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**NUMERICAL METHOD FOR SOLUTION OF TRANSIENT,
HOMOGENEOUS, EQUILIBRIUM, TWO-PHASE FLOWS
IN ONE SPACE DIMENSION**

by

Yong W. Shin and Arne H. Wiedermann

BASE TECHNOLOGY

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NOMENCLATURE

C	Sonic speed
E	Total energy, $i + u^2/2$
$\vec{F}$	Flux vector in the conservation equation defined by Eq. 6
g	Gravity
i	Internal energy
p	Pressure
Q	Heat addition to fluid per unit volume of fluid
s	Entropy
$\vec{S}$	Source vector in the conservation equation defined by Eq. 6
t	Time
Δt	Time step used in the numerical procedure
T	Temperature
u	Velocity of fluid
$\vec{U}$	Vector representing transport quantities defined by Eq. 6
x	Space coordinate along the tube
Δx	Nodal length used in the numerical procedure
β	$\Delta t / \Delta x$
ρ	Fluid density
θ	Inclination angle representing tube orientation
τ	Tube wall friction

Subscripts

A, B, C, P	Refers to nodal points used in the numerical procedure
1, 2	Refers to tentative points in t-x plane used in the numerical procedure
T, R	Refers to intercept points on current time plane of the characteristics
P'	Refers to mapped state of the solution state at point P
0	Refers to stagnation state

NUMERICAL METHOD FOR SOLUTION OF TRANSIENT, HOMOGENEOUS, EQUILIBRIUM, TWO-PHASE FLOWS IN ONE SPACE DIMENSION

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ABSTRACT

This report presents a solution method for transient, homogeneous, equilibrium, two-phase flows of a single-component fluid in one space dimension. The method combines a direct finite-difference procedure and the method of characteristics. The finite-difference procedure solves the interior points of the computing domain; the boundary information is provided by a separate procedure based on the characteristics theory. The solution procedure for boundary points requires information in addition to the physical boundary conditions. This additional information is obtained by a new procedure involving integration of characteristics in the hodograph plane.

Sample problems involving various combinations of basic boundary types are calculated for two-phase water/steam mixtures and single-phase nitrogen gas, and compared with independent method-of-characteristics solutions using very fine characteristic mesh. In all cases, excellent agreement is demonstrated.

I. INTRODUCTION

Design considerations for an LMFBR intermediate heat-transport system must include the potential for water/steam leakage from the high-pressure tubeside into the shellside sodium of the steam generator. Because the ensuing chemical reaction can produce large pressure and thermal loadings that propagate through the system, the amount and rate of leakage are important factors that determine the extent of these loadings. Hence, accurate means of determining these parameters are essential tools for designers to ensure that the system is structurally sound and can withstand the potential loadings, and/or to design an adequate relief system.

This report discusses a method that enables accurate computation of the complete tubeside transient from which time-dependent leak rates can be obtained. The method uses a direct finite-difference technique for computation of interior nodes. For boundary nodes, a special procedure is developed which

uses the characteristic equations. Basically, the procedure involves searching of intersections of the characteristics and the boundary conditions in the hodograph plane. Actually, integration of characteristics is necessary because the characteristics are severely curved due to the drastic change in acoustic impedance of the fluid. This is especially true when the characteristic crosses the liquid-saturation line where the sonic speed is essentially discontinuous.

Earlier attempts at similar tasks used certain approximations in the boundary treatments. For example, as was usually done by many others, averaging of impedances at both ends of characteristics was used by Galluzzo,¹ and Berry and Daley.² This approximation was adequate for some flow regimes and yielded reasonable results, but inadequate for flow regimes where the states are largely different between adjacent nodes. In such instances, the approximation introduced large errors at the boundary and caused a complete breakdown of the procedure. This was noted in particular at the break end, where the large gradient in the state exists.

Lyczkowski, Grimesey, and Solbrig³ introduced another type of approximation in their boundary-end treatment which was partially successful in predicting the Edwards problem.⁴ Although the interior transient result agreed reasonably well with the experimental data, a close examination of their work revealed that the pressure and temperature transients at interior points were not very sensitive to the boundary treatment. The blowdown rate is a sensitive parameter, and a comparison of the blowdown rates showed a significant discrepancy between the result of the present method and the approximate approach.

The method presented here is a new methodology that provides exact boundary treatment since the solution state at the boundary node is determined by a distinct point in the hodograph plane. No assumptions or approximations are needed because the available characteristics exactly supplement the information lacking at the boundaries.

II. GOVERNING DIFFERENTIAL EQUATIONS

The governing equations are the conservation equations of mass, momentum, and energy of one-dimensional compressible flows. The equations considered are for homogeneous, equilibrium, two-phase flows of a single-component fluid. The equations are written in a conservation form as

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x} \rho u = 0, \quad (1)$$

$$\frac{\partial}{\partial t} \rho u + \frac{\partial}{\partial x} (\rho u^2 + p) = \rho g \cos \theta - \tau, \quad (2)$$

and

$$\frac{\partial}{\partial t} \rho E + \frac{\partial}{\partial x} [u(\rho E + p)] = \rho g \cos \theta \cdot u + Q, \quad (3)$$

where θ is the inclination angle between the gravity force and the positive x axis, τ is the friction acting on the wall per unit length and unit cross-sectional area, Q is the heat addition to the fluid per unit volume, and E is the total energy defined by

$$E = i + \frac{1}{2} u^2. \quad (4)$$

These equations contain four dependent variables: ρ , p , i , and u . An additional equation required for solution is the equation of state written in a functional form,

$$p = p(\rho, i). \quad (5)$$

The three governing equations are in the form of the generalized transport equation,

$$\frac{\partial}{\partial t} \vec{U} + \frac{\partial}{\partial x} \vec{F} = \vec{S}(\vec{U}), \quad (6)$$

where $\vec{U}$, $\vec{F}$, and $\vec{S}$ are vectors with three elements each representing the transport quantity, flux, and source, respectively.

III. FINITE-DIFFERENCE PRODEEDURE

The procedure used for interior node solution is the explicit technique based on the two-step Lax-Wendroff scheme.⁵ It is briefly illustrated here for a uniform nodal spacing, $\Delta x = \text{constant}$. Figure 1 depicts a typical grid system, rectangular in the x - t coordinate plane. Points A, B, and C are at the current time t_0 where the solution (the values of dependent variables) is known, and point P is the interior point for which the solution is sought; P is at an advanced time $t_0 + \Delta t$.

Fig. 1
Grid System for Finite-
difference Procedure

In the first step, intermediate, half-step values are calculated for points 1 and 2 of Fig. 1 as

$$\vec{U}_1 = \frac{1}{2}(\vec{U}_A + \vec{U}_C) + \frac{1}{2}\beta(\vec{F}_C - \vec{F}_A) + \frac{1}{4}\beta\Delta x(\vec{S}_A + \vec{S}_C) \quad (7a)$$

and

$$\vec{U}_2 = \frac{1}{2}(\vec{U}_A + \vec{U}_B) + \frac{1}{2}\beta(\vec{F}_A - \vec{F}_B) + \frac{1}{4}\beta\Delta x(\vec{S}_A + \vec{S}_B), \quad (7b)$$

where $\beta = \Delta t/\Delta x$. These intermediate values $\vec{U}_1$ and $\vec{U}_2$ are used to calculate flux vectors $\vec{F}_1$ and $\vec{F}_2$. The equation of state of the form given by Eq. 5 is also used.

The second step yields the final solution at P:

$$\vec{U}_P = \vec{U}_A + \beta(\vec{F}_1 - \vec{F}_2) + \frac{1}{2}\beta\Delta x(S_1 + S_2). \quad (8)$$

This procedure is repeated for all interior nodes before time can be advanced. For boundary nodes (two end nodes of a tube), all values of the dependent variables must be given. Obtaining solutions at all interior and boundary nodes constitutes one-step computation by which the solution is advanced by one time step Δt . In general, the order of computation (i.e., either interior nodes first, followed by boundary nodes, or vice versa) is immaterial. In certain types of boundary conditions (e.g., nonreflecting condition), the boundary specification requires interior information at the advanced time. In this case, the interior-node computation must be performed first. It is therefore safe to do the boundary-node computation after completion of interior nodes.

The procedure is presented here for uniform nodal spacing. It is equally applicable to variable node spacings ($\Delta x \neq \text{constant}$). The advantage in the variable spacing is the saving in computation time by reducing the number of computational points in unimportant parts of the domain. The disadvantage is that it introduces errors resulting from numerical smearing due to the small Courant number in regions of large Δx . This is because the overall procedure is limited in its time step by the CFL (Courant-Frederick-Lewy) condition,

$$\Delta t \leq \Delta x / (c + |u|)_{\max}. \quad (9)$$

IV. BOUNDARY-NODE SOLUTION METHOD

Following the usual practice, a quasi-steady state is assumed to exist at the boundary: At each step of computation the flow condition at the boundary

using states at nodes A and B. Likewise, all terms on the right-hand side of Eqs. 10-12 are evaluated by interpolation. Since entropy is used as an independent property, two additional forms of steam tables are required:

$$i = i(p, s); \quad (14)$$

$$\rho = \rho(p, s). \quad (15)$$

In applying the characteristics, we use a finite-step integration procedure in the hodograph plane. The result is that the characteristics are often curved in the solution plane. The curvature is most pronounced when the state change is large, as in the case of a flashing transient.

A. Outflow Boundary

The physical configuration of an outflow boundary is shown in Fig. 2a. Flow direction u is positive; hence, both T and R characteristics are present as shown in Fig. 2b. T characteristic determines $\bar{s}$. The solution methodology is described in Fig. 3. The solution state is simply the intersection of the

Fig. 3. Solution Methodology for Outflow Boundary

R characteristic and the boundary condition in the hodograph plane ($s = \bar{s}$). The boundary condition is either the back pressure (outside pressure) or the sonic line. If R lies in the supersonic branch, a supersonic outflow is possible. In this case, a check is made on the interior state (state B of Fig. 2b) to determine this possibility. If B is in supersonic state, the supersonic outflow is admitted as a solution state. Otherwise, P lies on the sonic line.

When the outflow end is not fully open, as in a partial tube rupture (outflow with an orifice as shown in Fig. 4a), the sonic state at the orifice corresponds to a subsonic state in the interior. In the supersonic branch,

however, the corresponding state in the interior is supersonic. Hence, the sonic line maps to two lines, and these are now the boundary conditions for the orificed case. The region between the two mappings of the sonic line represents nonadmissible solution states. P cannot assume a state lying in this region.

Mapping of the back pressure is also indicated in Fig. 4b. The solution state is the intersection of the R characteristic with the mapping of the sonic line or the back pressure line, as illustrated in Fig. 4b. Except for the mapping, the procedure is identical to the fully open case discussed previously. Mapping of states is based on the constancy requirements of mass and total enthalpy,

$$\rho_P A_P u_P = \rho_{P'} A_{P'} u_{P'} \quad (16)$$

and

$$i_P + \frac{p_P}{\rho_P} + \frac{1}{2} u_P^2 = i_{P'} + \frac{p_{P'}}{\rho_{P'}} + \frac{1}{2} u_{P'}^2 \quad (17)$$

(a) PHYSICAL CONFIGURATION

(b) SOLUTION METHODOLOGY

Fig. 4
Outflow Boundary
with Orifice

B. Reservoir End

The boundary condition at the reservoir end is the energy line (line of constant total enthalpy) defined by

$$h_0 = i_P + \frac{p_P}{\rho_P} + \frac{1}{2} u_P^2, \quad (18)$$

where P is the solution state in the hodograph plane. Figure 5a shows the physical configuration where the flow accelerates from the high-pressure reservoir into the tube. The wave diagram shown in Fig. 5b indicates that the S characteristic applies. The solution state P is the intersection of the S characteristic and the energy line (see Fig. 6). If P is found in the negative-velocity region, it implies that a flow reversal exists.

State P may lie in the supersonic branch, as illustrated in Fig. 6. The solution state in this case is state C (intersection of energy line and sonic line) instead of the usual intersection of the S characteristic with the energy line. This treatment implies that the supersonic acceleration is not admitted as a solution state at the reservoir end. The possibility of a smooth-contoured

entrance region enabling such acceleration is excluded here, since geometric details are ignored. In most applications, the entrance region is geometrically abrupt (as in most designs of heat-exchanger plenum entrance to heat-transfer tubes) for which the limiting treatment as described above is considered suitable.

Fig. 5. Boundary at Reservoir End

Fig. 6. Solution Methodology at Reservoir End

C. Closed End

The boundary condition for a closed end is $u = 0$. Values of other dependent variables remain to be solved for. A similar procedure may be followed, and the intersection of characteristics and boundary conditions may be sought in the state plane at some representative entropy $\bar{s}$. Entropy $\bar{s}$ may be chosen by an average of entropies at terminal points of characteristics. Pertinent characteristics here are the S and T characteristics. The boundary condition is the vertical line $u = 0$ in the hodograph plane. Hence, the solution state P lies on the p axis at the point where the S characteristic intersects. Conceptually, this procedure is identical to the preceding boundary treatments and hence is not elaborated further.

An alternative procedure is described here which is simpler and whose accuracy is consistent with the finite-difference scheme used for interior nodes. This procedure uses the symmetry property of flow at the closed end: The flow on the left of the closed end of Fig. 7 is the mirror image of that on the right. To represent this in the finite-difference procedure, a fictitious point 1 is assumed for which flow properties are assigned that are identical to point 2, except that the flow direction is reversed. Thus only Eq. 8 is necessary. This method is used throughout the sample problem calculations in this report.

Fig. 7
Boundary at Closed End

V. STEAM-TABLE PROGRAM

In addition to Eqs. 5, 14, and 15, the following functional forms are required:

$$c = c(p, s) \quad (19)$$

and

$$\left(\frac{\partial p}{\partial i}\right)_\rho = \left(\frac{\partial p}{\partial i}\right)_\rho(p, s). \quad (20)$$

An existing steam-table program with flexibility in the choice of independent variables has been modified to include wider choices. The sonic speed c is computed on the basis of thermodynamic equilibrium between phases.

The equation of state used in the program is based on Keenan's formulation,⁷ which is valid to 1000 bars (100 MPa) and 1300°C. This formulation expresses the Helmholtz function in terms of temperature and specific volume for both liquid and vapor phases. For other pairs of independent variables, an iteration procedure is used, starting from given approximate values. Accuracy depends on the tolerance limits assigned to the iteration procedure. The steam-table program has been performed satisfactorily in all applications to date.

VI. SAMPLE PROBLEM CALCULATIONS

Several sample problems involving various combinations of boundary types were calculated and the results compared with reference solutions. These problems include two-phase water/steam mixture and single-phase nitrogen gas. The reference solutions are method-of-characteristics solutions that are considered nearly exact as they are obtained with a very fine mesh of characteristics.

A. Blowdown from Closed Tube: Fully Open End

The tube under consideration is 5.1 m long and is initially at a uniform state. At $t = 0$, one end of the tube is suddenly opened and the subsequent blowdown is examined. The pressure in the tube is higher than the ambient (atmospheric), and an outflow is maintained at the open end.

As shown in the wave diagram (Fig. 8b), a centered rarefaction system develops at the open end and propagates into the tube. These waves remain expansion waves as they reverberate between the two ends. The flow is subsonic in the entire region and duration. At the open end, a sonic state is maintained until the pressure approaches the back pressure. The dashed line in Fig. 8b represents the initial expansion characteristic and its reflections.

Fig. 8. Closed-tube Decompression: Fully Open End

The results of the water/steam-mixture decompression together with the reference solutions are shown in Figs. 9 and 10. A uniform initial state of 13.79 MPa, 335.4°C and 51% quality is considered. Since the reference solution is for no friction, the friction in the finite-difference solution is suppressed by considering an arbitrarily large-diameter tube. There is excellent agreement in this comparison. The cusps in the method-of-characteristics solutions are seen somewhat rounded off in the finite-difference solutions. Small overshoots appearing for the open end during early time in the finite-difference solutions are

due to numerical features typically observed during starting transients. As the deviation from reference solution is small, no special effort has been made to improve this point.

Fig. 9
Water/Steam-mixture
Decompression Result:
Pressure Comparison

Fig. 10. Water/Steam-mixture Decompression
Result: Blowdown-rate Comparison

Blowdown of nitrogen gas at 1.379 MPa and 37.8°C from the same tube was calculated, and the results are compared with reference solutions in Figs. 11 and 12. In all respects, the results resemble those of the water/steam-mixture case. The agreement between the two solution methods is similarly excellent.

Fig. 11. Nitrogen Decompression Re-
sult: Pressure Comparison

Fig. 12. Nitrogen Decompression Result: Blowdown-rate Comparison

B. Blowdown from Closed Tube: Orificed End

Effects of an orifice at the open end was studied for the foregoing water/steam-mixture decompression. Figure 13 describes the physical configuration and the wave diagram. The area reduction due to the orifice is 50%. The presence of the orifice creates regions of constant states at both the open end and the closed end, with the size of the region diminishing with time. The history plots at the open end (likewise at the closed end) exhibit intermittent finite plateaus (see Figs. 14 and 15), the lengths of which decrease with time. The outflow state is subsonic at all times.

Fig. 13. Closed-tube Decompression: Orificed End

C. Reservoir Compression for Water/Steam Mixture

As described in Fig. 16, a compression wave is sent into the tube from the reservoir end which reflects at the open end as a centered rarefaction-wave system. The strength of the shock wave considered here is not high enough to result in a sonic or supersonic outflow state. The results at the ends will therefore indicate step

changes in the history plots. The reservoir condition is 13.79 MPa, 335.4°C, and 51% quality. The tube is at a uniform state of 12.41 MPa, 327.2°C, and

50% quality. The initial entropies of the reservoir and the tube are approximately the same. The back pressure is set equal to the initial tube pressure, 12.41 MPa.

Fig. 14. Water/Steam-mixture Decompression with Orifice: Pressure Comparison

Fig. 15. Water/Steam-mixture Decompression with Orifice: Blowdown-rate Comparison

Fig. 16. Reservoir-evaluation Case

and that at the open end S_N'' . The intermediate points between the ends would be the states along the dotted line connecting the limiting states. The pressure history at the reservoir is shown in Fig. 18, where both cases with and without friction are presented. Similarly, the velocity history is shown in Fig. 19. Again, the friction tends to retard the flow. The step changes in the history plots are clearly indicated.

The hodograph plane is shown in Fig. 17. The solution states at the reservoir end lie on the energy line moving downward from the reservoir state. Intersections of the reverberating characteristics with the energy line are the solution states at the reservoir end; S_1 is the first solution state. In the absence of friction, S_N is the limiting solution state corresponding to the steady-state solution. When a steady-state solution is reached, the entire tube will be at S_N . The states at the open end are the intersections of the reverberating characteristics with the constant-pressure line ($p = 12.41$ MPa). When friction is included, the limiting state S_N is not attained. Rather, there exists a gradient with the limiting state at the reservoir $S_{N'}$

Fig. 17
Hodograph Plane for
Reservoir-evaluation
Case

Fig. 18
Pressure History at Reservoir End
for Reservoir-evaluation Case

Fig. 19
Velocity History at Open End
for Reservoir-evaluation Case

D. Reservoir Blowdown for Nitrogen Gas

Unlike the closed-tube blowdown case, the outflow state in the reservoir blowdown can be in the supersonic state. In fact, a certain finite region near the open end is in the supersonic state. This is indicated in the wave diagram shown in Fig. 20. The case considered here is a uniform state (both reservoir and tube) of nitrogen gas at 6.895 MPa and 37.8°C being exposed to an atmospheric back pressure at $t = 0$.

The blowdown rates are shown in Fig. 21 for the friction and no-friction cases. Again, the effect of friction is to retard the flow.

Figure 22 depicts the steady-state solution with friction, showing the gradients along the tube similar to the Fanno-line solutions.

Fig. 20

Reservoir-blowdown
Case for Nitrogen

Fig. 21
Mass-flow-rate History for Res-
ervoir Blowdown of Nitrogen

Fig. 22

Steady-state Distribution in
Tube for Reservoir Blowdown
of Nitrogen

VII. DISCUSSIONS AND CONCLUSION

The new numerical procedure presented in this report overcomes all difficulties encountered in earlier attempts noted in the literature. The procedure developed for the boundary types considered yields results in excellent agreement with reference solutions obtained by an independent method. The reference solutions were obtained by use of such a fine mesh of characteristics that the results may be regarded virtually exact.

In view of the excellent accuracy generally observed in the comparison, both the interior and boundary formulations are concluded to be an accurate method for computing transient, homogeneous, equilibrium flows. Formulations for other types of boundaries should be straightforward and may be developed by following basically the same procedures discussed in this report. An effort is under way to include other boundary types commonly encountered in typical piping networks.

The small discrepancy (typically, a few percent) observed during the starting transient is related to the mesh resolution in the finite-difference grid. Although the error is small and rather unimportant for all practical applications, control of the error is still desirable. Use of finer meshes in regions sensitive to the errors should minimize such errors.

APPENDIX

Derivation of Characteristic Equations

The set of governing equations written in an expanded form is

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0, \quad (\text{A.1})$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} = \rho g \cos \theta - \tau, \quad (\text{A.2})$$

and

$$\rho \left(\frac{\partial i}{\partial t} + u \frac{\partial i}{\partial x} \right) + p \frac{\partial u}{\partial x} = u\tau + Q. \quad (\text{A.3})$$

It is desirable to rewrite these equations in terms of the desired set of dependent variables, namely, p , s , and u . To do this, Eqs. A.1 and A.3 are combined, using

$$di = T ds + \frac{p}{\rho^2} d\rho \quad (\text{A.4})$$

to yield the first desired equation,

$$\frac{ds}{\partial t} + u \frac{\partial s}{\partial x} = \frac{1}{\rho T} (u\tau + Q). \quad (\text{A.5})$$

Equation A.5 is the particle-path equation.

A second equation is obtained by combining Eqs. A.1 and A.5:

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \rho c^2 \frac{\partial u}{\partial x} = \frac{1}{\rho} \left(\frac{\partial p}{\partial i} \right)_{\rho} (u\tau + Q), \quad (\text{A.6})$$

where the following thermodynamic relationship is used:

$$\left(\frac{\partial p}{\partial \rho} \right)_i = c^2 - \frac{p}{\rho^2} \left(\frac{\partial p}{\partial i} \right)_{\rho}. \quad (\text{A.7})$$

A set of three characteristic equations is sought in the following which are merely linear combinations of Eqs. A.2, A.5, and A.6. Equation A.5 is already a characteristic equation; hence, only Eqs. A.2 and A.6 are combined:

$$\begin{aligned}
& \alpha_1 \left[\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} \right] + \alpha_2 \left(\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \rho c^2 \frac{\partial u}{\partial x} \right) \\
& = \alpha_1 (\rho g \cos \theta - \tau) + \alpha_2 \frac{1}{\rho} \left(\frac{\partial p}{\partial i} \right)_\rho (u\tau + Q).
\end{aligned} \tag{A.8}$$

This equation can be rearranged to show the respective gradients of variables:

$$\begin{aligned}
& \alpha_2 \frac{\partial p}{\partial t} + (\alpha_1 + u\alpha_2) \frac{\partial p}{\partial x} + \rho\alpha_1 \frac{\partial u}{\partial t} + \rho(u\alpha_1 + c^2\alpha_2) \frac{\partial u}{\partial x} \\
& = \alpha_1 (\rho g \cos \theta - \tau) + \alpha_2 \frac{1}{\rho} \left(\frac{\partial p}{\partial i} \right)_\rho (u\tau + Q).
\end{aligned} \tag{A.9}$$

The vector $\vec{\lambda}$ normal to the characteristic in the x-t plane having components λ_1 and λ_2 in the t and x coordinates, respectively, must satisfy the conditions

$$\alpha_2 \lambda_1 + (\alpha_1 + u\alpha_2) \lambda_2 = 0 \tag{A.10.1}$$

and

$$\rho\alpha_1 \lambda_1 + \rho(u\alpha_1 + c^2\alpha_2) \lambda_2 = 0. \tag{A.10.2}$$

These are homogeneous equations in α_1 and α_2 ; hence, their determinant must vanish:

$$\begin{vmatrix} \lambda_2 & \lambda_1 + u\lambda_2 \\ \lambda_1 + u\lambda_2 & c^2\lambda_2 \end{vmatrix} = 0. \tag{A.11}$$

The solution to Eq. A.11 is substituted in Eq. A.10 to evaluate the α terms. Substitution of these values in Eq. A.9 finally yields the characteristic equations being sought,

$$\frac{\partial p}{\partial t} + (u + c) \frac{\partial p}{\partial x} + \rho c \left[\frac{\partial u}{\partial t} + (u + c) \frac{\partial u}{\partial x} \right] = \frac{1}{\rho} \left(\frac{\partial p}{\partial i} \right)_\rho (u\tau + Q) + c(\rho g \cos \theta - \tau) \tag{A.12}$$

and

$$\frac{\partial p}{\partial t} + (u - c) \frac{\partial p}{\partial x} - \rho c \left[\frac{\partial u}{\partial t} + (u - c) \frac{\partial u}{\partial x} \right] = \frac{1}{\rho} \left(\frac{\partial p}{\partial i} \right)_\rho (u\tau + Q) - c(\rho g \cos \theta - \tau). \tag{A.13}$$

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