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ABSTRACT

An extensive numerical study has been carried out for internal modes with toroidal mode number unity. These are internal kink modes, when the $q = 1$ surface falls within the plasma, and have a ballooning characteristic when $q_{axis} > 1$. Both modes show a dependence on the pressure and have a second region of stability at high β . A parameter survey has been conducted, varying the geometry, i.e., aspect ratio, ellipticity, triangularity, etc. and the current profiles, through the pressure and safety factor. The principal results show that the modes are dependent on the geometry and are strongly stabilized by high-order, noncircular effects. Broader pressure profiles and reduced shear are favorable for limiting the instability.

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I. INTRODUCTION

The $n = 1$ internal kink mode is a subtle and potential harmful instability occurring in tokamak systems. It is associated with plasmas whose boundary is held fixed and have $q_{axis} < 1$ such that the $q = 1$ surface falls within the plasma. This instability has small growth-rates, down by a factor of ϵ^2 , where ϵ is the inverse aspect ratio, from the more dangerous external kink modes. These small growth rates make it more difficult to analyze this mode and it represents a challenge to stability codes. Hitherto it has not received as much attention as the external modes since the reduced growth rate made it appear less dangerous. However, there is evidence that it is closely associated with the MHD oscillations, called fishbones, observed experimentally, which result in a loss of confinement.¹ This makes it an interesting topic for the same kind of extensive analysis, that was given to external kink modes and ballooning modes.²⁻⁴ This paper presents an extensive parameter survey of $n = 1$ internal modes in the context of ideal MHD. While the internal kink mode only exists when the $q = 1$ surface lies in the plasma, the system can be unstable to an internal perturbation even when the $q = 1$ surface is not present. The structure of this mode is reminiscent of the ballooning mode and in spite of the low toroidal mode number, unity, it will be given that name in this report. It is logical to combine the study of both types of modes, and address the issue of stability to internal $n = 1$ ideal MHD modes in axisymmetric toroidal systems. This is the subject of this paper.

We have generated several sequences of equilibria using the flux conserving technique to ensure that the pressure and safety-factor profiles remain constant. These are examined as a function of q_{axis} , which is varied by scaling the toroidal field. We use the 2-d ideal MHD stability analysis code PEST⁵ to examine these equilibria and map out regions of instability in a

plane formed by q_{axis} and the product of the inverse aspect ratio $\epsilon (= a/R)$, and $\beta_{pol} (= 8\pi \int p ds / \mu_0 I_t^2$, where the integral is an area integral over the plasma cross section and I_t is the total current in the toroidal direction). We find results similar to those reported in Refs. 6-8, which showed the validity of Bussac's analysis⁹ of the ideal internal kink mode at low β and also demonstrated the existence of a second stable region at high β . We also make contact with the work of Strauss *et al.*,¹⁰ which referred to ballooning modes for $q_{axis} > 1$. The major accomplishment of this work is to put all the pieces together and to extend the analysis to a variety of equilibria and establish the role of various parameters in determining stability to the $n = 1$ mode. Specifically, we examine geometric effects, i.e., aspect ratio and noncircularity; ellipticity, triangularity, and indentation. We also examine the role of the current profile by varying the pressure and safety-factor profiles. Analytic understanding of numerical 2-d equilibria is quite difficult and we must resort to empirical numerical methods. We do this to show the important role of the axis shift, and local shear in determining the unstable regions.

The plan of this paper is to describe the equilibrium and stability calculation in Sec. II. The results are presented in Sec. III, and in Sec. IV we present a discussion of the results and conclusions that may be reached.

II. EQUILIBRIUM DESCRIPTION

The $n = 1$ internal kink mode is a weak instability ($\delta w \sim \epsilon^4$) and is a sensitive test of numerical efforts to model it. The accuracy requirements of the equilibrium and stability analysis are quite stringent. To ensure that these requirements are adequately met, we have employed a flux coordinate equilibrium code¹¹ used in a fixed boundary form specifying the shape of the

outermost flux surface. This code distributes radial mesh points in equal intervals of $\psi^{1/2}$ giving a denser mesh near the magnetic axis where greater accuracy is indeed required. Generally, we have used 70 radial and 120 poloidal, mesh points. The convergence criterion has been $\epsilon < 10^{-8}$ where ϵ is the maximum amplitude of the relative change between iterations. With these criteria we feel confident that we have eliminated spurious results due to errors in the numerical equilibria.

The pressure and safety factor profiles were specified as

$$p(\psi) = p_0 (\tilde{\psi})^{\alpha_p} \quad (1)$$

$$q(\psi) = q_0 + (q_\ell - q_0) (1 - \tilde{\psi})^{\alpha_q} \quad (2)$$

with $\tilde{\psi} = (\psi_\ell - \psi) / (\psi_\ell - \psi_0)$ taking values from one to zero as ψ goes from ψ_0 (on axis) to ψ_ℓ (at the boundary). p_0 , q_0 , q_ℓ , α_p , and α_q are input quantities. Fixing the q -profile we can generate an FCT (flux conserving) sequence by varying p_0 . This gives us a family of equilibria with the same q -profile. We have generally arranged to generate this sequence with $q_0 = 1$. To study the internal kink mode we require equilibria with $q_0 < 1$. We obtain these from the original equilibrium by scaling the toroidal field while keeping the pressure and poloidal field constant. This keeps the poloidal β constant and varies q_0 keeping q_ℓ/q_0 roughly constant. Varying q_ℓ and α_q independently permits us to separate the influence of local and global shear.

The shape of the outermost surface is specified analytically through

$$x = x_0 - b + (a + b \cos \theta) \cos (\theta + \delta \sin \theta) \quad (3)$$

$$z \approx \kappa a \sin \theta$$

where x_0 is the major radius, a the minor radius, κ the ellipticity, δ the triangularity, and b the indentation which yields a bean shape.

The equilibria are mapped onto a finer mesh with 401 radial points and 64-128 θ divisions for analysis with the PEST stability code. In general, we find that a higher accuracy is required in the radial direction. This is related to the necessity of modeling the sharp discontinuity in the displacement vector associated with the internal kink mode at the rational surface. In the poloidal direction we use a Fourier representation and obtain good convergence with 25-30 harmonics.

We use the PEST II code (Refs. 5,12) to analyze the stability of these equilibria. Briefly, the PEST II code is based on a scalar form of the ideal MHD energy principle obtained by minimizing the functional

$$L \equiv \omega^2 K(\xi^*, \xi) - \delta W(\xi^*, \xi) \quad (4)$$

Here δW , K are, respectively, the change in the potential and kinetic energy due to the Lagrangian displacement

$$\xi(\mathbf{x}, t) = \xi(\mathbf{x}) e^{i\omega t} \quad (5)$$

If we are only interested in marginal stability, then two components of ξ can be eliminated analytically and a scalar equation is obtained. This procedure is described in detail in Ref. 5. The advantage is that we obtain a compact code which produces more accurate results quicker than with previous

codes.¹² A notable feature of this code is that by eliminating the magnetosonic continuum and shifting the continuum corresponding to the usual shear Alfvén spectrum upwards, it permits us to trace a mode from the unstable to the stable side of the marginal point. Indeed the marginal point is found more accurately by interpolation. Another significant feature of the code is the ability to tailor the radial mesh to the singular surfaces where $B \cdot \nabla$ is not invertible. This gives us the ability to use the finite elements more economically. This is particularly important here, as the internal kink mode has small growth rates and needs a very fine mesh for resolution of the mode. In general, it is appropriate to eliminate the mesh dependence when obtaining an eigenvalue. This is done by repeating the calculation with different meshes and extrapolating to the limit of an infinite number of points, Fig. 1. However, this process, called a convergence study, is costly and time consuming. Fortunately, the enhanced resolution of a tailored mesh with 100-200 points is adequate for determining the marginal point with an accuracy of better than 5%. By this we mean that the difference in the marginal point as obtained using a single run versus the value from a completely converged sequence is less than 5% (e.g., $q_{axis}^{crit} = 0.762$ vs. 0.766). This is shown in Fig. 2, where we plot the lowest eigenvalue as a function of q_{axis} . Finally, we note that the convergence in poloidal Fourier modes is so rapid that there is virtually no change ($< 0.5\%$) between the converged value and that obtained using 21 Fourier modes, which was the standard number used.

The outline of the computational procedure is as follows: we produce a family of equilibria with different values of β_{pol} representing a flux conserving sequence with specified pressure and safety factor profile. We map this onto a dense mesh of 401 constant- ψ surfaces and 64 constant- θ rays.

This is analyzed for marginal stability by scaling the toroidal field and hence varying q_{axis} as described above. In general, there may be several marginal points and these are obtained by systematically scanning q_{axis} at constant β_{pol} (see Fig. 2). The marginal values of q_{axis} at different β_p are connected to show the regions of instability. As observed in previous studies, e.g., Refs. 9 and 13, the appropriate pressure dependent parameter is $\epsilon\beta_{pol}$. Hence we show our results on a plane with $\epsilon\beta_{pol}$ as ordinate and q_{axis} as abscissa.

III. RESULTS

We have examined a variety of equilibria to determine the dependence of the $n = 1$ internal mode on geometry and equilibrium surface functions. The range of q_{axis} was chosen to cover both the $n = 1, m = 1$ internal kink mode as well as the $n = 1, m \geq 1$ ballooning type modes. Generally, when q_{axis} is less than unity and the $q = 1$ surface lies within the plasma, the unstable mode is dominated by the $m = 1$ Fourier harmonic and has the characteristic features of an internal kink mode (Fig. 3). When q_{axis} is greater than 1, the mode has large contributions from several $m > 1$ harmonics as well, and has the characteristics of a ballooning mode (Fig. 4). There is some overlap between the two types when $q_{axis} \approx 1$, especially at high β_{pol} . To isolate the influence of geometry we first examine the stability of several equilibria, characterized by having the same pressure and safety-factor profile, and varying the geometry. We then fix the geometry and vary the profiles to examine their influence.

A. Geometric effects

We choose equilibria where the pressure parameter α_p is chosen to be 1.5. The safety factor profile is such that $q_R/q_0 = 2.5$ and $\alpha_q = 1.5$. We then investigate three different equilibria with aspect ratio 10, 5, and 3. The results are shown in Fig. 5. We see a stabilizing effect due to decreasing aspect-ratio. This effect is more pronounced for the high- m ballooning type modes and consequently also affects the transition to the second stability region of the internal kink mode. We can measure the stabilization by comparing the maximum range of instability in $\epsilon\beta_{pol}$. The inset of Fig. 5 shows a linear dependence on the inverse aspect ratio.

To emphasize the dependence on other parameters, i.e., noncircularity, we find it useful to work at large aspect ratio and choose a profile which tends to increase the unstable region. This is accomplished by choosing $\alpha_p = 2.5$, $q_R/q_0 = 2.5$, and $\alpha_q = 1.5$. We then examine several equilibria, all with aspect ratio 10 and various noncircularities. Specifically, we compare the region of instability in a circle, an ellipse, a dee-shaped, and an indented bean-shaped plasma. This incremental method introducing higher harmonics of noncircularity should help to elucidate the relative dependence on the different harmonics. The different cross sections are shown in Fig. 6. The contours of marginal stability are shown in Fig. 7. As predicted (Refs. 13 and 14), we find that ellipticity is destabilizing and the region of instability increases for all values of q_{axis} . When triangularity is added, there is a dramatic reduction in the area of instability. In fact, the region of ballooning instability virtually disappears and the second stable region for internal kink modes becomes more easily accessible. When indentation is added, one notes a further dramatic reduction in the region of instability. When the indentation is increased to approximately twice the value shown here,

the unstable region vanishes entirely. This certainly augurs well for the development of internal kink stable designs.^{21,22}

B. Profile effects

Having observed the strong stabilization associated with small aspect ratio and higher shaped harmonics, to examine the influence of the current profiles on the $n = 1$ internal mode, we fixed the geometry to be circular with large aspect ratio.¹⁰ This gives us the ability to study a variety of profile effects without getting too close to marginal situations. We remind ourselves of the basic parameters that determine the current profile. These are a) α_p - the exponent in the pressure profile. Smaller values of α_p yield broader profiles while large values give peaked profiles, b) q_{ℓ}/q_0 - the ratio of q_{edge} to q_{axis} when q_{axis} equals unity. This is a measure of the global shear and varies slightly when q_{axis} is changed by scaling the toroidal field, and c) α_q - the exponent of the q -profile, varying which permits us to modify the local shear while keeping the global shear constant. We recognize that this choice of parameterization is not necessarily likely to yield current profiles characteristic of tokamak operation, but is chosen for its simplicity (only three parameters), and its ability to reasonably mock a variety of diffuse current profiles. We note that even with this simple choice the number of equilibria needed to cover all possible choices is quite large. When we multiply that by the requirement of scanning in $\alpha_{\text{pol}}-q_{\text{axis}}$ space, we quickly appreciate the need to restrict the number of independent parameters. Table I shows the parameter values that were used in this study. Two values were used for each parameter and were chosen to cover the conventional range of current profiles. With this choice of equilibria we can study the role of various parameters extensively. For example, we study the effect of global shear by

comparing equilibria with q_x/q_0 equal to 2.5 and 4. Furthermore we can do so with peaked or broad, pressure and safety factor profiles, thus eliminating the possibility of having stumbled on a singular situation, and obtaining a better perspective on the effects of global shear.

Pressure profile: Here we have a single parameter α_p to be varied. The results are shown in Figs. 8a - d. In general, we note that larger α_p , which makes the profile more peaked tends to increase the region of instability. The effect is more pronounced for q_{axis} greater than one. In Fig. 8a we show the typical features. Note that we have an extra equilibrium not described in Table 1, with $\alpha_p = 2$. This shows a continuous stabilization of the high- m ballooning mode as α_p is decreased and the pressure profile gets broader. At α_p of 2.5 there is a continuous band of instability at constant $\epsilon\beta_{pol}$ of 0.6 for $q_{axis} \gtrsim 0.8$. As α is decreased to 2, a region of stability appears for $1.1 \leq q_{axis} \leq 1.5$, and when α_p is further reduced to 1.5, the region of instability with $q_{axis} > 1$ becomes an island. Further reduction of α_p would eliminate this completely. Further, comparing Figs. 8a - b with 8c - d we note that when the global shear is larger, there is a substantial shift upward of the lower margin of stability for q_{axis} less than to unity. At low shear this effect is barely noticeable.

Safety factor profile: Here we have two parameters, α_q which determines $dq/d\psi$ and q_x/q_0 the total shear. We first examine the results which show the influence of q_x/q_0 (Figs. 9a - d). As before, we note that there are basically two regions of instability, which may or may not be connected. In general, increasing q_x/q_0 tends to increase the size of the regions of instability. This is partly due to the increased region of the plasma that is accessible to the $q = 1$ surface. As q_{axis} is reduced from unity, the rational surface moves further out until it leaves the plasma. With higher q_x/q_0 , the

q_{axis} when this occurs is lower, and the available parameter range in q_{axis} increases. However this does not account for the change in the vertical direction. Here we see that increasing the shear makes the second region of stability less accessible. However, the first threshold of instability generally moves upward. This is more pronounced for broader current profiles (Figs. 9a - b) than for peaked profiles (Figs. 9c - d).

On the other hand, varying α_q which varies the local shear keeping the global shear constant does not affect the instability quite as strongly (Figs. 10a - d). There is a modest change for the internal kink mode, $q_{axis} < 1$. The high- m ballooning mode, on the other hand, is destabilized when the shear is increased (Figs. 10a - b). Note that when α_q is small (1.5) the shear increases uniformly. When it is larger, then in the region closer to the axis the shear is reduced while the outside has it increased (Fig. 11). However, when the pressure profile is more peaked, the situation is reversed (Figs. 10c - d), and the low shear case is more unstable.

IV. DISCUSSION

The results presented in this paper are fairly extensive and combine the effects of two different modes. It is therefore useful to first identify the various regions in $\epsilon\beta_{pol}$ vs q_{axis} space. Figure 12 shows a canonical description of the instability space for $n = 1$ internal modes. We distinguish between the internal kink mode which is present when the $q = 1$ surface lies in the plasma and an internal ballooning type mode, occurring when $q_{axis} > 1$. The regions of instability corresponding to these modes may or may not be connected. We further distinguish each mode's instability space by its threshold for stability. Thus there is a low- β limit corresponding to the threshold for instability (A,C), and a subsequent high- β limit marking the

transition to the second stable region (B,D). The region marked A has received a lot of attention (Refs. 6-9,13-17) analytically and numerically. The main results have been elucidated by Bussac et al.,⁹ Kerner⁶ has numerically verified them, and other workers have agreed with this conclusion. We have duplicated Kerner's result with our code and generally agree that this region (A) is adequately described in Refs. 9, 13, and 14. The region of second stability, marked B, has been demonstrated earlier in Refs. 7 and 8 independently. We have examined several equilibria and observed that this stabilization is due to the shift of the magnetic axis, squeezing the surfaces closer to the stabilizing conducting wall. Figure 13 shows contours of the shift as seen at the $q = 1$ surfaces in the $\epsilon\beta_{bol} - q_{axis}$ plane for a typical case, showing how the shift sets the upper stability limit. Presumably, the noncircularity associated with the shift provides the stabilizing force. In the region $0.55 < q_{axis} < 0.7$ we see clearly that a shift of 0.009 stabilizes the mode. At larger values of q_{axis} , the rational surface is closer to the magnetic axis and the reduced region of unstable plasma helps to stabilize it even more rapidly. As q_{axis} gets even closer to unity, then the mode has more of a ballooning characteristic and other effects stabilize it as discussed below. This picture of a critical shift is observed in all the equilibrium studied.

As indicated earlier when $q_{axis} > 1$, the mode is basically a ballooning type mode. To clarify this, we point out that this mode has a broad radial support and has several high- n Fourier harmonics of comparable strength (Fig. 4). Though the term "ballooning" has been reserved for localized high- n modes, we extend it here to refer to internal modes which tend to localize on rational surfaces and have contributions from several poloidal harmonics regardless of the toroidal mode number. This mode is a pressure driven mode

and appears at some finite value of β . As the pressure is increased, a second stable region appears. This behavior has been observed earlier, in particular we recall the results of Ref. 16. This mode has also been analyzed by several authors (Refs. 18-20). We use the results of Greene and Chance (Ref. 20) who call attention to the role of local shear in driving this mode. They define this to be

$$S = - \frac{\mathbf{B} \times \nabla \psi}{|\nabla \psi|^2} \cdot \nabla \times \frac{\mathbf{B} \times \nabla \psi}{|\nabla \psi|^2} .$$

They assert that the ballooning modes are influenced by the pressure gradient and magnetic field line curvature, which are destabilizing and the local shear which is stabilizing. In effect, one concludes that if there are points of vanishing S in the region of bad curvature, then the pressure gradient combines with the bad curvature to drive the mode. Conversely, if S can be made finite everywhere in the plasma, then the mode may be stabilized. Recognizing that we are examining low- n modes rather than the high- n modes for which this result was derived, we cannot expect this to be strictly correct here. Surprisingly, when we plot the local shear as a function of the surface for $x > x_{\text{mag. axis}}$ at $z = 0$ (Fig. 14), we notice that as β increases the point of vanishing shear approaches the boundary of the plasma. Further we note that stabilization occurs even before the $S = 0$ point leaves the plasma. This is not surprising since the low n internal modes tend to be less unstable than the high n -modes and consequently should be more easily stabilized.

The results showing the effect of noncircularity (Fig. 7) confirm the analysis of Refs. 13 and 14, which shows that the elliptic term is destabilizing and higher harmonics, e.g., triangularity and indentation are strongly stabilizing. In fact our results demonstrate this very clearly. We

can also study the axis-shift and local shear to understand the effects of noncircularity. Basically, one observes the same effects reproduced, as noncircularity causes the $S = 0$ point to leave the plasma and stabilize the ballooning mode. The effective magnetic shift is much higher for an indented plasma compared to a circular one, at the same β_{pol} . Consequently, one sees a rapid stabilization of the internal kink mode. This feature has been studied in detail^{21,22} and forms the basis for a new experiment.²³

The effects of shaping the current profile are more subtle in that the relationship between the profile parameters (here q_p , q_l , and q_l/q_0), and the axis shift, and local shear is difficult to establish. Nevertheless we can make some general statements. When the pressure profile is more peaked, we increase the region of instability. However, this effect is more significant for the $q_{axis} > 1$ ballooning type mode rather than the internal kink mode. When modifying the safety factor profile, the ratio q_l/q_0 seems to be more important. Increasing it increases the region of instability and also shifts the onset of the internal kink mode to higher values of β_{pol} . Varying the functional form has a smaller effect on the kink mode. It does however affect the ballooning mode and depending on the pressure profile can increase or reduce the region of instability.

This study has concentrated on the $n = 1$ internal mode and has effectively combined analysis of the internal kink mode and a low- n ballooning type mode. It confirms the analysis at low β of the internal kink mode and establishes the stabilization at high β . Some of these results have been reported elsewhere but not to this degree of completeness. We feel that the parameter survey presented here should prove valuable to the experimental community as well as provide grist for the analytic mill. The shear stabilization described here in detail for a variety of equilibria confirms the validity of Ref. 20 in application to realistic two-dimensional

equilibria. In conclusion we note that we have restricted this study to $n = 1$ internal modes. It is conceivable that higher values, e.g., $n = 2$, may complicate the picture and should be studied.

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TABLE I

	$q_{\lambda}/q_0 = 2.5$				$q_{\lambda}/q_0 = 4.0$			
α_p	1.5	1.5	2.5	2.5	1.5	1.5	2.5	2.5
α_q	1.5	2.5	1.5	2.25	1.5	2.5	1.5	2.5

FIGURE CAPTIONS

FIG. 1. (a) Radial mesh point distribution, uniform, and tailored to increase the density near the rational surface and the magnetic axis.

(b) Convergence of the lowest eigenvalue for three different values of q_{axis} as a function of $1/M^2$, where M is the number of radial finite elements. The eigenvalues (filled circles) are obtained using the uniform mesh. One example is shown of the effect of the tailored mesh (open circle) when the number of elements is fixed at 100.

FIG. 2. Growth-rate vs. q_{axis} to show the comparison between the converged results, dashed lines, and the results using a tailored mesh with 100 elements, solid curve.

FIG. 3. (a) Radial profile of the dominant Fourier harmonics of ξ_ψ for an internal kink mode, for an equilibrium with aspect ratio = 10, $q_1/q_0 = 4$, $\alpha_p = 1.5$, $\alpha_q = 2.5$, $\epsilon\beta_{pol} = 0.213$, and $q_{axis} = 0.9$.

(b) Contours of δW for this mode in the x-z plane. The broken contours show regions of negative δW . The rational surfaces are superposed.

FIG. 4. (a) Radial profile of the dominant Fourier harmonics of a ballooning mode for the same equilibrium sequence as in Fig. 3 with $\epsilon\beta_{pol} = 1.02$ and $q_{axis} = 1.8$.

(b) Contours of δW for the ballooning mode. As before the broken contours show regions of negative δW and the rational surfaces are superimposed.

FIG. 5. Contours of marginal stability for different aspect ratios with the p and q profiles fixed such that $q_{\ell}/q_0 = 2.5$, $\alpha_p = 1.5$, and $\alpha_q = 1.5$. The region of instability lies within the contour. The inset shows an approximate linear dependence of the unstable region on the aspect ratio.

FIG. 6. The different cross-sectional shapes showing the outermost flux-surface. All have aspect ratio of 10.

FIG. 7. Marginal stability contours showing the effect of noncircularity. p and q are given by $\alpha_p = 2.5$, $q_{\ell}/q_0 = 2.5$, and $\alpha_q = 1.5$.

FIG. 8. Marginal stability contours showing dependence on α_p , the pressure parameter at various values of q_{ℓ}/q_0 and α_q : a) $q_{\ell}/q_0 = 2.5$, $\alpha_q = 1.5$; b) $q_{\ell}/q_0 = 2.5$, $\alpha_q = 2.5$; c) $q_{\ell}/q_0 = 4$, $\alpha_q = 1.5$; and d) $q_{\ell}/q_0 = 4$, $\alpha_q = 2.5$.

FIG. 9. Marginal stability contours showing dependence on the global shear q_{ℓ}/q_0 at various values of α_p and α_q : a) $\alpha_p = 1.5$, $\alpha_q = 1.5$; b) $\alpha_p = 1.5$, $\alpha_q = 2.5$; c) $\alpha_p = 2.5$, $\alpha_q = 1.5$; and d) $\alpha_p = 2.5$, $\alpha_q = 2.5$.

FIG. 10. Marginal stability contours showing dependence on local shear α_q at various values of α_p and q_{ℓ}/q_0 : a) $\alpha_p = 1.5$, $q_{\ell}/q_0 = 2.5$; b) $\alpha_p = 1.5$, $q_{\ell}/q_0 = 4$; c) $\alpha_p = 2.5$, $q_{\ell}/q_0 = 2.5$; and d) $\alpha_p = 2.5$, $q_{\ell}/q_0 = 4$.

FIG. 11. Plot of q' vs. ψ with q_l/q_0 fixed.

FIG. 12. Schematic diagram of $n = 1$ internal mode marginal stability identifying the different regions where different physical effects dominate the mode.

FIG. 13. Contours of the axis shift for the $q = 1$ surface when $\alpha_p = 1.5$, $q_l/q_0 = 2.5$ and $\alpha_q = 1.5$. The broken curve shows the margin of stability, above it the internal kink mode is stable.

FIG. 14. Curves of the local shear parameter S for $z = 0$ and $x > x_{axis}$ vs. ψ . Each curve corresponds to a different value of β_{pol} . The broken curves represent the stable cases. Aspect ratio = 10, $q_l/q_0 = 2.5$, $\alpha_q = 1.5$, and $\alpha_p = 2.5$.

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Fig. 1

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Fig. 2

Fig. 3

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Fig. 4

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Fig. 5

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Fig. 6

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Fig. 7

Fig. 8

Fig. 9

Fig. 10

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Fig. 11

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Fig. 12

Fig. 13

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Fig. 14

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