useful information.

4.3.3 SUMMARY OF TEST RESULTS

4.3.3.1 Theoretically Calculated Fundamental Periods

Theoretically calculated fundamental periods were compared with fundamental periods determined by ambient transient, steady-state

mechanical oscillator, explosive, and earthquake-induced vibrations. In practically all cases studied, the analytical procedures used to compute the fundamental periods were based on mathematical models that were significantly inferior to present state-of-the-art capability. For high-rise buildings, one-dimensional models were used that ignore torsion and soil/structure interaction. Mathematical models used to estimate fundamental periods of nuclear plant structure reviewed in some cases assumed rigid body behavior and in no case used an adequate representation of soil/structure interaction, nor did the procedures approach present Nuclear Regulatory Commission licensing requirements. Primarily because of inadequate mathematical modeling, variations of 100 to 200% between the theoretically computed and the experimentally measured modal periods by ambient transit, steady-state, and explosive-induced vibratory test procedures resulted. The computed fundamental periods were found to be in error both above and below the measured values. When the computed periods are compared to periods measured for structures during a strong earthquake, the variation can be expected to be as high as 100 to 300%. It is postulated that variations between computed fundamental periods for conventional nuclear power plant structures, based on present licensing practice, and values that might be measured in the SSE environment would normally be as great as 100%. It is further postulated that if the best analytical modeling procedures are used, the variation can be reduced to about 50%.

4.3.3.2 Ambient Transient Vibratory Tests

The results of the tests indicate that when ambient transient vibratory test methods are carefully applied to multistory buildings having a depth of embedment of about 10% of the aboveground height, or less, modal period determinations can be made that are in excellent

agreement with those determined by steady-state vibratory test methods. Modal damping determinations (1 to 2%) are less certain, particularly when modes are closely spaced. However, levels of excitation are usually three orders of magnitude below that resulting in a strong earthquake environment. Therefore, modal period determinations for multistory buildings in a strong seismic environment can be expected to be 50 to 200% higher, and damping will be at least a factor of 3 to 5 higher than determined from ambient transient vibration tests. Ambient transient tests on more massive and more deeply embedded structures, which are characteristic of nuclear power plant structures, have not produced reliable results and are not recommended.

4.3.3.3 Steady-State Vibratory Tests

Steady-state vibratory tests of multistory buildings produce response accelerations at least one order of magnitude lower than result in a strong earthquake environment, and for massive, embedded structures, the response may be two orders of magnitude lower. For multistory buildings having a depth of embedment of about 10% of the aboveground height, or less, more reliable damping and slightly more reliable modal period can be obtained with steady-state tests than with ambient transient tests. Modal period determinations for multistory buildings in a strong seismic environment, however, will usually be 50 to 200% higher, and damping will be a factor of 3 to 5 higher than determined by steady-state vibratory tests. More massive structures, similar to some nuclear power plant structures, which have a depth of embedment of about 10% of the aboveground height, or less, and which are supported on rock having nearly strain independent properties, should also yield reasonable modal data when tested with steady-state vibratory procedures. However, steady-state vibratory tests

may provide little useful information for massive structures when the depth of embedment is greater than 10 to 20%, and/or the supporting media is soil with nonlinear stress/strain characteristics.

4.3.3.4 Explosive-Induced Vibratory Tests

Buried explosives can be used to simulate a strong earthquake environment for testing nuclear power plant structures. However, charge weights of 100 to 500 tons buried at distances of 1200 to 1500 ft from the structure are needed to provide realistic response. For these conditions, the duration of strong motion will probably not exceed 2 sec, and the free-field response spectra will be deficient below frequencies of 2 to 3 Hz. Much greater charge weights with several time delays would be required to provide more realistic durations of strong motion, and to extend the free-field response spectra into lower frequencies at Regulatory Guide 1.60 spectra levels. Because of the large area that would be subjected to strong ground motion, and the expense of the test, this test procedure can seldom be used to test commercial nuclear power plant structures. Tests at low charge weights, such as one ton or under, are not recommended.

4.3.3.5 Measured Response to Natural Earthquake Excitation

The best experimental information on the response of structures to a strong earthquake environment can be obtained by placing strong motion recorders at selected points in the structure and in the free-field soil environment near the structures. However, the disadvantages of this method of testing are the expense of instrumenting many structures in order to make certain that some measurements are obtained, and the long delay before meaningful data can be collected. Also, present instrumentation procedures for multistory buildings do

not usually provide sufficient information for an adequate analysis of torsional response, or the influence of soil/structure interactions on the

response of the structure. The same shortcomings exist for normal instrumentation of nuclear power plants.

CHAPTER 5

EQUIVALENT RESPONSE CYCLES FOR FATIGUE ANALYSES

5.1 EARTHQUAKE-INDUCED FATIGUE IN NUCLEAR PLANTS

An important aspect of the fatigue resistance of any structure is the number of load cycles applied to the structure during its design life, as well as its internal environment during these repeated load applications. These factors can vary considerably, depending on the particular fatigue design application. For example, aircraft structures can be subjected to many hundreds of thousands of load cycles during their lifetime. This requires that such structures be designed for an adequate fatigue life under "high-cycle" load conditions. In addition, the fatigue life of such structures is affected by the fact that they are typically operating at normal to low temperatures. In contrast to this, the total number of earthquake-induced load cycles applied to nuclear plants will be about 1000. Even when combined with the cyclic effects of thermal loads, the total is considerably less than experienced by aircraft. Therefore, nuclear plants are designed for "low-cycle" fatigue effects. Furthermore, these fatigue effects must be evaluated in light of the high temperatures present within the nuclear reactor.

There are a number of important differences in the basic characteristics of high-cycle and low-cycle fatigue effects. For example, in high-cycle fatigue, the strength of the element, rather than its deformations, is emphasized. This is because the applied stresses are quite small so that the materials behave almost linearly up to failure. In contrast to this, low-cycle fatigue effects are typically represented using strains or deformations since, for this case, the element is typically stressed well

into its plastic range. Furthermore, under low-cycle fatigue conditions, the stress-strain curves may change during successive load cycles. For example, steel materials may exhibit the Bauschinger effect (i.e., a reduction in elastic limit stresses due to residual internal stresses introduced by reversals of large loads), while reinforced concrete elements will undergo microcracking that could reduce their stiffness, depending on the amount of reinforcing steel available (Newmark and Rosenblueth, 1971).

Finally the importance of fatigue in nuclear plants is compounded by the high temperatures present within the nuclear reactor. The behavior of structural and equipment elements in high temperature-low cycle fatigue is not fully understood and is an area of active research in many universities and research organizations. The prediction of the fatigue resistance of equipment within the nuclear reactor is further complicated by the presence of creep effects, although the duration of most earthquake motions is typically not sufficient to cause excessive creep.

5.2 SUMMARY OF CUMULATIVE DAMAGE LAWS

The assessment of the integrity of an element in fatigue is typically based on the application of various cumulative damage theories. Theories of this type are based on the classical linear damage rule first proposed by Palmgren (1924) and later by Langer (1937) and Miner (1945). This rule is expressed mathematically as:

$$\sum_i \frac{n_i}{N_i} = 1 \quad (5-1)$$

where

n_i = Number of cycles at i th strain level applied to the element by the design loading

N_i = Number of cycles at i th strain level required to cause failure of the element

Equation 5-1 assumes that at any stage of the loading history of the material, the percentage of the fatigue life used is proportional to the cycle ratio at that loading condition. Thus, if a strain range is applied for n_i cycles at a condition where failure would occur if N_i cycles are applied, the percentage of life used is n_i/N_i . It is well known that Equation 5-1 is just an approximation and may lead to erroneous predictions of fatigue life. To illustrate, Manson (1965) points out that the behavior of materials can change during cumulative fatigue loading, as evidenced by observations of cyclic hardening and softening of many materials. In addition, Equation 5-1 is based on the assumption that the order in which the various strain cycles are applied is irrelevant. As noted by Newmark and Rosenblueth (1971), this assumption is contradicted by vast experimental data on many different materials; therefore, the theory is inadequate for predicting fatigue effects in general circumstances. However, Newmark and Rosenblueth also point out that the approach may be more reasonable when the loading process is stochastic with a poor correlation between successive cycles, which is often the case in earthquake applications.

Many current design requirements are based on modified versions of Equation 5-1. For example, Section III and Division 2 of Section VIII of the ASME Boiler and Pressure Vessel Code (ASME, 1974) require a cyclic analysis for fatigue; these sections of the code are based on an elastic behavior for materials and are therefore limited to temperatures below the creep range. For high temperature conditions within nuclear reactors, Code Case 1592 of

Section III contains combined creep-fatigue requirements given as:

$$\sum_i \frac{n_i}{N_i} + \sum_k \frac{t_k}{T_k} \leq D \quad (5-2)$$

where

n_i, N_i = As defined under Equation 5-1. It is noted that design values of N_i as a function of strain range are provided in Code Case 1592 for different materials (Fig. 5-1)

t_k = Total time of application of k th design load condition

T_k = Allowable time of application of stresses from k th design load condition

D = Parameter with a value ≤ 1 that depends on the relative amounts of creep and fatigue present

It is seen that Equation 5-2 is of the same form as Equation 5-1, except for a creep term on the left-hand side, $\sum_k t_k/T_k$, and a modification of the right-hand side to account for creep effects. When there is no creep present, Equations 5-1 and 5-2 are identical.

It is noted that other fatigue damage laws have been developed; an excellent summary of many such theories is provided by Stallmeyer and Walker (1968). A common feature of Equations 5-1 and 5-2 and many of the theories described by Stallmeyer and Walker is that they consider cyclic straining of the element, despite the fact that earthquakes actually produce randomly varying strains. Therefore, it becomes necessary to utilize a random strain history in the element, estimated to correspond to one or more criteria earthquake events, as a basis for defining "equivalent" cyclic strains.

5.3 DETERMINISTIC PROCEDURES FOR ESTIMATING EQUIVALENT RESPONSE CYCLES

There are two sets of deterministic procedures for estimating equivalent response cycles: the first set consists of procedures developed in the aircraft industry for analyzing fatigue effects in aircraft structural elements, while the

(a) 304 SS and 316 SS

(b) 304 SS and 316 SS — elastic analysis

(c) Ni-Fe-Cr (Alloy 800H)

(d) Ni-Fe-Cr (Alloy 800H) — elastic analysis

FIGURE 5-1. DESIGN FATIGUE STRAIN-RANGE CURVES IN CODE CASE 1592 (ASME, 1974)

second set corresponds to ASME code requirements.

5.3.1 CYCLE-COUNTING METHODS DEVELOPED FOR AIRCRAFT APPLICATIONS

In past applications in the aircraft industry, a number of different methods have been developed for counting equivalent cycles from a random strain history. As noted previously, aircraft structures are subjected to high-cycle fatigue, as opposed to low-cycle fatigue effects induced in nuclear plants by earthquakes.

Therefore, the applicability of the cycle-counting methods to nuclear plants has not been proven, although there is nothing in the literature that indicates these methods are inapplicable to low-cycle fatigue conditions.

The methods include the counting of peaks, mean crossing peaks, level crossings, ranges, range means, and range pairs, as well as the fatigue meter and rain-flow methods. These procedures are summarized in Table 1 of Ts'ao and Werner (1975) and have been described in a number of publications, such as

Schijve (1963), Jacoby (1967), and Dowling (1972).

As one basis for evaluating the various methods of cycle counting, sequences shown in Figure 5-2 can be utilized. These three sequences consist of small reversals superposed on a large cycle (Fig. 5-2a), large reversals superposed on a large cycle (Fig. 5-2b), and a large cycle with no superposed reversals (Fig. 5-2c). The peak-count method will give the same results for the Figure 5-2a and 5-2b sequences, even though the Figure 5-2b sequence is likely to be more damaging (Ts'ao and Werner, 1975). Furthermore, the mean crossing peak-count approach gives the result that the Figure 5-2a and 5-2c sequences are equivalent, which is unconservative for cases where the small reversals in Figure 5-2a do significant damage.

The range and range-mean counting methods have the characteristic that, if small reversals are counted, the large ranges are broken up and counted as several smaller ones (Ts'ao and Werner, 1975). This gives the unrealistic result that small excursions do negative damage, and the calculated damage can be decreased by including them, as shown in Figure 5-3. For example, in the sequence shown in Figure 5-2a, the large cycle on which the smaller reversals are superposed is not recognized by the range-count approach; therefore the calculated damage for the Figure 5-2a sequence could be less than for Figure 5-2c. The range-pair and rain-flow counting methods give more reasonable results when applied to these sequences; these methods treat the small reversals as interruptions of the larger strain ranges, and the damages for the large and small strain ranges are simply added.

The effect of strain sequences are not considered by either the peak, mean crossing peak, level crossing, or fatigue meter counting

FIGURE 5-2. SEQUENCES THAT CAUSE PROBLEMS FOR SEVERAL CYCLE-COUNTING METHODS

FIGURE 5-3. EFFECTS OF IGNORING ALL EXCURSIONS SMALLER THAN A SPECIFIED VALUE

methods. These methods all give the same number of cycles for the two sequences shown in Figures 5-4a and 5-4b, even though Dowling (1972) indicates the Figure 5-4b sequence to be much more damaging. As still another example of the ability of the various methods to account for sequencing effects, the sequences of Figures 5-5a and 5-5b are provided. These sequences are identical except that the smaller peaks have opposite signs. It has been shown experimentally that the Figure 5-5a sequence can result in more plastic straining than the sequence of Figure 5-5b; the range-pair and rain-flow counting methods correctly predict

that the Figure 5-5a sequence will be more damaging than Figure 5-5b, while the application of the peak- and level-crossing counting methods indicates the same number of cycles for each of these sequences.

FIGURE 5-4. SEQUENCES FOR WHICH SEVERAL COUNTING METHODS GIVE IDENTICAL RESULTS

FIGURE 5-5. SEQUENCE CHANGES THAT CAN AFFECT THE FATIGUE LIFE

Still another difficulty with many counting methods is selection of an appropriate method of accounting for mean strain in combination with the various cycles counted. For some regular sequences, it may be possible to define a single mean strain for various sections of the strain time history; however, for more irregular time histories, the division of the time history into sections becomes somewhat arbitrary. Perhaps the least difficulty is encountered if the mean strain of each cycle is determined. This can be done when cycles are defined according to the rain-flow counting method, in which the mean strain of each cycle is simply the average of the most positive and negative peaks occurring during that cycle.

In summary, considerable difficulties may result from the use of many of the different cycle counting methods described herein. The range-pair and rain-flow methods appear to provide the most consistent results for the widest range of conditions and, as shown from

a comparison by Dowling (1972), the two methods are quite similar.

5.3.2 ASME CODE REQUIREMENTS

The use of Equation 5-1 for computing cumulative fatigue loading is specified in Subsection NB-3222-4(e) of Section III, ASME Boiler and Pressure Vessel Code (ASME, 1974). This subsection does not consider the problem of defining equivalent cycles from a random response time history; however, it does address the problem of defining equivalent stress or strain levels associated with each group of cycles by referring the reader to Subsection NB-3216. As noted previously, this portion of the ASME code is limited to lower temperatures not typical of the interior conditions in nuclear reactors; as a result, the approaches described in this section of the code are of interest for those other portions of the nuclear plant not subjected to high temperature effects.

In Subsection NB-3216, two different approaches for defining stress levels are provided. For the case where the directions of the principal stresses do not change, the engineer is required to compute the time variation of the principal stress differences $S_{ij} = \sigma_i - \sigma_j$ ($i, j = 1, 2, 3$). The extremes of the range through which S_{ij} fluctuates is then determined, the magnitude of these extremes for each S_{ij} being designated as S_{rij} . The alternating stress is then computed as $S_{aij} = 0.5 S_{rij}$ for each principal stress difference, and the alternating stress intensity S_a is then taken as the largest of the various S_{aij} values. This value of S_a is then used to enter the fatigue curves for the element to define the number of cycles to failure (the denominator in Eq. 5-1).

The second approach outlined in Subsection NB-3216 deals with the case where the orientations of the principal stress are variable. For this case, the time histories of each of the

six stress components (σ_{ij} , $i, j = 1 - 3$) are first computed, taking account of structural discontinuities, thermal effects, etc. Then a point in time is chosen where the conditions correspond to one of the extremes for the cycle (either maximum or minimum algebraically); differences between each of the stress components at this time and the corresponding stress components at all other points in time are obtained. These stress component differences are then used to compute principal stresses at each time, and the resulting principal stress differences $S_{ij} = \sigma_i - \sigma_j$ ($i, j = 1, 2, 3$) as a function of time can be computed. The largest absolute magnitude of S_{ij} is then identified and the alternating stress intensity S_a is then computed as half of this magnitude.

5.4 PROBABILISTIC PROCEDURES FOR ESTIMATING EQUIVALENT RESPONSE CYCLES

The theory of probability is applied to the estimation of equivalent load cycles resulting from earthquake events for fatigue design purposes. This entails (1) the estimation of the expected number and size of the earthquake events that will occur during the life of the nuclear plant, and (2) the evaluation of the equivalent cyclic response of the nuclear plant elements to these various events.

5.4.1 ASSESSMENT OF NUMBER OF OCCURRENCES OF EARTHQUAKE EVENTS

In most nuclear plant applications, the estimation of equivalent fatigue cycles is based on a prescribed number of occurrences of the Safe Shutdown Earthquake (SSE) and the Operating Basis Earthquake (OBE) during the prescribed life of the plant. However, the basis for selection of the number of events is typically not defined in the PSAR for the plant and, in many instances, the resulting risk (i.e., probability of failure) associated with fatigue effects induced by these multiple earthquake events may not be consistent with the

risk associated with other aspects of the seismic design criteria. To circumvent this problem and define risk-consistent equivalent response cycles, probabilistic methods can be employed.

For probabilistic methods used to estimate the risk associated with the occurrence of multiple earthquake events at an example site in the western United States, peak ground acceleration of 0.40 g and 0.20 g are assumed for the SSE and OBE, respectively. Computation of the probability of occurrence of a single earthquake event assumes that the arrival rate of the earthquake events in the vicinity of the site is Poisson distributed (AA, 1975a).

For estimating the probability of occurrence of multiple events, the probability of n earthquakes with an arrival rate μ is given by Ts'ao (1974):

$$P(n, \mu) = \frac{\mu^n}{n!} \exp(-\mu) \quad (5-3)$$

The probability of no earthquake occurrences during the time period of interest is obtained by letting $n = 0$, i.e.,

$$P(0, \mu) = \exp(-\mu)$$

and for exactly one earthquake ($n = 1$), two earthquakes ($n = 2$), etc. is

$$P(1, \mu) = \mu \exp(-\mu)$$

$$P(2, \mu) = \frac{\mu^2}{2!} \exp(-\mu)$$

The probability of having n or more earthquakes within the time period specified is expressed as the summation of these expressions:

$$P'_n = \sum_{m=n}^{\infty} \frac{\mu^m}{m!} \exp(-\mu) \quad (5-4)$$

The probability of having one or more earthquakes occur during the specified time period is:

$$P_f = 1 - P(0, \mu) = 1 - \exp(-\mu) \quad (5-5)$$

From these equations, the probabilities of occurrence of multiple earthquake events having

ifferent peak acceleration levels at an example site in the western United States are presented in Table 5-1. The probability of occurrence of multiple events that include, but are not restricted to, the OBE and SSE is depicted in Figure 5-6. Figures 5-6a and 5-6b represent two alternate approaches for showing the inter-relationship between the probability of occurrence, peak acceleration, and number of earthquake events that might occur during the design life of the nuclear plant.

The results shown in Table 5-1 and Figure 5-6 provide a basis for defining multiple earthquake events for fatigue design purposes, using acceptable risk levels as a means for defining these events. These risk levels should be defined from regulatory agency requirements and cost-vs-risk optimization studies that incorporate assessments of the uncertainties in such factors as the applied loads, material properties, analysis techniques, etc. For purposes of an illustrative example here, acceptable risk levels can be assumed. If such risk

levels correspond to a probability of occurrence of multiple earthquake events of, say, 0.5%, then Table 5-1 indicates that one SSE event (with a probability of exceedance of 0.143%) and three OBE events (with a probability of exceedance of 0.264%) would be acceptable; i.e., assuming the SSE and OBE events to be independent, the total probability for these multiple events would be the sum of the probabilities for each ($0.143 + 0.264 = 0.407\%$) which is less than the assumed acceptable probability level of 0.5%. A more conservative criteria would require a lower probability of occurrence, or more cycles of stress, be considered. Thus if a 0.2% probability is required, one SSE and four OBE events should be considered.

Although most current practices consider only SSE and OBE events in defining multiple earthquake events, fatigue design criteria does not necessarily have to be restricted to only those acceleration levels. For example, Figure 5-6 indicates that significant earthquake-induced fatigue loads can be caused not only by

TABLE 5-1. PROBABILITY OF OCCURRENCE OF MULTIPLE OBE AND SSE EVENTS AT EXAMPLE SITE IN WESTERN UNITED STATES

No. of Earthquake Events, n	OBE		SSE	
	Probability of Occurrence of Exactly n Events, $P(n, \mu)$, Percent	Probability of Occurrence of n or more Events, P'_n , Percent	Probability of Occurrence of Exactly n Events, $P(n, \mu)$, Percent	Probability of Occurrence of n or more Events, P'_n , Percent
0	76.46	100.00	99.86	100.00
1	20.53	23.54	1.43×10^{-1}	1.43×10^{-1}
2	2.76	3.02	1.02×10^{-4}	1.47×10^{-4}
3	2.47×10^{-1}	2.64×10^{-1}	4.85×10^{-8}	4.51×10^{-5}
4	1.65×10^{-2}	1.75×10^{-2}		
5	8.89×10^{-4}	9.60×10^{-4}		
6	3.98×10^{-5}	7.11×10^{-5}		
7	1.52×10^{-6}	3.13×10^{-5}		
8		2.98×10^{-5}		
9				
10				
	Small	Small	Small	Small

- Notes (1) Probability levels based on plant design life of 50 years
(2) SSE and OBE correspond to peak accelerations of 0.40 g and 0.20 g respectively, for this example site
(3) $P(n, \mu)$ and P'_n are computed from Equations 5-3 and 5-4, respectively

(a) Number of occurrences vs. probability for fixed peak acceleration level

(b) Number of occurrences vs. probability for fixed probability level

FIGURE 5-6. PROBABILITY OF OCCURRENCE OF MULTIPLE EVENTS HAVING DIFFERENT PEAK ACCELERATION LEVELS -- EXAMPLE SITE IN WESTERN UNITED STATES

the OBE and SSE events but by other events as well. Therefore, it may be appropriate to consider additional earthquake acceleration levels when specifying equivalent cycles, such as the 1-1/2 OBE and 1/2 OBE levels. The inclusion of these additional levels of ground shaking would result in improved estimates of equivalent cycles, although the number of additional levels that can be considered must be limited so that the resulting assessment of equivalent cycles will not be overly complicated. The basis for selection of the resulting combination of multiple earthquake events should be based on two constraints: (1) the total of the probabilities associated with multiple occurrences of each earthquake event should correspond to a preselected overall probability level for fatigue loading; and (2) the fatigue loads corresponding to the selected combination of events should be reasonably conservative when compared to loads from other combinations of events; this degree of conservatism should be consistent with cost and acceptable risk constraints for the plant.

5.4.2 EVALUATION OF RESPONSE CYCLES

Probabilistic procedures for defining equivalent response cycles induced by the application of earthquake ground motions are based on a knowledge of the response characteristics of the element being evaluated. These responses can be estimated from random input motions based on procedures outlined in Appendix A of Ts'ao and Werner (1975).

It is assumed that the response is stationary and ergodic. When the response is stationary, its probabilistic and statistical characteristics do not vary with time. If the response is also ergodic, ensemble average response characteristics at any discrete time t_1 can be replaced by a time average of a single

representative member of the ensemble.* Each of these assumptions has been widely applied in earthquake-related problems. For example, Housner and Jennings (1964) developed a series of artificial earthquake records based on the assumption of a stationary random process. Their approach was subsequently shown to be valid for representing motions whose duration of strong shaking is long when compared to the initial rise and the decaying tail of the ground motion history.†

Most lightly damped structural systems can be represented as narrow-band random processes, where the number of cycles can be approximated by the number of times the response crosses a given threshold level. This approximation is exact when the response is cyclic, since each threshold crossing corresponds to a cycle.

To illustrate one approach along these lines, let $x(t)$ and $\dot{x}(t)$ be the displacement and velocity histories of an element. The number of times the displacement crosses a threshold level, $x = s$, within a time T is then:

$$n(s) = v_s^+ T \quad (5-6)$$

*This presumes that each member of the ensemble of response histories is statistically equivalent to any other member.

†The time-dependent variation of earthquake motions consists of an initial segment of rapid increase to a high level, a segment of uniformly strong shaking at these high levels for a certain duration, and an attenuating final portion of the motion. If the central portion of this ground shaking process is reasonably long when compared to the initial and final portions, then an assumption of stationarity is satisfactory; this is generally appropriate for investigating the response of structures to strong earthquake motions. However, the assumption is less appropriate for modeling smaller earthquake motions or for use in studies where the less intense but longer tails of the accelerogram may be important (Jennings et al., 1969).

where $n(s)$ is the number of threshold crossings and v_s^+ is the rate of crossing of the threshold level. Mathematically, v_s^+ is expressed as:

$$v_s^+ = \int_0^\infty \dot{x} P(s, \dot{x}) d\dot{x} \quad (5-7)$$

where $P(x, \dot{x})$ is the joint distribution of the response displacement x and velocity $\dot{x}$. The distribution of $P(x, \dot{x})$ is dependent on the nature of the earthquake motions. For example, if these motions can be described as filtered white noise whose accelerations have a Gaussian distribution with zero mean, $P(x, \dot{x})$ for a single-degree-of-freedom system also has a Gaussian distribution, i.e.,

$$P(x, \dot{x}) = \frac{1}{2\pi\sigma_x\sigma_{\dot{x}}} \exp\left\{-\frac{1}{2}\left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{\dot{x}}{\sigma_{\dot{x}}}\right)^2\right]\right\} \quad (5-8)$$

An approximate alternate approach that simplifies the subsequent evaluation of accumulated damage (Eq. 5-13) has been developed by Crandall and Mark (1963). This approach is based on the rate of positive zero crossing, v_0^+ , which is obtained by substituting Equation 5-8 into 5-7, setting $s = 0$, and performing the indicated integration. This results in the following expression for v_0^+ :

$$v_0^+ = \frac{1}{2\pi} \frac{\sigma_{\dot{x}}}{\sigma_x} \quad (5-9)$$

which, in turn, is used to compute the average number of occurrences of the threshold response, $x = s$, within the given time period, T , as:

$$n(s) = v_0^+ T P(s) ds \quad (5-10)$$

In Equation 5-10, $v_0^+ T$ represents the total number of peaks during the time period T , and $P(s)$ is the probability density function of the peak response.

The only real effort involved in the use of Equation 5-10 is the development of an appropriate representation of $P(s)$. There are a

number of approaches currently available for doing this. For example, the ensemble of response time histories can be divided into several levels denoted as $a, b, c, \dots$ in Figure 5-7. The peak distribution function for each time history is obtained by counting and plotting the number of peaks that exceed each level; then, based on an assumption of ergodicity the ensemble peak distribution function can be obtained as the mean of the distribution function from each time history. Alternately, Crandall and Mark (1963) and Shinozuka (1966) have shown that, when the response history, $x(t)$, is a narrow-band, Gaussian, stationary random process with zero mean, $P(s)$ can be considered to be Rayleigh distributed and is expressed as:

$$P(s) = \frac{s}{\sigma_s^2} \exp\left[-0.5 \frac{s^2}{\sigma_s^2}\right] \quad 0 < s < \infty \quad (5-11)$$

In Equation 5-11, the standard deviation of the peak response, $\sigma(s)$, can be estimated from processing of the ensemble of response time histories.

Once $n(s)$ is obtained according to Equation 5-10, it can be directly used in a cumulative damage law (such as Eq. 5-1, for example) to estimate the fatigue damage. For example, if $N(s)$ is defined as the number of cycles to failure at a constant stress amplitude, s , the fatigue damage caused by that stress level is:*

$$\frac{n(s)}{N(s)} = v_0^+ T \frac{P(s)}{N(s)} ds \quad (5-12)$$

The accumulated damage caused by all stress levels is then obtained by integrating Equation 5-12, i.e.:

$$D = \int_0^\infty \frac{n(s)}{N(s)} ds = v_0^+ T \int_0^\infty \frac{P(s)}{N(s)} ds \quad (5-13)$$

*The approach shown is based on stress cycles, although a similar procedure can be used to consider strain cycles, as used in low-cycle fatigue applications.

may include structures that require unprecedented levels of reliability (e.g., nuclear power plants), or structures that should resist unusual environmental conditions (e.g., certain military structures). Since experience with these structures is limited, the engineering intuitive judgment is more prone to errors, and scientific methods are needed for direction.

The experiences gained for ordinary structures can be extended to provide a framework of design criteria applicable to unconventional structures. For example, consider the development of design specifications for nuclear power plants. Assuming that current design specifications of ordinary structures are adequate, the formulations developed here can be used to estimate needed increases in the safety or load

factors in order to meet the higher reliability required for these structures.

3.7 DEVELOPMENT OF PROBABILISTIC DESIGN PROCEDURES

The critical examination and further development of the current deterministic design procedures are needed at this time, but perhaps the enduring value of this study is its contribution toward the formulation of a probabilistic design procedure involving rational criteria of safety and serviceability. Towards that end, the major contribution here is the method developed for the evaluation of extreme combined loads, and in the formulations of safety and load factors for the case of multiple load combinations.

(a) Subdivision of ensemble of N response time-histories into threshold levels

(b) Superposition of peak response density functions from individual records in ensemble

FIGURE 5-7. CONSTRUCTION OF PEAK RESPONSE DENSITY FUNCTION

In the conventional fatigue failure tests, the number of cycles to failure, $N(s)$, is determined for all stress levels and a S - N curve is derived for design purposes. The S - N curve is most commonly expressed as:

$$N(s) s^b = c \quad (5-14)$$

where b and c are constants determined from tests. By substituting Equation 5-14 into Equation 5-12, along with an appropriate representation of $P(s)$ (either from Eq. 5-11 or the approach indicated in Fig. 5-7), the fatigue damage can be readily estimated.

resulting time-dependent response at a given degree of freedom, say $\ddot{X}_j(t)$, is obtained as the inverse transform of the product of $\ddot{X}_g(i\omega)$ and $H_j(i\omega)$.

The transformations between the time and frequency domains are typically carried out using Fast Fourier Transformation Algorithms (Cooley and Tukey, 1965). These algorithms significantly reduce the number of computations as compared with the direct Fourier Transform method and make solutions in the frequency domain suitable for application to large structural systems and to parametric studies.

2.5.1.3 Nonproportional Damping

A third approach for determining the system response using a general damping matrix is through the use of nonproportional damping procedures. There are a number of basic concepts that separate this approach from conventional normal mode methods. First, the nonproportional damping is introduced into the equations of motion prior to transformation to normal mode form, i.e., the resulting mode shapes and frequencies are a function of the damping of the system as well as its mass and stiffness properties. Second, the system with nonproportional damping may be made to vibrate freely in a set of uncoupled "modes" in which all points in a system have different phase angles. Therefore, the mode shape is such that the relative position of any two points along the structure may vary with time, and the node points (i.e., points of zero displacement) of the structure are not stationary. Finally, for a system with nonproportional damping, the mode shapes and frequencies will be complex and will occur in conjugate pairs (for an underdamped mode). Orthogonality relationships between eigenvectors of different modes, which permit a decoupling of the equations of motion, can be obtained in terms of the

real and imaginary parts of these quantities (Hurty and Rubenstein, 1964).

2.5.2 APPROXIMATE METHODS FOR DEFINING EQUIVALENT MODEL DAMPING

In current practice, the most widely used approach for defining modal damping in nuclear plants is through the use of approximate methods based on certain assumptions regarding the weighting of the damping factors of the various elements within the structure. All of these methods are to some extent based on intuition and, as yet, no comparisons are known to have been made between responses using the approximate modal damping values and the rigorous solution of the structural response.

The various approximate methods are:

- Weighted Energy Method, where the i th mode damping ratio associated with each structure element, including the soil, is weighted according to the relative amount of strain energy stored in that element when the structure is vibrating in its i th mode (Whitman, 1970; Roesset, Whitman, and Dobey, 1973; Roesset, 1973).
- Weighted Modal Response Method, where the i th mode damping ratio is computed by weighting the damping in each element according to its mode shape, modal mass, or product of its mode shape and mass (Johnson and McCafferty, 1969).
- Transfer Function Method, where the modal damping ratio is computed by equating the modal transfer function to the exact system transfer function (corresponding to a unit harmonic base excitation) at all modal frequencies and a predetermined location in the structure (Tsai, 1974).

CHAPTER 6

SEISMIC ISOLATION

6.1 INTRODUCTION

The difficulties and high costs involved in seismic qualification and requalification of the various Category I systems and equipment in nuclear power plants have raised the question as to whether it is feasible and economical to consider seismic isolation as a design alternative. Considerable design and construction experience for shock isolation of nonnuclear structures and equipment has been gained over the past 20 years, and much of the technology developed is directly applicable to the seismic isolation of nuclear power plant structures and equipment. The basic purpose of an isolation approach to design is to reduce the maximum acceleration experienced during an earthquake by the isolated system to some predetermined level that the equipment and supporting structures are capable of resisting without special design considerations. The reduced seismic environment may lower costs sufficiently to pay for the isolation system and structural modifications to accommodate it, as well as provide additional system reliability. Since large breeder reactor power plants are under conceptual design development, the feasibility of seismic isolation as a viable design alternative needs to be addressed.

A two-phase investigation was conducted to determine the feasibility of providing seismic isolation for nuclear power plant systems that utilize liquid metal as the heat transfer media. In the first phase, several concepts for seismic isolation of an example reactor system were developed and evaluated, and from these, a preferred isolation concept was selected. This selected concept was further developed in the second phase of the study, using the general arrangement of a 1000-MWe Prototype Large

Breeder Reactor (PLBR) power plant designed conceptually by Atomics International and Burns and Roe (AI/B&R, 1977). An assessment of the performance of the PLBR isolation system was provided by comparing the dynamic response of the nonisolated PLBR to the response of the isolated PLBR for a common seismic input motion.

6.1.1 OBJECTIVES OF SEISMIC ISOLATION

The conventional approach for reactor seismic design utilizes conservative design forces, excessive structural strength, and over-qualification of equipment to provide a safety margin. The alternative design approach offered by the concept of seismic isolation is accomplished by the introduction of a compliant element between the structure and foundation or between the equipment and structural support. Such an element, capable of experiencing a large deflection, is designated as an isolator. If the isolator is sufficiently flexible, the inertia of the structure or equipment mass will retard the motion of the mass, thus allowing the isolator to accommodate the foundation or support motion during an earthquake. If viewed from a fixed reference point, the structure or equipment mass would appear to stand still in space with the isolator being deformed by the motion of the points of support. Thus, the mass is effectively "isolated" from the acceleration of the support points.

Therefore, the objective of a seismic isolation system is to attenuate the response of the structure or equipment to a level not requiring special design consideration. In conventional design concepts, the structural connections and equipment function are the points

of vulnerability. For isolation concepts, the focus of vulnerability falls upon the isolator elements, with the structural connections and equipment considered to be less vulnerable due to the attenuated response. The uncertainties associated with assessing the seismic safety of a conventional nuclear power plant would likewise be de-emphasized, and the question of seismic safety would be concerned with the reliability of the isolation system.

The use of isolation as a design concept has been used for several years to protect the equipment and personnel within hardened military facilities from the severe ground motion effects induced by nuclear weapons (Crede, 1963). The time duration of such ground motion is usually very short, thus the term "shock isolation" is utilized. However, some ground shock environments resemble earthquake motion. Thus, most of the shock isolation concepts and isolator designs are applicable to the case of seismic isolation. Much of the design and construction experience acquired over the past twenty years in the various ballistic missile programs has been documented (Crawford et al., 1974; AA, 1978). Concepts have ranged from isolation of entire structures (Kfoury, 1963) within underground cavities to isolation of commercial grade equipment on platforms within blast resistant aboveground structures (Boyd and Huang, 1977).

6.1.2 RESPONSE CHARACTERISTICS OF A SEISMIC ISOLATION SYSTEM

The effectiveness of a seismic isolation system can be illustrated with a single-degree-of-freedom dynamic system. The isolator is represented by a linear spring and viscous damping device. The supported structure or equipment is assumed to be a rigid body. The peak response of this system for seismic ground motion may be presented in the form of a response spectrum that is similar to RG 1.60

criteria. The desired performance objectives of a seismic isolation system may be superimposed upon the response spectrum by indicating the maximum acceptable acceleration and displacement response. These response bounds are established by the capability of the equipment (as associated anchorages) to resist lateral loads and the available "rattlespace" or flexibility limitations of connected piping and other umbilicals. The intersection of these bounds with the response spectrum establish the frequency bounds of the isolation system for which seismic isolation of the structure or equipment can be accomplished. These bounds create an isolation frequency "window" for the design of the isolation system. In general, the system response within the frequency window may be characterized by a nearly constant pseudovelocity (relative) response level, $\dot{\delta}$, thus, the acceleration of the isolated mass may be approximately expressed as $\ddot{X} \cong \omega \dot{\delta}$, where ω is the circular natural frequency of the isolation system. Usually this response level can be attenuated only by the introduction of additional system damping. This importance of damping is the distinguishing difference between seismic isolation and shock isolation design. For short duration shock motions, damping is ineffective in reducing response levels. Damping devices are often included in shock isolation systems only to reduce the residual response time after the shock motion has subsided. However, for a seismic isolation system, the incorporation of specialized damping devices into the design concept is necessary to achieve the performance objectives.

To ensure that the isolated mass returns to the rest position after a seismic event, the isolator elements must be elastic springs. Due to the constant velocity response characteristic of a seismic isolation system, the amount of active isolator spring material required to achieve isolation is independent of system

frequency. For any idealized isolator element, the volume of active material may be shown to be proportional to the expression

$$\frac{W_T}{g} \frac{E}{\sigma^2} \dot{\delta}^2$$

where W_T is weight of structure or equipment to be isolated, E is elastic constant of isolator spring, σ is the elastic stress level, and g is gravity. Thus, for a given level of response $\dot{\delta}$ (associated with a given effective damping), the amount of spring material required can be minimized only by reduction of isolated structure or equipment weight, W_T , or by use of spring materials with a high allowable (elastic) stress level, σ . Isolator configurations which utilize all of the available spring material at a constant allowable stress level are required. Once the configuration and materials are established for isolator design, the total cost of the isolation system will be a function of the system weight alone. Thus, weight reduction in the structure or equipment design becomes the dominant concern for the isolation system design.

6.1.3 SEISMIC ISOLATION OF A FLEXIBLE STRUCTURE

The idealization of an isolated structure or equipment item as a rigid body is valid only if the structure or equipment item has sufficient stiffness relative to the isolator stiffness. To ensure that the performance objectives of the isolation system are met, the entire structure or equipment must be subjected to a constant acceleration level. Thus, any amplification due to structure or equipment flexibility must be reduced to a negligible level. The general behavior of a flexible structure with a base isolation mount can be illustrated using a cantilever beam with base shear spring. Three general regions of response behavior may be identified. Given that the base is extremely stiff, the cantilever behaves as if the base were

fixed. If the base spring is very flexible, the beam behaves as a rigid body in translation. An intermediate region of response modification, characterized by combined flexible and rigid body response modes, occurs for a range of spring stiffness. For this response modification to be avoided by the isolated structure, the structure must have a minimum required stiffness.

In general, an isolation system must accomplish both horizontal and vertical isolation of a structure to achieve the performance objectives. If a base mount configuration of isolators is utilized to achieve isolation for a cantilever beam, the rotational response of the structure as a rigid body must be accommodated. For such a system, the vertical isolators are loaded by both vertical and horizontal base motion. Since the rocking response increases the displacement and acceleration response at locations away from the center of rotation, a center-of-gravity mount configuration is required to stay within the established response bounds. This configuration requires that the horizontal isolator groups be located and assigned stiffnesses to decouple the rotational and horizontal response modes of the isolated structure. The structure must also have sufficient stiffness to prevent response modification due to the structure flexibility.

6.2 EVALUATION OF SEISMIC ISOLATION CONCEPTS

To assess the feasibility of seismic isolation of a nuclear power plant, it is necessary to examine concepts for integrated isolation system configurations, with compatible isolator spring elements. The two basic isolation approaches differ chiefly in the location of the seismic separation (i.e., internal or external to the containment shell) and in the resulting total isolated weight. First, external isolation concepts are evaluated here qualitatively. Next,

internal isolation concepts are presented based on the designs and technology developed for shock isolation in military programs. It should be noted that internal isolation of a reactor system requires that any piping or conduit that crosses the interface between the isolated system and nonisolated structure, must accommodate the relative displacement response of the isolation system during a seismic event. Due to the difficulty of designing acceptable flexible joints for high pressure piping (> 1000 psi), only reactor systems with low pressure (< 200 psi) piping are acceptable candidates for internal isolation concepts. Thus, the internal isolation of Pressurized Water Reactor (PWR) or Boiling Water Reactor (BWR) systems is not considered practical, and the isolation of such systems is limited to external schemes. However, reactor systems that employ liquid metal as the heat transfer media are possible candidates for internal isolation schemes.

Only metallic isolator elements are considered here, and for these, the principal potential system candidates can be classified as systems based on axially compressible isolator units, systems based on torsion bars or torque tube isolator units, and systems based on spring beam isolator units. The torsion tube internal isolation concept was selected as the preferred system. The relationships governing the sizing of the active elements are developed and units sized to isolate an example reactor system and associated supporting structures. The conceptual layout of the isolators within the containment is also presented to further demonstrate the technical feasibility of internal seismic isolation.

6.2.1 EXTERNAL ISOLATION CONCEPTS FOR PWR BUILDING STRUCTURES

The seismic isolation of the entire nuclear island of a PWR power plant complex is currently (1979) being incorporated in plants

designed for South Africa and Iran by European firms. The basic concept involves the support of an upper base mat on elastomer bearings and friction plates. The pertinent details of this concept are shown in Jolivet and Richli (1977) and AA (1979). Only horizontal base isolation is considered, which is achieved by the lateral flexibility of the bearing and sliding of the friction plates to limit the horizontal acceleration transmitted to the nuclear island.

Another concept which has been proposed for PWR (and BWR) isolation is based upon a concept developed in New Zealand for horizontal base isolation of commercial buildings (Skinner et al., 1975a). This concept utilizes elastomer bearings for deadweight support but also couples a hysteretic damper element in a parallel spring configuration. Several different hysteretic damper units have been evaluated and tested (Skinner et al., 1975b, 1977) resulting in selection of a torsional-beam damper assembly as preferred. The basic details of this concept applied to commercial buildings are shown in Skinner et al. (1975a) and AA (1979). The development of the concept for horizontal isolation of a reactor structure has been presented in Skinner et al. (1976) and AA (1979).

A qualitative evaluation (Hadjian, 1978) of both concepts has identified several concerns. Among these concerns are the flexibility of the upper base mat, spatial variation of ground motion, and the lack of vertical isolation. The behavior of these systems for extreme wind loading (i.e., tornado loading) may be identified as an additional concern.

Both concepts utilize rubber/metal laminated bearings that are stiff vertically but provide the necessary shear flexibility required for horizontal isolation. The design and fabrication of rubber bearings for civil engineering applications is well documented (Derham and Thomas, 1979) with some installations in place

for 20 years without replacement. Several low-rise commercial buildings (Lee and Medland, 1979; Kelly, 1979; Delfosse, 1979) have been constructed in New Zealand and France, which utilize rubber bearings for horizontal seismic base isolation. For these applications, the bearings were made sufficiently stiff to preclude rocking and vertical response modes. Such bearings are designed to be approximately 400 to 500 times stiffer in the vertical direction than in the horizontal direction. For isolation systems with a horizontal (rigid body) frequency in the range 0.5 to 0.7 Hz, the corresponding vertical frequency of the structure on the rubber bearings would be in the range of 10 to 15 Hz. Within this frequency range, R.G. 1.60 vertical design spectra have the same amplification as horizontal design spectra.

For piping system design, the vertical component of motion is just as important as horizontal motion, due to the spatial configuration of piping loops. Furthermore, large diameter (liquid metal) piping systems will have primary modes of response within the 10 to 15 Hz range. The vertical component of motion is also an important design consideration for reactor core supports and vessel design.

Thus, for future U.S. plants, it is doubtful that only horizontal base isolation would be advantageous. If external isolation is considered as a seismic design option, then an individual containment structure would be the minimum configuration considered for total isolation. The ground mounted support for the isolation system can be visualized as a reinforced concrete basin of dimensions adequate to accommodate the lower 100 ft of the containment structure plus clearance below and on all sides for the isolator installation. The horizontal

isolation consists of a circular array of isolator struts paired to form equilateral triangles with the bases at the ground mounted support and the apexes at the seismic isolated structure. The horizontal isolation struts would be installed in several tiers with the number of struts in each tier and the circumferential locations selected to maintain the center of isolation system restraint at the center of gravity of the seismic isolated mass. The vertical isolation struts are arranged in a horizontal array of vertical tripods with each tripod base supported on the ground mounted structure and the apexes of the tripods at the base of the seismic isolated structure. The tripod bases would form equilateral triangles with minimum side lengths to ensure that, at the maximum horizontal displacement, the line of action of the vertical load on each tripod will remain within the limits of the base triangle so that the vertical loads will never produce a net overturning moment on the vertical isolation system. Otherwise, the development of the overturning moment would require an additional restoring force from the horizontal isolation system and could also increase the maximum displacement of the isolated structure.

For a typical PWR power plant, the total isolated weight of the containment structure and contents would be approximately 285×10^6 lb, while the internal weight of the Nuclear Steam Supply System (NSSS) and supporting structure would be approximately 40×10^6 lb. Since the volume of active isolator material is directly proportional to the weight of the structure, it would appear extremely doubtful that external total isolation of a containment structure could be economically justified if internal isolation of a reactor system is feasible. The additional excavation and foundation cost alone would eliminate the external concept as a viable alternative.

6.2.2 INTERNAL ISOLATION CONCEPTS FOR EXAMPLE REACTOR SYSTEM AND SUP- PORTING STRUCTURES

Many shock-isolation concepts have been developed since 1958 in various military programs for isolating complete internal structures as well as individual pieces of equipment from the ground shock induced by nuclear weapons (Kfoury, 1963). Most of these concepts are directly applicable to the case of seismic ground motion with minor changes in conceptual arrangement. To demonstrate the applicability of these concepts for internal isolation of a reactor system, an example is presented here. The principal equipment components of a reactor primary heat transfer system are large pressure vessels and pumps interconnected with spatial piping loop configurations. Due to thermal expansion considerations, the equipment components and piping must be supported with a minimum of support points on a common structure to avoid excessive relative motion of the components. Thus, the support structure for the reactor vessel and primary heat transfer system must be considered as the minimum "package" for seismic isolation. Smaller equipment components, such as fans, motor control centers, transformers, etc., can be isolated on separate platforms, if desired. A typical plan and cross section of the example building are shown in AA (1979). The structural configuration lends itself very well to seismic isolation. The containment structure is a rugged external shell to which horizontal isolators may be attached, while a double floor system may be arranged to provide space for vertical isolators. Various isolation methods for internal isolation are described next.

6.2.2.1 Elastomeric Spring Concept

Springs and mountings made of natural or synthetic rubber are widely used for vibration and shock isolation. Rubber sandwich pads

have been used in several ICBM bases to shock isolate cable trays, piping, and large water chillers. Large rubber shear sandwich isolators have been used for years to absorb the energy of ships striking the fender systems of piers and docks. Advantages of rubber springs include relatively high energy storage per unit volume, high damping, and less sound transmission than steel. Disadvantages include rapid deterioration at temperatures above about 150 to 250° F, depending on type of rubber; lack of resistance to oil in some types; drift or creep under long-continued loading; and deterioration from radiation depending on the type of rubber and type and level of radiation. Strict quality control procedures must be practiced during the fabrication of rubber sandwich isolators to assure the use of the proper elastomer type, correct durometer hardness, and bonding method.

6.2.2.2 Coil Spring Concept

The simplest isolation concept is to base-mount the structure on coil springs that resist both horizontal and vertical base motions. This concept was used to isolate the entire NORAD Combat Operations Center, a three-story building complex weighing approximately 25×10^6 lb. Base-mounted equipment platforms utilizing coil springs were also used extensively in the Safeguard ABM program. The principal disadvantage of this configuration is that the horizontal response is a coupled lateral displacement and rotation of the isolated structure. This should be avoided in the case of large masses, where small angles of rotation can result in large translations at extreme locations far from the center of gravity.

This isolation system concept was used in the Titan I Propellant Terminal. All spring units have simple universal joints at each end allowing only axial load in the spring. Since there is no lateral load on the springs, they

can be much smaller. Stability of the configuration is maintained by requiring the center of resistance of the springs to coincide with the system center of gravity, thus the rotational response is suppressed. The horizontal springs are designed to function only in compression even though the unit as a whole functions both in tension and compression. The vertical response is such that the vertical springs remain in compression. Since all springs remain in compression they may be preset which has the advantage of equalizing the stresses in the springs and permitting the use of a higher apparent allowable stress. The principal advantages of coil springs are their long-standing use for shock isolation and the fact that design feasibility has been demonstrated on Titan and Minuteman systems for similar requirements. Disadvantages are their small structural damping, large mass (5 to 10 times comparable liquid springs), and requirement of complex detail design to allow flexibility in all directions. Also, for large displacements long springs are required, making it necessary to increase the size of the containment structure provided the horizontal springs cannot be placed under floors or in pockets where they will not interfere with equipment or operations. Fabrication limitations (e.g., hot coiling, heat treatment, etc.) restrict the maximum spring wire diameter to 3-in. bar and the maximum coil diameter to less than 2 ft. Even if counterwound, triple-nested configurations are considered, the outside coil of a nested assembly largely controls design and spring characteristics. Thus, the amount of load carried by an individual spring assembly is limited by practical manufacturing considerations.

6.2.2.3 Coned or Tapered Disc Spring Concept

Conical discs (Belleville springs) or radially tapered disc springs may be assembled

in cylinders or cages and used as a unit in the same manner as the coil spring concept. The conical disc spring unit is made up of a series of individual cones stacked in alternate directions. The stack is housed in a sleeve which allows the cones to work in a series configuration, thus providing for large displacements. The radially tapered disc spring is a flat disc spring having the thickness proportional to the radius. Radially tapered disc springs are the most efficient in terms of energy storage capacity per unit weight of metal, but are not widely used due to their higher manufacturing costs. Disc springs owe their high efficiency to a relatively uniform stress distribution that permits a high average utilization of the strength of the metal throughout the disc, and to the fact that the critical stress is compressive. Both forms are of interest for the proposed application because of their greater total load capacity as compared to helical coil springs. The peculiar advantage of the conical disc spring is the fact that the stress and deflection produced in the spring are not proportional to the applied load, because the change in form upon deflection changes the load-deflection and load-stress relationships. This makes it possible to secure almost any desired variation of spring rate and to obtain a considerable range of deflection under almost constant load. The principal disadvantages of both the conical and the tapered disc springs are the fabrication costs and the length of assemblies required to handle large displacements. They also require universal joints or ball joints at each end as do most of the other concepts. Fabrication considerations limit the maximum useful disc thickness to about 1.5 in. in order to achieve full hardness penetration during heat treatment. The production equipment capacity of known current disc spring manufacturers limits the disc diameter to approximately 24 in.

6.2.2.4 Pendulum Concept

If the internal structure is suspended to form a pendular configuration, both horizontal and vertical flexibility may be provided with the same units. Horizontal isolation is accomplished by providing swivel joints at the point of attachment of the pendulum arms so that gravity provides the horizontal restoring force (or stiffness). Liquid or pneumatic springs are incorporated in the pendulum arms to provide the vertical flexibility. A liquid spring is a device that provides a high spring force in a small space envelope. Variable vertical damping may be incorporated in the isolator design. Disadvantages are that the compressibility and damping vary with temperature. Elastomer seals are a problem, and leakage will occur if periodical inspection and maintenance procedures are not carefully followed. Since most pendulum systems have low horizontal natural frequencies, they are displacement sensitive and require greater clearances, or "rattlespace," than other systems. They exhibit very little damping in the horizontal mode and thus require the addition of sway dampers. Pendulum isolators were used in most of the ICBM missile silos and used extensively in the Safeguard Program. Since, in general, only peripheral support may be provided for suspended platforms, extensive stiffening of the support structure is required for heavy, long-span configurations.

6.2.2.5 Spring Beam Concept

Spring beams were first used for the Tital I Power Houses, Control Centers, and Equipment Terminals. Each unit consists of two high strength steel beams supported by short pendulums (giving lateral flexibility). The pendulums hang from T struts that are rigidly mounted to the foundation slab. The bottom end of the pendulums are fastened to the spring beams at points away from the ends

toward the center with sufficient overhang to allow the required vertical flexibility. The isolated structure is supported on struts that rest on the end of the spring beams. This concept has an economic advantage because of the simplicity of fabrication which can be accomplished by any qualified steel fabricator. The major disadvantage is the large space requirement to accommodate the spring beam configuration.

6.2.2.6 Torsion Tube Concept

Torsion bar springs are commonly used when large loads must be supported. A variant of the torsion bar is the torsion tube which utilizes a tube (or bar) within a tube connected at one end and both loaded and reacted at the other. Torsion bars and torsion tubes have been used for shock isolation of transportation vehicles for many years. Studies in military programs concerning shock isolation of structures in deep buried cavities indicate combination torsion bar and tube units to be both feasible and economical. Although lever arms and kinematic linkages are necessary to utilize torsion tubes as horizontal and vertical springs, the torsion tube has the unique characteristic of having the spring axis normal to the direction of displacement. Thus, it can be used advantageously when the space in the direction of travel is limited. Like the spring beam units, the torsion tubes represent conventional technology and once installed require little or no maintenance. The major disadvantage of the torsion tube concept is the length of unit required and the resulting difficulty of replacing a unit should this become necessary.

6.2.3 ISOLATOR ELEMENT CONCEPTS

If isolator springs are classified by active material, there are four classifications: metal, liquid, pneumatic, and elastomeric springs.

The susceptibility of most elastomers and elastomer seals to temperature and radiation damage would militate against the use of pneumatic, liquid, or elastomer springs inside the containment shell of most power reactor plants. While elastomers and seals can be engineered to reduce temperature and radiation damage, the reliability of these elements would always be an open question. Thus, only metal spring configurations are considered further here.

In comparison of the coil spring concept to the disc spring concept, the disc spring concept would be preferred due to the fewer units required. Thus, the basic concepts considered are the coned or tapered disc spring concept, the spring beam concept, and the torsion tube concept. To determine if any of these concepts are feasible, preliminary sizing of the active elements of the isolators must be accomplished.

6.2.3.1 Coned-Disc Isolator Strut

The basic isolator unit configuration consists of a disc spring assembly housed in a cylindrical, pin ended, axially loaded strut. For horizontal isolation, the struts would be double acting with the disc springs in compression under either tensile or compressive loading. End connections could be tongue and yoke with the spherical pivots. (Commercial spherical pivots which accommodate up to 1000 kips of load are available.) Vertical shock isolator units would be of similar construction except that they need act only in compression and would be of the single acting type.

6.2.3.2 Spring Beam Isolator Unit

This isolator unit consists of steel plate and shapes. The steel plate and shapes used for fabrication may be obtained at the lowest price per pound for a given steel. Bracing is provided to carry the lateral restoring forces created at the pendulums through the spring

beams and struts to the seismic isolated floor. Since the horizontal frequency is dependent only on the length of the pendulum arm, the design of the system to limit displacements (i.e., short pendulum arms) can be a difficult task.

6.2.3.3 Torsion Tube Isolator Units

This configuration utilizes nesting of tubes to conserve space. It has the additional advantage of concentrating the reactions from both torsion tubes on one anchor plate. The torque arms are interconnected to form a four-bar linkage for the torsion tubes acting as horizontal springs. The pantograph type kinematic linkage provides a constant stiffness in any horizontal direction.

6.2.4 APPLICATION OF TORSION TUBE CONCEPT TO INTERNAL ISOLATION OF EXAMPLE REACTOR SYSTEM

Since the nested torsion tube concept allows maximum loading of an isolator element with minimum spatial interference, the torsion tube concept was chosen for a more detailed study. Because the torsion tube concept is relatively simple, a positive result will leave fewer questions unanswered than other concepts. Also, prior studies conducted for military programs have indicated that this concept has the lowest cost potential.

To demonstrate technical feasibility of the concept, the tube units will be sized for an example reactor system weight (including supporting structure) and seismic design criteria. The weight of the example system will be taken as $W_T = 40 \times 10^6$ lb. The reactor system and supporting structure will be treated as a rigid body in translation only for both the vertical and horizontal modes. The center of resistance of the isolator array will be assumed to coincide

with the system center of gravity. The secondary interaction of vertical and horizontal motion on the vertical isolators will not be considered for this preliminary sizing. Thus, the isolation system may be treated as independent single-degree-of-freedom spring-mass systems in the vertical and horizontal directions. The natural frequency of both the vertical and horizontal systems will be taken as 0.8 Hz, which will be assumed to be within an isolation frequency window formed by the response limits (peak displacement and peak acceleration). The seismic design criteria to be used for preliminary sizing of the isolators is given in Table 6-1.

6.2.4.1 Horizontal Isolators

The kinematics of the horizontal isolator assembly is illustrated in Figure 6-1. The pantograph type linkage provides a constant stiffness in any horizontal direction. Note that a pair of tubes is required at each circumferential location and that not all tubes will be fully stressed for a given level and direction of horizontal response of the isolated system. The applied torque for a given tube will range between $T = 0$ and $T = Pa$ depending upon the direction of response. Thus, each tube must be sized to resist stresses for $T = Pa$. The torsion tube design parameters are illustrated in Figure 6-2, with notations listed in Table 6-2.

TABLE 6-1. DESIGN VALUES FOR ISOLATORS

Direction	f_n , Hz	Total Δ , in.	Max Force, kips	δ in.	$\dot{\delta}$ in./sec	$\omega\delta$ g's
Horizontal	0.8	9	24,000	9	46	0.6
Vertical	0.8	21	56,000	6	31	0.4

TABLE 6-2. DESIGN PARAMETER NOTATION

Parameter	Description	Parameter	Description
δ	Relative peak displacement of isolator	G	Shear modulus of steel, taken as 11.5×10^6 psi
$\dot{\delta}$	Relative peak velocity of isolated mass	g	Acceleration of gravity
$\ddot{X}$	Absolute peak acceleration of isolated mass	ω	Natural frequency of isolation system, radians/sec
N	Number of torsion tubes required	D_i	Outside diameter of tube
K_δ	Effective lateral spring constant of single isolator unit	d_i	Inside diameter of tube
K_ϕ	Torsional spring constant of single torsion tube unit	t_i	Tube thickness $[(D_i - d_i)/2]$
$\bar{M}_T$	Isolated mass	J_i	Polar area moment of inertia of tube
F_T	Peak total reaction force of isolator array	τ_i	Maximum shear stress in tube material
P	Force normal to torque arm	τ_{dwi}	Dead weight stress
T	Torque resisted by single torsion tube unit	τ_{mr}	Torsional modulus of rupture
T_{dw}	Dead weight torque	τ_a	Allowable design stress
ϕ	Angular rotation of torsion tube assembly	i=1,	Subscript denoting inner tube
a	Length of torque arm	i=2,	Subscript denoting outer tube
ℓ	Length of torsion tube assembly	s	Maximum uniform spacing of isolator centerlines necessary to accommodate N isolator units or N/2 pairs of isolator units

(a) Horizontal isolator linkage kinematics at various circumferential locations

$$P, \delta, P = P_t / \sqrt{2}, \delta = \Delta / \sqrt{2}$$

$$P_t / \Delta = K_t = \frac{P_t / \sqrt{2}}{\Delta / \sqrt{2}} = \frac{P}{\delta} = K_\delta$$

(c) Tangential motion

$$\delta, P, P \cong \frac{P_r}{\sqrt{2}}, \delta \cong \frac{\Delta}{\sqrt{2}}$$

$$\frac{P_r}{\Delta} = K_r = \frac{P_r / \sqrt{2}}{\Delta / \sqrt{2}} = \frac{P}{\delta} = K_\delta$$

(b) Radial motion

(d) Motion normal to torque arm

FIGURE 6-1. KINEMATICS OF HORIZONTAL TORSION TUBE LINKAGE

FIGURE 6-2. TORSION TUBE DESIGN PARAMETERS

The rotational response of each nested tube will be limited to 20 deg or less, thus, small angle approximations are valid for the preliminary sizing.

The following relationships govern the sizing of the horizontal torsion tube isolators:

$$F_T = \frac{N}{2} K_\delta \delta; \quad \delta \approx \phi a \quad (6-1)$$

$$T = Pa = \frac{2F_T}{N} a = K_\phi \phi; \quad K_\phi = \frac{G}{l} \frac{J_1 J_2}{J_1 + J_2} \quad (6-2)$$

$$K_\delta = \frac{K_\phi}{a^2} = \frac{G}{2a^2 l} \frac{J_1 J_2}{J_1 + J_2} \quad (6-3)$$

$$\omega^2 = \frac{N/2 K_\delta}{\bar{M}_T} = \frac{NG (J_1 J_2)}{2 \bar{M}_T a^2 l (J_1 + J_2)} \quad (6-4)$$

$$F_T = \bar{M}_T \ddot{X} \approx \bar{M}_T \omega \dot{\delta} = \frac{\bar{W}_T \omega \dot{\delta}}{g} \quad (6-5)$$

$$\tau_i = \frac{TD_i}{2J_i} \quad (6-6)$$

$$J_i = \frac{\pi}{32} (D_i^4 - d_i^4) \quad (6-7)$$

The selection of size of tubing is based on engineering judgment, and once the isolator is sized, the space required is checked against that available. In the interest of economy, commercial pipe sizes and wall thicknesses should be used. It is not anticipated that the inside and outside tubes can be perfectly balanced using only commercial sizes, but the

closer their maximum torque capacity, the more efficient their use of material. The outside diameter of the inner tube and the inside diameter of the outer tube must be close enough to be welded at the end but with sufficient clearance to allow for normal tolerances for the particular pipe.

A basic seven-step preliminary design procedure using Equations 6-1 through 6-7 for sizing the tube isolator units is outlined in Appendix A of AA (1979). The application of this design procedure yielded the results tabulated in Table 6-3 for the horizontal isolator design (computational details in Appendix A of AA, 1979). Since the minimum isolator spacing is less than 4 ft, there is ample room for grouping of units to provide space for penetrations and entrances.

6.2.4.2 Vertical Isolators

The governing relationships for the design of the vertical isolators are essentially the same as those for the horizontal. However, since the vertical isolators have to carry the dead weight as well as the dynamic loading, the expression for the additional dead weight stress within each tube is

$$\tau_{dwi} = \frac{W_T a D_i}{N 2 J_i} \quad (6-8)$$

and the total stress is

$$\tau_i = \tau_{dwi} + \frac{F_T a D_i}{N 2 J_i} \quad (6-9)$$

Since the four-bar linkage is not used for the vertical isolators, then

$$F_T = N K_\delta \delta \quad (6-10)$$

and

$$T = \frac{F_T}{N} a \quad (6-11)$$

The ratio of dead weight stress to total stress is

$$\frac{\tau_{dwi}}{\tau_i} = \frac{1}{1 + \frac{\omega \dot{\delta}}{g}} \quad (6-12)$$

TABLE 6-3. PRELIMINARY SIZING RESULTS FOR HORIZONTAL TORSION
TUBE ISOLATION OF EXAMPLE REACTOR SYSTEM

Note W_T = Total weight of structure = 40×10^6 lb
 f_n = Natural frequency of isolator system = 0.8 Hz
 $w_n \delta$ = Response acceleration of the structure before isolation
= 0.6g

Description	Symbol	Result
Inner tube diameter	D_1	10.75 in
Outer tube diameter	D_2	12.75 in
Inner tube thickness	t_1	1.0 in
Outer tube thickness	t_2	0.562 in
Torque arm length	a	30 in
Tube assembly length	ℓ	15 ft
No. of isolator pairs	$N/2$	96
Max. available space	s	4.6 ft/pair
Material specification	-	AISI 4300

6.2.5 CONCLUSIONS

The evaluations of seismic isolation concepts considered here result in the following conclusions for reactor systems with low-pressure heat transfer systems

- Both vertical and horizontal seismic isolation are required to achieve the performance objectives.
- Based on a qualitative assessment, external isolation concepts will not be cost effective compared to internal isolation concepts.
- Experience gained in military shock-isolation programs is directly applicable to internal reactor system isolation.
- Metal isolator elements are preferable to elastomer elements for internal isolation schemes.
- An isolation system based on torsion tube isolator units is the preferred concept for internal isolation of reactor systems and supporting structures

- Preliminary design sizing of torsion tube isolators for an example reactor weight of 40×10^6 lb indicates that the concept is technically feasible and that the layout of the units can be accomplished without major configuration changes in nuclear plant structures.

6.3 DEVELOPMENT OF PLBR SEISMIC ISOLATION SYSTEM

In order to determine the feasibility of seismic isolation support for future breeder reactors, it is important to have a concept of the structure and system layout for such facilities. The general arrangement and engineering features of a Prototype Large Breeder Reactor (PLBR) 1000-MWe power plant, developed conceptually by Atomics International and Burns and Roe (AI/B&R, 1977), have been selected as a guideline for the determination of isolation feasibility. The basic structure and system layout are given in AA (1979).

6.3.1 APPLICATION OF TORSION TUBE/ HYSTERETIC DAMPER ISOLATION CONCEPT TO PLBR

The torsion tube concept of internal isolation to the PLBR can be such that the entire containment structure for the reactor and primary heat transfer system would be isolated within the confinement annulus formed by the containment shell and shield building. A slight increase in confinement annulus space approximately equivalent to the shield building shell thickness (3 ft) would be necessary to accommodate the horizontal tube isolators. Additional space (approximately 8 ft) between the containment slab and the base mat, to accommodate the vertical isolators, would be required. An estimate of the total weight of the isolated containment, based on data provided in the PLBM development study (AI/B&R, 1977), was computed to be approximately $W_T = 200 \times 10^6$ lb, which is a factor of five greater than the example isolation system weight considered previously. Approximately 30% of the total isolated PLBR containment weight is allocated to the reactor assembly, shielding, and central concrete supports. Thus, three independent concentric rings of radially placed isolators are envisioned. This would allow sizing of the vertical isolators to accommodate the dead weight distribution. The horizontal isolators would be placed in the confinement annulus at appropriate locations to ensure that the rigid body horizontal translation mode is decoupled from the rotational mode. In general, two circumferential tiers of horizontal isolator reactions, one tier at the containment base slab level and the other tier at the operating floor level, are envisioned.

The internal isolation concept limits the containment penetrations to the intermediate heat transfer system piping, the diverse decay heat removal system piping, power and control conduits, and other umbilicals which can be

designed with sufficient flexibility to accommodate the differential motion of the isolated structure. Thus, the fixed in-place reactor fuel transfer system developed for the PLBR is not compatible with the internal isolation concept. An alternate system of fuel transfer will have to be utilized for the seismic isolated PLBR. Both the Clinch River Breeder Reactor Plant (CRBRP) and the Fast Flux Test Facility (FFTF) have utilized ex-vessel fuel transfer machines (like the CRBRP-EVTM or the FFTF-CLEM) which enter the containment structure through a large penetration of a rail system. Penetration closures can be used for fuel transfer machine access.

6.3.1.1 Dynamic Response Characteristics

Given that the isolated PLBR containment structure has adequate stiffness to ensure that the free-free vibration modes approximate the flexible structure behavior, the seismic response of the isolated structure will be governed by rigid body dynamics. The horizontal isolators would be grouped in pairs using a four-bar linkage that provides an equal stiffness (spring constant) in any horizontal direction for each of the circumferential isolator pairs. In addition, the computed location of the confinement center of gravity in the horizontal plane is very near the reactor center, and the distribution of mass about any vertical plane through the containment center of gravity is approximately symmetric. Thus, the rigid body motion of the isolated containment structure may be approximated with the planar model shown in Figure 6-3. The resultant spring constants of each horizontal isolator tier or each vertical support ring are represented simply by the sum of the isolator stiffnesses within each tier or ring. Further, since the vertical isolators are closely spaced around the circumference of each support ring, the overturning resistance of the body may be expressed as

proportional to a rotational spring constant, defined by assuming that the vertical spring stiffness is uniformly distributed around each support ring circumference. The equations of motion for the rigid body response of the isolated confinement structure are given in Figure 6-3. As can be noted, coupling between the horizontal and vertical modes will occur unless the following relationship is satisfied:

$$h K_B = e K_T \quad (6-13)$$

where e and h are the vertical locations of the horizontal isolator tiers, above and below the center of gravity, respectively, while K_T and K_B represent the resultant spring constants of the top and bottom isolator tiers, respectively. Equation 6-13 may be used to determine the number of isolator pairs required in each tier. The number of isolators required within each vertical support ring is determined by the distribution of dead weight.

EQUATIONS OF MOTION:

$$\begin{bmatrix} J_c & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & H \end{bmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{u}_c \\ \ddot{z} \end{pmatrix} + \begin{bmatrix} (K_\theta + h^2 K_B + e^2 K_T) & (-h K_B + e K_T) & 0 \\ (-h K_B + e K_T) & (K_B + K_T) & 0 \\ 0 & 0 & K_v \end{bmatrix} \begin{pmatrix} \theta \\ u_c \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -H \ddot{x}_g \\ -H \ddot{z}_g \end{pmatrix}$$

FIGURE 6-3. RIGID BODY MOTION OF ISOLATED PLBR CONTAINMENT

6.3.1.2 Seismic Design Criteria Selection

The seismic response spectra selected for the sizing of isolator elements were constructed using the procedures described in Regulatory Guide 1.60 with a maximum SSE horizontal ground acceleration of 0.5 g and a maximum SSE vertical ground acceleration of 0.33 g. These criteria are sufficiently high to cover any site that would be considered for a nuclear plant. Tripartite plots of horizontal and vertical spectra are presented in Figure 6-4. A system meeting these criteria can be designed to meet lesser criteria at a reduced cost. The response limitations or dynamic performance boundaries for the isolated containment structure are also indicated in Figure 6-4. The displacement response is assumed to be limited to 12 in. in both the vertical and horizontal directions. This bound was selected as a nominal estimate of the design flexibility of an expansion loop for PLBR piping (36 in. dia.) crossing the isolation system interface. The acceleration bounds selected were 0.75 g and 0.5 g for the horizontal and vertical directions, respectively. The choice of these bounds is based upon a qualitative assessment of the resistance of equipment designed without seismic considerations and the horizontal force factors given in the Uniform Building Code (UBC, 1976) for the design and anchorage of equipment for essential facilities.

The important influence of damping on system response should be noted. For an isolation system with 2% critical damping, the horizontal response can be kept within the response limits only for a unique system frequency. To create a frequency window, the horizontal acceleration bound would have to be increased to approximately 1.0 g. For the selected response limits, a system with 5% critical damping is required in order to obtain a reasonable frequency window. If the horizontal acceleration bound was further restricted to 0.5 g, a

(a) Regulatory guide 1.60 horizontal spectra

(b) Regulatory guide 1.60 vertical spectra

FIGURE 6-4. SEISMIC DESIGN CRITERIA

system with 5% critical damping can be kept within the displacement limit only for a unique system frequency. In the vertical direction a system with 5% critical damping has a reasonable frequency window even if the vertical acceleration bound is restricted to 0.33 g. Thus, it is concluded that 5% critical damping must be achieved to keep the isolation system response within the selected limits. Since only 2% critical damping can be justified for a torsion tube isolator without testing of a prototype unit, the addition of energy/absorbing devices to the system must be considered as necessary to

achieve the required damping level. Dampers utilizing fluids and seals or elastomer elements are not considered acceptable for application within the PLBR confinement. Friction devices would have questionable reliability, thus metallic hysteretic dampers are the logical choice.

It should be noted that considerable friction or Coulomb damping is possible, due to the linkage joints and bearing area supporting the isolator tubes at the lever end. However, the magnitude of this effective damping is difficult to estimate a priori, and can only be determined by testing a full scale prototype. Thus, for conceptual design, damper units must provide the additional 3% critical damping above the nominal level of 2% critical damping.

6.3.2 SELECTION OF DAMPER ELEMENTS

Hysteretic energy devices have been used for several years as reusable shock absorbers in military applications (Jackson et al. 1972). Recent applications are for automobile bumper systems, highway barriers, and aircraft crash seats. One type of hysteretic energy absorber is the "torus" device which rolls a series of toroidal wires, or single helix wire, compressed between two concentric tubes (Schrader, 1965). Stroking of the device causes the wires to roll and absorb energy by cyclic plastic straining. These devices are commercially available (trade name TOR-SHOK). Another type of device utilizes ring elements aligned on toroidal retainers which are compressed between concentric tubes. Stroking of the device rolls the rings which absorb energy by cyclic plastic straining. Another type of cyclic strain energy absorber is rotary and is also commercially available (trade name ROTO-SHOK). It consists of a set of tubes which are axially aligned and compressed between two concentric drums. Relative rotation of the device rolls the tubes which absorb energy by cyclic plastic straining. All

of these devices produce relatively constant resisting force or torque and thus may be idealized as true elasto-plastic elements.

The integral incorporation of controlled inelastic deformation (Fintel and Khan, 1969) and ductile restraints (Casper, 1970) has been suggested for design of multistory concrete structures using "soft-story" or base isolation concepts. The use of yielding structural support elements to protect the porcelain columns of large high voltage power apparatus from earthquake damage has also been reported (Gilmour and Hitchcock, 1971).

Several different types of hysteretic damper configurations have been developed in recent years for application to seismic base isolation of structures (Skinner et al., 1975b, 1977; Kelly and Tsztoo, 1978). These damper units are all structural configurations, rather than a kinematic device. The force/deflection behavior of these units may be idealized as a bilinear element. The torsion-beam damper has been shown to be a convenient form for incorporation into base isolation schemes. These units are easily fabricated from mild steel and have been extensively tested. Based upon the test data, sizing relationships have been developed for the damper unit design. Since this design data is available within the published literature and the torsion-beam configuration is compatible with the torsion tube isolator concept, it was selected as the preferred damper element to be incorporated into the PLBR isolation system. A four-bar linkage would be used for the horizontal damper pair.

The physical dimensions and bilinear stiffness parameters of the torsion-beam damper units selected for application to the PLBR isolation concept are given in Table 6-4. These units will provide acceptable performance for the given values of nominal (maximum) lever arm deflection, D . Once the basic isolation

TABLE 6-4. TORSION BEAM DIMENSIONS AND PARAMETERS (Skinner et al., 1976)

Direction	D, in.	r, in.	t, in.	b, in.	ℓ, in.	F' _d , lb	tan θ'/F' _d , in. ⁻¹	tan φ'/F' _d , in. ⁻¹
Vertical	6	33	3.75	15	18.75	128 x 10 ³	0.297	0.019
Horizontal	12	66	4.5	18	22.5	111 x 10 ³	0.297	0.019

system parameters are chosen, the mean effective frequency and mean effective viscous (linear) damping factor for the bilinear system can be computed from the relationships given by Skinner et al. (1975a). The effect of the four-bar linkage on the bilinear behavior of a damper pair has not been established.

6.3.3 DEVELOPMENT OF HORIZONTAL ISOLATION SYSTEM

The governing design relationships for sizing the horizontal isolators for PLBR isolation

are essentially the same as those used to size the isolator elements for the PWR isolation system. However, the governing relationships must be modified to account for the addition of the damper units. The expressions for the natural frequencies and the parameters for sizing the horizontal torsion tube isolators are given in AA (1979). Sizing results of an example horizontal isolation system for a PLBR is presented in Table 6-5, in accordance with a detailed discussion in AA (1979).

TABLE 6-5. PRELIMINARY SIZING RESULTS FOR HORIZONTAL TORSION TUBE/BEAM DAMPER ISOLATION OF PLBR

Note: W_T = Total weight of structure = 200×10^6 lb
 f_n = Natural frequency of isolator system = 0.8 Hz
 $\omega_n \delta$ = Response acceleration of the structure before isolation = 0.6g

Description	Symbol	Result
Damper yield level	$\frac{N_d F'_d}{2W_T}$	0.1
No. damper pairs	$N_d/2$	180
Damper Frequency	f_d	0.54 Hz
Inner tube diameter	D_1	14.0 in.
Outer tube diameter	D_2	16.0 in.
Inner tube thickness	t_1	0.938 in.
Outer tube thickness	t_2	0.656 in.
Torque arm length	a	30 in.
No. isolator pairs	$N/2$	140
Upper tier	$N_T/2$	72
Lower tier	$N_B/2$	68
Tube assembly length	ℓ	18.3 ft
Maximum available spacing	s	8.9 ft/pair
Material specification	--	AISI 4300

6.3.4 DEVELOPMENT OF VERTICAL ISOLATION SYSTEMS

The general preliminary design procedures for sizing the vertical isolators for PLBR isolation follows the same sequence of design steps used to size the vertical isolators for the PWR isolation system. The governing relationships must also be modified to account for the addition of damper units. However, the damper units do not support any of the dead weight of the isolated containment structure. Further, the layout of the isolators in the three support ring configuration is assumed to be sufficient to ensure that the isolators are uniformly loaded. Thus, the stress in each tube due to the dead weight is

$$\tau_{dwi} = \frac{W_T aD_i}{N2 J_i} \quad (6-14)$$

Both the isolators and dampers carry the additional dynamic loading in a parallel spring configuration. Thus, the total stress in an isolator unit is

$$\tau_i = \tau_{dwi} + \frac{aD_i}{2 J_i} P \quad (6-15)$$

where the total dynamic force resisted by the isolator units is

$$NP = \frac{NK_\delta}{NK_\delta + N_d K_d} F_T \quad (6-16)$$

The ratio of dead weight stress to total stress is

$$\frac{\tau_{dwi}}{\tau_i} = \frac{1}{1 + \frac{w\delta}{g} \left(1 - \frac{w_d^2}{w^2}\right)} \quad (6-17)$$

The total volume of active material required for the vertical isolators may be expressed as

$$V_T = \frac{W_T g G}{4} \left(\frac{1}{\sqrt{w^2 - w_d^2}} + \frac{\delta}{w g} \sqrt{w^2 - w_d^2} \right)^2 \times \left[\left(\frac{D_1}{\tau_1} \frac{D_2}{\tau_2} \right) \left(\frac{A_1 + A_2}{J_1 + J_2} \right) \right] \quad (6-18)$$

The sizing results for selecting an example vertical isolation system for a PLBR are presented in Table 6-6, in accordance with a detailed discussion in AA (1979). The total number of isolators and dampers are distributed to each support ring based upon the distribution of dead load supported by each ring. For the PLBR containment the total weight of 200×10^6 lb is allocated as follows:

Inner support ring: 60×10^6 lb
Center support ring: 50×10^6 lb
Outer support ring: 90×10^6 lb

6.3.5 CONCLUSIONS

The development of a PLBR seismic isolation system considered here results in the following conclusions.

- Consideration of the PLBR conceptual design (AI/B&R, 1977) indicates that, due to the required equipment cells, the entire primary heat transfer system, reactor vessel, shielding, and support structures within the containment vessel should be isolated as a single unit within the confinement shell.
- The fixed in-place fuel transfer concept developed for the PLBR is not compatible with an internal seismic isolation concept. Thus, an ex-vessel fuel transfer machine will have to be considered for an isolated PLBR.
- The addition of damper devices is required to keep the isolation system response within the bounds imposed by the seismic design criteria. An effective critical damping level of 5% should be achieved by the isolation system.
- Hysteretic damping elements are recommended for damper units. The

TABLE 6-6. PRELIMINARY SIZING RESULTS FOR VERTICAL TORSION
TUBE/BAM DAMPER ISOLATION OF PLBR

Note W_T = Total weight of structure = 200×10^6 lb
 f_n = Natural frequency of isolator system = 0.965 Hz
 $w_n \delta$ = Response acceleration of the structure before isolation
= 0.48g

Description	Symbol	Result
Damper yield level	$\frac{N_d F'_d}{2W_T}$	0.1
No. damper pairs	$N_d/2$	78
Damper Frequency	f_d	0.54 Hz
Inner tube diameter	D_1	22.0 in
Outer tube diameter	D_2	24.0 in
Inner tube thickness	t_1	1.125 in
Outer tube thickness	t_2	0.969 in
Torque arm length	a	48 in
Total isolators	N	200
Inner spt. ring	N_i	60
Center spt. ring	N_c	50
Outer spt. ring	N_o	90
Tube assembly length	l	27.5 ft
Material specification	--	AISI 4300

torsional-beam damper concept is the preferred damper configuration due to fabrication simplicity and compatibility with the torsion tube concept.

- e. The torsion-beam damper units may be sized for preliminary design using available published test and design data.
- f. Preliminary design sizing of the torsion tube isolators for a PLBR containment weight of 200×10^6 lb indicates that vertical and horizontal seismic isolation is technically feasible and that the layout of the units can be accomplished without major configuration changes in the PLBR conceptual design.

6.4 ASSESSMENT OF PLBR ISOLATION SYSTEM

An assessment of the internal seismic isolation system developed for the PLBR using a vertical and horizontal array of torsion tube isolators coupled with torsion beam dampers is conducted from three standpoints: (1) an analytical comparison of the response of both the fixed base PLBR and isolated PLBR when subjected to the same input ground motion, (2) the feasibility of accommodating large relative displacements for large diameter pipe crossing the isolation system interface, (3) an estimate of system costs and further development requirements. A detailed account of this assessment is provided in AA (1979). Based on this assessment the following conclusions

regarding the PLBR isolation system are obtained:

- a. A comparison of the horizontal floor response spectra for the fixed-base PLBR and seismic isolated PLBR, computed from the response of representative dynamic models subjected to the same ground motion input, indicates that the PLBR isolation system behaves as a narrow band filter tuned to the mean effective system frequency. The filtering behavior of the isolation system effectively removes the wide-band frequency content associated with seismic ground motion. The resulting floor response time history for the isolated PLBR is a pure sinusoid of varying amplitude which will not produce an amplified response for equipment (or subsystems, such as piping) with natural frequencies greater than 3 Hz.
- b. For equipment (and piping) with natural frequencies greater than 3 Hz, the seismic qualification (and design) of equipment within an isolated PLBR containment can be accomplished by demonstration of structural adequacy for static forces equivalent to a constant peak floor acceleration level. For a 0.5 g SSE ground motion input, the specification of a constant 1 g acceleration level for equipment located within the isolated containment would be a conservative design requirement. Functional qualification and/or fragility testing of critical equipment can be conducted using a single harmonic input waveform with frequency equal to the isolation system frequency.
- c. For equipment (or phenomena) with natural frequencies less than 3 Hz, amplification will occur within the isolated PLBR containment. Extreme amplification will occur within the frequency range 0.7 to 0.9 Hz. The effect of excessive fluid sloshing is identified as a potential problem.
- d. Additional mechanical damping devices should be considered to attenuate the low frequency (0.5 to 1.0 Hz) response of the isolation system and to prevent low level vibration during normal operation.
- e. Analysis (0.5 g SSE input criteria) indicates that 10 in. of displacement must be accommodated by piping and other umbilicals crossing the annular interface between the fixed confinement shell and the isolated containment structure. Evaluation of inverted U-loop piping configurations with gimbaled flexible joints (bellows) indicate that 10 in. of displacement can be accommodated by large (36 in.) diameter Class 2 piping using current Class 2 high temperature design criteria and sodium piping technology.
- f. From the results of the study it is concluded that the seismic isolation approach considered for the PLBR is technically feasible and that economic feasibility is the major question to be resolved in future studies.
- g. For an isolated PLBR design based on any type of isolator spring element, the seismic isolation costs will be nearly proportional to the isolated weight. For the torsion tube/damper

concept considered herein, a preliminary cost bound of \$0.10/lb of isolated weight is estimated. Substantial weight reduction is anticipated if the PLBR conceptual design is reconsidered, and excess concrete is eliminated from the containment structure.

- h. Cost/benefit studies are required to determine the economic feasibility of the isolation concept. These studies require the input of engineers experienced in reactor design and construction to assess the cost considerations associated with system integration, weight reduction, equipment qualification, construction, operation, maintenance, and inspection of the isolated reactor system.
- i. Once the overall economic feasibility of the isolation concept has been

established, prototype isolator and damper units will need to be fabricated and subjected to full-scale structural testing (low-cycle fatigue) to demonstrate concept reliability.

- j. Scale model tests of the isolated containment using a biaxial shake table could be conducted to further demonstrate the effectiveness of the isolation system. It is important to note that if a power reactor is designed and constructed with an isolated containment, the isolation system could be subjected to full-scale SSE response levels using appropriate dynamic testing methods. It is doubtful that full-scale SSE verification testing can ever be provided for an unisolated system.

REFERENCES

- Agbabian Associates (AA). (1972a) User's Guide for PRDEPS, DEPS, and DPSOUT Codes, U-7128-2-2175. El Segundo, CA: AA, Jan.
- . (1972b) User's Guide for TRI/SAC Code, R-7128-4-2314. El Segundo, CA: AA, May.
- . (1975a) Statistical and Probabilistic Considerations in Defining Seismic Input Criteria, TOP-AA104. Washington, DC: Energy Res. and Dev. Admin., May.
- . (1975b) Evaluation of Earthquake Ground Motion Characteristics at Nuclear Plant Sites, TOP-AA107. El Segundo, CA: AA, Jun.
- . (1975c) Direction of Approach of Seismic Input Motions and Superposition of Modal and Directional Dynamic Responses, TOP-AA105. El Segundo, CA: AA, May.
- . (1975d) User's Guide for FEDRC Code, U-7523-2-3868. El Segundo, CA: AA, May.
- . (1976) User's Guide for TRI/SAC Code, 2nd rev. ed. R-7128-4-4102. El Segundo, CA: AA, May.
- . (1978) Designing Facilities to Resist Nuclear Weapons Effects: Shock Isolation Systems, TM 5-858-4. El Segundo, CA: AA, Dec.
- . (1981) User's Guide for STARS Code, U-4700-5261. El Segundo, CA: AA, Dec.
- Amer. Concrete Inst. (ACI). (1971) Building Code Requirements for Reinforced Concrete, ACI-318-71. Detroit: ACI.
- Amer. Nat'l Standards Inst. (ANSI). (1974) Quality Assurance Requirements for the Design of Nuclear Power Plants, ANSI-N45.2.11-1974. New York: ANSI.
- Amer. Soc. of Civil Eng. (ASCE). (1972) Safety and Reliability of Metal Structures. New York: ASCE.
- Amer. Soc. of Mechanical Eng. (ASME). (1974) ASME Boiler and Pressure Vessel Code, (Section III: "Nuclear Power Plant Components" and Section VIII: "Pressure Vessels," Div. 1 and 2). New York: ASME.
- Ang, A.H-S. (1972) "Probabilistic Bases of Safety, Performance, and Design," Proc. ASCE Spec. Conf. on Safety and Reliability of Metal Structures, Pittsburgh, Nov.
- . (1973) "Structural Risk Analysis and Reliability-Based Design," Jnl Struct. Div., ASCE, 99:ST9, Sep, pp 1891-1910.
- . (1977) "Risk, Reliability, and Safety Analyses in Engineering Design," Structural and Geotechnical Mechanics, ed. W.J. Hall. Englewood Cliffs, NJ: Prentice-Hall, pp 24-54.
- Ang, A.H-S. and Amin, M. (1968) "Reliability of Structures and Structural Systems," Jnl Eng. Mech. Div., ASCE, 94:EM2, Apr, pp 671-691.
- Ang, A.H-S. and Cornell, C.A. (1974) "Reliability Bases of Structural Safety and Design," Jnl Struct. Div., ASCE, 100: ST9, Sep, pp 1755-1769.
- Ang, A.H-S. and Ellingwood, B.R. (1971) "Critical Analysis of Reliability Principles Relative to Design," Proc. 1st Int. Conf. on Applications of Statistics and Probability to Soil and Structural Eng. Hong Kong: Hong Kong Univ. Press.
- Ang, A.H-S. and Newmark, N.M. (1971) Development of a Transmitting Boundary for Numerical Wave Motion Calculations, DASA Report 2631. Urbana, IL: Newmark Consulting Engineers, Apr.
- Ang, A.H-S. and Tang, W.H. (1975) Probability Concepts in Engineering Planning and Design (Vol 1): Basic Principles. New York: Wiley.
- Archer, J.S. (1963) "Consistent Mass Matrix for Distributed Mass Systems," Jnl Struct. Div., ASCE, 89:ST4, Aug, p 61.
- Army Engineer Waterways Exp. Station (AEWES). (1972) Earthquake Resistance of Earth and Rock-Fill Dams: Feasibility of Simulating Earthquake Effects on Earth and Rock-Fill Dams Using Underground Nuclear Events, Misc. Paper-S-71-17, Rpt. 3. Vicksburg, MS: AEWES, Sep. (AD 748 816)
- Atomics International and Burns and Roe (AI/B&R). (1977) Liquid Metal Fast Breeder Reactor Conceptual Plant Design 1000 MWe, FBR-77-1,-2,-3,-4. Canoga Park, CA: AI and Oradell, NJ: B&R, May. (TID-27701-1,-2,-3,-4)
- Bathe, K.J. (1971) Solution Methods for Large Generalized Eigenvalue Problems in Structural Engineering, SESM 71-20. Berkeley: Univ. of Calif.

- Belytschko, T. and Chiapetta, R.L. (1973) A Computer Code for Dynamic Stress Analysis of Media/Structure Problems with Nonlinearities (SAMSON), Vol. I-IV, AFWL-TR-72-104. Kirtland AFB, NM: AF Weapons Laboratory, Feb.
- Benjamin, J.R. and Cornell, C.A. (1970) Probability, Statistics and Decision for Civil Engineers. New York: McGraw-Hill.
- Bergstrom, R.N.; Chu, S.L.; and Small, R.J. (1971) "Seismic Analysis of Nuclear Power Plant Structures," Jnl Power Div., ASCE, 97:2, Mar.
- Biggs, J.M. (1964) Introduction to Structural Dynamics. New York: McGraw-Hill.
- Biggs, J.M. and Roesset, J.M. (1970) "Seismic Analysis of Equipment Mounted on a Massive Structure," Seismic Design for Nuclear Power Plants, ed. R.J. Hansen. Cambridge, MA: MIT.
- Bleiweis, P.B. (1970) The Dynamic Response of the Enrico Fermi Nuclear Power Plant During Blasting, M.S. thesis, UCLA.
- Bleiweis, P.B. et al. (1973) "Simulation of Strong Motion Earthquake Effects Using Explosive Blasts," Nuclear Eng. and Des., 25:1, Jun, pp 126-149.
- Blume, J.A. (1969) "Response of Highrise Buildings to Ground Motion from Underground Nuclear Explosion," Bull. Seismol. Soc. Amer., 59:6, Dec, pp 2343-2370.
- Blume, J.A.; Sharpe, R.L.; and Kost, G. (1971) Earthquake Engineering for Nuclear Reactor Facilities, JAB-101. San Francisco: URS/J.A. Blume and Assoc., Jan.
- Bolotin, V.V. (1969) Statistical Methods in Structural Mechanics. San Francisco: Holden-Day.
- Bosshard, W. (1975) On Stochastic Load Combination. Stanford, CA: The John A. Blume Earthquake Eng. Center.
- Boyd, M.A. and Huang, C.C. (1977) "Experiences on Shock Isolation of Equipment in the SAFEGUARD System," Shock & Vib. Bull., 47, Pt. 1. Washington, DC: Naval Res. Lab., Sep.
- Carder, D.S. (1936) "Vibration Observations," Earthquake Investigations in California 1934-1935. Washington, DC: USGPO, 75 pp.
- Carder, D.S. and Cloud, W.K. (1959) "Surface Motions from Large Underground Explosions," Jnl Geophys. Res., 64:10, Oct.
- Caspe, M.S. (1970) "Earthquake Isolation of Multistory Concrete Structures," ACI Jnl, 67:11, Nov, pp 923-933.
- Caughey, T.K. (1960) "Classical Normal Modes in Damped Linear Systems," Jnl Applied Mechanics, ASME, Jun, pp 269-271.
- Cherry, S. and Brady, A.G. (1965) "Determination of Structural Dynamic Properties by Statistical Analysis of Random Vibrations," Proc. 3rd World Conf. on Earthquake Eng., Auckland and Wellington, New Zealand, Jan, Vol. 2, pp 50-67.
- Chuang, K.P. and Isenberg, J. (1973) A Study of Implicit and Explicit Step-by-Step Integration Techniques for Finite Element Applications, R-7243-5-2954. El Segundo, CA: Agbabian Assoc., Jun.
- Cloud, W.K. (1963) "Period Measurements of Structures in Chile," Bull. Seismol. Soc. Amer., 53:2, Feb, pp 359-379.
- Clough, R.W. (1969) "Analysis of Structural Vibrations and Dynamic Response," Recent Advances in Matrix Methods of Structural Analysis and Design, ed. R.H. Gallagher. Huntsville, AL: Univ. of Alabama Press.
- Clough, R.W. and Bathe, K.J. (1972) "Finite Element Analysis of Dynamic Response," Proc. 2nd U.S.-Japan Seminar on Matrix Methods of Structural Analysis and Design, Tokyo, Aug.
- Clough, R.W. and Penzien, J. (1975) Dynamics of Structures. New York: McGraw-Hill.
- Cooley, J.W. and Tukey, J.W. (1965). "An Algorithm for the Machine Calculation of Complex Fourier Series," Mathematics of Computation, 19:297, Apr, pp 297-301.
- Cornell, C.A. (1969) "A Probability-Based Structural Code," ACI Jnl, 66:12, Dec, pp 974-985.
- Corps of Eng. Dept. of the Army (COE). (1977) Hardness Verification, TM 5-858-6. Washington, DC: COE, Jul.
- Costantino, C.J. et al. (1968) Stress Waves in Layered Arbitrary Media (SLAM Code Free-Field Study), Vols. I-IV, SAMSO-TR-68-181. Chicago: IIT Research Institute.
- Crandall, S.H. and Mark, W.D. (1963) Random Vibration. New York: Academic Press.

- Crawford, R.E.; Higgins, C.J.; and Bultmann, E.H. (1974) The Air Force Manual for Design and Analysis of Hardened Structures, AFWL-TR-74-102. Albuquerque, NM: Civil Nuc. Systems Corp., Oct. (AD B004-152L).
- Crawford, R. and Ward, H.S. (1964) "Determination of the Natural Periods of Buildings," Bull. Seismol. Soc. Amer., 54:6A, Dec, pp 1743-1756.
- Crede, C.E. (1963) "Shock Isolation of Structure Contents," Shock & Vib. Bull. 32, Pt. 3. Washington, DC: Naval Res. Lab., Dec.
- Davenport, A.G. (1967) "Gust Loading Factors," Jnl Struct. Div., ASCE, 93:3, Jun, pp 11-34.
- Delfosse, G.C. (1979) Aseismic Building Isolation Systems: Better Protection against Earthquake Damage, ASME Paper 79-PVP-54, presented at Pressure Vessels and Piping Conference, San Francisco, Jun.
- Derham, C.J. and Thomas, A.G. (1979) The Design and Use of Rubber Bearings for Vibration Isolation and Seismic Protection of Structures, ASME Paper 79-PVP-58, presented at Pressure Vessels and Piping Conference, San Francisco, Jun.
- Der-Kiureghian, A. (1977) "Reliability-Based Design under Multiple Load Combinations," Trans. 4th Int. Conf. on Struct. Mech. in Reactor Tech., San Francisco, Aug.
- . (1978) "Second-Moment Combination of Stochastic Loads," Jnl Struct. Div., ASCE, 104:ST10, Oct, pp 1551-1567.
- Der-Kiureghian, A. and Ang, A.H-S. (1975) A Line Source Model for Seismic Risk Analysis, Struct. Res. Series 419. Urbana, IL: Univ. of Illinois.
- Desai, C.S. and Abel, J.F. (1972) Introduction to the Finite Element Method. New York: Nostrand Reinhold.
- Dibaj, M. and Penzien, J. (1969) Nonlinear Seismic Response of Earth Structures, EERC 69-2. Berkeley: Univ. of Calif., Earthquake Eng. Res. Ctr., Jan.
- Dowling, N.E. (1972) "Fatigue Failure Predictions for Complicated Stress/Strain Histories," Jnl of Materials, 7:1, Mar, pp 71-87.
- Drake, L.A. (1972) "Love and Rayleigh Waves in Nonhorizontally Layered Media," Bull. Seismol. Soc. Amer., 62:5, Oct, pp 1241-1258.
- Earth Sciences (ES). (1968) Ambient Vibration Survey and Mathematical Analysis of the Carolinas-Virginia Tube Reactor. Pasadena, CA: ES, Sep.
- Ellingwood, B.R. and Ang, A.H-S. (1972) Probabilistic Study of Safety Criteria for Design, UILU-ENG-72-2008. Urbana, IL: Univ. of Illinois, Jun.
- . (1974) "Risk-Based Evaluation of Design Criteria," Jnl Struct. Div., ASCE, 100: ST9, Sep, pp 1771-1778.
- Englekirk, R.E. and Matthiesen, R.B. (1967) "Forced Vibration of an Eight-Story Reinforced Concrete Structure," Bull. Seismol. Soc. Amer., 57:3, Jun, pp 421-436.
- Fintel, M. and Khan, F.R. (1969) "Shock-Absorbing Soft Story Concept for Multi-story Earthquake Structures," ACI Jnl, 66:5, May, pp 381-390.
- Fletcher, R. and Powell, M.J.D. (1963) "A Rapidly Convergent Descent Method for Minimization," Computer Jnl, 6, pp 163-168.
- Foss, K.A. (1958) "Coordinates Which Uncouple the Equations of Motion of Damped Linear Dynamic Systems," Jnl Appl. Mech., 80, pp 361-364.
- Ghosh, S. and Wilson, E. (1969) Dynamic Stress Analysis of Axisymmetric Structures under Arbitrary Loading, EERC 69-10. Berkeley: Univ. of Calif., Earthquake Eng. Res. Ctr., Sep.
- Gilmour, R.M. and Hitchcock, H.C. (1971) "Use of Yield Ratio Response Spectra to Design Yielding Members for Improving Earthquake Resistance of Brittle Structure," Bull. New Zealand Soc. Earthq. Eng., Apr, pp 285-293.
- Grigoriu, M. (1975) On the Maximum of Sum of Random Process Models, Internal Document No. 1, Structural Load Analysis and Specification Project, Dept. of Civil Eng. Cambridge, MA: MIT.
- Gumbel, E.J. (1958) Statistics of Extremes. New York: Columbia Univ. Press.
- Hadjian, A.H. (1978) "Engineering of Nuclear Power Facilities for Earthquake Loads," Nuc. Eng. and Des., 48, pp 21-47.

- Hart, G.C. and Ibañez, P. (1973) "Experimental Determination of Damping in Nuclear Power Plant Structures and Equipment," *Nuc. Eng. and Des.*, 25:1, Jun, pp 112-125.
- Hart, G.C.; Lew, M.; and DiJulio, R. (1973) "High Rise Building Response: Damping and Period Nonlinearities," *Proc. 5th World Conf. on Earthquake Eng.*, Rome, Jun, Vol. 2.
- Hart, G.C. and Vasudevan, R. (1975) "Earthquake Design of Buildings: Damping," *Jnl Struct. Div.*, ASCE, 101:ST1, Jan, pp 11-30.
- Hasselmann, T.K. (1972) "Method for Constructing a Full Modal Damping Matrix from Experimental Measurements," *AIAA Jnl*, 10:4, Apr, pp 526-527.
- Hoerner, J.B. and Jennings, P.C. (1969) "Modal Interference in Vibration Tests," *Jnl Eng. Mech. Div.*, ASCE, 95:EM4, Aug, pp 827-839.
- Holtz, H. (1973) Prony's Method and Detailed Approaches to Exponential Approximation, Aerospace Corporation, Report No. ATR-73(9990)-5, Jun.
- Hornbuckle, J.D. et al. (1969) "Forced Vibration Tests and Analysis of the CVTR Reactor Seismic Response," *ANS Trans.*, 12:2, Nov, p 780.
- Housner, G.W. and Jennings, P.C. (1964) "Generation of Artificial Earthquakes," *Jnl Eng. Mech. Div.*, ASCE, 90:EM1, Feb, pp 113-150.
- Hudson, D.E. (1961) A New Vibration Exciter for Dynamic Tests of Full Scale Structures. Pasadena, CA: Calif. Inst. Tech., Earthquake Eng. Res. Lab., Sep.
- . (1962) Synchronized Vibration Generators for Dynamic Tests of Full Scale Structures. Pasadena, CA: Calif. Inst. Tech., Earthquake Eng. Res. Lab., Nov.
- . (1965) "Equivalent Viscous Friction for Hysteretic Systems with Earthquake Like Excitations," *Proc. 3rd World Conf. on Earthquake Eng.*, Auckland and Wellington, New Zealand, Jan, Vol. 2, pp 185-202.
- . (1970) "Dynamic Tests of Full-Scale Structures," *Earthquake Engineering*, ed. R.L. Wiegel. Englewood Cliffs, NJ: Prentice-Hall.
- . (1976) "Dynamic Tests of Full-Scale Structures," *Proc. Dynamic Response Structures, Instrumentation, Testing Methods, and System Identification*, UCLA, Mar. (suppl. to Chapt. 7 in *Earthquake Engineering*, ed. R.L. Wiegel)
- Hudson, D.E.; Alford, J.L.; and Housner, G.W. (1954) "Measured Response of a Structure to an Explosive-Generated Ground Shock," *Bull. Seismol. Soc. Amer.*, 44:3, Jul, pp 513-527.
- Hudson, D.E.; Alford, J.L.; and Iwan, W.D. (1961) "Ground Accelerations Caused by Large Quarry Blasts," *Bull. Seismol. Soc. Amer.*, 51:2, Apr, pp 191-202.
- Hurty, W.C. and Rubinstein, M.F. (1964) *Dynamics of Structures*. Englewood Cliffs, NJ: Prentice-Hall.
- Ibañez, P. (1970) Analog-Simulated Earthquake Response of the Experimental Gas-Cooled Reactor, M.S. thesis, UCLA.
- Ibañez, P. et al. (1970) San Onofre Nuclear Generating Station Vibration Tests, ENG-7037. Los Angeles: UCLA Nuc. Energy Lab., Aug.
- Iemura, H. and Jennings, P.C. (1974) "Hysteretic Response of a Nine-Story Reinforced Concrete Building," *Int. Jnl of Earthquake Eng. and Struct. Dynamics*, 3:2, Oct-Dec, pp 183-202.
- Institute of Electrical and Electronic Engineers (IEEE). (1973) *IEEE Guide for Seismic Qualification of Class I Electric Equipment for Nuclear Power Generating Stations*, Rev. 1, Jun.
- Jackson, E.C.; Miller, A.B.; and Bernreuter, D.L. (1972) "A Shock-Isolation System for 22 Feet of Vertical Ground Motion," *Shock and Vib. Bull.* 42, Pt. 2, Washington, DC: Naval Res. Lab., Jan.
- Jacobsen, L.S. (1930). "Steady Forced Vibration as Influenced by Damping," *Trans.*, ASME, APM 52-15, pp 169-181.
- . (1960) "Damping in Composite Structures," *Proc. 2nd World Conf. on Earthquake Eng.*, Tokyo, Jul, Vol. 2, pp 1029-1044.
- Jacoby, G.H. (1967) *Fatigue Life Estimation Processes under Conditions of Irregularly Varying Loads*, AFML TR67-215. Wright-Patterson AFB, Ohio: AF Materials Lab., Aug.

- Jennings, P.C. (1963) Response of Simple Yielding Structures Subjected to Earthquake Excitation. Pasadena: Calif. Inst. Tech., Earthquake Eng. Res. Lab., Jun.
- . (1968) "Equivalent Viscous Damping for Yielding Structures," Jnl Eng. Mech. Div., ASCE, 94:EM1, Feb, pp 103-116.
- Jennings, P.C. and Bielak, J. (1973) "Dynamics of Building-Soil Interaction," Bull. Seismol. Soc. Amer., 63:1, Feb, pp 9-48.
- Jennings, P.C.; Housner, G.W.; and Tsai, N.C. (1969) "Simulated Earthquake Motions for Design Purposes," Proc. 4th World Conf. on Earthquake Eng., Santiago, Chile, Jan.
- Jennings, P.C. and Kuroiwa, J.H. (1968) "Vibration and Soil-Structure Interaction Tests of a Nine-Story Reinforced Concrete Building," Bull. Seismol. Soc. Amer., 58:3, Jun, pp 891-916.
- Jennings, P.C. et al. (1969) "Simulated Earthquake Motions for Design Purposes," Proc. 4th World Conf. on Earthquake Eng., Santiago, Chile, Jan.
- (1972) "Forced Vibrations of a Tall Steel-Frame Building," Int. Jnl of Earthquake Eng. and Struct. Dynamics, 1:2, Oct-Dec, pp 107-132.
- Johnson, T.E. and McCaffery, R.J. (1969) "Current Techniques for Analyzing Structures and Equipment for Seismic Effects." Presented at ASCE National Meeting on Water Resources, New Orleans, Feb.
- Jolivet, F. and Richli, M. (1977) "Aseismic Foundation System for Nuclear Power Stations," Paper K9/2, Trans. 4th Int. Conf. on Struct. Mech. in Reactor Tech., San Francisco, Aug.
- Kapur, K.K. and Shao, L.C. (1973) "Generation of Seismic Floor Response for Equipment Design," Proc. ASCE Specialty Conf. on Struct. Design of Nuclear Plant Facilities, Chicago, Dec.
- Kausel, E.; Roesset, J.M.; and Waas, G. (1975) "Dynamic Analysis of Footing on Layered Media," Jnl Eng. Mech. Div., ASCE, 101:EM5, Oct.
- Keightley, W.O.; Housner, G.W.; and Hudson, D.E. (1961) Vibration Test of the Encino Dam Intake Tower. Pasadena, CA: Calif. Inst. Tech., Earthquake Eng. Res. Lab., Jul.
- Kelly, J.M. (1979) "Aseismic Base Isolation," Proc. 2nd U.S. National Conf. on Earthquake Eng., Stanford Univ., Aug.
- Kelly, J.M.; Eiginger, J.M.; and Derham, C.J. (1977) A Practical Soft Story Earthquake Isolation System, UCB/EERC-77/27. Berkeley: Univ. of Calif., Earthquake Eng. Res. Ctr., Nov.
- Kelly, J.M. and Tsztoo, D.F. (1978) Development of Energy-Absorbing Devices of Aseismic Base Isolation Systems, UCB/EERC-78/01. Berkeley: Univ. of Calif., Earthquake Eng. Res. Ctr., Jan. (PB 284 978)
- Kfoury, N.F. (1963) "Shock Isolation for Hardened Missiles and Underground Structures," Shock and Vib. Bull. 32, Pt. 3. Washington, DC: Naval Res. Lab., Dec.
- Kishida, H. and Takano, A. (1970) "The Damping in the Dry Sand," Proc. 3rd Japan Earthquake Eng. Symposium, Tokyo.
- Kovacs, W.D. (1968) "An Experimental Study of the Response of Clay Embankments to Base Excitation," Ph.D. dissertation, Univ. of Calif., Berkeley.
- Krizek, R.J. and Franklin, A.G. (1967) "Energy Dissipation in a Soft Clay," Proc. Symposium on Wave Propagation and Dynamic Properties of Earth Materials, Albuquerque, NM.
- Langer, B.F. (1937) "Fatigue Failure from Stress Cycles of Varying Amplitude," Jnl of Applied Mechanics, ASME, Vol. 59, Dec, pp A160-A162.
- Lee, D.M. and Medland, I.C. (1979) "Base Isolation Systems for Earthquake Protection of Multi-Story Shear Structures," Earthquake Eng. and Struct. Dynamics, 7:6, pp 555-568.
- Leistner, H.G. et al. (1969) Structure/Medium Interaction and Design Procedure Study (FEAT), Vols. I-IV, SAMSO-TR-69-313. Seattle: The Boeing Co., Oct.
- Lind, N.C. (1971) "Consistent Partial Safety Factors," Jnl Struct. Div., ASCE, 97:ST6, Jun, pp 1651-1670.
- Lysmer, J. (1975) FLUSH, A Computer Program for Approximate 3-D Analysis of Soil/Structure Interaction Problems, EERC-75-30. Berkeley: Univ. of Calif., Earthquake Eng. Res. Ctr., Nov.

- Lysmer, J. and Kuhlemeyer, R.L. (1969) Finite Dynamic Model for Infinite Media, *Jnl Eng. Mech. Div.*, ASCE, 95:EM4, Aug.
- Lysmer, J. and Waas, G. (1972) "Shear Waves in Plane Infinite Structures," *Jnl Eng. Mech. Div.*, ASCE, 98:EM1, Feb.
- Lysmer, J. et al. (1974) LUSH: A Computer Program for Complex Response Analysis of Soil-Structure Systems, EERC 74-4. Berkeley: Univ. of Calif., Earthquake Eng. Res. Ctr.
- Manson, S.S. (1965) "Fatigue: A Complex Subject -- Some Simple Approximations," *Jnl of Experimental Mechanics*, 5:7, Jul, pp 193-226.
- Masri, S.F. and Safford, F.B. (1976) "Dynamic Environment Simulation by Pulse Techniques," *Jnl Eng. Mech. Div.*, ASCE, 101:EM1, Feb, pp 151-169.
- Matthiesen, R.B. and Smith, C.B. (1966) A Simulation of Earthquake Effects on the UCLA Reactor Using Structural Vibrators, NEL-105. Los Angeles: UCLA, Oct.
- . (1969) Forced Vibration Tests of the Carolinas-Virginia Tube Reactor (CVTR), Report 69-8. Los Angeles: UCLA, Feb.
- Matthiesen, R.B. et al. (1970) San Onofre Nuclear Generating Station Supplementary Vibration Tests, ENG-7095. Los Angeles: UCLA Nuc. Energy Lab., Dec.
- Mayes, R.L. and Mowbray, N.A. (1975) "The Effect of Coulomb Damping on Multi-Degree of Freedom Elastic Structures," *Int. Jnl of Earthquake Eng. and Struct. Dynamics*, 3:3, Jan-Mar, pp 275-286.
- McGuire, R.K. and Cornell, C.A. (1974) "Live Load Effects in Office Buildings," *Jnl Struct. Div.*, ASCE, 100:ST7, Jul, pp 1351-1366.
- McLamore, V.R. et al. (1971) "Ambient Vibration of Two Suspension Bridges," *Jnl Struct. Div.*, ASCE, 97:ST10, Oct, pp 2567-2582.
- Medearis, K., Assoc. (1975) A Comparative Study of Structural Response to Explosion-induced Ground Motions, ASCE Research Activity. New York: ASCE.
- Miner, M.A. (1945) "Cumulative Damage in Fatigue," *Jnl Applied Mech.*, ASME, Vol. 67, Sep, pp A159-A164.
- Nat'l Oceanic and Atmospheric Administration (NOAA). (1973) San Fernando, California Earthquake of February 9, 1971, 3 Vols. Washington, DC: USGPO.
- Newmark, N.M. (1959) "A Method of Computation for Structural Dynamics," *Jnl Eng. Mech. Div.*, ASCE, 85:EM3, Jul, pp 67-94.
- . (1969) "Design Criteria for Nuclear Reactors Subjected to Earthquake Hazards," Presented at IAEA Panel on Aseismic Design and Testing of Nuclear Facilities. Tokyo: Japan Earthquake Eng. Promotion Soc.
- . (1971) "Seismic Response of Reactor Facility Components," *Proc. Symp. on Seismic Analysis of Pressure Vessel and Piping Components*, ASME, pp 1-24.
- . (1972) "Earthquake Response Analysis of Reactor Structures," *Nuc. Eng. and Des.*, 20:2, Jul, pp 303-322.
- Newmark, N.M.; Blume, J.A.; and Kapur, K.K. (1973) "Seismic Design Spectra for Nuclear Power Plants," *Jnl Power Div.*, ASCE, 99:PO2, Nov, pp 287-303.
- Newmark, N.M. and Hall, W.J. (1969) "Seismic Design Criteria for Nuclear Reactor Facilities," *Proc. 4th World Conf. on Earthquake Eng.*, Santiago, Chile, Jan, Vol. 2.
- Newmark N.M. and Rosenblueth, E. (1971) *Fundamentals of Earthquake Engineering*. Englewood Cliffs, NJ: Prentice-Hall.
- Newmark, N.M. et al. (1973) "Design Response Spectra for Nuclear Power Plants," Presented at ASCE Structural Engineering Meeting, San Francisco, Apr.
- Nielsen, N.N. (1964) *Dynamic Response of Multistory Buildings*. Pasadena, CA: Calif. Inst. Tech., Earthquake Eng. Res. Lab., Jun.
- Oden, J.T. (1972) *Finite Elements of Nonlinear Continua*. New York: McGraw-Hill.
- Olsen, B.E.; Singleton, N.R.; and Bohm, G.J. (1973) "Determination of the Dynamic Characteristics of a Pressurizer Water Reactor Primary Loop," *Proc. 2nd Int. Conf. on Pressure Vessel Technology -- Part 1 -- Design Analysis*, ASME, Oct, pp 437-455.
- Otnes, R. (1974) Personal Communication to S.D. Werner, Agbabian Assoc., El Segundo, CA, Nov.

- Palmgren, A. (1924) "Die Lebensdauer von Kugellagern," ZUDI, 68:14, Apr, pp 339-341.
- Peir, J.C. and Cornell, C.A. (1973) "Spatial and Temporal Variability of Live Loads," Jnl Struct. Div., ASCE, 99:ST5, May, pp 903-922.
- Plichon, C.E. (1970) "Dynamic Analysis of Nuclear Power Plant Behavior to Seismic Excitation," Nuc. Eng. and Design (Holland), 12:55.
- Prause, R.H. and Ahlbeck, D.R. (1972) "Seismic Evaluation of Electrical Equipment for Nuclear Power Stations," Shock and Vib. Bull., 42, Pt. 2, pp 11-20.
- Prause, R.H. et al. (1973) "Seismic Evaluation of an Electrical Equipment Enclosure." Presented at ASME Design Eng. Tech. Conf., Cincinnati, Sep.
- Przemieniecki, J.S. (1968) Theory of Matrix Structural Analysis. New York: McGraw-Hill.
- Reddy, D.P. and Ts'ao, H.S. (1975) Nonlinear Analysis of Norris Dam for Seismic Loads, R-7523-2-3894. El Segundo, CA: Agbabian Assoc., Jul.
- Richart, F.E.; Hall, J.R.; and Woods, R.D. (1970) Vibrations of Soils and Foundations. Englewood Cliffs, NJ: Prentice-Hall.
- Roesset, J.M. (1973) Personal Communication to S.D. Werner, Agbabian Assoc., El Segundo, CA, May.
- Roesset, J.M.; Whitman, R.V.; and Dobry, R. (1973) "Modal Analysis for Structures with Foundation Interaction," Jnl Struct. Div., ASCE, 99:ST3, Mar, pp 399-416.
- Rosenblueth, E. and Esteva, L. (1972) "Reliability Basis for Some Mexico Codes," Probabilistic Design of Reinforced Concrete Buildings, SP-31. Detroit: Amer. Concrete Inst.
- Rubenstein, M.F. (1966) Matrix Computer Analysis of Structures. Englewood Cliffs, NJ: Prentice-Hall.
- Safford, F.B. et al. (1977) "Air Blast and Ground Shock Simulation Testing of Massive Equipment by Pulse Techniques," Presented at 5th Int. Symp. on Military Application of Blast Simulation, Stockholm, Sweden, May.
- Schijve, J. (1963) Fatigue of Aircraft Structures, eds. W. Barrios and E.L. Ripley. New York: Pergamon Press, pp 115-148.
- Schmitt, R.C. (1970) Evaluation and Comparison of Structural Dynamics Investigations of the Carolinas-Virginia Tube Reactor Containment, IN-1372. Idaho Falls, ID: Idaho Nuclear Corp., May.
- Schnabel, P.B. et al. (1972) SHAKE, A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites, EERC-72-12. Berkeley: Univ. of Calif., Earthquake Eng. Res. Ctr., Dec. (PB 220 207)
- Schrader, E.W. (1965) "Torus Absorbs Impact Energy," Design News, Nov.
- Seed, H.B. and Idriss, I.M. (1970) Soil Moduli and Damping Factors for Dynamic Response Analysis, EERC 70-10. Berkeley: Univ. of Calif., Earthquake Eng. Res. Ctr., Dec.
- Shannon & Wilson/Agbabian Associates (SW/AA). (1972) Soil Behavior Under Earthquake Loading Conditions: State-of-the-Art Evaluation of Soil Characteristics for Seismic Response Analyses. Seattle, WA: SW & El Segundo, CA: AA. (TID 26444)
- . (1975) Procedures for Evaluation of Vibratory Ground Motions of Soil Deposits at Nuclear Power Plant Sites, NUREG-75/072. Washington, DC: U.S. Nuclear Regulatory Commission.
- Shinozuka, M. (1966) "Application of Stochastic Processes to Fatigue, Creep and Catastrophic Failures," Seminar in the Application of Statistics in Structural Mechanics, Dept. Civil Eng., Columbia Univ., Nov.
- Singh, A.K.; Chu, S.L.; and Singh, S. (1973) "Influence of Closely Spaced Modes in Response Spectrum Method of Analysis," Proc. ASCE Specialty Conf. on Structural Design of Nuclear Plant Facilities, Chicago, Dec.
- Skinner, R.I.; Beck, J.L.; and Bycroft, G.N. (1975a) "A Practical System for Isolating Structures from Earthquake Attack," Earthquake Eng. and Struct. Dynamics, 3:3, pp 297-309.
- Skinner, R.I.; Bycroft, G.N.; and McVerry, G.H. (1976) "A Practical System for Isolating Nuclear Power Plants from Earthquake Attack," Nuc. Eng. and Des., 36, pp 287-297.

- Skinner, R.I.; Heine, A.J.; and Tyler, R.G. (1977) "Hysteretic Dampers to Provide Structures with Increased Earthquake Resistance," Proc. 6th World Conf. on Earthquake Eng., New Delhi, India, Jan.
- Skinner, R.I.; Kelly, J.M.; and Heine, A.J. (1975b) "Hysteretic Dampers for Earthquake-Resistant Structures," Earthquake Eng. & Struct. Dynamics, 3:3, pp 287-296.
- Skreiner, K.M. et al. (1974) "New Seismic Requirements for Class I Electrical Equipment," Presented at IEEE PES Winter Meeting, New York, Jan.
- Smith, C.B. and Matthiesen, R.B. (1969) Forced Vibration Tests of the Experimental Gas-Cooled Reactor, Report No. 69-42. Los Angeles: UCLA, Aug.
- . (1971) "Simulating Strong Motion Earthquake Effects on Nuclear Power Plants," Presented at Nuclear Safety Program Annual Information Meeting, Oak Ridge National Laboratory, Oak Ridge, TN, Feb.
- Smith, C.B. et al. (1971) Response of San Onofre Nuclear Generating Station to Earthquakes, UCLA-ENG-7151. Los Angeles: UCLA, Jul.
- Stallmeyer, J.E. and Walker, W.H. (1968) "Cumulative Damage Theories and Application," Jnl Struct. Div., ASCE, 94:ST12, Dec, pp 2739-2750.
- Stoykovich, M. (1973) "Seismic Design and Analysis of Nuclear Plant Components," Proc. ASCE Specialty Conf. on Struct. Design of Nuclear Plant Facilities, Chicago, Dec.
- Stubbs, I.R. and McLamore, V.R. (1973) "The Ambient Vibration Survey," Proc. 5th World Conf. on Earthquake Eng., Rome, Jun, Vol. 1,
- Thomson, W.T. (1965) Vibration Theory and Application. Englewood Cliffs, NJ: Prentice-Hall.
- Trifunac, M.D. (1970) Wind and Microtremor Induced Vibrations of a Twenty-Two Story Steel Frame Building, EERL-70-01. Pasadena, CA: Calif. Inst. Tech., Earthquake Eng. Res. Lab.
- . (1972) "Comparisons between Ambient and Forced Vibration Experiments," Earthquake Eng. and Struct. Dynamics 1:2, Oct-Dec, pp 133-150.
- Trifunac, M.D. and Udawadia, F.E. (1974) "Variations of Strong Earthquake Ground Shaking in the Los Angeles Area," Bull. Seismol. Soc. Amer., 64:5, Oct, pp 1429-1454.
- Tsai, N.-C. (1972) "Spectrum Compatible Motions for Design Purposes," Jnl Eng. Mech. Div., ASCE, 98:EM2, Apr, pp 345-356.
- . (1974) "Modal Damping for Soil-Structure Interaction," Jnl Eng. Mech. Div., ASCE, 100:EM2, Apr, pp 323-341.
- Tsai, N.-C. et al. (1971) Seismic Analyses of Structures and Equipment for Nuclear Power Plants, BC-TOP-4. San Francisco: Bechtel Corp.
- Ts'ao, H.S. (1974) "The Seismic Risk of the Puget Sound Area," Proc. 69th Annual Meeting, Seismological Soc. of America, Las Vegas, Mar.
- Ts'ao, H.S. and Werner, S.D. (1975) Determination of Equivalent Response Cycles for Use in Earthquake-Induced Fatigue Analyses, SAN/1011-106. El Segundo, CA: Agbabian Assoc., May.
- Uniform Building Code (UBC). (1976) Whittier, CA: Int. Conf. of Building Officials.
- U.S. Atomic Energy Commission (USAEC). (1973a) "Design Response Spectra for Seismic Design of Nuclear Plants," Regulatory Guide 1.60, Rev. 1. Washington, DC: GPO, Oct.
- . (1973b) "Damping Values for Seismic Design of Nuclear Power Plants," Regulatory Guide 1.61. Washington, DC: GPO, Oct.
- . (1973c). "Design Limits and Loading Combinations for Seismic Category 1 Fluid System Components," Regulatory Guide 1.48. Washington, DC: GPO, May.
- . (1974) "Combination of Modes and Spatial Components in Seismic Response Analysis," Regulatory Guide 1.92. Washington, DC: GPO, Dec.
- U.S. Coast and Geodetic Survey (USCGS). (1936) Earthquake Investigations in California, 1934-1935, Special Pub 201. Washington, DC: USGPO.
- U.S. Energy Res. & Dev. Admin. (ERDA). (1973) Quality Assurance Program Requirements, RDT Standard RDT-F-2-2. Oak Ridge, TN: Oak Ridge Nat'l Lab., Aug.

- . (1974) Quality Verification Program Requirements, RDT Standard RDT-F-2-4T. Oak Ridge, TN: Oak Ridge Nat'l. Lab., Dec.
- U.S. Nuc. Regulatory Comm. (USNRC). (1975a) "Quality Assurance Requirements for the Design of Nuclear Power Plants," Regulatory Guide 1.64, Rev. 1. Washington, DC: USNRC, Feb.
- . (1975b) Standard Review Plan for the Review of Safety Analysis Reports for Nuclear Power Plants, LWR Edition, 4 Vols. Washington, DC: USNRC, Sep. (NUREG 75/087)
- Ward, H.S. and Crawford, R. (1966) "Wind Induced Vibrations and Building Modes," Bull. Seismol. Soc. Amer., 56:4, Aug, pp 793-813.
- Wen, Y.K. (1977a) "Statistical Combination of Extreme Loads," Jnl Struct. Div., ASCE, 103:ST5, May, pp 1079-1094.
- . (1977b) "Probability of Extreme Load Combination," Trans. 4th Int. Conf. on Struct. Mech. in Reactor Tech., San Francisco, Aug.
- Werner, S.D. (1976) Seismic Soil/Structure Interaction Analysis Guideline, Vol. II: State of the Art Procedures, TOP-AA111. El Segundo, CA: Agbabian Assoc., Apr.
- Werner S.D. and Van Dillen, D. (1977) "Use of Analytical and Statistical Techniques to Assess In-situ Soil-Test Procedures," Proc. 6th World Conf. on Earthquake Eng., New Delhi, India, Jan.
- Whitman, R.V. (1970) "Soil-Structure Interaction," Seismic Design for Nuclear Power Plants, ed R.J. Hansen. Cambridge, Mass: MIT, pp 245-269.
- Wilkinson, J.H. (1965) The Algebraic Eigenvalue Problem. Oxford: Clarendon Press.
- Wilson, E.L.; Farhoomand, I.; and Bathe, K.J. (1973) "Nonlinear Dynamic Analysis of Complex Structures," Earthquake Eng. and Struct. Dynamics, Vol. 1, pp 241-252.
- Wilson, E.L. and Penzien, J. (1972) "Evaluation of Orthogonal Damping Matrices," Int. Jnl Numerical Methods in Engineering, 4:1, Jan.
- Zienkiewicz, O.C. (1971) The Finite Element Method in Engineering Science. London: McGraw-Hill.