BENDING STRESS AND FREQUENCY
CALCULATIONS FOR THE BEARING-PACK SHAFT*

Jeff Barnwell
Don W. Dareing

Maurer Engineering, Inc.
Houston, Texas

ABSTRACT

The bearing-pack shaft investigation was conducted to analyze bending and cycle fatigue stresses and resonance. The bending stresses produced were considered to be minimal, and the cycle fatigue stress was less than the material's endurance level. Resonating frequencies were estimated to be sufficiently high to preclude problems.

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INTRODUCTION

The purpose of this bearing-pack shaft investigation is three-fold. As stated in the task description section of the contract, the objectives included:

1. analysis of the bending stresses produced in the shaft by deviation forces acting on the drill bits;
2. fatigue analysis of the highly cycled bearing-pack shaft; and
3. frequency analysis of the bearing-pack-turbine shaft assembly to determine if resonances may occur.

The results, and the solution methods used to reach them, are given in the following sections. Conclusions concerning this design check of the bearing-pack shaft are given in the final Conclusion-Discussion section.

BENDING STRESS ANALYSIS

The bending stress analysis considers the portion of the bearing-pack shaft below the bottom internal radial bearing. The bearing assembly clamps or fixes the shaft to the housing which stiffens and supports the remaining portion of the shaft. The bottom portion is therefore more vulnerable to lateral loads at the drill bit.

Figure 1 shows the bearing-pack assembly and identifies the portion of the shaft that is stress analyzed according to simple beam theory.

Figure 1

Thus, the bearing shaft is considered fixed (no deflection or longitudinal slope) at the first radial bearing (PN79-498). Then as the transverse and axial loads cause deflection of the bit-end, the portion of the shaft between the bearing and the point of application of the loads can be considered a cantilever beam, composed of two distinct sections, each possessing a different section modulus. This configuration is shown in Figure 2.

Figure 2

The simple support shown in Figure 2 at position (2) is representative of the lower seal leak sleeve (PN79-463) and the distance, δ_o , is the radial clearance between the sleeve and the shaft.

Here, it is recognized that the stiffness of the section between point (2) and the point of application of the load is much greater than that of the portion between points (1) and (2). So, by considering the stiffer section as rigid, we can apply loads at point (2) and add a moment to account for the difference. The configuration is shown in Figure 3.

Figure 3

However, a complication arises when the deflection of the shaft exceeds δ_0 , the radial clearance. When $\delta = \delta_0$, the reaction force, R , must be included. The final configuration would then be as shown in Figure 4.

Figure 4

The differential equations of beam bending were applied to this model, solved in closed form, and programmed on the computer. The program has inputs for bending length, stiffened length, inside and outside diameters of the bending section, axial bit-force components, and values of radial clearance. The program then varies the values of the transverse force between 0-10,000 pounds for any given set of input values. Output consists of the lateral deflection, sleeve reaction, axial stress, bending and maximum stresses at position (1) and (2).

In the actual utilization of the program, several parameters, such as bit weight and sleeve clearance, were varied to determine their effects on stress magnitude. The radial clearance was set at .0025", which is the ideal (no tolerance) distance. Increases in bit forces move the

curves toward higher stress levels (Graphs 1-3). Their general shapes remain the same. The stress at position (1) rises with increasing transverse force until the point where deflection is equal to the radial clearance. From this point, the stress at (1) falls with increases in transverse force. However, the relation between stress at position (2) and transverse force is always positive and linear. The maximum stress predicted occurs at position (2) with a 50,000 pound transverse force. The level was 28,833 psi (compression).

Variations in radial clearance affect stress magnitude as follows. By using dimensions and tolerances from design drawings, the radial clearance was found to vary from .0025" to .0045". Referring to Graph 4, it is seen that increasing the clearance does increase the stresses at position (1). But even with the maximum .0045" clearance, these do not approach the maximum level of position (2) stresses. (NOTE: there is a small range, corresponding to low transverse forces, where position (1) stresses are higher than position (2) levels.)

FATIGUE ANALYSIS

The results of the bending stress analysis were then used in conjunction with failure theory to give an estimate of the safety factor for fatigue failure. The critical stress was found on the outer surface of the shaft. This stress is a uniaxial state of stress and varies periodically with each revolution of the shaft. Maximum and minimum stress levels are as follows:

$$\sigma_{\max} = +25,067 - 3,767 = 21,300 \text{ psi}$$

$$\sigma_{\min} = -25,067 - 3,767 = -28,834 \text{ psi}$$

Since these are limits on cyclic stresses, Soderberg's theory of fatigue failure was used in conjunction with maximum-normal-stress safety factor determination to yield an equivalent safety factor in fatigue.

$$\sigma_{av} = \frac{\sigma_{max} + \sigma_{min}}{2} = 3,767 \text{ psi}$$

$$\sigma_{range} = \frac{\sigma_{max} - \sigma_{min}}{2} = 25,067 \text{ psi}$$

Now, finding an equivalent normal stress:

$$\begin{aligned}\sigma_{equiv} &= \sigma_{av} + \left(\frac{\sigma_{yp}}{\sigma_{endurance}} \right) \sigma_r \\ &= 3,767 + \left(\frac{120,000}{47,000} \right) 25,067 \\ &= 67,768 \text{ psi}\end{aligned}$$

$$fs = \frac{\sigma_{yp}}{\sigma_{equiv}} = \frac{120,000}{67,768} = 1.77$$

It was observed that there would also be a torsional shearing stress. However, at maximum horsepower, the amount of stress produced was insignificant when compared to the tension-compression figures given earlier.

FREQUENCY ANALYSIS

The stiffened section consisting of the box connection and bit was considered as rigid with moment of inertia I . The bearing shaft (excluding the lower box connection) was taken as flexible with length L_1 , polar moment of area J_1 , mass moment of inertia I_1 , and torsional stiffness $\frac{GJ_1}{L_1}$. The turbine rotor shaft was approximated as having L_1 similar parameters. However, it was noted that the mass moment of inertia of the bearing-shaft was much less than those of the bit end box and the turbine rotor shaft. Also, the stiffness of the turbine shaft was much greater than that of the bearing shaft. So, the system was modeled as a massless shaft with torsional rigidity GJ_1/L_1 , with a rigid disk attached on each end.

By using the approach of modal analysis for a 2-degree-of-freedom system, the natural frequencies of vibration were found. The fundamental frequency is obviously 0 cps, corresponding to rigid body rotation. The second natural frequency was found to be $665 \frac{\text{rad}}{\text{sec}}$. Refer to the appendix for derivation of the governing equations.

1944-1945

The first part of the year was spent in the
field, working on the project. The weather was
very good, and the work was done in a
pleasant atmosphere. The results of the
work were very satisfactory, and the
project was completed on time. The
second part of the year was spent in the
office, working on the project. The weather was
very good, and the work was done in a
pleasant atmosphere. The results of the
work were very satisfactory, and the
project was completed on time.

The third part of the year was spent in the
field, working on the project. The weather was
very good, and the work was done in a
pleasant atmosphere. The results of the
work were very satisfactory, and the
project was completed on time. The
fourth part of the year was spent in the
office, working on the project. The weather was
very good, and the work was done in a
pleasant atmosphere. The results of the
work were very satisfactory, and the
project was completed on time.

CONCLUSIONS

A review of the bearing-pack design reveals the following conclusions.

Bearing Shaft Stress

The stresses produced by bending are minimal when considered as static stresses and compared to the yield point of the material. In increasing bit force, linear increases in maximum stress levels were noted. In addition, the deflection has an effect on maximum stress levels at the fixed-bearing end, but the critical stress is always found at the lower box end of the bearing-shaft.

Fatigue Analysis

The maximum cyclic stress was much lower than the endurance level for the material. When Soderberg's theory of fatigue failure was applied, a safety factor of 1.77 was found. Although this did not account for stress concentrations, it was assumed all notches were sufficiently radiused and polished.

Frequency Analysis

Resonance should present no problem since the estimate by the model was 106 rps (=6,420 RPM), which is much higher than any potential use of the tester assembly.

GRAPH #1

GRAPH #2

GRAPH #3

GRAPH #4

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APPENDIX

- a-1 Bending Stress Derivations
- a-2 Bending Stress Computer Printouts
- a-3 Characteristic Equation Calculations
 Frequency Analysis

a-1

In general

$$EI y'' = M_0 + (F_T - R) x + F_a (\delta - y)$$

$$EI y'' + F_a y = M_0 + (F_T - R) x + F_a \delta$$

First Case $\delta \leq \delta_0$, therefore $R = 0$

$$EI y'' + F_a y = M_0 + F_T x + F_a \delta$$

Complimentary Solution

$$y_c = A \sin \alpha x + B \cos \alpha x$$

$$\text{where } \alpha^2 = \frac{F_a}{EI}$$

Particular Solution

$$y_p = \frac{1}{F_a} (M_0 + F_T x + F_a \delta)$$

Total Solution (Case 1)

$$y = A \sin \alpha x + B \cos \alpha x + \frac{M_0}{F_a} + \frac{F_T}{F_a} x + \delta$$

$$y' = A\alpha \cos \alpha x - B\alpha \sin \alpha x + \frac{F_T}{F_a}$$

Boundary Conditions are

$$y(L) = 0$$

$$y'(L) = 0$$

$$0 = A \sin \alpha L + B \cos \alpha L + \frac{M_0}{F_a} + \frac{F_T}{F_a} L + \delta$$

$$0 = A \cos \alpha L - B \sin \alpha L + \frac{1}{\alpha} \frac{F_T}{F_a}$$

also note

$$y'(0) = A\alpha + \frac{F_T}{F_a}$$

So

$$M_0 = \ell F_T + \ell F_a \left(A\alpha + \frac{F_T}{F_a} \right)$$

$$M_0 = \ell F_T + \alpha \ell F_a A + \ell F_T$$

$$M_0 = 2\ell F_T + \alpha \ell F_a A$$

Substituting gives

$$A \sin \alpha L + B \cos \alpha L = -2\ell \frac{F_T}{F_a} - \alpha \ell A - \frac{F_T}{F_a} L - \delta \quad (1)$$

$$A \cos \alpha L - B \sin \alpha L = -\frac{1}{\alpha} \frac{F_T}{F_a} \quad (2)$$

which are equal to

$$A (\alpha \ell + \sin \alpha L) + B \cos \alpha L = -2\ell \frac{F_T}{F_a} - L \frac{F_T}{F_a} - \delta \quad (1)$$

$$A \cos \alpha L - B \sin \alpha L = -\frac{1}{\alpha} \frac{F_T}{F_a} \quad (2)$$

or

$$A (\alpha \ell + \sin \alpha L) + B \cos \alpha L = -(L + 2\ell) \frac{F_T}{F_a} - \delta \quad (1)$$

$$A \cos \alpha L - B \sin \alpha L = -\frac{1}{\alpha} \frac{F_T}{F_a} \quad (2)$$

Note also

$$y(0) = \delta$$

$$\delta = B + \frac{M_0}{F_a} + \delta$$

$$0 = B + 2\ell \frac{F_T}{F_a} + \alpha\ell A \quad (3)$$

Three equations to be solved simultaneously are

$$(\alpha\ell + \sin\alpha L) A + \cos\alpha L B + \delta = - (L + 2\ell) \frac{F_T}{F_a} \quad (1)$$

$$\cos\alpha L A - \sin\alpha L B + 0 = - \frac{1}{\alpha} \frac{F_T}{F_a} \quad (2)$$

$$\alpha\ell A + B + 0 = - 2\ell \frac{F_T}{F_a} \quad (3)$$

which can be written in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} A \\ B \\ \delta \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Once A, B, and δ are determined, M and M_0 follow.

Second Case $\delta = \delta_0$, therefore $R = 0$

$$EI y'' + F_a y = M_0 + (F_T - R) x + F_a \delta_0$$

$$y_c = A \sin \alpha x + B \cos \alpha x$$

$$y_p = \frac{1}{F_a} [M_0 + (F_T - R) x + F_a \delta_0]$$

$$y = A \sin \alpha x + B \cos \alpha x + \frac{1}{F_a} [M_0 + (F_T - R) x + F_a \delta_0]$$

$$y' = A \alpha \cos \alpha x - B \alpha \sin \alpha x + \frac{F_T - R}{F_a}$$

Boundary Conditions

$$y(L) = 0$$

$$y'(L) = 0$$

$$0 = A \sin \alpha L + B \cos \alpha L + \frac{M_0}{F_a} + \frac{F_T - R}{F_a} L + \delta_0 \quad (1)$$

$$0 = A \cos \alpha L - B \sin \alpha L + \frac{1}{\alpha} \frac{1}{F_a} (F_T - R) \quad (2)$$

recall

$$M_0 = \ell F_T + \ell \theta F_a$$

but

$$\theta = y'(0) = A\alpha + \frac{F_T - R}{F_a}$$

therefore

$$M_0 = \ell F_T + \ell (\alpha F_a A + F_T - R)$$

$$M_0 = 2\ell F_T - \ell R + \alpha \ell F_a A$$

substituting into (1) gives

$$\begin{aligned} 0 = A \sin \alpha L + B \cos \alpha L + 2\ell \frac{F_T}{F_a} - \frac{\ell R}{F_a} + \alpha \ell A \\ + \frac{F_T}{F_a} L - \frac{R}{F_a} L + \delta_0 \end{aligned} \quad (1)$$

or

$$\begin{aligned} 0 = (\alpha \ell + \sin \alpha L) A + \cos \alpha L B - \left(\frac{L + \ell}{F_a} \right) R \\ + (2\ell + L) \frac{F_T}{F_a} + \delta_0 \end{aligned} \quad (1)$$

Also Note $y(0) = \delta_0$

$$\delta_0 = B + \frac{M_0}{F_a} + \delta_0 \quad (3)$$

$$0 = B + \frac{F_T}{F_a} 2\ell - \frac{\ell}{F_a} R + \alpha\ell A$$

the three equations are

$$(\sin\alpha L + \alpha\ell) A + \cos\alpha L B - \left(\frac{L + \ell}{F_a}\right) R = - (2\ell + L) \frac{F_T}{F_a} - \delta_0 \quad (1)$$

$$\cos\alpha L A - \sin\alpha L B - \frac{1}{\alpha F_a} R = - \frac{1}{\alpha} \frac{F_T}{F_a} \quad (2)$$

$$\alpha\ell A + B - \frac{\ell}{F_a} R = - \frac{F_T}{F_a} 2\ell \quad (3)$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} A \\ B \\ R \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

M and M_0 follow

RUN

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BENDING LENGTH, INCHES?10.4375

STIFFENED LENGTH, INCHES?21.875

INSIDE DIAMETER, INCHES?1.8125

OUTSIDE DIAMETER, INCHES?4.493

AXIAL BIT FORCE COMPONENT, LBS?30000

RADIAL CLEARANCE, INCHES?.0025

TRANSVERSE FORCE LBS	LATERAL DEFLECTION INCHES	SLEEVE REACTION LBS	AXIAL STRESS PSI	BENDING STRESS (1)PSI	MAX STRESS (1)PSI	BENDING STRESS (2)PSI	MAX STRESS (2)PSI
0.	0.0000	0.	2260.	0.	2260.	0.	2260.
500.	0.0013	0.	2260.	1850.	4110.	1244.	3504.
1000.	0.0025	256.	2260.	3393.	5653.	2489.	4749.
1500.	0.0025	2324.	2260.	2764.	5024.	3747.	6007.
2000.	0.0025	4391.	2260.	2135.	4395.	5005.	7265.
2500.	0.0025	6458.	2260.	1506.	3766.	6263.	8523.
3000.	0.0025	8526.	2260.	876.	3136.	7521.	9781.
3500.	0.0025	10593.	2260.	247.	2507.	8779.	11039.
4000.	0.0025	12661.	2260.	-383.	1877.	10037.	12297.
4500.	0.0025	14728.	2260.	-1011.	1249.	11295.	13555.
5000.	0.0025	16795.	2260.	-1640.	620.	12553.	14813.
5500.	0.0025	18863.	2260.	-2270.	-10.	13811.	16071.
6000.	0.0025	20930.	2260.	-2898.	-638.	15070.	17330.
6500.	0.0025	22998.	2260.	-3528.	-1268.	16327.	18587.
7000.	0.0025	25065.	2260.	-4158.	-1898.	17585.	19845.
7500.	0.0025	27133.	2260.	-4786.	-2526.	18844.	21104.
8000.	0.0025	29200.	2260.	-5417.	-3157.	20101.	22361.
8500.	0.0025	31268.	2260.	-6045.	-3785.	21359.	23619.
9000.	0.0025	33335.	2260.	-6674.	-4414.	22618.	24878.
9500.	0.0025	35402.	2260.	-7304.	-5044.	23875.	26135.
10000.	0.0025	37470.	2260.	-7932.	-5672.	25134.	27394.

PROGRAM STOP AT 1040

USED 5.87 UNITS

RUN

BEARPACK 09:46CDT 09/25/79

BENDING LENGTH, INCHES?10.4375

STIFFENED LENGTH, INCHES?21.875

INSIDE DIAMETER, INCHES?1.8125

OUTSIDE DIAMETER, INCHES?4.493

AXIAL BIT FORCE COMPONENT, LBS?40000

RADIAL CLEARANCE, INCHES?.0025

TRANSVERSE FORCE LBS	LATERAL DEFLECTION INCHES	SLEEVE REACTION LBS	AXIAL STRESS PSI	BENDING STRESS (1)PSI	MAX STRESS (1)PSI	BENDING STRESS (2)PSI	MAX STRESS (2)PSI
0.	0.0000	0.	3013.	0.	3013.	0.	3013.
500.	0.0013	0.	3013.	1846.	4859.	1238.	4251.
1000.	0.0025	245.	3013.	3398.	6411.	2477.	5491.
1500.	0.0025	2311.	3013.	2770.	5783.	3734.	6748.
2000.	0.0025	4377.	3013.	2141.	5154.	4991.	8004.
2500.	0.0025	6443.	3013.	1512.	4526.	6248.	9261.
3000.	0.0025	8509.	3013.	884.	3897.	7505.	10518.
3500.	0.0025	10574.	3013.	255.	3269.	8762.	11775.
4000.	0.0025	12640.	3013.	-374.	2639.	10018.	13031.
4500.	0.0025	14706.	3013.	-1002.	2011.	11275.	14289.
5000.	0.0025	16772.	3013.	-1631.	1382.	12532.	15545.
5500.	0.0025	18838.	3013.	-2260.	754.	13789.	16802.
6000.	0.0025	20904.	3013.	-2888.	125.	15046.	18059.
6500.	0.0025	22970.	3013.	-3517.	-504.	16303.	19316.
7000.	0.0025	25036.	3013.	-4145.	-1132.	17560.	20573.
7500.	0.0025	27102.	3013.	-4774.	-1761.	18816.	21829.
8000.	0.0025	29168.	3013.	-5404.	-2390.	20072.	23085.
8500.	0.0025	31234.	3013.	-6031.	-3017.	21330.	24344.
9000.	0.0025	33300.	3013.	-6660.	-3647.	22586.	25600.
9500.	0.0025	35366.	3013.	-7289.	-4276.	23843.	26856.
10000.	0.0025	37431.	3013.	-7916.	-4903.	25101.	28114.

PROGRAM STOP AT 1040

USED 5.97 UNITS

RUN

BEARPACK 09:48CDT 09/15/79

BENDING LENGTH, INCHES?10.4375

STIFFENED LENGTH, INCHES?21.875

INSIDE DIAMETER, INCHES?1.8125

OUTSIDE DIAMETER, INCHES?4.493

AXIAL BIT FORCE COMPONENT, LBS?50000

RADIAL CLEARANCE, INCHES?.0025

TRANSVERSE FORCE LBS	LATERAL DEFLECTION INCHES	SLEEVE REACTION LBS	AXIAL STRESS PSI	BENDING STRESS (1)PSI	MAX STRESS (1)PSI	BENDING STRESS (2)PSI	MAX STRESS (2)PSI
0.	0.0000	0.	3767.	0.	3767.	0.	3767.
500.	0.0013	0.	3767.	1841.	5608.	1232.	4998.
1000.	0.0025	234.	3767.	3403.	7170.	2466.	6233.
1500.	0.0025	2298.	3767.	2775.	6542.	3722.	7488.
2000.	0.0025	4363.	3767.	2147.	5913.	4977.	8744.
2500.	0.0025	6427.	3767.	1519.	5286.	6233.	9999.
3000.	0.0025	8491.	3767.	891.	4657.	7488.	11255.
3500.	0.0025	10556.	3767.	263.	4029.	8744.	12511.
4000.	0.0025	12620.	3767.	-366.	3401.	9999.	13766.
4500.	0.0025	14685.	3767.	-994.	2773.	11255.	15022.
5000.	0.0025	16749.	3767.	-1622.	2145.	12510.	16277.
5500.	0.0025	18814.	3767.	-2250.	1517.	13766.	17533.
6000.	0.0025	20878.	3767.	-2878.	888.	15021.	18788.
6500.	0.0025	22943.	3767.	-3505.	261.	16278.	20045.
7000.	0.0025	25007.	3767.	-4134.	-367.	17533.	21300.
7500.	0.0025	27072.	3767.	-4763.	-996.	18788.	22555.
8000.	0.0025	29136.	3767.	-5390.	-1624.	20044.	23811.
8500.	0.0025	31201.	3767.	-6018.	-2252.	21300.	25067.
9000.	0.0025	33265.	3767.	-6647.	-2880.	22555.	26322.
9500.	0.0025	35330.	3767.	-7275.	-3508.	23811.	27578.
10000.	0.0025	37394.	3767.	-7903.	-4136.	25067.	28833.

PROGR

P AT 1040

USED 5.86 UNITS

RUN

BEARPACK 09:51CDT 09/25/79

BENDING LENGTH, INCHES?10.4375

STIFFENED LENGTH, INCHES?21.875

INSIDE DIAMETER, INCHES?1.8125

OUTSIDE DIAMETER, INCHES?4.493

AXIAL BIT FORCE COMPONENT, LBS?50000

RADIAL CLEARANCE, INCHES?.0035

TRANSVERSE FORCE LBS	LATERAL DEFLECTION INCHES	SLEEVE REACTION LBS	AXIAL STRESS PSI	BENDING STRESS (1) PSI	MAX STRESS (1) PSI	BENDING STRESS (2) PSI	MAX STRESS (2) PSI
0.	0.0000	0.	3767.	0.	3767.	0.	3767.
500.	0.0013	0.	3767.	1841.	5608.	1232.	4998.
1000.	0.0026	0.	3767.	3683.	7449.	2463.	6230.
1500.	0.0035	740.	3767.	4639.	8405.	3704.	7470.
2000.	0.0035	2804.	3767.	4011.	7777.	4959.	8726.
2500.	0.0035	4869.	3767.	3383.	7149.	6215.	9981.
3000.	0.0035	6933.	3767.	2755.	6521.	7470.	11237.
3500.	0.0035	8998.	3767.	2127.	5893.	8726.	12493.
4000.	0.0035	11062.	3767.	1498.	5265.	9981.	13748.
4500.	0.0035	13127.	3767.	870.	4637.	11237.	15004.
5000.	0.0035	15191.	3767.	242.	4009.	12493.	16259.
5500.	0.0035	17256.	3767.	-386.	3381.	13748.	17515.
6000.	0.0035	19320.	3767.	-1014.	2752.	15004.	18770.
6500.	0.0035	21385.	3767.	-1642.	2125.	16260.	20026.
7000.	0.0035	23449.	3767.	-2270.	1496.	17515.	21281.
7500.	0.0035	25513.	3767.	-2898.	868.	18770.	22537.
8000.	0.0035	27578.	3767.	-3527.	239.	20026.	23792.
8500.	0.0035	29643.	3767.	-4155.	-389.	21281.	25048.
9000.	0.0035	31707.	3767.	-4783.	-1016.	22537.	26304.
9500.	0.0035	33771.	3767.	-5411.	-1644.	23793.	27560.
10000.	0.0035	35836.	3767.	-6039.	-2273.	25048.	28815.

PROGRAM STOP AT 1040

USED 5.89 UNITS

RUN

BEARPACK 09:53CDT 09/25/79

BENDING LENGTH, INCHES?10.4375

STIFFENED LENGTH, INCHES?21.875

INSIDE DIAMETER, INCHES?1.8125

OUTSIDE DIAMETER, INCHES?4.493

AXIAL BIT FORCE COMPONENT, LBS?50000

RADIAL CLEARANCE, INCHES?.0045

TRANSVERSE FORCE LBS	LATERAL DEFLECTION INCHES	SLEEVE REACTION LBS	AXIAL STRESS PSI	BENDING STRESS (1) PSI	MAX STRESS (1) PSI	BENDING STRESS (2) PSI	MAX STRESS (2) PSI
0.	0.0000	0.	3767.	0.	3767.	0.	3767.
500.	0.0013	0.	3767.	1841.	5608.	1232.	4998.
1000.	0.0026	0.	3767.	3683.	7449.	2463.	6230.
1500.	0.0040	0.	3767.	5524.	9291.	3695.	7462.
2000.	0.0045	1246.	3767.	5875.	9641.	4941.	8708.
2500.	0.0045	3311.	3767.	5246.	9013.	6197.	9963.
3000.	0.0045	5375.	3767.	4618.	8385.	7452.	11219.
3500.	0.0045	7440.	3767.	3991.	7757.	8708.	12475.
4000.	0.0045	9504.	3767.	3362.	7129.	9963.	13730.
4500.	0.0045	11569.	3767.	2734.	6501.	11219.	14986.
5000.	0.0045	13633.	3767.	2106.	5873.	12475.	16241.
5500.	0.0045	15697.	3767.	1479.	5245.	13731.	17497.
6000.	0.0045	17762.	3767.	850.	4617.	14986.	18753.
6500.	0.0045	19826.	3767.	222.	3989.	16242.	20008.
7000.	0.0045	21891.	3767.	-406.	3360.	17497.	21264.
7500.	0.0045	23956.	3767.	-1035.	2731.	18752.	22519.
8000.	0.0045	26020.	3767.	-1663.	2104.	20008.	23775.
8500.	0.0045	28084.	3767.	-2291.	1475.	21263.	25030.
9000.	0.0045	30149.	3767.	-2919.	848.	22520.	26286.
9500.	0.0045	32213.	3767.	-3547.	219.	23775.	27541.
10000.	0.0045	34278.	3767.	-4174.	-408.	25031.	28798.

PROGRAM STOP AT 1040

USED 5.97 UNITS

a-3

Equations of motion are

$$\ddot{\theta}_0 J_0 = -K\theta_0 + K\theta_1$$

$$\ddot{\theta}_1 J_{m1} = K\theta_0 - K\theta_1$$

or in matrix form

$$[M] [\ddot{\theta}] + [k] [\theta] = 0$$

$$\begin{bmatrix} J_0 & 0 \\ 0 & J_{m1} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_0 \\ \ddot{\theta}_1 \end{bmatrix} + \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

which gives the following characteristic equation

$$\Delta \omega^2 = \det \begin{vmatrix} K - \omega^2 J_0 & -K \\ -K & K - \omega^2 J_{m1} \end{vmatrix} = 0$$

The solutions to this equation are

$$\omega_1 = 0$$

and

$$\omega^2 = K \left(\frac{J_o + J_{m1}}{J_o J_{m1}} \right)$$

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