139
4/11/80

DR. 1037

LA-8289-MS

Informal Report

MASTER

**Radial Focusing and Space Charge in an
Alternating-Phase-Focusing Linac**

University of California

LOS ALAMOS SCIENTIFIC LABORATORY

Post Office Box 1663 Los Alamos, New Mexico 87545

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

RADIAL FOCUSING AND SPACE CHARGE IN AN ALTERNATING-PHASE-FOCUSING LINAC

by

Shigemi Inagaki

ABSTRACT

Using the strong focusing theory for the phase-energy space, it is possible to find the space-charge limit of an Alvarez linac or to find the appropriate combination of the synchronous phases for an alternating-phase-focusing (APF) linac.

In a linac design, it is common to use the integral form to compute the transverse dynamics. This report will show that the use of the differential form and of the periodicity condition gives the space-charge term and the appropriate combination of the synchronous phases for the radial oscillation.

I. INTRODUCTION

In the design of the Pion Generator for Medical Irradiation (PIGMI) alternating-phase-focusing (APF) structure, D. A. Swenson of the Los Alamos Scientific Laboratory (LASL) used the following equations¹ for the transverse dynamics,

$$\Delta x = d_n x' \quad , \quad (1)$$

$$\Delta x' = \frac{-\pi q V_n T \sin \phi_n}{W_0 \beta^3 \gamma^3 \lambda} x \quad , \quad (2)$$

where q is the charge, n denotes the number of gaps, V_n is the peak voltage in the n th gap, T is the transit time factor, ϕ_n is the phase of the rf when it arrives at the n th gap, W_0 is the rest energy of the synchronous

particle, β and γ are the relativistic velocity and mass factors corresponding to the particle energy during the n th drift, and λ is the wavelength.

Equation (2) is derived from the integral

$$\Delta p_r = \int_{\text{cell}} e E_r - v(B\psi) dt . \quad (3)$$

where p_r is the radial momentum, E_r is the radial electric field, v is the particle velocity, B is the magnetic field, and ψ is the particle phase.

To find the space-charge effect in the Alvarez linac, we use the longitudinal oscillation equation²

$$\beta^{-3} \frac{d}{ds} \left(\beta^3 \frac{dX}{ds} \right) + k_\ell^2 \left[(1 - \mu_\ell)X - \frac{X^2}{2|\phi_s|} \right] = 0 , \quad (4)$$

where X is $(\psi - \psi_s)$, k_ℓ is the longitudinal wave number, μ_ℓ is the longitudinal space-charge factor, ϕ is the particle phase, and s denotes the synchronous particle. The transverse oscillation equation is

$$\beta^{-1} \frac{d}{ds} \left(\beta \frac{dy}{ds} \right) + k_t^2 (1 - \mu_t) y = 0 , \quad (5)$$

where t denotes the transverse plane. These expressions are averaged over a cell.

However, in the strong focusing theory it is essential that the periodical configuration of the focusing and defocusing elements results in net focusing over a period, but there will be strong variations within the period. Therefore, it may be possible to have a larger current limit, better phase stability, or a simpler configuration in a model where an integrated formula is used.

The author applied this concept to find both the current limit for an Alvarez-type proton linac³ and the longitudinal stability of the APF structure.⁴

We propose now to study the APF linac from a different point of view by using the differential form of the equations.

Using $d\ell$, the number of rf cycles needed by the synchronous particles to transverse the distance dz ,

$$dz = v_s dt_s = \beta_s \lambda d\ell \quad , \quad (6)$$

and assuming a constant velocity in a cell, the longitudinal oscillation equation

$$\frac{d}{dz} \left[\gamma_s^3 \beta_s^3 \frac{d}{dz} (\psi - \psi_s) \right] = - \frac{2\pi}{\lambda} \cdot \frac{e}{m_0 c^2} \left[E_z(r, z, t) - E_z(r, z, t_s) \right] \quad , \quad (7)$$

can be rewritten periodically as

$$\frac{d^2 X}{d\ell^2} = -n(\ell) X \quad (8)$$

and

$$n(\ell) = (G - S') F \quad . \quad (9)$$

In Eqs. (8) and (9) $X = \psi - \psi_s$, F is defined by

$$F = - \frac{2\pi e \lambda E_m}{m_0 c^2 \gamma_s^3 \beta_s} \sin \psi_s \quad , \quad (10)$$

S' is derived from the space-charge effect, and G is the average field in an element divided by E_m .

The solution can be expressed by

$$X(\ell) = \begin{bmatrix} X(\ell) \\ X'(\ell) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} X(\ell_0) \\ X'(\ell_0) \end{bmatrix} = M(\ell|\ell_0)X(\ell_0) \quad , \quad (11)$$

The term, $M(\ell|\ell_0)$, has the following values.

$$M(\ell|\ell_0) = \begin{bmatrix} \cos \alpha & n^{-1/2} \sin \alpha \\ (-n)^{1/2} \sin \alpha & \cos \alpha \end{bmatrix} \quad \text{for } n > 0 \quad , \quad (12)$$

$$M(\ell|\ell_0) = \begin{bmatrix} 1 & \ell - \ell_0 \\ 0 & 1 \end{bmatrix} \quad \text{for } n = 0 \quad , \quad (13)$$

and

$$M(\ell|\ell_0) = \begin{bmatrix} \cosh \beta & (-n)^{-1/2} \sinh \beta \\ (-n)^{1/2} \sinh \beta & \cosh \beta \end{bmatrix} \quad \text{for } n < 0 \quad , \quad (14)$$

with

$$\alpha = n^{1/2}(\ell - \ell_0) \quad , \quad (15)$$

and

$$\beta = (-n)^{1/2}(\ell - \ell_0) \quad . \quad (16)$$

Using the periodicity of the rf length, one can find the stability region by applying the condition

$$-1 < 1/2 \text{ Tr} M < 1 \quad . \quad (17)$$

This formalism gives not only the space-charge limit but also sets limitations on the combinations of field strengths or rf lengths in an Alvarez linac or an APF linac.

II. TRANSVERSE OSCILLATIONS

The transverse oscillation equation in a linac is .

$$\beta \frac{d}{dz} \left(\gamma \beta \frac{dr}{dz} \right) = \frac{e}{m_0 c^2} (E_r - c \beta B_\phi) \quad . \quad (18)$$

If one neglects the magnetic term and makes approximations for $\partial E_z / \partial z$ and ρ by taking their values on the axis, Eq. (18) becomes

$$\beta \frac{d}{dz} \left(\gamma \beta \frac{dr}{dz} \right) = - \frac{er}{2m_0 c^2} \left[\frac{\partial E_z(0, z, t)}{\partial z} - \frac{\rho(0, z, t)}{\epsilon} \right] \quad , \quad (19)$$

where ρ is the charge density and ϵ is the dielectric constant.

By assuming a constant beta value and by using the independent variable $d\ell$, Eq. (18) gives

$$\frac{d^2 r}{d\ell^2} = -n_t(\ell)r \quad (20)$$

and

$$n_t(\ell) = (G_t - S'_t)F_t \quad , \quad (21)$$

where F_t , G_t , and S'_t are defined by

$$F_t = \frac{e}{2m_0 c^2} \cdot \frac{\beta_s^2 \lambda^2}{\gamma \beta^2} \left(\frac{\overline{\partial E}}{\partial z} \cos \psi \right)_1, \quad (22)$$

$$G_t = \left(\frac{\overline{\partial E}}{\partial z} \cos \psi \right)_1, \quad (23)$$

and

$$S'_t = \left(\frac{\frac{\rho}{\epsilon}}{\overline{\partial E}} \cos \psi \right)_1. \quad (24)$$

In these expressions the field gradient along z is replaced by the average value, by adding length at the exit and the entrance of the two drift tubes. The recovering force of the transverse oscillation is the electric field gradient while that of the longitudinal oscillation is the electric field. Thus, the transfer matrix is

$$R(\ell) = \begin{bmatrix} r(\ell) \\ r'(\ell) \end{bmatrix} = M_t(\ell | \ell_0) R(\ell_0). \quad (25)$$

III. APPLICATION TO AN APF STRUCTURE

One application of this formalism is a simple APF structure in which a superperiod consists of two gaps in two rf lengths,

$$\sum_{i=1}^8 \ell_i = 2. \quad (26)$$

We make the following assumptions: $|\partial E_z / \partial z|$ is symmetrical relative to the gap center; the gap length, ℓ , is 0.25; and the effective length of $\partial E / \partial z$ is ℓ equal to 0.2. For a fixed synchronous phase in the first gap, it is possible to change the phase in the second gap. Table I shows some parameters for the phase changes.

In Fig. 1, ℓ is the rf length between the centers of the two gaps.

Figures 2 and 3 show the stable phase region corresponding to a synchronous phase in the first gap of -30° or -60° .

Figure 4 shows the stable phase region for $S'_t = 0.1$. The realistic value of the average field gradient, F_t , is approximately 1.

IV. CONCLUSION

The electric field derivatives at the exit and the entrance of the drift tubes show that the radial focusing of an APF linac is obtained. If the field gradient is too strong, the band structure will be so dominant that the stable region will be significantly reduced.

TABLE I
PHASE-CHANGE PARAMETERS

	ℓ_i	$(\partial E / \partial z)_i$	ψ_i
1	0.2	1	$\psi_{s1} - 45^\circ$
2	0.05	0	ψ_{s1}
3	0.2	-1	$\psi_{s1} + 45^\circ$
4	$\ell - 0.45$	0	
5	0.2	1	$\psi_{s2} - 45^\circ$
6	0.05	0	$\psi_{s2} = (\ell - 1) \cdot 360^\circ + \psi_{s1}$
7	0.2	-1	$\psi_{s2} + 45^\circ$
8	$1.55 - \ell$	0

Fig. 1: APF structure with two gaps.

Fig. 2. Stable phase region for -30° synchronous phase in the first gap.

Fig. 3. Stable phase region for -60° synchronous phase in first gap.

Fig. 4. Stable phase region for $S'_t = 0.1$.

ACKNOWLEDGMENTS

I thank R. Gluckstern for his encouragement to continue this work. I also appreciate the hospitality of the LASL AT-1 group during my stay at LASL. My visit to LASL was supported by the Yoshida Foundation for Science and Technology, Tokyo, Japan.

REFERENCES

1. D. A. Swenson, "Alternating Phase Focused Linacs," Part. Accl. 7, 61 (1975).
2. R. L. Gluckstern, "Space Charge Effects," in Linear Accelerator, (North-Holland Publishing Company, 1961) p. 830.
3. S. Inagaki, "Space-Charge Limit for a Drift-Tube Linac from the Stability Point of View of Longitudinal Periodicity," Los Alamos Scientific Laboratory report LA-8288-MS (March 1980).
4. S. Inagaki, "Space-Charge Limit for an Alternating-Phase-Focusing Structure," Los Alamos Scientific Laboratory report LA-8287-MS (March 1980).