MASTER

UV FREE ELECTRON LASERS FOR SYNCHROTRON RADIATION SOURCES*

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ABSTRACT

We discuss the possibility of operating a free electron laser in an electron storage ring similar to those being designed or built as synchrotron radiation sources. The laser output power is proportional to the total power radiated as synchrotron radiation in the ring. The electron beam properties make the storage ring particularly suitable for operating a laser in the UV region. Small signal gain per pass larger than one at wavelengths near $600 \text{ \AA}$ can be obtained in a 500 MeV storage ring.

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I. INTRODUCTION

The experiments made at Stanford^{1,2} starting from 1975 have shown that a free electron laser can be build successfully. They have also shown that the experimental results agree with the free electron laser theory^{3,6} when one considers the main parameters of the system, while perhaps some more work is needed to explain some details, like the fine structure of the light pulse.

The case of a free electron laser operating in a storage ring differs from the Stanford experiments where a linear accelerator was used as the electron beam source. The difference is that in the Stanford case each electron interacts only once with the radiation field in the optical cavity, and after this interaction, is thrown away, while in the storage ring case each electron interacts at each revolution. Since we want to operate the system over millions or billions of revolutions and the laser performance depends strongly on the electron beam characteristics at the laser entrance, the effect of this interaction on the electron beam has to be taken into account.

The extension of the free electron theory to the storage ring case⁷ has shown the great importance of synchrotron radiation in determining the laser output power. As will be discussed in more details in Section III, the output laser power can be assumed to be proportional to the total synchrotron radiation power emitted in the storage ring. This limits the laser efficiency to rather low values (of the order of one percent or less). On the other hand, an electron storage ring is the only electron source existing today capable of providing a beam with the characteristics

that are necessary to obtain a very high gain per pass. A very high gain per pass is needed to operate a laser at short wavelengths, say $2000 \text{ \AA}$ or below.

All this makes the electron storage ring-free electron laser system an interesting choice to provide continuously tunable laser radiation in the region below $2000 \text{ \AA}$, for low power applications such as material research. The same ring would also be a good synchrotron radiation source, thus providing both the extended synchrotron radiation spectrum and a narrow laser line. The intensity per unit wavelength in this line is greater than that of the synchrotron radiation by a factor of the order of 10^8 .

In this paper we want to discuss the main properties of a storage ring-free electron laser system. In Sections II and III, we summarize the main laser properties, and in Section IV, the electron beam properties. In Section V we will apply these results to the case of a model storage ring, optimized for the operation of the laser in the UV region. This model ring may allow to obtain a very high gain per pass, of the order of ten, in the wavelength region between $500 \text{ \AA}$ and the visible.

The use of a storage ring to operate a free electron laser in the visible and IR region has also been considered.⁸ In the IR region systems based on linacs or microtrons might be competitive with the storage ring system. However, the storage ring represents the only possibility in the UV region.

II. FREE ELECTRON LASER DESCRIPTION

In this section we want to summarize the main properties of a free electron laser operating in a storage ring. All this formulae can be

found in references (3-7) and (9).

We consider only the case of a constant period wiggler magnet, similar to that used in the Stanford experiments. Variable geometry wigglers, having the magnetic field and/or the period varying either in the longitudinal or transverse direction, have been studied theoretically by several authors.^{10,11} They have shown that variable wigglers can increase the laser efficiency by one or two order or magnitudes. Limiting ourselves to the most pessimistic case we believe to underestimate the laser performance. However, we think that a free electron laser would be of interest for research applications also in this pessimistic case.

A simple scheme of a free electron laser operating in a storage ring is given in Fig. 1. The optical cavity is shown in Fig. 2 and is assumed to be a confocal spherical cavity of length L_c . The wiggler magnet has N_w periods of length λ_w . We also assume that n_B electron bunches are circulating in the ring and that the distance between two bunches is equal to twice the cavity length L_c .

The laser radiation wavelength, λ , is related to the wiggler magnet period λ_w and to the electron energy γ (measured in rest energy units $m_0 c^2$) by⁹

$$\lambda = \frac{\lambda_w}{2\gamma}(1 + K^2) \quad (\text{II.1})$$

where K , the wiggler parameter is

$$K = \frac{e B_0^* \lambda_w}{2\pi m_0 c^2} \quad (\text{II.2})$$

In (II.2) B_0^* is the r.m.s. wiggler magnetic field, equal to the field

itself for a helical wiggler, or to the peak field divided by $\sqrt{2}$ for a transverse wiggler. As can be seen from (II.1) the laser can be tuned by changing either the electron energy γ or the wiggler parameter, K .

An important laser parameter is the small signal gain, G . In the case of electron and photon bunches of equal transverse area, Σ , and equal length, ℓ , with uniform electron and photon distribution G is given by⁹

$$G = 32(2)^{\frac{1}{2}} \pi^2 (\lambda^3 \lambda_w)^{\frac{1}{2}} \frac{K^2}{(1 + K^2)^{3/2}} \frac{I_p}{\Sigma I_A} N_w^3 f(x) \quad (\text{II.3})$$

where $I_p = eNc/\ell$ is the peak bunch current, N being the number of electrons per bunch; $I_A = ec/r_0 = 1.7 \times 10^4 \text{ A}$, r_0 being the classical electron radius and e the electron charge. The function $f(x)$, the gain function, is given by⁹

$$f(x) = [\cos x - 1 + \frac{1}{2} x \sin x] / x^3 \quad (\text{II.4})$$

where

$$x = 4\pi N_w (\gamma_0 - \gamma_R) / \gamma_R \quad (\text{II.5})$$

with γ_0 the initial electron energy

and

$$\gamma_R^2 = \lambda_w (1 + K^2) / 2\lambda \quad (\text{II.6})$$

for given λ , λ_w and K .

The function $f(x)$ is an odd function of x ; the electron lose or gain energy according to the sign of x ; it is zero for $\gamma_0 = \gamma_R$, has a maximum value near to 7×10^{-2} for $x = 2.5$ or $(\gamma_0 - \gamma_R) / \gamma_R \approx 1/5 N_w$, and drops off to zero for larger values of x .

The expression (II.3) of the small signal gain is only valid for a monochromatic electron beam of zero angular divergence. It gives a good approximation as long as the electron beam energy spread, σ_E , and angular spread, σ_θ , satisfy the conditions⁹

$$\sigma_E < 1/2N_w \quad (\text{II.7})$$

$$\sigma_\theta < 1/[\gamma(2N_w)^{\frac{1}{2}}] \quad (\text{II.8})$$

In the case when (II.7), (II.8) are satisfied but the electron and photon distribution are gaussian instead of uniform, the small signal gain can still be written as in (II.3) if we assume that

$$I_p = eNc/(2\pi\sigma_w^2)^{\frac{1}{2}} \quad (\text{II.9})$$

and

$$\Sigma = 2^{3/2} \pi (w^2 + \sigma_\perp^2) \quad (\text{II.10})$$

where σ_w is the r.m.s. bunch length, assumed equal for electrons and photons, and $\sigma_\perp$, w are the transverse r.m.s. electron and photon bunch dimensions. For a confocal spherical cavity of length L_c , w^2 is given by¹²

$$w^2 = \lambda L_c / 8\pi \quad (\text{II.11})$$

The electron r.m.s. radius can be expressed in terms of the transverse beam emittance $\epsilon_\perp$ and the storage ring betatron oscillation envelope function at the wiggler location, β_s , by^{13,14}

$$\sigma_\perp^2 = \beta_s \epsilon_\perp / \pi \quad (\text{II.12})$$

To derive (II.10) we have neglected the variation of w and $\sigma_\perp$ along the wiggler. This is a good approximation if the wiggler length is half the cavity length or shorter and the function β_s is larger than the cavity

length. For $N_w \lambda_w = L_c/2$ and $N_w \lambda_w = \beta_s$, w^2 and σ_1^2 vary by 25% from the center to the end of the wiggler.^{12,13}

For given λ and L_c , w^2 is determined by (II.11). It is then convenient, in order not to reduce the gain, to choose $\sigma_1^2 \leq w^2$ or

$$\epsilon_1 \leq \frac{\pi w^2}{\beta_s} \quad (\text{II.13})$$

The condition (II.8) on the beam angular spread can also be written, using the beam emittance, as

$$\epsilon_1 \leq \frac{\pi \beta_s}{2 N_w \gamma^2} \quad (\text{II.14})$$

From (II.13), (II.14) and using (II.11), we obtain

$$\epsilon_1^2 \leq \frac{\pi}{8} \frac{L_c}{(N_w \lambda_w)(1 + K^2)} \lambda^2 \quad (\text{II.15})$$

The quantity $L_c/(N_w \lambda_w)(1 + K^2)$ is of the order of one. The condition (II.15) defines the beam emittance needed to operate a laser at a wavelength λ . Assuming $L_c/(N_w \lambda_w)(1 + K^2) = 1$, this can also be written in the simplified form

$$\epsilon_1 \leq 0.6 \lambda \quad (\text{II.16})$$

The condition (II.16) can be used to establish a connection between the laser wavelength and the electron beam accelerator. A linear accelerator at 50 MeV gives a beam emittance of the order of 10^{-6} m-rad, decreasing like the square root of the energy. A similar emittance is also produced by a microtron. These accelerators can be used to produce laser light at wavelength longer than $1 \mu\text{m}$ but not in the UV region. A storage ring emittance can be of the order of 10^{-7} - 10^{-8} m-rad for a 500 to 1000 MeV ring and this machine can be used to drive a laser in the UV region.

III. THE LASER POWER AND LINEWIDTH

For a laser oscillator, the small signal gain must be larger than the laser cavity losses, L . The losses are due to diffraction, to the light absorption on the mirrors and to the light extracted from the cavity. The diffraction losses are smaller than 10^{-4} if the ratio

$$F = \frac{g^2}{L_c \lambda} \quad (\text{III.1})$$

is larger than one,¹² where g is the half aperture of the wiggler magnet which we assume to be the smallest diaphragm in the cavity.

The reflectivity of the mirrors is very near to one (larger than 99%) for $\lambda > 2000 \text{ \AA}$ and decreases to values of the order of 90% near $1200 \text{ \AA}$ and smaller values at shorter wavelengths. Mirrors suitable for optical cavity (with a reflectivity of the order of 50% or more for normal incidence) in the $500 \text{ \AA}$ region have yet to be developed.

Assuming that the gain is larger than the losses and that the laser starts to oscillate the maximum amount of energy that can be extracted from an electron in a single passage of the wiggler is given by⁹

$$\eta = \left(\frac{\Delta \gamma}{\gamma} \right)_{\text{MAX}} \approx \frac{1}{2N_w} \quad (\text{III.2})$$

corresponding to the width of the gain curve as described by the function $f(x)$ defined in (II.4). This quantity also represents the efficiency in energy transfer from the electron to the electromagnetic field. The maximum output laser power for an electron beam of current I is obtained using (III.2) and is given by

$$P_L = \eta (I m_0 c^2 \gamma) \quad (\text{III.3})$$

Not all the electrons passing the wiggler will lose the same amount of energy. In fact, an electron beam which is monoenergetic at the laser entrance, will have at the exit an energy spread, $\left(\frac{\Delta\gamma}{\gamma}\right)_s$, of the same order of magnitude of the energy loss,⁹ so that its maximum value is

$$\left(\frac{\Delta\gamma}{\gamma}\right)_s \approx \frac{1}{2N_w} \quad (\text{III.4})$$

For an electron beam with a non zero energy spread, the gain is obtained folding the expression (II.3) with the electron energy distribution. The effect of the energy spread is to reduce the gain, and, when the spread becomes of the order of $1/2N_w$ or larger, the gain becomes nearly zero. This electron energy spread produced by the laser introduces an important problem in the operation of a free electron laser in a storage ring. In this case, in fact, the same electron bunch has to interact, hopefully for for something like $10^8 - 10^{10}$ revolutions, with the radiation field in the laser cavity. For a laser operated with a linear electron accelerator each electron bunch traverses the wiggler only once and the electron distribution function at the wiggler entrance is determined only by the accelerator characteristics. Instead, in the storage ring case, the same electron bunch is repetitively interacting with the laser field, and its characteristics are determined also by this interaction.

This problem has been studied by Renieri⁷ who has shown that the interaction is such that the energy spread would continue to grow and the effective gain would become zero. The only mechanism opposing this process is the synchrotron radiation damping which tends to decrease the energy spread. An equilibrium between these two processes can be reached such

that the laser output power is determined by the strength of the radiation damping process, which is proportional to the total amount of synchrotron radiation energy, U_o , lost by the electrons in one revolution. Hence for a storage ring case equation (III.3) must be substituted by

$$P_L = \eta(U_o i_{av}) \quad (III.5)$$

The effective gain is also reduced so that to obtain a value of η near to $1/2N$ one needs a gain calculated for the unperturbed beam larger than the losses by a factor of 5-10.^{7,8}

The synchrotron radiation energy loss per revolution is a strong function of electron energy. For a bending radius, ρ , in the ring one has¹³

$$U_o = \frac{4}{3} \pi \frac{r_o}{\rho} \gamma^4 m_o c^2 \quad (III.6)$$

so that to increase P_L is very convenient to increase γ .

The space-time structure of the laser beam reflects that of the electron beam, hence it consists of pulses separated by a distance equal to the distance between the electron bunches and of length less or equal than the electron bunch length. The photon bunch length defines the radiation line width. For a bunch of length $2\sigma_n$ one has

$$\delta\omega = \frac{2\pi c}{2\sigma_n} \quad (III.7)$$

or

$$\frac{\delta\omega}{\omega} = \frac{\lambda}{2\sigma_n} \quad (III.8)$$

The photon bunch also has an inner structure, consisting of lines spaced by the inverse of the separation between bunches. The photon

bunch is the envelope of this line, whose width is proportional to the inverse of the correlation time,¹⁵ and can be much smaller than the value (III.8).

IV. THE ELECTRON BEAM PROPERTIES

We have seen in Section II that the laser small signal gain depends on the properties of the electron beam, like energy spread and bunch dimension. In this section we want to collect and discuss the formulae describing these properties. These formulae strictly apply only to a storage ring without a laser. We will indicate what electron beam properties will change as a result of the interaction with the laser field and what are expected to remain unchanged.

In describing the electron beam properties we will make the following simplifying assumptions:

1. the momentum dispersion function is zero in the laser straight section, hence the transverse beam dimension does not depend on the energy spread;
2. the horizontal and vertical betatron oscillations are fully coupled; the horizontal and vertical betatron wave numbers, ν_x , ν_y , and the horizontal and vertical beta functions are equal in the laser region so that the beam is round and can be characterized by one transverse r.m.s. radius, σ_1 ;
3. the momentum compaction factor, α , is chosen to adjust the transverse dimension and the energy spread to the laser requirements;
4. the synchrotron radiation damping times for the horizontal and

vertical betatron oscillations, τ_x , τ_y , and for the synchrotron oscillation, τ_E , are adjusted in such a way that $\tau_x = \tau_y = 2\tau_E$. The beam transverse emittance is given by¹³

$$\epsilon_1 = \pi \sigma_{EO}^2 \frac{R}{v_x} \quad (IV.1)$$

where R is the average ring radius, and σ_{EO} is the zero current energy spread. The quantity σ_{EO} is determined only by the effect of synchrotron radiation and is given by¹³

$$\sigma_{EO}^2 = 1.92 \times 10^{-13} \frac{v^2}{\rho} \quad (IV.2)$$

where ρ , in meter, is the electron radius of curvature in the bending magnets.

At zero, or very small, current the electron bunch length and energy spread are determined by synchrotron radiation. However we can expect that at the very large current needed to obtain a high laser gain the length and energy spread are determined by the microwave instability and are larger than those at zero current. In this case the r.m.s. bunch length can be written as¹⁶

$$\left(\frac{\sigma_R}{R}\right)^3 = \frac{(2\pi)^{\frac{1}{2}} e I_0 \alpha \left|\frac{Z}{n}\right|_{eff}}{F v_s^2 m_0 c^2 \gamma} \quad (IV.3)$$

where R is the average machine radius, I_0 is the average current per bunch ($I_0 = eNc/2\pi R$), α is the momentum compaction factor, v_s the synchrotron oscillation frequency measured in units of the revolution frequency,

F is a form factor (of the order of 6 for a Gaussian bunch) and $|Z/n|_{\text{eff}}$ is an effective longitudinal coupling impedance.¹⁶ When σ_n is known we can calculate the energy spread from¹³

$$\sigma_E = \frac{v_s}{\alpha} \frac{\sigma_n}{R} \quad (\text{IV.4})$$

and the peak current from (II.9) and the definition of I_o , as

$$I_p = \sqrt{2\pi} I_o \left(\frac{R}{\sigma_n} \right) \quad (\text{IV.5})$$

If the momentum dispersion function is zero in the laser straight section, we can expect the transverse beam dimension to remain unchanged when the laser is operating and to be given by

$$\sigma_x^2 = \frac{\beta_s e_{\perp}}{\pi} \quad (\text{IV.6})$$

with $e_{\perp}$ given by (IV.1).

We can instead expect that the energy spread and the bunch length will increase when the laser is operating.⁷ If the condition (II.7) is satisfied when the laser is off, with σ_E given by (IV-3), (IV-4) we can expect that σ_E will grow to a value of the order of $1/2N_w$ and σ_n to the corresponding values.

V. A MODEL STORAGE RING-LASER SYSTEM

In this section we want to apply the results of Sections II, III, and IV to one particular free electron laser-storage ring system. We want to show that using a state of the art storage ring it is possible to obtain small signal gains of the order of ten. Such a value of the gain should allow us to operate the laser also in the wavelength region below 1000 Å, and to use mirrors which reflect only 50% of the light.

Let us define a "model storage ring" with a 500 MeV energy, whose parameters are given in Table I. These parameters are not the result of an optimization study, but can be used to allow us to obtain an estimate of the magnitude of the quantities involved. They define, however, a ring that can be built using our present storage ring experience.

To design the laser-storage ring system we choose

$$\sigma_{\perp}^2 \cong w^2 = \frac{\lambda L_c}{8\pi}$$

at the value of K , equal to $\sqrt{2}$, which optimizes the gain as a function of K . Assuming for the wiggler magnet a period of 4 cm and a gap aperture of 4 cm, we have, for an electron energy of 500 MeV

$$\lambda = 6.27 \times 10^{-8} \text{ m.}$$

With three bunches the distance between bunches is 20.9 m and the cavity length must be $L_c = 10.45 \text{ m}$ giving

$$w^2 = 2.6 \times 10^{-8} \text{ m}^2.$$

The electron beam properties, at an energy of 500 MeV, are given in Table III. The condition $\sigma_{\perp}^2 \approx w^2$ is satisfied for $\lambda = 6.27 \times 10^{-8} \text{ m}$. At longer wavelength the photon bunch radius is larger than the electron radius and at shorter wavelength is smaller. The value of β_s has been chosen so that the variation of $\sigma_{\perp}$ in the wiggler is small, less than 25%, and the condition (II.8) on σ_{θ} is satisfied. The beam energy spread, when the laser is off and the average current is one Ampere per bunch, is 10^{-3} which satisfies (II.7).

Some of the laser characteristics are given in Table IV. The radiation wavelength and the small signal gain as a function of K for constant

electron energy are given in Fig. 3. For $K = 0.7$, corresponding to a wavelength of $300 \text{ \AA}$, one still has a gain of one, while for $K = 2$, one has $\lambda = 1000 \text{ \AA}$ and $G = 10$. It is also possible to operate at different energies, thus increasing the flexibility of the system. At lower energies the transverse emittance, and σ_1^2 , decrease like γ^2 , while σ_r and σ_E increase like $\gamma^{1/3}$. This mode of operation might be convenient in the short wavelength region, where $\sigma_1^2 > w^2$, or also to reduce the magnetic field required for a given wavelength.

The value of the ratio $g^2/L_c\lambda$ is very high, so that diffraction losses are negligible and the cavity losses are determined only by the mirror reflectivity.

The laser output power is of the order of 17W. It is possible to increase this power by increasing the synchrotron radiation damping rate. This can be done by adding an auxiliary wiggler radiator.⁸ In this way the laser power can be brought to the level of 10^2 or 10^3 W.

As we already said in Section II, it is also possible to increase the laser power by the use of special wigglers with either a transverse or a longitudinal gradient.^{10,11} In this case one might expect that the laser power is still given by the expression (III.5) with η having a value as high as 0.5-1. This would allow to increase the output power by more than one order of magnitude.

TABLE I.

Energy, E	500 MeV
Average Radius, R	10 m
Radius of curvature, ρ	3.3 m
Straight Section Length	6 m
Radial wave number, ν_x	3.4
Vertical wave number, ν_y	3.4
Radial beta function in long straight, β_{sx}	16 m
Vertical beta function in long straight, β_{sy}	16 m
Number of bunches, n_B	3
Momentum compaction factor	0.12
Synchrotron wave number, ν_s	2×10^{-2}
Harmonic number, h	9
R.f. acceptance, $\mathcal{E}_{RF}$	3.7×10^{-2}
Peak r.f. voltage	1.15 MeV

TABLE II.

Wiggler period, λ_w	4 cm
Number of periods, N_w	150
Total length, L_w	6 m
Gap aperture	4 cm
r.m.s. magnetic field, B^* , for $K=1$	2.7 K Gauss

TABLE III.

R.m.s. Transverse emittance, ϵ_1	1.54×10^{-8} m.rad
R.m.s. beam radius, σ_1	1.71×10^{-4} m
R.m.s. beam angular divergence, σ_θ	2.8×10^{-5}
Average bunch Current, I_0	1A
R.m.s. bunch length, σ_z	6.3 cm
R.m.s. energy spread, σ_E	1×10^{-3}
Peak Current, I_p	396A
Synchrotron radiation loss per turn, U_0	1.67 KeV
Synchrotron oscillation damping time, τ_s	6.2×10^{-2} s

TABLE IV.

Small signal gain at $\lambda = 627 \text{ \AA}$, G	6.0
Laser output power, P_L	16.7 W
Line width, $\Delta\nu/\nu$, at $\lambda = 1000 \text{ \AA}$	8×10^{-7}
Ratio, $g^2/L_c\lambda$, at $\lambda = 1000 \text{ \AA}$	380

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Figure Captions

Fig. 1. Scheme of a storage ring-free electron laser system: (1) optical cavity mirrors; (2) bending magnets; (3) radio frequency cavity; (4) electron trajectory; (5) wiggler magnet.

Fig. 2. Geometry of the confocal cavity resonator. The mirror radius of curvature, ρ , is equal to the cavity length, L_c .

Fig. 3. The laser radiation wavelength and the small signal gain versus the wiggler parameter K , for fixed electron energy $E = 500$ MeV. The storage ring parameters used in the calculation are those of Table I.