MASTER

Methods of Mathematical Programming
To Program Planning and R&D Strategies

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Presented at the
Energy Policy Analysis Forum
Assessment of Energy Research and Development Strategies
Washington, D.C. (U.S.A.)

December 1979

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ABSTRACT

The intention of this paper is to discuss how some methods of mathematical programming might effectively be used to guide R&D planning with respect to

- total budget;
- the budget distribution between supply and demand technologies;
- the budget distribution among competing technologies;
- the budget distribution in light of the need to diversify and avoid risk.

The discussion is limited, out of necessity, to special frameworks such as linear programming, mixed integer programming and quadratic programming since they are most characteristic of the state-of-the-art energy system formulations. Market penetration, the treatment of vintage stock and the introduction of uncertainty in these modeling frameworks are key features which lend credibility to the well-known techniques applied for technology assessment, cost-benefit analysis and multi-criteria analysis.

ACKNOWLEDGMENT

I wish to thank Ling Ling Juang and Joe Rabinowitz for their useful comments and editorial assistance on the original draft.

Thanks are due to Mary Lou Weis for editing and coordinating the preparation of this report and also to Betty Knittweis, Susan White DePace, Gwen Weaver and Sharon Zuhoski for their hard work in typing the many drafts required to produce this document.

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I. INTRODUCTION

Technical and policy options available to decisionmakers in the energy field have specific and distinct effects on the efficiency, cost, and environmental attributes of the energy system that must be analyzed and understood through policy analysis. The United States energy system has a wide variety of available energy resources and conversion technologies, available and emerging from the research and development stage, which provide flexibility in interfuel substitution in the longer term.

The intention of this paper is to discuss how methods of mathematical programming might effectively be used to guide R&D planning with respect to:

- total budget constraints;
- the budget distribution between supply and demand technologies;
- the budget distribution among competing technologies;
- the budget distribution in light of the needs for diversification (risk aversion).

From the general R&D perspective these questions form essential components of "portfolio" or "cost-benefit" analysis and are required for determining the optimal allocation of limited funds in R&D strategy development. Government R&D policy aims at affecting both the first date of commercial availability and the rate of introduction (market penetration) of new technologies. Further, it is important in forming policy to identify those technologies which **do not** require government assistance to assure that limited resources are allocated to marginal or high risk technologies. The primary concern can be stated: "Given the limited funds and perceived criteria (e.g., national security, minimum system cost, minimum environmental effects) what is the optimal distribution of R&D funds and what is the optimal long-term R&D planning strategy?" What are the impacts of alternative strategies? Evidently, these questions suggest an optimization framework.

Although several established mathematical programming models for energy systems analysis exist (e.g., The Stanford University PILOT/WEM^{1,2} system, ETA-MACRO³ or EIA's Mid-range Energy Forecasting System (MEFS), formerly known as PIES⁴), we will use the Time-Stepped Energy System Optimization Model⁵ of Brookhaven National Laboratory in most discussion instances due to the greater familiarity of the author.

Mathematical programming is the generic term which refers to optimization of a mathematically well-defined (quantifiable) objective function subject to constraints, i.e., the allocation of scarce resources to best satisfy demands. Mathematical programming can be further subdivided according to the restrictions on the variables, the form of the objective function, and the type and form of the constraints. Another dimension is realized when we distinguish between stochastic mathematical programming and deterministic programming. Stochastic programming¹⁹ includes mathematical programming techniques which account for uncertainty in the coefficients. The motivation for this stems from certain complex decision processes; if randomness is ignored and an expected value (deterministic programming) is used in lieu of the random variable then the analyst can be seriously misled. This mistake is usually called the "Fallacy of Averages."¹⁹ It implies that one is not usually justified in using expected values for coefficients when they are really random variables. Multi-stage programming and the chance-constrained programming are the two of the more notable stochastic programming techniques. Unfortunately, for actual decision problems associated with energy R&D program planning, the actual size of the formulation of the multi-stage technique is staggering and impractical.

The chance-constrained formulation has two desirable properties. First, the problem size and structure is the same as the deterministic version. Second, only the unconditional probabilities of the right hand side coefficients need to be known. The principal weakness is that the model only indirectly evaluates the economic consequences of violating a constraint. Further, selection of the probability should be part of the optimization and is not. Although the focus of this paper is the application of mathematical programming techniques to R&D strategies and program planning, it will be useful to first provide a brief categorization of some of the important areas of mathematical programming and their distinguishing characteristics. Practitioners of mathematical programming techniques in modeling usually operate under the assumptions that optimization of some quantifiable objective, with appropriate constraints, often provides a useful framework for analyzing physical systems, even if the physical system does not optimize the objective quantified. The belief that the physical system optimizes a quantifiable objective is equivalent to the belief that the physical system acts to optimize a "common good," as defined by the objective function, and that every component of the physical

system is aware of the decisions and constraints on the rest of the system. This is usually an idealistic (heroic) assumption with the possible exception of business environments where such a single-minded goal can be enforced (profit maximization, risk aversion).

In spite of this heroic assumption the advantage of using mathematical programming techniques for traditional technology assessment,⁶ cost-benefit analysis,⁷ multi-criteria analysis^{8,9} have been well documented.

Briefly, mathematical programming formulations of energy systems are most useful as normative tools for evaluating the effects of policy changes on a reference case. Normative tools are meant to provide information on the magnitude and direction of policy changes. These types of models provide information as to how the energy system should evolve given the stated objectives and constraints, not how it will develop. Use of such models to forecast future events without coupling to a macro-economic model with feedback would be a serious misuse of the model. The economic information available in the dual relationships to mathematical programming models is very useful in assisting cost-benefit analysis. If a linear programming context is used then sensitivity analysis can easily be performed to provide insights into the behavior of the energy system near a desirable solution. In the context of multiobjective analysis, for example, the effects of increasing the contribution of clean renewables, up to the maximum realistic potential, was weighed against the objectives to minimize annualized system cost, reduction of non-renewable resource usage and an environmental indicator. The methodology appears in [7]. Suffice it to say that mathematical programming, specifically linear programming, has been found to be a powerful tool for these types of analyses. However, mathematical programming techniques, by virtue of the optimization framework, tend to give "all-or-nothing" behavior and this type of behavior is the most visibly criticized aspect of these modeling frameworks. Very little has been done to integrate energy systems models with market penetration algorithms to alleviate the problem. These features and others identified, when integrated, provide a credible framework for purposes of R&D assessments and strategy development. Finally, the optimization framework lends itself to the analysis of the energy system under multiple criteria.

TRADITIONAL AREAS

Linear Programming:

Objective Function - Linear: $\sum_i c_i x_i$

Constraints - Linear: $\sum_j a_{ij} x_j \leq b_i$

Variables - Continuous: $x_j \in (\alpha_j, \beta_j)$

Implicit Assumption: The physical system being modeled is basically a linear system.

Integer Programming:

Objective Function - Linear: $\sum_i c_i d_i$

Constraints - Linear: $\sum_j a_{ij} d_j \leq b_i$

Variables - The variables may take on only integer values: d_j are integers.

Implicit Assumptions: The physical system being modeled is basically a linear system but all the decision variables are integer. The variables could reflect, for example, the number of 500 megawatt coal-fired power plants to be installed in a particular region.

Mixed Integer Programming:

Objective Function - Linear: $\sum_i c_i x_i + \sum_j c_j d_j$

Constraints - Linear: $\sum_j a_{ij} x_j + \sum_j f_{ij} d_j \leq b_i$

Variables - the x_j 's, are continuous and the d_j 's are integer.

Implicit Assumptions: The physical system contains decision variables which are integral, as in the number of power plants of specific size, and variables which are continuous as in the amount of fuel consumed by each plant selected.

Nonlinear Programming:

- Objective Function - nonlinear: $f(x_1, x_2, \dots, x_n)$
- Constraints - nonlinear: $g_i(x_1, x_2, \dots, x_n) = 0$.
- Variables - either continuous or integer, or both.

Implicit Assumptions: This is the most general form of traditional mathematical programming in that it contains the fewest restrictions on the form of the objective function and constraints.

General techniques for nonlinear programming include quadratic programming, a quadratic objective function with linear constraints, and convex programming, a convex function with convex solution space. The special case, called quadratic programming, can be solved almost as efficiently as linear programs and provides flexibility and additional realism to energy modeling. The technique is not often used because efficient commercial codes for large problems have not yet been developed.

OTHER AREAS (Listed Only)

Dynamic Programming

The common characteristic of all dynamic programming models is the expression of decision problem by means of a recursive formulation. The recursion is backward in time or phase in that it commences calculations with the last period (phase) of the horizon and moves toward the first period or phase.²⁰

Stochastic Programming

Already discussed above.

Probabilistic (Markov Processes)

See reference [20] for a thorough discussion.

Other Hybrid Processes of the above.

See reference [20].

The list above is not meant to be exhaustive or complete but meant to serve as benchmark for discussions to follow.

R&D strategies must be analyzed in the context of their impact on the totality of the energy-economy-environmental system; the analysis must often include not only national and regional considerations but often site specific considerations. Investment costs for experimental coal gasification plants,

for example, can vary substantially depending on coal transportation costs and the proximity of the plants to the main gas pipelines or users. Viewed from this perspective, "no existing single model can be relied upon to give unqualified answers to the R&D strategy questions posed."

II. OVERVIEW OF TESOM

We provide in this section an overview of TESOM, a representative example of a state-of-the-art mathematical programming energy model. The Brookhaven Time-Stepped Energy System Optimization Model, TESOM, is a single region, time-stepped (sequential) linear programming model that was developed for the quantitative evaluation of national energy technologies and policies within a dynamic system's framework. The model is designed to examine interfuel substitution in the context of time dependent constraints on the availability of competing resources and technologies (stocks) and their associated costs.

TESOM is structured around the Reference Energy System (RES) that depicts the flow of energy from a resource to the point of actual end-use. Figure 1 is a sample RES for the U.S. in 1977. The RES is a specialized format for representing the detailed technological structure of the energy system along with resource consumption and associated emissions. Conceptually, the RES is a network representation of the flow of energy from resource extraction, through all intermediate conversion processes, to the satisfaction of final energy service needs (e.g. space heating, air conditioning, etc.). The RES provides a complete thermodynamically consistent energy accounting system, in physical units, for energy flow through energy conversion devices: the extraction or importation of primary energy resources and products, refining and the various stages of energy conversion, transportation, distribution, storage, consumption of fuels and electricity by end-use technologies corresponding to particular energy service demands. Within the RES, emphasis is placed on the comprehensive technological structure relating the energy flows which enter the system (oil, gas, coal, uranium, solar, etc.) to the nonsubstitutable, functional energy services which are the final product of the flow (space conditioning, motive power, process heat, lighting etc.).

The time-stepped model optimally allocates energy resources and products and selects the optimal mix of supply, conversion, and demand technologies according to selected energy, environmental, or economic criteria (usually least cost) to satisfy a set of energy service demands. Energy service demand representations are specified as exogenous requirements by year or annual growth rates. Resource supply representations are specified as long or short term supply curves or fixed prices and availabilities by year. The TESOM model provides a "vintage" representation of the nation's energy system in that the optimal levels of the decision variables for any time period are determined from:

- The optimal levels established for previous periods;
- The retirement and deterioration rates, the lifetimes, and the associated costs of vintage capital stocks;
- Economic and technological factors affecting the feasible levels of the decision variables for the period under investigation (e.g., decline rates, supply elasticities, cumulative resource availabilities, market penetration considerations).

Mathematically, the model is formulated as a sequence of expanding linear programming formulations of the RES--one for each time period. For a given time period, the solutions derived for earlier periods along with assumptions regarding retirement rates, average lifetimes, age-dependent conversion efficiencies, plant factors, O&M costs, and the capital charges for the stocks-in-place are incorporated into the sequenced formulation. Energy demands are satisfied in accordance with the supply expansions and increased penetrations that are attainable for the period, the net resource and stock availabilities from the evolution of the energy system to date, and the objective function criteria selected. In meeting future energy demands, TESOM has a memory of the net availabilities from the past augmented by that which is attainable in the near-term. The updating and sequencing procedures are repeated until solutions for the entire time horizon are determined.

The input information required for the model falls into seven general time-dependent categories: (1) supply and demand efficiencies; (2) costs; (3) environmental impact factors; (4) resource availabilities (entered explicitly or generated by submodels); (5) basic energy demands (entered explicitly

REFERENCE ENERGY SYSTEM, YEAR 1977

TOTAL ENERGY, 1977

73.84×10^{18} Btu

AVAILABLE ENERGY, 1977

65.49×10^{18} Btu

57.24×10^{18} Btu

50.67×10^{18} Btu

29.73×10^{18} Btu

14.59×10^{18} Btu

or computed with growth rates); (6) permitted optimistic market penetration levels; and (7) demand load characteristics of all electric service demands.

The supply efficiencies, which relate intermediate energy forms to primary resources, include efficiencies for refining, electric conversion, transmission and distribution of electricity and by-product heat, and synthetic fuel manufacturing. A refining efficiency represents a combined extraction, cleaning, and transportation efficiency and is defined as the ratio of the useful energy content of the fuel delivered to the user to the energy content at the mine-mouth or wellhead. In addition to these supply efficiencies, effective end-use utilization efficiencies must be specified for each utilizing device. Finally, efficiency and electric generation plant factor decay rates must be specified for all conversion equipment as a function of age.

The costs considered in TESOM include charges for investment in capital equipment, operation and maintenance of the stock, transmission and distribution of electricity, and resource consumption. Capital recovery costs are associated with each individual supply category and end-use device. Also specified are maximum utilization factors for electric generation plant types. The actual capacity factor is determined by the model and is related to the load factor of the demands served by each technology.

Environmental coefficients are associated with each process in the various fuel paths from primary resource supply to end-use. The total emissions for each process are calculated and summed.

There are six main categories of constraint equations used in TESOM.

(1) Energy conservation process equations balance the energy content of fuels into a process with the energy content of fuels out from a process including energy losses.

(2) Supply equations limit the amount of a given resource, at a given price, in terms of the energy content of the primary fuel (mine-mouth and well-head) available from the supply category.

(3) Demand equations specify the level of nonsubstitutable functional end-use energy demands.

(4) Electrical supply and demand load characterization equations limit the electrical generating capacity of plant types as well as accounting for peak electric demands and energy balances. The load characterization equations

relate the type and quantity of electric generating capacity to the demand load characteristics of each demand type by season and time of day.

(5) Environmental constraints limit emissions of various types, such as sulfur dioxide or particulates.

(6) Market penetration equations limit the technology penetration of markets and define the range of permissible fuel usage or permitted capacity of a particular type of energy conversion process.

TESOM provides a time dependent representation of the electric sector. Certain of the energy service demands may be satisfied by electricity. However, the simple summation of the electricity inputs to these end-use devices does not determine the generating capacity which must be installed. Rather, a set of demand types (e.g., base and intermediate loads, off-peak, heating, cooling, etc.) are defined. Each demand type has its own set of characteristics regarding its stochastic behavior and its seasonal (winter, summer, spring-fall) and daily (day, night) loading. Required capacity is governed by the highest total peak demand which occurs during any time of the year and day. By appropriately loading the electric energy service and, subsequently, loading these onto the various season-day combinations, capacity is simply the maximum of the individual season-day peaks with allowances for transmission and distribution losses and reserve margins. This feature permits the introduction of load management considerations into the problem formulation as the load duration curve is, in part, exogenously determined from the detailed demand characteristics and their implications for the electric system.

The model contains a number of features designed to smooth the intertemporal transitions indicated by the sequence of solutions. Among these are the mechanisms for pricing and adjusting the availabilities of vintage stocks and an improved market penetration algorithm which avoids the all-or-nothing (bang-bang) characteristic of linear programming models. The annualized capital charges for the technology imposes a cost on the system as long as the age of a vintage stock is within its economic lifetime. Thus, fixed costs associated with previous investments are incurred regardless of whether or not it is optimal to operate, partially or fully, an older technology. "Unrealistic" displacements of relatively inefficient vintages are discouraged when the associated variable costs of alternative stocks are combined with the fact that immediate write-offs are precluded.

The procedure for market penetration requires, as input, "optimistic" penetration levels for each technology in each year. The penetration algorithm then incorporates the marginal values, implementation rates, and lag times from previous periods as well as the technological and market characteristics for the current period into the determination of more realistic bounds for the activity levels. The user-specified bounds are adjusted endogenously to account for the previous penetration and attractiveness of the technology.

SPECIAL FEATURES - Time Dependent Adjustments

Inventory Characterizations Over Time. For the initial time period, preferably a historical year, TESOM requires, as input, specifications for each kind of conversion device averaged over all stock in place. These "averaged" specifications included economic lifetime, capital cost, operation and maintenance cost, conversion efficiency, fixed charged rate, and years remaining in the economic lifetime. For following periods, generally future years, similar specifications are required, but they are expected to characterize new stock introduced in that period rather than an average of vintaged stock.

Investment costs are specified in constant dollars and are annualized (rented) using a capital recovery factor which depends on the devices' fixed charge rate and economic life. Specifically, if FCR is the fixed charge rate (interest rate) specified by device and year of introduction and LIFE is the economic life also specified by device and year of introduction then the capital recovery factor is given by

$$\frac{FCR}{1 - (1 + FCR)^{-LIFE}}$$

Investment costs may also be specified as time dependent to reflect learning curves for advanced technologies; that is, initial investments may be high but may decline to some asymptotic value. The model ensures that annualized capital charges for equipment introduced in earlier periods continue to be paid over the economic lifetime of the device. This discourages premature retirement of equipment. Operation and maintenance (O&M) charges are accrued by installed devices only if they are utilized (at a rate proportional to their usage).

Capital charges cease at the end of the economic life of the device even though some of the devices installed are available for use past their economic life. For a given device type let

f = a fraction such that $f \cdot \text{LIFE}$ is the number of years of potential operation beyond the economic lifetime;

t = the set of periods where $t \in \{[0,1,\dots]\}$;

Y_t = the year representing period t ;

$A_{t,t-k}$ = the amount of stock installed in period $t-k$ available in period t ;

$\Delta T_{t,t-k} = Y_t - Y_{t-k}$ = the number of years between periods;

α = a scale factor to be used in association with the tanh function.

When $t-k \neq 0$,

$$A_{t,t-k} = \begin{cases} A_{t-k,t-k} & \text{if } \Delta T_{t,t-k} \leq \text{LIFE} \\ 0 & \text{if } \Delta T_{t,t-k} \leq (1+f) \cdot \text{LIFE} \\ A_{t-k,t-k} \cdot \gamma_{t,t-k} & \text{otherwise} \end{cases}$$

where

$$\gamma_{t,t-k} = 1 - \tanh \left(\frac{\alpha \cdot (\Delta T_{t,t-k} - \text{LIFE})}{f \cdot \text{LIFE}} \right).$$

Since the devices "introduced" in the initial period represent an "average" of vintage stock already in place, the availability of these devices over time is computed in a slightly different manner.⁹

To represent the aging of devices, the TESOM characterization of energy conversion technologies incorporates operation and maintenance cost growth rates, efficiency decay rates, and, for electricity generating plants, maximum availability factor; the maximum availability factor is defined to be the maximum fraction of the year that a given type of device could be in operation. All of these growth/decay rates are assumed to be linear.

Market Penetration Adjustments. Market penetration, the gradual substitution over time of a new commodity for one or more existing commodities in satisfying basic consumption or production needs, is a process which must be captured in some form if the projection framework for evaluating evolving energy systems is to have credibility. Hurter and Rubenstein¹⁰ provide a general

literature search and review of work in the area while Condap and Kydes¹¹ discuss market penetration representations in relation to an important new class of energy equilibrium models, of which a typical example is the U.S. Department of Energy Long-range Energy Analysis Program (LEAP)¹² model. This class of models incorporates market penetration in an integrated framework.

The TESOM formulation permits a natural marriage of the traditional stand-alone market penetration algorithms with an integrated energy system's framework by incorporating cost effectiveness, time lags, and market shares, determined from the TESOM model, as the key coupling mechanisms to link with the traditional approaches.

For each type of energy conversion device (e.g., electricity generating plants, end-use devices) TESOM adjusts the optimistic market penetration rates through time. Marginal values are generated for each technology in the penetration algorithm by imposing small lower bounds for each year. To keep the problem size manageable, those device types which remain at the lower bound are deleted from the inventory rather than carried forward. If two conversion devices are equally attractive, "within noise" (as specified by the user), then the new capacities installed for each technology are proportional to the ratios of capacity expansion potential implied by the optimistic penetration rates. The time lags, marginal values, and current market shares are used to adjust the market penetration trajectory.

To define the market penetration optimistic upper bound adjustment algorithm for a given device, let

π_t = the marginal value for the technology in period t ;

U_t = the user-specified optimistic upper bound on fuel consumption (production in the case of processing plants) over all vintage stock in period t ;

AT_t = the measure of the technology's cost attractiveness for period t ;

a = the user-specified weight for the importance of cost attractiveness on behavior;

β = the user-specified weight for the importance of time lag on behavior;

τ = the upper bound on marginal values above which the effects are unchanged for market penetration (required to normalize the penetration function);

σ = the value below which marginal values are considered "noise" in the system;

I_t' = the optimistic upper bound on the fuel consumption (production) by stock that can be introduced in period t ;

I_t = the adjusted upper bound on fuel consumption (production) by newly introduced (period t) stock;

$\gamma_{t,t-k}$ = fraction of capacity still in existence as described in an earlier section;

$E_{t,t-k}$ = conversion efficiency in period t of period $t-k$ vintage stock;

P = the most recent positive optimistic increment. See below.

The optimistic new stock upper bound is computed as

$$I_t = \begin{cases} U_t & \text{if } t = 0 \\ U_t - \left(\sum_{i=0}^{t-1} I_i \frac{*E_{i,i}}{E_{t,i}} * \gamma_{t,i} \right) & \text{if } t \geq 0. \end{cases}$$

The parameter P , which is initialized to zero before period 0, is set equal to I_t if $I_t \geq 0$.

The cost attractiveness indicator is derived from technology's marginal value in the previous period as follows:

$$AT_t = \begin{cases} \tau & \text{if } \pi_{t-1} \geq \tau \\ -\tau & \text{if } \pi_{t-1} \leq -\tau \\ 0 & \text{if } |\pi_{t-1}| \leq \sigma \\ \pi_{t-1} & \text{otherwise} \end{cases}$$

Finally the adjusted incremental upper bound is calculated as

$$I_t = P * \tanh(\alpha * AT_t + \beta * \Delta I_{t,t-1}).$$

The parameter P rather than I_t is used in the above calculation so that the overall penetration of the technology can continue to grow, up to the optimistic upper bound, even though the optimistic increment may be less than or equal to zero.

By fixing the availability of old stock as described in an earlier section and restricting the fuel consumption (production) by new stock to the adjusted increment upper bound, TESOM controls the market penetration of the technology for each vintage stock.

Surrogates for Expectation and Uncertainty

Although TESOM is basically a deterministic model it currently contains three modeling features which incorporate uncertainty and expectation.

Consumer fuel price expectation. Resource prices are functionally dependent on the current period price plus a weighted average of the price trend realized in the last two periods. That is, if $P(t)$ is the price of the resource in period t ,

$$r_t = \frac{P(t) - P(t-1)}{\Delta T_{t,t-1}}$$

and W is the expectation trend by consumers of continued cost escalation, then the perceived cost on which consumers respond is

$$\tilde{p}(t) = p(t) + w \cdot r_t .$$

This can be viewed as a first-order adjustment to consumer uncertainty of prices; $w = 0$ is the case of perfect myopia. The case $w = 0$ implies that consumers ignore recent price trends and only current prices are used for decision making.

Distribution of market shares for nearly competitive technologies. In a national energy model, such as TESOM, differences in regional transportation costs, coefficients of performance, and weather conditions imply that when technologies are nearly equally competitive, they ought to capture the same fraction of their respective market growth potential in the next period. This behavior is approximated in TESOM (and can equally well be approximated in other LP modeling frameworks) by adding ratio constraints when competitive technologies are equal within some prescribed tolerance level indicated by shadow prices.

For each competitive technology, let

x_i^t = new capacity actually built in time period t of conversion device type i (to be computed);

c_i^t = new maximum capacity growth potential in time period t for conversion device type i (input).

Then, if competing technologies, i and j , are "equally attractive within noise" in period $t - 1$, the following constraint is added for period t :

$$\frac{x_i^t}{c_i^t} - \frac{x_j^t}{c_j^t} = 0.$$

Obsolescence of conversion devices. Conversion devices are assumed to be available for use through their economic life with decreasing conversion efficiencies and increasing O&M costs. Devices are retired in an approximately exponential manner after their economic life while the remaining surviving units may continue to be used in a manner consistent with their age.

The inventory characterization of energy conversion devices and the time dependent adjustments described are all intended to reduce or eliminate the "flip-flop" or "all-or-nothing" behavior of linear programming (optimization) formulations.

Two additional problems persist with modeling market penetration of new technologies in single region mathematical programming energy models. State-of-the-art, calibrated market penetration models, have generally not been integrated with large-scale energy models to provide "sensible" time dependent growth potential nor have the regional variations in transportation costs been captured to provide reasonable long-term steady-state market shares of technologies on a national scale. Setting market penetration rates in an optimization framework is usually developed in a largely ad-hoc manner.

Proposed New Adjustment to Market Penetration

Suppose prices of competing products (oil space heat, gas space heat, resistance space heat, etc.) are defined as independent random variables from the same type of distribution with means $\bar{p}_i$ and variances σ_i^2 respectively. The market share of the i -th product in an N product competitive case is defined by

$$S_i = \int_{-\infty}^{\infty} f_i(x_i) dx_i \int_{-\infty}^{\infty} \prod_{j=1}^N f_j(x_j) dx_j$$

and can be analytically determined when the probability density functions, $f_i(x_i)$ are either exponential¹³, Gamma¹³ or Weibull¹³ but must be approximated numerically for normally distributed random variables.¹³ This expression provides equilibrium market shares and assumes that the real prices are not changing and no new competitors appear before equilibrium is reached.

Calibration of this approach is likely to be difficult because new product competitors usually enter the market before equilibrium is reached; the effects of new technologies may be difficult to factor out. One potential approach to calibration would be to consider the market shares for each technology at end-use for an historic year and to seek the variance of each product distribution if the mean national prices, market shares, and type of distribution are known. Assuming that an adequate calibration of this problem can be achieved, the question remains "at what rate and over what time frame can we optimistically expect the products to reach equilibrium?" This question has been studied by a number of authors including Mansfield,¹⁴ Blackman,¹⁵ Fisher and Pry¹⁶ and Peterka¹⁷ and empirical results are available for several technologies.

In principal, we could use, for example, Fisher and Pry's technique which has been calibrated for several industries to predict the optimistic take-over rate and time of each technology. The authors assume that a technology-based substitution will proceed to completion if the challenger attains a few percent of the market and the fractional rate of substitution by the challenger is proportional to the fraction left to the defender. If $f(t)$ is the challenger's share by time Δt , then

$$f'(t)/f(t) = \theta (1 - f(t))$$

or

$$f(t) = [1 + \exp - ((t - t_{0.5}))]^{-1}.$$

where $t_{0.5}$ is the time of 50% substitution. Fisher and Pry anchor their curves to the half replacement and define "take-over time," Δt , as the time for a challenger to increase its share from 10% to 90%. It can be shown that

$$t = 4.4/\theta.$$

The method can be extended to several technologies. We would constrain, however, the market share to that derived from probabilistic considerations, previously defined, to capture the interregional differences in costs or pricing.

* Elapsed time after product captures at least 3% of the market.

This approach attempts to incorporate the state-of-the-art models of market penetration and the interregional price variations expressed probabilistically within an integrating energy system model to generate realistic maximum (and possibly minimum) market penetration rates through time for competing technologies.

III. PROGRAM PLANNING AND PORTFOLIO ANALYSIS

Now that the basic analytical framework and potential refinements have been described, we focus our attention on the initial question of how mathematical programming can be used to guide R&D planning. For R&D planning, investment decisions usually require "lumpy" investment rates. For example, decisions are usually made to build entire pipelines, synthetic oil plants or nuclear reactors, not just parts of them.

Suppose the incorporation of uncertainty or expectation in the success rate for commercialization of advanced technologies and maximization of "expected" energy utilization are two primary concerns. Suppose also that for each technology, s , we can provide N levels of "lumpy" investments, each with different expectations of success. Another analysis is required to produce the relations between expected production y_s^j and investment R_s^j . This might be the case, for example, if R&D test plants for production of a particular synthetic are contemplated. The selection of several different contractors (redundancy) may be the cause for several "lumpy" choices which also presumably results in an increased probability of successful commercialization and synthetic production by a prescribed year. Formally, suppose that the technology s is commercialized, given an R&D investment level of R_s^j ($j = 0, 1, 2, \dots$) dollars, with an expected production (yield, not utilization) of y_s^j .

Further, suppose, for simplicity, that we consider only supply-side technologies; that is, technologies which produce usable energy in existing end-use technologies. The TESOM formulation could be augmented as follows:

$$\sum_j e_j x_j + \sum_s e_s y_s = \text{Demand} \quad : \text{demands must be satisfied}$$

(the e_j are efficiencies)

$$y_s \leq \sum_{k=0}^n y_s^k \cdot \delta_s^k \quad : \text{Utilization may not exceed production.}$$

$$\sum_{k=0}^n \delta_s^k = 1$$

δ_s^k are binary (0,1).

} : these two equations attempt to restrict the choices.

$$R_s - \sum_{k=1}^n R_s^k \delta_s^k = 0 \quad : \text{select at most one spending level}$$

$$\sum_{s=1}^N R_s \leq \text{Budget} \quad : \text{budget constraint}$$

$$y_s^0 \leq y_s^1 \leq y_s^2 \leq \dots \leq y_s^n \quad : \text{greater investments give greater expected production from supply technology "s" (inputs)}$$

This formulation has become a mixed integer programming problem and solution costs, on average, can increase exponentially at approximately 2^{N-1} times the cost of the continuous counterpart; N is the number of binary integer variables (δ 'S). These computing costs quickly become prohibitive for the mixed integer formulations and continuous approximations must be sought. A major advantage and disadvantage of using mathematical programming techniques becomes apparent. It is usually easy to model the investment decision process using mathematical programming techniques but it is also often infeasible to solve these problems at reasonable cost or time using current state-of-the-art algorithms.

If we maximize $\sum_{s=1}^N y_s$, the expected energy consumption from all advanced energy sources, then the restriction that δ_s^j be integer 0 or 1 probably need not be enforced since selection of y_s^j monotonic increasing with j biases the solution toward integer values of δ_s^j . If the integer restriction must be enforced then the formulation becomes prohibitive in cost.

If we use this linear programming formulation and assumptions embedded in the formulation it becomes quite easy to do trade-off analysis among the desired objectives; goal programming¹⁸ becomes a viable option.

At this point it is crucial to define the important conflicting objectives and goals before attempting to complete the analytical framework. Alternative quantifiable measures of "best" must be well-defined before trade-off analysis can be performed with respect to them.

Potentially Useful Objective Functions

Concerns which dominate R&D planning include the distribution of limited R&D funds to maximize some set of desirable impacts. Some of the desirable objectives include:

- Minimization of oil imports or total oil usage. This is often used as a surrogate for "security."
- Minimization of annualized system cost. This presumably provides incentives to consumers to make the long-term switch.

Choices are available as to the level of inclusion of annualized charges in the annualized system cost function. If the Federal government absorbs the entire R&D cost, the cost would appear as government expenditures in a macro-economic model. If they are included, then the time length of the "loan" and the interest rate must be specified and each of these implies a different Federal policy.

- It may be desirable to invest in such a way that the sum of the marginal rates of oil reduction per unit investment is maximized.
- We may wish to maximize the production of useful energy from new technologies.
- We may wish to review the trade-offs in such a way as to maximize the rate of energy benefit evenly between supply and demand technologies.
- An implicit goal in program planning may be to spend as much of the R&D budget allotment as possible since unspent money in one year may mean a lower budget the next.

A System's View of Portfolio Analysis

It is now useful to provide a system's view of R&D strategy development. The process is sketched briefly in a phased procedure given below and the diagrams which follow.

Phase I requires that all R&D options, all measures of benefit and cost, all budget and other quantifiable constraints be carefully identified and formalized. This requires a close working relationship between the policymakers, who know the political realities and limitations, the engineers and economists

("technology assessors") who can accurately characterize the associated technologies, their uncertainties and costs, and the budgetary personnel. A Forum activity or workshop with carefully selected participants might be the appropriate setting to initialize the process. It is also appropriate in Phase I to identify the "reference case," an energy-economy scenario, against which the alternative policy options are to be measured in the cost-benefit analysis.

Identification and analysis of the reference case scenario also serve to identify those technologies, either established or emerging, which are likely or desirable candidates for substitution relative to the criteria identified. Minimization of particulate emissions, for example, might indicate that coal-based technologies should be reduced whereas this runs counter to other goals such as minimization of U.S. national dependence on oil, especially imported products.

PORTFOLIO ANALYSIS

- PHASE I - IDENTIFY ALL R&D OPTIONS; DEFINE THE CRITERIA AND OBJECTIVES TO MEASURE "BENEFITS" AND "COSTS" OF R&D OPTIONS. IDENTIFY BUDGET AND OTHER QUANTIFIABLE CONSTRAINTS.
- PHASE II - DEFINE R&D OPTIONS WHICH OBVIOUSLY REQUIRE NO GOVERNMENT INTERVENTION OR SPENDING.
 - DETERMINE IMPACT ON ENERGY SYSTEM IF NO GOVERNMENT INTERVENTION. RE-EXAMINE PRIORITIES AND CRITERIA. HAVE CONSTRAINTS BEEN FORGOTTEN?
- PHASE III - PRIORITIZE REMAINING OPTIONS ACCORDING TO VARIOUS CRITERIA WITHOUT REGARD TO BUDGET.
- PHASE IV - PRIORITIZE WITH RESPECT TO BUDGET CONSTRAINTS.
- PHASE V - SELECT A GOOD SOLUTION.
 - DO SENSITIVITY ANALYSIS TO GET TRADE-OFFS.
 - SELECT FINAL PRIORITIZED LIST.
 - REVIEW ANALYSIS AND MAKE RECOMMENDATIONS. HAVE CONSTRAINTS BEEN FORGOTTEN?

PORTFOLIO ANALYSIS - PHASES I AND II

PHASE III - PRIORITIZE WITHOUT R&D BUDGET CONSTRAINTS

TECHNOLOGICAL ASSESSMENT HIERARCHY

PHASE IV AND V - PRIORITIZE WITH BUDGET CONSTRAINT

Phases II and III require analysis to determine which technologies are obviously excellent choices and require no government R&D subsidy to prompt eventual commercialization; i.e., the technologies are judged to be socially cost effective and sufficiently low in capital R&D investments so that costs do not form a barrier to commercialization. Those that would require continued government subsidies and do not satisfy enough of the other policy objectives can be deleted from further consideration. "Back of the envelope" calculations should always precede any detailed analysis of technologies to delete obvious poor choices (those that are orders of magnitude unacceptable) and possibly other technologies which are "certain to emerge." Linear programming models such as TESOM can be used to introduce each technology into the reference case at some small but nonzero level (b_i) through time to determine at what year the technology merits commercialization and what subsidies may be required to make the technology cost effective prior to that date. One can then price the value of the technology with respect to every other objective function for each year by simply changing objective functions and using the repricing option in commercial LP packages. If $COST, O_2, \dots, O_N$ are the N objectives to be optimized, where $COST$ is the annualized system cost, and the optimization is made by minimizing $COST$, then repricing with respect to the other objective functions gives the quantity $\partial O_i / \partial b_i$ evaluated at the minimum value of $COST$, as well as the value of all the remaining objective functions where $COST$ is minimized. The variable b_i denotes the bound on advanced technology i being tested for attractiveness and having a small fixed bound b_i .

A second optimization step is usually required to guarantee that the O_i at optimality for $COST$ are "efficient" (i.e., other objectives cannot be improved without worsening $COST$). This is accomplished by minimizing $\sum_i O_i(x)$ subject to the usual energy system constraints and with $COST(x) \leq COST(\min) + \epsilon$.

Relative weights of importance would have to be elicited from each policymaker for each trade-off ($\partial O_i / \partial b_i$) and/or value of the objective functions to ultimately derive a preliminary priority indicator. Better performance of this technique is realized if the weights permitted are standardized, discrete, and well separated.

A set of better approaches exists. See Schank⁹ or Zionts and Wallenius.²¹ Schank identifies and tests an efficient semi-automated procedure using linear and nonlinear objective functions which are functions of the objectives

originally quantified within the context of BESOM constraints to derive a "good solution." The procedure begins by first formulating a reference case to simulate some energy scenario. Upper and lower bounds are relaxed to fractions of the reference case to allow meaningful trade-offs and small nonzero lower bounds are forced on all advanced technologies. The dual solution provides a priority ordering of the technologies with respect to the composite objective function. Trade-offs with respect to the original objectives can be accomplished as previously described.

The approach by Zionts is interactive and requires that all the objectives be quantified. The number of objectives should be kept relatively small (no more than six) to keep the procedure efficient. A preliminary solution with alternative efficient adjacent solutions is offered and the user is requested to respond for each alternative if he prefers, rejects, or is indifferent to the new solution. If at least one preferred solution is stated, a new convex combination of the weights is generated for the composite objective function and new constraints added based on the responses; a new solution with alternatives is again presented. Alternative solutions which have already been offered are deleted to avoid potential infeasibility. In the end, the procedure terminates when no new preferred options are found. The solution is then presented and a ranking is made by using the duals with respect to this composite function. The individual trade-offs with respect to the original objectives and parametric analysis can be performed on the weights of the final objective function to determine the sensitivity of the priority ordering to the choice of weights. This problem has not been solved satisfactorily for multiple decision makers. At this point, advanced technologies which do not enter the basis (that is, those which enter at the lower bound) and whose shadow prices with respect to the combined objective are below some prescribed threshold (say \$2.50/MMBtu equivalent) are removed from further consideration.

Determination of obvious good choices are those that are highly desirable from a consumer's viewpoint (cost effective, secure, socially desirable) and commercially acceptable. Determination of commercial acceptability can be made through the use of dynamic sector-specific, optimization models which include the cost of R&D investments and commercialization. The Iron and Steel Industry model which considers the industry in considerably more detail than national or even regional models is one example. Another potential approach would include

the use of a market penetration model, such as Peterka.¹⁷ Peterka uses aggregate assessments of capital and operating costs to forecast substitution of primary energy resources. His main premise is that each basic industry corresponding to these fuels increases its production capacity through internally generated revenues once it has established itself as a viable conversion technology. The R&D cost can be amortized over some reasonable period, consistent with that industry.

The market shares formulation for the i^{th} industry is derived from the relation

$$\alpha_i(t) \cdot \dot{x}_i(t) = x_i(t) [p(t) - c_i(t)],$$

where for each industry type

$\alpha_i(t)$ = capital cost per unit capacity,

$x_i(t)$ = productive capacity,

$x(t) = \sum_i x_i(t)$ total productive capacity,

$\dot{x}_i(t)$ = rate of change of capacity over time,

$p(t)$ = average market price of processed energy types,

$c_i(t)$ = total operating costs per unit of capacity,

$f_i(t) = x_i(t)/x(t)$,

$r_{ij}(t) = c_i(t) - c_j(t)$.

Then the market share for industry i at

$$f_i(t) = \frac{1}{1 + \sum_{j \neq i} \frac{f_{j0}}{f_{i0}} \exp \left\{ \int_{t_0}^t r_{ij}(\tau) d\tau \right\}},$$

where $f_{j0} = f_j(t_0)$ and t_0 is some point in time when all industries are commercially operational. Note that the $r_{ij}(t)$ could be used to reflect absorbed R&D costs in the early years. New technologies which survive to capture a mid-term nonzero market share are potential candidates for choices which do not need government intervention since the industry will support its development. The same method can be applied to determine the poor choices by eliminating the R&D cost from the r_{ij} . This brings us to the risky ventures.

Phase IV and V require that the technologies be grouped according to whether they are supply or demand types and according to whether they are competitors (substitutes) or complements. This a priori partition of technologies decomposes the prioritization task into smaller, more easily solved components and also lends itself to the questions of diversification, risk aversion, and sensitivity analysis. Two approaches may be considered depending on the availability of a good Mixed Integer Programming (MIP) package or not. Assume first that a good MIP package exists and add eight classes of additional accounting rows as follows:

$$g_{(s)} = \sum_{k \in I_s(s)} y_k ; \text{ the expected utilization from supply category (s).}$$

$$g_{(d)} = \sum_{k \in I_s(d)} y_k ; \text{ the expected consumption}$$

from demand category $I_s(d)$.

$$g_s = \sum_s g_{(s)} : \text{ the expected utilization of supply side technologies.}$$

$$g_d = \sum_d g_{(d)} ; \text{ the expected consumption from demand side technologies;}$$

$$L(s) = \sum_{k \in I_s(s)} \sum_{j=1} \delta_k^j : \text{ the number of technologies selected from supply group ;}$$

$$L_s = \sum_s L(s) : \text{ the number of new supply technologies selected;}$$

$$L(d) = \sum_{k \in I_s(d)} \sum_j \delta_k^j : \text{ the number of technologies selected from demand group (d);}$$

$$L_d = \sum_d L(d) : \text{ the number of demand technologies selected;}$$

We list and describe a few potentially useful measures of risk aversion and diversification.

- maximize $g_s + g_d$: maximize total expected consumption from potential new technologies;
- maximize $[(g_s + g_d) - \alpha Z]$ with

$$\begin{aligned} L_s - L_d &\leq Z \\ -L_s + L_d &\leq Z \quad \text{with } Z \leq 0: \end{aligned}$$

maximize total consumption from all advanced sources while attempting to select evenly from both supply and demand side options; α is the relative importance assigned to having even distribution between supply and demand sides versus the objective maximizing consumption.

- maximize $[(g_s + g_d) - \alpha Z]$ with

$$\begin{aligned} L(s) &\leq Z \\ -L(s) &\leq Z \\ L(d) &\leq Z \\ -L(d) &\leq Z \quad \text{with } Z \leq 0: \end{aligned}$$

- minimize (total imported oil) + αZ with

$$\begin{aligned} L_s - L_d &\leq Z \\ -L_s + L_d &\leq Z \quad \text{with } Z \geq 0: \end{aligned}$$

minimize total imported oil while attempting to diversify through selection of both supply and demand side options.

- maximize expected consumption of new technologies while attempting to select roughly equal numbers of technologies from each group at the bottom of the technology hierarchy chart.

Clearly, additional objectives can be defined which link expected new energy consumption to reduction of oil usage. The above criteria, as well as those previously listed, require the use of multi-objective analysis techniques to help prioritize the R&D technologies, assist in program planning, and perform strategic energy analysis.

If a good MIP package is not available then heuristic techniques using the continuous counterpart can be used with parameterization on the total budget. This technique has been successfully applied to R&D strategies for conservation measures.

IV. Concluding Remarks

Optimization techniques can and have been effectively used as normative tools and provide a natural framework for including policymakers at several critical decision points, including the formation of the relevant criteria, costs and objectives for multicriteria analysis. In an interactive mode, the policymaker must help in the decision phase in the trade-off analysis and this forces the policymaker and analyst to communicate information relevant to the model.

The primary focus of this paper has been the discussion of techniques which can be embedded within certain sequentially dynamic LP formulations of the energy system to avoid, in a realistic way, the "bang-bang" behavior of typical LP's.

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