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IN HOLLOW CYLINDERS***

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Paper presented at the

Fifth COMPUMAG Conference
Fort Collins, Colorado

June 3-6, 1985

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*Work supported by the U.S. Department of Energy.

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Abstract - The early experiments with the FELIX (Fusion Electromagnetic Induction Experiments) facility have been devoted to obtaining data which can be used to validate eddy current computer codes. This paper describes experiments on field variation inside conducting cylinders.

INTRODUCTION

The first wall, blanket, limiter, and other electrically conducting components of a fusion reactor are pervaded by magnetic fields. Changes in these magnetic fields, in either normal operation or off-normal events, produce eddy currents with accompanying forces, torques, and voltages. A quantitative understanding of these electromagnetic effects is vital to the design of a fusion reactor.

Existing eddy current codes, and new ones as they are developed, must be validated against experimental data representing geometry and operation similar to those of a fusion reactor. Consequently, the early experiments with the FELIX [1,2] facility have been devoted to obtaining data which can be used to validate eddy current computer codes. This paper describes experiments on field variation inside conducting cylinders.

The FELIX facility at Argonne National Laboratory (ANL) includes:

1. A solenoid magnet producing fields up to 1 T.
2. A dipole magnet surrounding the solenoid and producing fields up to 0.5 T perpendicular to the solenoid field.
3. A switching circuit capable of discharging the dipole field with a time constant as low as 10 ms.
4. A cylindrical experimental volume within the fields of the two magnets of dimensions 1.2 m axially by 0.9 m diameter.
5. A non-conducting test-piece support frame centered on a support tube perpendicular to both magnetic fields and providing, through phosphor bronze leaf springs, restoring torques of up to 9 kN m/rad.
6. Instrumentation suitable for measuring currents, magnetic fields, angular displacements, temperatures, and stresses.
7. Computerized data acquisition with provision for recording values at 2048 times on as many as 30 data channels.

HOLLOW CYLINDER EXPERIMENTS

A series of experiments was performed to provide data to investigate the limits of applying two-dimensional eddy current codes to three-dimensional geometries. The test pieces were hollow aluminum cylinders with axis horizontal (perpendicular to the dipole field and parallel to the solenoid field direction). Each cylinder was mounted in the support frame

and had two full-length slits located diametrically opposite one another. The cylinder could be rotated so that the slits were at any angular position. When the slits were located in a horizontal plane, they did not impede the eddy-current flow, and two eddy-current loops covered the full top and bottom halves of the cylinder just as though no slits were present. With the slits at the top and bottom of the cylinder, only smaller eddy-current loops, each covering one quadrant of the cylinder, were possible.

The dimensions of the three cylinders are shown in Table I. The dimensions were chosen so that all three would have about the same L/R time constant and that the cylinder with the smallest thickness could be approximated closely by a two-dimensional shell, and the other two would be more essentially three-dimensional.

TABLE I

Dimensions of Split-Cylinder Test Pieces

Test Piece	Length (cm)	Thickness (cm)	Outer Diam. (cm)	Inner Diam. (cm)
Large	120	0.48	27.3	26.34
Medium	60	1.27	13.97	11.43
Small	60	2.54	10.16	5.08

Nearly all of the experiments described below were conducted with slits in the horizontal plane (i.e., unimpeded eddy current pattern), no solenoid field, dipole field of 100 mT and 40 ms decay time. The vertical component of field was measured with a Hall probe at several positions on the cylinder axis. For the large cylinder, with half-length 60 cm, the field was measured at 0, 10, 20, 30, 40, 45, 50, 55, and 60 cm. For the other two cylinders, with half-length 30 cm, the field was measured at 0, 10, 20, 25, and 30 cm. At each position the field was measured at 2048 different times; through the time period of interest the measurements were made every 2 ms. Experiments with other operating conditions (10 ms and 100 ms decay time; 25 mT and 50 mT dipole magnet field; non-zero solenoid magnet field) have not yet been analyzed.

DATA ANALYSIS

The field measurements are presented in two ways, as total field and as induced field. Figure 1 shows the raw data for a typical case, the large cylinder with slits horizontal (CLH) with measurements at 0 cm. The Hall probe was found to have a non-zero signal (typically -0.2 mT to -0.7 mT out of a total field of 100 mT) after the field had decayed. The Hall probe signal was averaged over the time period 420 ms to 620 ms (see Fig. 1), and that average value subtracted from all measured values. The resulting field is called the total field.

The induced field is the difference between the total field and the dipole magnet field, measured by the Hall probe when the cylindrical test piece is absent. Before the measured dipole magnet field is subtracted, it is normalized by the ratio of the initial total field to initial dipole magnet field, both

*Work supported by the U.S. Department of Energy.

Fig. 1. Raw data for magnetic field at the center of the large cylinder with slits in a horizontal plane (CLH). A: time period 420 to 620 ms, used to determine offset. B: Time period -260 to -60 ms, used to normalize the subtraction of the dipole magnet field values. C: Time period 3 to 120 ms, used for exponential fit to the data.

averaged over the time period of -260 ms to -60 ms. See Fig. 1. This normalization is needed because the field with no test piece was measured at the central position while the field is 2% lower at 30 cm, the end of the medium or small cylinder, and 10% lower at 60 cm, the end of the large cylinder

Fit to Exponentials

To put the experimental results in a form convenient for comparison with the results of eddy-current code computations, they should be expressed as the parameters of a function fitted to the data. Suitable functions are combinations of two exponentials if the cylinder can be characterized by a single L/R time. For the induced and total fields B_{ind} and B_{total} ,

$$B_{ind} = C[\exp(-\gamma_1 t) - \exp(-\gamma_2 t)] \quad (1)$$

$$B_{total} = (B_0 + C) \exp(-\gamma_1 t) - C \exp(-\gamma_2 t), \quad (2)$$

where γ_1 and γ_2 are respectively the reciprocals of the dipole magnet decay time and the cylinder L/R time, B_0 is the initial field amplitude, and C is a constant.

Equations (1) and (2) hold if the decay begins at $t = 0$. For the more general case, in which the decay begins at an unknown time t_0 , the field can be written as

$$B = C_1 \exp(-\gamma_1 t) + C_2 \exp(-\gamma_2 t) \quad (3)$$

$$t_0 = \ln(-C_2/C_1)/(\gamma_2 - \gamma_1). \quad (4)$$

The experimental results will be characterized by performing a least-squares fit of (3) to the data on the time period 7 ms to 120 ms. However, it is well known that an exponential equation of this type in which all four parameters are to be fitted is extremely ill conditioned [3]. Consequently four fitting procedures to one, two, or three parameters were used, as summarized in Table 2.

Figure 2 shows the sum of square deviations for the FIT3 procedure applied to the total field, as a function of the specified γ_1 . Note the sharp minimum

TABLE 2

Least-Square Fits to Two Exponentials:
 $B = C_1 \exp(-\gamma_1 t) + C_2 \exp(-\gamma_2 t)$

Procedure	Unknowns	Application
FIT3	C_1, C_2, γ_2	1. Fitting results for known decay rate γ_1 . 2. Repeated use to find γ_1 and to find sensitivity to γ_1 .
FIT2	C_1, C_2	1. Fitting results when dipole magnet decay rate and test piece L/R time are both known. 2. Repeated use to find joint sensitivity to γ_1 and γ_2 .
FIT1	$C_1, \gamma_1 (C_2 \equiv 0)$	Fitting exponential tail to find γ_1 for subsequent FIT3.
FIT0	$C_1 (C_2/C_1 \text{ known})$	Fitting data by normalizing a predefined curve.

Fig. 2. Sum of squared deviations (SSD) for a three-parameter fit to the total field, as a function of dipole magnet field decay rate γ_1 . Fits are shown for the three cylinders with slits in a horizontal plane (CSH, CMH, CLH), for the large cylinder with slits in a vertical plane (CLV), and for no test piece.

for the field with no test piece present. That value ($\gamma_1 = 0.0252 \text{ ms}^{-1}$, $\tau = 39.7 \text{ ms}$) was used in all subsequent analysis. Note also that the minimum is somewhat broader for the large cylinder with slits in the

vertical phase (CLV) and broader yet for the large, medium, and small cylinders with slits in a horizontal planes (CLH, CMH, CSH). For the induced fields, the FIT3 procedure gives still broader minimum, but all results are consistent with $\gamma_1 = 0.0252 \text{ ms}^{-1}$.

RESULTS

Total Field

Figure 3 shows the total field at the center of the large cylinder (CLH) and the field at the same position with no cylinder. Asymptotically the field decays exponentially. The effect of the cylinder is to delay the decay. This time delay can be determined by extrapolating the asymptotic time variation back to the original value of the field.

Fig. 3. Total field at the center of the large cylinder (CLH), indicated by pluses, and in the same location with no test piece, indicated by squares. The lines show the curves fitted to the data over the range 3 to 120 ms by the FIT3 procedure.

The delay varies from cylinder to cylinder and within each cylinder varies with axial position, as shown in Fig. 4. Table 3 gives the total field parameters for all cylinders and all positions.

Induced Field

The induced field is a more sensitive indicator of eddy current effects than is the total field. Over the central region of each cylinder, the induced field signal was strong enough to permit the FIT3 procedure. Near the ends the induced field is much weaker but fitting by the FIT0 procedure to a time pattern determined at the center of the cylinder is still meaningful.

If the eddy currents in the cylinder could be characterized by a single current pattern and a single L/R time (γ_1^{-1}), then the FIT3 AND FIT0 procedure should give the same results, γ_2 should be the same at all axial positions, and the ratio of induced field at different axial positions should remain constant over time. Deviations from this behavior could be due either to the effects of finite (versus infinite) cylinder length or finite (versus infinitesimal) cylinder thickness.

Results of the fittings are shown in Table 4. For the three-parameter fit, t_0 is found by (4), and the amplitude C is given by

Fig. 4. Delay of the field decay at different axial positions within the three cylinders.

$$C = C_1 \exp(-\gamma_1 t_0) = -C_2 \exp(-\gamma_2 t_0)$$

The values of t_0 in Table 4 are measured from the beginning of the range used for fitting. The value $t_0 = -3.1 \text{ ms}$ was used to define the time scale used throughout this paper.

The variation of amplitude with x for each cylinder is shown in Fig. 5. Figure 6 shows the induced field data in CLH at 45 cm, along with the two fitted curves. Visually the curves appear to fit about equally well, although Table 4 indicates that the sum of squared deviations (SSD) is 44% higher for the FIT0 procedure. By contrast the one-parameter fits to CMH and CSH at 20 cm, shown in Fig. 7, seem distinctly worse than the three-parameter fits, shown in Fig. 8. The SSD ratios are 2.14 and 1.71 for CMH and CSH, respectively.

Fig. 5. Amplitude of the induced magnetic field at different axial positions within the three cylinders, expressed as a fraction of the amplitude at the center of the cylinder.

Due to the FELIX circuitry, there is a small downward drift in field just before the exponential decay begins, as can be seen in Fig. 3 between -20 ms and 0 ms. This drift creates eddy currents and induced field, as can be seen over the same time range in Figs. 6, 7, and 8.

TABLE 3

Total Field on Cylinder Axis
 $(\gamma_1 = 0.0252 \text{ ms}^{-1}, \tau = 39.68 \text{ ms}, \text{ Three-Parameter Fit})$

x (cm)	B ₀ (mT)	C (mT)	Time Delay (ms)	γ_2 (ms ⁻¹)	C ₂ (mT)	SSD (mT ²)
<u>No Cylinder</u>						
0	103.0	103.0	0.	3.157	--	6.46
<u>Large Diameter Cylinder (Vertical Slits)</u>						
0	103.4	112.8	3.45	3.434	--	25.97
<u>Large Diameter Cylinder</u>						
0	103.0	131.2	9.62	0.1019	-29.4	4.81
10	102.8	128.7	8.92	0.1005	-26.2	8.61
20	102.0	126.4	8.51	0.0991	-24.6	10.38
30	101.0	130.9	10.29	0.1043	-30.4	2.19
40	99.5	119.6	7.29	0.1142	-21.5	4.67
45	98.5	115.3	6.24	0.1184	-17.9	1.70
50	97.2	108.0	4.19	0.1089	-11.3	3.26
55	95.2	99.4	1.72	0.1052	-5.8	3.31
60	93.4	94.7	0.54	0.1097	-2.4	2.79
<u>Medium Diameter Cylinder</u>						
0	102.6	155.7	16.55	0.0803	-54.2	2.72
10	102.9	155.7	16.43	0.0816	-54.5	1.87
20	102.1	140.7	12.73	0.0899	-39.2	1.78
25	100.6	124.2	8.36	0.1045	-24.8	3.58
30	101.2	106.5	2.02	0.1055	-6.1	4.66
<u>Small Diameter Cylinder</u>						
0	99.6	155.4	17.64	0.0763	-56.5	12.62
10	103.3	156.9	16.59	0.0768	-54.1	26.20
20	102.0	154.5	16.47	0.0847	-55.9	12.62
25	101.8	143.4	13.58	0.0900	-41.4	49.73
30	100.6	108.0	2.82	0.1907	-12.0	18.20

Note: B₀ is the initial value of the field and SSD is the sum of squared deviation.

TABLE 4

Induced Field on Cylinder Axis
 $(\gamma_1 = 0.0252 \text{ ms}^{-1}, \tau = 39.68 \text{ ms})$

x (cm)	<u>Three-Parameter Fit</u>				<u>One-Parameter Fit</u>	
	τ_0 (ms)	C (mT)	γ_2 (ms ⁻¹)	SSD (mT ²)	C (mT)	SSD (mT ²)
<u>Large Diameter Cylinder</u>						
0	-3.16	27.80	0.0977	12.97	27.80	12.97
10	-3.64	25.80	0.0960	26.60	25.56	27.09
20	-3.76	24.47	0.0942	27.88	24.03	28.41
30	-2.48	29.80	0.1000	15.12	30.10	16.43
40	-3.18	19.79	0.1079	4.26	16.20	7.42
45	-3.44	16.69	0.1110	6.49	17.57	9.33
50	-4.52	11.13	0.0973	8.34	11.07	9.49
55	--	--	--	--	3.88	15.70
60	--	--	--	--	1.02	5.32
<u>Medium Diameter Cylinder</u>						
0	-2.96	52.22	0.0778	6.28	52.22	6.28
10	-2.84	51.91	0.0791	9.62	52.42	9.73
20	-3.34	38.40	0.0864	10.08	40.69	21.60
25	--	--	--	--	26.43	21.80
30	--	--	--	--	5.86	8.24
<u>Small Diameter Cylinder</u>						
0	-3.10	55.21	0.0741	32.60	55.21	32.60
10	-3.18	53.08	0.0743	38.67	53.22	38.85
20	-2.26	50.71	0.0823	6.85	54.01	11.72
25	--	--	--	--	45.39	10.10
30	--	--	--	--	9.71	1.42

TABLE 5
Comparison of L/R Times for the Hollow Cylinder
(all values in ms)

Cylinder	Experimental Values		Finite Thickness	Infinitesimal Thickness	
	Center	2/3 Point		Mean Radius	Inner Radius
Large	10.24	9.27	10.14	10.26	10.08
Medium	12.85	11.57	12.05	12.86	11.57
Small	13.50	12.15	12.61	15.43	10.29

Fig. 6. Induced magnetic field 45 cm from the center of the large cylinder (CLH). Squares indicate the experimental data, pluses the three-parameter fit (FIT3), and crosses the one-parameter fit (FIT0), which fits the data by normalizing the curve found at the central position.

Fig. 7. Induced magnetic field 20 cm from the center of the medium cylinder (CMH), indicated by pluses, and the small cylinder (CSH), indicated by squares. Lines show the one-parameter fit.

Comparison with Analytical Results

An infinitely long hollow cylinder with infinitesimal thickness w , radius a , and electrical resistivity ρ has an L/R time constant τ

$$\tau = \mu_0 w a / 2\rho. \quad (5)$$

A cylinder with finite thickness has L/R time constant

$$\tau = \mu_0 / k^2 \rho, \quad (6)$$

Fig. 8. Induced magnetic field 20 cm from the center of the medium cylinder (CMH), indicated by pluses, and the small cylinder (CSH), indicated by squares. Lines show the three-parameter fit.

where k^2 is a constant found by solving the appropriate eigenvalue equation involving Bessel and Neumann functions. Davey [4] has calculated the value of k^2 for the three FELIX cylinders. He has also calculated the finite length corrections and found them to have negligible effect on the results.

Experimental values of τ can be found as the reciprocal of γ_2 . In Table 5 the experimental values are listed for the center of the cylinders and for the point two-thirds of the way to the end (40 cm for CLH, 20 cm for CMH and CSH). For the thin cylinder equations both the mean and inner radii are used.

CONCLUSION

At each position on the cylinder axis the induced field is fit well by the difference of two exponentials, with time constants corresponding to the decay time of the applied field and a single L/R time for the cylinder. Axial variation of that L/R time ($= \gamma^{-1}$) is of the order of the expected effect of the finite cylinder length.

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