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TMRBAR - A Code to Calculate Plasma Parameters
for Tandem-Mirror Reactors Operating in the MARS Mode

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August 30, 1983

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Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under Contract W-7405-Eng-48.

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TMRBAR - A CODE TO CALCULATE PLASMA PARAMETERS FOR
TANDEM MIRROR REACTORS OPERATING IN THE MARS MODE

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1. Introduction

The purpose of this report is to document the plasma power-balance model currently used by LLNL to calculate steady-state operating points for tandem-mirror reactors. The code developed from this model, TMRBAR, has been used to predict the performance and define supplementary heating requirements for drivers used in the Mirror Advanced Reactor Study (MARS)¹ and for the Fusion Power Demonstration (FPD) study.² The equations solved included particle and energy balances for central cell and end cell species, quasineutrality at several cardinal points in the end cell region, as well as calculations of volumes, densities and average energies based on given constraints of beta profiles and fusion power output. Alpha particle ash is treated self-consistently, but no other impurity species is treated. Profiles of vacuum magnetic field strength and electrostatic potential for the MARS baseline fusion driver are shown on Figs. 1 and 1b. Figure 1c shows an isometric view of the end plug of MARS. The large field peak corresponds to the midplane of the choke coil; the two sets of field peaks correspond to the mirrors of the anchor and plug yin yangs. The magnetic field profile is generated by the EFFI³ code, and the potential profile is generated by interpolating between the potential at the cardinal points generated from the plasma power balance code to be described here.

The plasma parameters for the baseline MARS driver are shown in Table 1. The remainder of the report will describe the equations used to calculate these parameters. The basic information flow of the model is shown on Fig. 2. Input parameters include magnetic field strength at cardinal points, fusion power outputs, lengths of central cell, transition region, anchor and plug, beta values at several points in the machine, alpha particle concentration in central cell, and trapped to passing density ratios. Initial guesses for temperatures and potentials are also necessary. Outputs include temperatures, potentials, plasma radii, and end cell sustaining powers.

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Figure 1(a). $B(z)$ and Lengths Definitions (not to scale).

Figure 1(b). $\phi(z)$ Definitions (not to scale).

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FIGURE KEY

- A. CENTRAL CELL BLANKET MODULE
- B. CENTRAL CELL SOLENOIDAL COIL
- C. END CELL CHOKE COIL
- D. TRANSITION COIL
- E. ANCHOR YIN-YANG COIL SET
- F. 475 keV PLUG SLOSHING ION BEAM
- G. PLUG YIN-YANG COIL SET

Figure 1c. Isometric view of MARS end plug.

Table 1. Plasma Parameters for Baseline MARS Reactor

Parameter (Units)	Value
<u>Central Cell</u>	
Density, n_c (cm^{-3})	3.3×10^{14}
Density profile exponent, γ	4 (quartic)
Ion temperature, T_c (keV)	28
Electron temperature, T_{ec} (keV)	24
Plasma radius, r_c (cm)	49
Vacuum magnetic field, $B_{c,vac}$ (T)	4.7
Beta, $\langle \beta_c \rangle$ (volume averaged)	0.28
Floating potential, ϕ_e (keV)	171
Cold fueling current, I_c (kAmps)	2.1
Fuel ion confinement parameter, $(n\tau)_i$ (sec/cm^3)	5.13×10^{14}
Pastukhov fuel ion confinement $(n\tau)_{i,\parallel}$ (sec/cm^3)	1×10^{16}
Electron confinement parameter, $(n\tau)_e$ (sec/cm^3)	5.3×10^{14}
First wall radius, r_{FW} (cm)	60
Central cell length, L_c , (effective) (m)	131
Alpha concentration, $C_{\alpha,c}$	0.06
Fusion power P_{fus} (MW)	2615
Choke coil peak field, B_{max} (T)	24
<u>Transition Region</u>	
Length (trans + anchor), L_t (m)	15.5
Beta $\langle \beta_t \rangle$	0.04
Vacuum magnetic field at midplane, $B_{t,vac}$ (T)	4
Passing DT ion density at midplane, $n_{pass,t}$ (cm^{-3})	1.7×10^{13}

Table 1. (Continued)

Parameter (Units)	Value
<u>Transition Region</u>	
ϕ_t (keV)	47
Alpha concentration, $C_{\alpha,t}$	0.055
g_{DT}	2.75
g_{α}	3.13
D-T trapping current, $I_{trap,DT}$ (kAmp)	1.7
Vacuum magnetic field at peak, B (T)	7
Alpha concentration at peak, C_{α}	0.057
Passing DT ion density at peak, n_{pass} , (cm^{-3})	3.5×10^{13}
ϕ (keV)	26
Neoclassical D-T ion flux, $I_{neo,DT}$ (amps)	78
Neoclassical alpha flux, $I_{neo,\alpha}$ (amps)	7.4
<u>Plug Region</u>	
Sloshing ion injection energy, $E_{inj,a}$ (keV)	475
Peak to average sloshing density ratio	2.7
Vacuum magnetic field at barrier midplane, $B_{b,vac}$ (T)	2.97
Midplane plug (barrier, pt. b) beta, $\langle\beta_b\rangle$	0.5
Passing D-T ion density at barrier midplane, $n_{pass,b}$ (cm^{-3})	4.8×10^{12}
Alpha concentration at point "b", $C_{\alpha,b}$	0.03
Plug length, L_b (m)	6.56
Inner plug mirror field, B_{mp} (T)	7.2
Outer plug mirror field, B_{ma} (T)	7.5

Table 1. (Continued)

Parameter (Units)	Value
<u>Plug Region (continued)</u>	
Hot electron energy, E_{eh} (keV)	820
Warm electron temperature, T_{ew} (keV)	123
Cold electron fraction at "b", F_{ec}	0.03
Barrier potential dip, ϕ_b (keV)	121
Potential drop at pt. a', $\phi_{a'}$ (keV)	34
Potential drop at inner mirror, ϕ_{mp} (keV)	25
Vacuum field at point a', $B_{a',vac}$ (T)	5.7
Beta at pt. a, $\langle\beta_a\rangle$	0.39
Beta at point a', $\langle\beta_{a'}\rangle$	0.32
Effective mirror ratio of sloshing ions $R_{eff,s}$	1.4
G	4.75
9DT	2.75
g_α	3.13
Ion confining potential, ϕ_c (keV)	154
<u>Anchor Region</u>	
Beta $\langle\beta_{anc}\rangle$	0.5
Anchor length, L_{anc} (m)	2.6
Hot ion average energy, E_{ih} (keV)	500
Hot ion density, n_{ih} (cm ⁻³)	7.1×10^{13}
Anchor midplane magnetic field, $B_{anc,vac}$ (T)	3.9
Hot ion confinement parameter, $(n\tau)_{anc}$ (sec/cm ³)	6.4×10^{13}

Table 1. (Continued)

Parameter (Units)	Value
<u>Power Balance</u>	
Absorbed sloshing ion neutral beam power, P_{slosh} (MW)	5.7
Absorbed ECRH power at potential peak (pt. a), $P_{\text{ech,a}}$ (MW)	6.6
Absorbed ECRH power at barrier midplane (pt. b), $P_{\text{ech,b}}$ (MW)	77
Synchrotron radiation power, P_{sync} (MW)	38
Neutron wall loading, Γ (MW/m ²)	4.2
Power entering halo, P_{halo} (MW)	77
Absorbed Anchor ICRH power, P_{anc} (MW)	11.4
Q ($\equiv$ fusion power/absorbed injected power)	26

Figure 2. Information flow and calculational procedure of power balance model.

The remainder of this document will be organized as follows. Section II discusses the calculation of the density of electrons and ions (both central cell and end cell) which reach a given axial location in the end cell. This is an important element in the present physics analysis, because the axial potential profile as well as end cell energies, sustaining powers, and plasma stability are sensitive to these densities. Section III deals with the equations used in the model, and explains and justifies the assumptions made in deriving them. This major section of the report, considers the equations of quasineutrality, global particle and energy balance, particle and energy balance of end cell hot ions and electrons, and alpha particle removal and barrier ion drift pumping self-consistency. Section IV outlines the additional physics constraints which are adhered to which are not explicitly mentioned in Section III. Section V reviews the past history of the code. Section VI lists suggestions for further improvement of the models and the TMRBAR code.

II. Axial Dependence of Ion and Electron Densities

A set of equations are necessary to relate the densities of central cell D-T ions, alpha particles, and electrons to those in the end cell. These density mappings are an important element in the physics analysis of tandem mirrors, since they impact the axial potential profiles, end cell particle energies, sustaining powers and plasma stability. The particle species which reside strictly in the end cell (hot and warm electrons, hot ions) also have axial dependences which will be discussed.

II.1 Cold Electron Density

For the mapping of the central cell electrons into the end cell, we use a Maxwell-Boltzmann distribution of the form:

$$n_{ec}(z) = n_{ec}(0) \exp [-\phi(z)/T_{ec}] \quad (1)$$

Here, $n_{ec}(0)$ is the central cell electron density, $\phi(z)$ is the potential (positive) measured relative to the central cell floating potential ϕ_e , and T_{ec} is the central cell electron temperature. The subscript "c" distinguishes these "cold" electrons from the hot ("h") and warm ("w") electrons, all which co-exist in the plug region. Using the Boltzmann relation here implies that the cold electrons are quite collisional, and near thermodynamic equilibrium. Equation (1) is applicable to points to the left

of the barrier minimum potential, point "b" on Fig. 1. Between points a' and b, the cold electron behavior becomes something other than Maxwell-Boltzmann, because of the heating of cold electrons by the hot electrons and directly by ECRH, and because the thermal barrier reflects cold electrons. Modeling the latter effect, and leaving the heating effects for further model improvements, we use for the cold electron density mapping at the bottom of the barrier⁴:

$$n_{ec}(b) = n_{ec}(0) \frac{[\exp(-\phi_b/T_{ec}) - \exp(-\phi_e/T_{ec})]}{[1 - \exp(-\phi_e/T_{ec})]} \quad (2)$$

Note that in the limit where ϕ_b approaches ϕ_e , all electrons are cut off, as one would expect physically.

II.2 Ion Density Mappings

For the DT fuel ions and alpha particles, a different method for computing the density mapping is needed. There are two species of cooler ions in the end cell, those which communicate freely with the central cell, the so-called passing ions; and those which are reflected at a field and potential in the end cell, the trapped ions. Much of the physics of end cell potential formation concerns itself with identifying and estimating the source of these trapped ions, and the development of energy efficient means of keeping their density at an acceptably low level. In the model, the trapped ion density distributions are not explicitly calculated. Instead, they are related to the passing density by the input parameter g_j for ion species "j":

$$g_j = \frac{n_{pass,k} + n_{trap,k}}{n_{pass,k}} \quad (3)$$

where $n_{pass,k}$ is the passing ion density at cardinal point k, and $n_{trap,k}$ is the corresponding trapped density. Usually the species of passing or trapped particles is understood from the context. Otherwise an additional subscript is added. In general, g is a function of axial position. However, the Fokker-Planck cases (which actually compute the trapped ion distribution function) so far examined for a pumped transition, anchor, and plug region seem to indicate that g is fairly independent of z , except perhaps near the high field mirror point, where it must approach unity. Since none of the

cardinal points of interest lie near this mirror, we shall take g to be the same at all cardinal points. The g values for alpha particles and D-T ions are adjusted to obtain consistent rates of trapping and removal of these trapped species, as discussed later in Sec. III.2.2.1.

The problem which remains is to calculate n_{pass} for a given species of ions. The basic idea is to take a Maxwellian distribution in the central cell and determine which regions in velocity space will have particles that can reach a given location in the end cell. A velocity space integration over those regions will yield the passing density at that point. A calculation of this kind assumes that the time it takes for passing ions to collide is long compared to the time it takes to travel axially from the central cell to the end cell and back. The criterion which is used to determine whether a particle reaches a given z location is that it have a positive value of $v_{\parallel}^2$, where $v_{\parallel}$ is the component of particle velocity along the field line. We know that particles conserve total energy ϵ :

$$\epsilon = e\phi + \frac{1}{2}mv^2 \quad (4a)$$

and magnetic moment, μ :

$$\mu = \frac{\frac{1}{2}mv_{\perp}^2}{B} \quad (4b)$$

where ϕ is the electrostatic potential measured relative to the potential at the outer plug mirror, B is magnetic field, m is particle mass, v is the particle speed, and $v_{\perp}$ is the component of particle velocity perpendicular to B . The positivity of $v_{\parallel}^2$ translates into an inequality in (ϵ, μ) , space:

$$\epsilon > \epsilon(v_{\parallel} = 0) = e\phi(z) + \mu B(z) \quad (5)$$

where ϕ and B are their local values. Particles located at a given B and ϕ cannot exist below this line.

The distribution of passing ions in (ϵ, μ) space will not always occupy the entire region above this line, since part of the distribution may have already been cut off by another line of the form of Eq. (5) with either larger ϕ or B values which occurred at a z value closer to the central cell. Furthermore, particles that have non-zero parallel velocity at the confining potential peak (pt "a") will be lost from the machine, hence no particles will exist above the line defined at this point. Figure 3 shows these aspects on an (ϵ, μ) plot. In the central cell, the region between the Eq. (5) line which defines the central cell and the loss region will be filled with particles (Fig. 3a). As the $B_{\max}$ field is approached, the potential will remain (approximately) constant, but the distribution will be cut off until it reaches $B_{\max}$, as shown in Fig. 3b. Within most of the end cell region, B and ϕ are below $B_{\max}$ and ϕ_e , and correspondingly no more of the distribution is cut off (Fig. 3c). However, the passing density still varies as a function of B and ϕ because the velocity space volume element is a function of the local B and ϕ . As the potential rises to its maximum value at point "a", specifically when $\phi = \phi_e$, the distribution is cut off further (Fig. 3d) until the confined region for passing ions vanishes ($n = 0$) at the potential peak. The region between the $(B_{\max}, \phi_e)$ line and the (B, ϕ) line will typically fill up with so-called "trapped" particles. The degree to which this occurs will depend upon the strength of the pumping which is used to remove them. Reference 4 discusses these aspects in further detail.

For completeness, we give the formula for the passing density of a given ion species here. For multiply charged species, (i.e., alphas, impurities) one should replace all potentials with their charge times the potential. In the central cell ($B = B_c$), the passing density is just the central cell density, by definition. From this point where $B = B_c$ to the choke coil field peak, $B = B_{\max}$, the passing density is given by:⁵

(a) Central cell

(b) Choke coil mirror

(c) Region between choke coil and point where $\phi = \phi_e$ in end cell.

(d) Region between $\phi = \phi_e$ and potential peak (pt. a)

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Figure 3. Velocity space at several z locations.

$$\begin{aligned}
n_{\text{pass}} &= n_c \left[S \left(0, 1, 1, -\phi/T_c, \frac{\phi}{T_c}, \frac{\phi_A}{T_c} \right) + \left(1 - \frac{B}{B_A} \right) \right. \\
&\quad \left. (x) S \left(0, 1, \left(1 - B/B_A \right), \left(\frac{\phi_A}{T_c} \frac{B}{B_A} - \frac{\phi}{T_c} \right), \phi_A/T_c, \frac{\epsilon}{T_c} \right) \right] \\
&\quad (x) \left[S \left(0, 1, 1, -\phi_e/T_c, \phi_e/T_c, \phi_A/T_c \right) + \left(1 - B_c/B_A \right) \right. \\
&\quad \left. (x) S \left(0, 1, \left(1 - \frac{B_c}{B_A} \right), \left(\frac{\phi_A}{T_c} \frac{B_c}{B_A} - \frac{\phi_e}{T_c} \right), \frac{\phi_A}{T_c}, \frac{\epsilon_c}{T_c} \right) \right]^{-1}.
\end{aligned} \tag{6}$$

From the choke coil field peak to the end of the machine, n_{pass} is given by:

$$\begin{aligned}
n_{\text{pass}} &= n_c \left[S \left(0, 1, 1, -\phi/T_c, \phi_{\text{max}}/T_c, \phi_A/T_c \right) \right. \\
&\quad \left. - S \left(0, 1, \left(1 - B/B_{\text{max}} \right), \frac{\phi_{\text{max}}}{T_c} \frac{B}{B_{\text{max}}} - \frac{\phi}{T_c}, \frac{\phi_{\text{max}}}{T_c}, \frac{\epsilon^*}{T_c} \right) \right. \\
&\quad \left. + \frac{B}{B_A} S \left(0, 1, \left(\frac{B_{\text{max}} - B}{B_A} \right), -\frac{\phi_A}{T_c} \left(\frac{B_{\text{max}} - B}{B_A} \right) + \frac{\phi_{\text{max}}}{T_c} - \frac{\phi}{T_c}, \frac{\phi_A}{T_c}, \frac{\epsilon^*}{T_c} \right) \right] \\
&\quad (x) \left[S \left(0, 1, 1, -\phi_e/T_c, \phi_e/T_c, \phi_A/T_c \right) + \left(1 - \frac{B_c}{B_A} \right) \right. \\
&\quad \left. (x) S \left(0, 1, \left(1 - \frac{B_c}{B_A} \right), \left(\frac{\phi_A}{T_c} \frac{B_c}{B_A} - \frac{\phi_e}{T_c} \right), \frac{\phi_A}{T_c}, \frac{\epsilon_c}{T_c} \right) \right]^{-1}.
\end{aligned} \tag{7}$$

Where $\epsilon^* = \phi_e + B_{\text{max}} \mu^*$, $\mu^* = \phi_c / (B_{\text{max}} - B_a)$, and $\epsilon = \epsilon^*$ if $B > B_a$,
 $= \infty$ if $B < B_a$.

In these formulae, ϕ is the local potential measured relative to that at the outer plug mirror, T_c and n_c are the central cell temperature and density. The field B_{max} is the choke coil field, B_a is the field at point a, and ϕ_{max} is the potential at the choke coil. The function S is given by

$$S(A,B,C,D,x_1,x_2) \equiv R_e \int_{x_1}^{x_2} dx e^{-x} (Ax+B) \sqrt{Cx+D} \quad (8)$$

and can be evaluated in general in terms of the incomplete gamma function.

II.3 Hot Species in End Plugs

There are hot electron and ion species in the plug which must be mapped from one cardinal point to another in that region. For the hot electrons, a mirror confined distribution is assumed, with a term which models the expelling effect of the barrier potential dip. We need the mapping from point "b" to elsewhere in the plug. This is given by

$$n_{eh}(z) = n_{eh}(b) \left(\frac{B_{ma,vac} - B_{pla}(z)}{B_{ma,vac} - B_{b,pla}} \right)^v \left[1 + \frac{\phi_b - \phi(z)}{0.2 E_{eh}} \right]^{-1/2} \quad (9)$$

where $\phi(z)$ is measured down from the ϕ_e potential, $B_{pla}(z)$ is the beta reduced field at the mapped point, $B_{ma,vac}$ is the field at the outer plug mirror, and E_{eh} is the hot electron energy. The factor 0.2 is the ratio of parallel to total energy in the hot electrons at point b. The power v in this equation is currently set equal to unity, and additional Fokker-Planck calculations of ECRH heated electrons will be necessary to evaluate v . Results from TMX-U suggest $v > 1$.

For the sloshing ions, a density mapping which has free parameters is used, with the values of free parameters based on results from Fokker-Planck calculations of sloshing ions confined in the plug. To map from point "b" to point "a'":

$$n_{slosh}(a') = n_{slosh}(b) \left(\frac{B_{pla}(a')}{B_{b,pla}} \right)^\alpha \left[1 + \beta \frac{(\phi_b - \phi(a'))}{E_{inj,a'}} \right] \quad (10)$$

where $E_{inj,a'}$ is the sloshing ion energy at its injection point. From Fokker-Planck calculations $\alpha = 1.24$ and $\beta = 1$. The sloshing density (all D) at point b is related to the passing D-T density by

$(G-g) n_{\text{pass},b}$, where g is given by Eq. (3) and G is defined as sloshing plus passing plus trapped divided by passing density. The sloshing density at point "a" is also required, and is obtained by replacing a' by a in Eq. (10), where

$$E_{\text{inj},a} = E_{\text{inj},a'} - (\phi_c + \phi_{a'}) \quad (11)$$

III. Plasma Equations

This section presents the equations of quasineutrality, particle and energy balances, and auxiliary calculations required to determine, for example, plasma cross sectional areas, average energies not computed through energy balance, etc, given input β values and the desired fusion power output required.

III.1 Quasineutrality

The general statement of quasineutrality at any axial location is that:

$$n_e \approx \sum_j^{\text{ions}} z_j n_j \quad (12)$$

where n_j is the density of ion species 'j', and z_j is its charge state. This relation is called "quasineutrality" because the equality is only approximate. If n_e exactly equalled the sum of $z_j n_j$, then there would be no potential produced in the plasma (with zero potential at the boundaries). Strictly speaking one has to solve Poisson's equation instead of Eq. (12). However, when potentials have scale lengths which are long compared to a Debye length, in which case the $\nabla^2 \phi$ term in Poisson's equation is small, Eq. (12) is valid. Another way of looking at this is to say that it takes only very small departures from exact charge neutrality to produce the potentials that exist in typical tandem mirror reactors (~100 kV), therefore the correction introduced by solving Poisson's equation is quite small.

Quasineutrality is solved for at each of the cardinal points in the end cell. These cardinal points are labeled in Fig. 1. The basic form of the quasineutrality relation will differ depending on the species of particles which exist at the evaluated point. For points which have only passing and

trapped thermal ions, and passing cold electrons, specifically points t, mp, and HO:

$$n_e = g_{DT} n_{pass,DT} + 2 g_\alpha n_{pass,\alpha} \quad (13)$$

where the n_{pass} is given by Eq. (7) for both alphas and D-T ions, the g values are defined by Eq. (3) and the cold electrons are modeled by the Maxwell-Boltzmann law Eq. (1). Given the magnetic field profile, the model is used to compute the potential, relative to the central cell potential.

For the cardinal points which include sloshing ions and hot electrons along with the thermal ions and cold electrons, the relations become a bit more complicated. The bottom of the thermal barrier, point "b" is such a point. In addition, there is another variable which enters the problem, F_{ec} , which is the ratio of cold electron density to total electron density (hot+cold) there. The lower F_{ec} , the better the performance. The minimum F_{ec} is set by the most restrictive of (1) stability of ion two stream modes, (2) hot electron microstability, and (3) fueling requirements of hot electrons. We believe that an F_{ec} of 0.03 satisfies all these constraints for MARS. Writing quasineutrality in terms of the cold electron density and F_{ec} , we get:

$$\begin{aligned} \frac{n_{ec,b}}{F_{ec}} = & (G_{DT} - g_{DT}) n_{pass,DT,b} + g_{DT} n_{pass,DT,b} \\ & + 2 g_\alpha n_{pass,\alpha,b} \end{aligned} \quad (14)$$

where the passing densities are again given by Eq. (7), but now the cold density is given by the cutoff distribution Eq. (2), to model the reflection of the majority of cold electrons by the thermal barrier.

The quasineutrality relation at point a', the sloshing ion injection point, also includes hot electrons and sloshing ions. This point differs from the others in that we are not given the magnetic field value a-priori. This is because the field at the potential peak (pt a), B_a , is an input variable,

and the fields at "a" and "a'" are not independent. Conserving magnetic moment and energy of sloshing ions between a' and a we get:

$$B_{a',pla} = \frac{B_{a,pla}}{\left[1 - \frac{(\phi_{a'} + \phi_c)}{E_{inj,a'}} \right]} \quad (15)$$

This relation makes quasineutrality at point a' a more complicated function of the local potential than would be inferred by just the mapping equations. The quasineutrality relation becomes of the form:

$$n_{ec,a'} + n_{eH,a'} = g_{DT} n_{pass,DT,a'} + 2 g_{\alpha} n_{pass,\alpha,a'} + n_{slosh,a'} \quad (16)$$

where the densities are calculated using the previously presented mapping equations.

Finally, quasineutrality at point "a" is used to compute the warm electron density there, instead of the potential. The potential is instead calculated based on requirements on axial end loss to be discussed later. At point "a":

$$n_{ew,a} = n_{slosh,a} - n_{eH,a} \quad (17)$$

using the appropriate mappings. There is no passing ion density at "a" since these ions are reflected by the potential which occurs there. Here we ignore any cold electron density at point "a"; a collisionless mapping formula (we can't use Eqs. (1) or (2) because the electrons are not Maxwell-Boltzmann) predicts it to be considerably lower than the warm density at "a". Neglecting the cold electron density at "a" also implies that there are no cold electron sources due to ionization of neutral particles. This is a good approximation for the MARS reactor (at least in the steady state) since the sloshing beams are injected at point "a'", the deeply trapped barrier ions are pumped with a drift pump (not charge exchange beams), and the halo parameters provide good shielding from cold gas.

III.2 Particle and Energy Balance Equations

The model includes steady state particle and energy balances for the central cell electrons, DT ions, alpha particles, warm electrons, and sloshing ions. An energy balance relation for the hot electrons is included, but the particle balance for this species is replaced by a specification of the cold electron fraction at the barrier midplane. A justification for this replacement will be made in this section.

III.2.1 Global Charge Balance

Subtracting the electron particle balance equation from the ion particle balance equation in the central cell yields an equation which represents a global quasineutrality and is solved for ϕ_e :

$$I_{\text{burn}} + I_{\text{tran}} + I_{\text{slosh,e}} + I_{\text{neo,DT}} = 0 \quad (18)$$

where I_{burn} is the loss of DT ions due to plasma burnup, and is given by:

$$I_{\text{burn}} = \frac{1}{2} \frac{q}{10^3} \overline{n_c^2} \langle \sigma v \rangle_{\text{DT}} V_{\text{cc}} \quad (19)$$

where q is the elementary charge, n_c is the central cell DT density, $\langle \sigma v \rangle_{\text{DT}}$ is the fusion reaction rate parameter, and V_{cc} is the central cell volume.* The bar above the square of the density represents the radial average. For a profile which varies as $n \sim n_0 (1 - (r/a)^{\gamma})$, this average is

$$\frac{\overline{n^2}}{n_0^2} = \frac{\gamma^2}{(\gamma+1)(\gamma+2)} \quad (19a)$$

We take $\gamma = 4$ for most of the reactor cases we consider. The reaction rate parameter is a fit to standard $\langle \sigma v \rangle_{\text{DT}}$ tables, and is given by

*Currents are expressed in kiloamperes of particles, densities in cm^{-3} and lengths in cm.

$$\langle \sigma v \rangle_{DT} = 3.05 \times 10^{-13} \exp(-17.7/T_c^{0.348}) \quad (20)$$

$$/ (1 + 0.1554 Y - 0.1418 Y^2 - 0.05 Y^3 + 0.0364 Y^4)$$

where $Y \equiv T_c^{1/3}$. The term $I_{slosh,e}$ describes a net source of 'cold' electrons. The sloshing ion neutral beam injected at 'a' will produce cold electrons which enter the central cell, while the sloshing ions which are produced leak directly out the ends. This term is numerically equal to the sloshing ion absorbed current, to be discussed later in this section. The term I_{tran} is the difference between the central cell DT ion particle loss and electron particle loss and is given by:

$$I_{tran} = \frac{q}{10^3} n_c^{-2} \left[\frac{1}{(n\tau)_{c,i}} - \frac{(1 + 2 C\alpha)^2}{(n\tau)_{c,e}} \right] V_{cc} \quad (21)$$

where $C\alpha$ is the central cell alpha concentration, $(n\tau)_{ce}$ and $(n\tau)_{c,i}$ are the net (radial plus axial) confinement $n\tau$ values for electrons and DT ions respectively. The $(n\tau)$ for electrons is purely axial, and given by the Pastukhov formula:

$$(n\tau)_{c,e} = \frac{3.6 \times 10^9}{1n\Lambda_{ee}} g(R) T_{ec}^{3/2} \frac{\phi_e}{T_{ec}} e^{-\phi_e/T_{ec}} \quad (22a)$$

$$R = \frac{\beta_{a,vac} \sqrt{1 - \beta_a}}{\beta_{c,vac} \sqrt{1 - \beta_c}} \quad (22b)$$

$$g(x) = \sqrt{1 + \frac{1}{x}} \ln \left\{ \frac{\sqrt{1 + \frac{1}{x}} + 1}{\sqrt{1 + \frac{1}{x}} - 1} \right\} \quad (22c)$$

$$\Lambda_{ee} = 4.66 \times 10^{13} T_{ec} \sqrt{n_c} \quad (22d)$$

In these equations, β_a and β_c are the centerline beta values at point "a" and central cell ("c") respectively. For ions, transport is a combination of axial and radial; the total $(n\tau)$ being given by:

$$(n\tau)_{c,DT}^{-1} = (n\tau)_{c,ax,DT}^{-1} + (n\tau)_{c,rad,DT}^{-1} \quad (23)$$

The axial component is also of the Pastukhov form, but this time for ions, and given by:

$$(n\tau)_{c,ax,DT} = \frac{3.1 \times 10^{11}}{\ln \Lambda_{ii}} \frac{A_{DT}}{(1+4 \frac{C_a}{C_c})} \frac{IR}{I(\phi_c/T_c)} \frac{T_c^{3/2}}{T_c} \frac{\phi_c}{T_c} \exp(\phi_c/T_c) \quad (24a)$$

where

$$I(x) = 1 + \sqrt{\frac{\pi}{4}} \frac{1}{x} e^x \operatorname{erfc}(\sqrt{x}) \quad (24b)$$

$$\text{and } \Lambda_{ii} = 2 \times 10^{15} \sqrt{\frac{A_{DT} T_{ec} T_e}{n_c}} \quad (24c)$$

The parameter IR accounts for the increase in axial confinement due to the presence of radial transport, and is taken to be about 1.5, as suggested by recent Fokker-Planck calculations. The radial confinement $n\tau$ product for the DT ions is computed knowing the required removal rate of alpha particle ash. From the steady state alpha particle balance:

$$f_{Th} \frac{1}{4} \frac{\overline{n_\alpha^2}}{n_c} \langle \sigma v \rangle_{DT} - \frac{n_\alpha}{\tau_{\alpha,rad}} - \frac{n_\alpha}{\tau_{\alpha,neo}} = 0 \quad (25)$$

Here, n_α is the alpha density $\tau_{\alpha,rad}$ is the drift pump induced radial confinement time, and $\tau_{\alpha,neo}$ is the intrinsic neoclassical loss of alphas. This confinement time is defined in Eq. (26). The factor f_{Th} is the fraction of alpha particles which thermalize before being lost. This fraction is currently taken to be 0.7, a value suggested by results of Fokker-Planck codes run some time ago. We can obtain $(n\tau)_{c,rad,DT}$ by noting that the drift pumping is non-selective between alphas and D-T ions, (i.e., $\tau_{\alpha,rad} = \tau_{DT,rad}$) and solving for $\tau_{\alpha,rad}$ in Eq. (25). Having defined the radial loss of DT's and alphas, one can go to formulae relating the drift pump diffusion coefficient to the required oscillating magnetic field required in the drift pump coil. We do not do this in the present calculation, although recent calculations estimate the oscillation field amplitude to be around 500 gauss.

The term $I_{neo,DT}$ is the radial D-T particle current due to neoclassical losses. This can be expressed as $I_{neo,DT} \sim \frac{q}{10^3} n_c v_c / \tau_{neo}^{DT}$. This term is a

radial loss in addition to the induced transport from the drift pump coils, caused by the non-axisymmetry of the end cell. The neoclassical confinement time, τ_{neo} , can be written in terms of a diffusion coefficient; D_{neo} :

$$\tau_{neo} = \frac{r^2}{4 D_{neo}} \quad (26a)$$

where r is the central cell radial coordinate. The diffusion coefficient has the form⁶

$$D_{neo} \approx \frac{(\Delta r)^2}{\tau_{eff}} \quad (26b)$$

where:

$$\Delta r = 0.04 \left(\frac{3}{B_{\theta 0}} \right) \left(\frac{\delta \phi_{\theta 0} + T_c}{124} \right) r \quad (26c)$$

from the numerical results of orbit codes applied to the MARS magnet set. Here, $B_{\theta 0}$ and $\delta \phi_{\theta 0}$ are the magnetic field and potential drop at the point in the transition where the geodesic curvature, θ_0 , is a maximum. The effective collision time, τ_{eff} , is given by:

$$\tau_{eff} = 4 \sqrt{\tau_{\theta i} \tau_{bi} R_c} \quad (26d)$$

where $\tau_{\theta i}$ is the Spitzer-Chandrasekhar deflection time,

$$\tau_{\theta i} = 1.64 \times 10^{10} \frac{\sqrt{\frac{m_i}{m_D}} T_c^{3/2}}{Z_i^2 n_c (1 + Z_\alpha^2 C_\alpha)} \quad (26e)$$

and τ_{bi} is the ion bounce time:

$$\tau_{bi} = (L_c + L_t) / (2 T_c / m_i)^{1/2} \quad (26f)$$

where the subscript "i" refers to DT ions here, and in Eq. (25) for alphas.

III.2.2 Ion Energy Balance

The next major equation presented is the central cell ion energy balance. It is assumed that thermalized alphas and DT ions have the same temperature T_c . This equation is used to solve for a central cell ion temperature, T_c . It can be written as:

$$P_{fus,i} - P_{i+e} - P_{tran,rad} - P_{tran,neo} + P_{icrh} - P_{tran,ax} = 0 \quad (27)$$

where $P_{fus,i}$ is the fusion produced alpha power deposited into thermal ion species, and given by:

$$P_{fus,i} = \frac{q}{10^3} \frac{1}{4} n_c^2 \langle \sigma v \rangle_{DT} U_{gain} V_{cc} \quad (28a)$$

where U_{gain} is the net energy gained by a D-T ion, and is

$$U_{gain} = U_\alpha f_{\alpha i} - 2 \bar{E}_{fus} \quad (28b)$$

where $f_{\alpha i}$ is the fraction of the hot alpha energy, U_α , (3500 keV) which goes to thermal ions, and $\bar{E}_{fus}$ is the average energy of a reacting fuel ion, approximated by:

$$E_{fus} = 45 + \frac{3}{2} T_c \quad (28c)$$

For the value of $f_{\alpha i}$, the following form is used

$$f_{\alpha i} = 0.95 - f_{\alpha e} \quad (28d)$$

where $f_{\alpha e}$ is a fit to Fokker-Planck calculations:

$$f_{\alpha e} = 0.88 \exp \{-T_{ec}/67.4\} \quad (28e)$$

In Eq. (28d), we account for a prompt hot alpha loss, due to birth in the loss cone, of 5%. The other terms in (28a) have been defined previously.

The term P_{i+e} is the equilibration term between thermal ion and electron species, and is of the form:

$$P_{i+e} = \frac{q}{10^3} \frac{n_c^2}{2} \frac{(T_c - T_{ec})}{(n\tau)_{ei}} V_{cc} \left(1 + \frac{5}{2} C_\alpha\right) (1 + 2 C_\alpha) , \quad (29a)$$

where:

$$(n\tau)_{ei} = \frac{10^{13} A_{DT}}{\ln A_{ei}} T_{ec}^{3/2} , \quad (29b)$$

and:

$$A_{ei} = 9.34 \times 10^{13} T_{ec} \sqrt{n_c} . \quad (29c)$$

In Eq. (29a), electron-thermal alpha equilibration is accounted for in the term containing $5/2 C_\alpha$.

The radial power loss due to drift pumping $P_{tran,rad}$ is given by

$$P_{tran,rad} = \frac{q}{10^3} \frac{n_c^2}{(n\tau)_{c,rad,DT}} V_{cc} E_{loss,i} \quad (30)$$

where $(n\tau)_{c,rad,DT}$ is derived from Eq. (25), and $E_{loss,i}$ is the average energy of an ion lost radially. Presently, we are taking it equal to $3/2 T_c$, but there is some evidence from Fokker-Planck code results that it is actually lower than T_c . This is because the process which tends to trap DT ions affects those of low energy ($I_{trap} \sim 1/T_c^{3/2}$). The power loss from the central cell should be representative of the energy of the trapped ion leaving the central cell, times the trapping current. This will reduce the power drain on the central cell species, thereby making ignition requirements less stringent. An effect of increased trapping current, to be discussed shortly, will negate this improvement.

The term $P_{tran,neo}$ is the energy loss attributable to neoclassical processes. It is expressed as $I_{neo,DT}$ given by Eq. (18) times the average ion loss energy, taken as $3/2 T_c$ here also. No work has been done to determine a more accurate value of this loss energy.

The axial power loss for DT ions, $P_{\text{tran,ax}}$ is given by:

$$P_{\text{tran,ax}} = \frac{q}{10^3} \frac{\overline{n_c^2}}{(\overline{n\tau})_{c,\text{ax,DT}}} (\phi_c + T_c) V_{cc} \quad (31)$$

where the $(\overline{n\tau})$ product is given by Eq. (24). In Eq. (27), we have also allowed for an external ion heat source P_{icrh} . The subscript of this term suggests the use of ion-cyclotron resonance heating (ICRH). We occasionally input a non-zero value for this term to investigate the characteristics of reactors with sub-ignited central cells. This is particularly useful when looking at machines with short central cells.

III.2.2.1 Ion Trapping and Pumping Self-Consistency

While on the subject of ion particle and energy balances, it is appropriate to present certain consistency relations between the rate at which ions trap in the transition and barrier regions, and the rate at which they are removed. The removal rate has been defined by alpha pumping requirements, (see Eq. (25)) and the trapping rate is given by the Futch-LoDestro formula.⁷ For consistency, these rates must be equal. In order to achieve consistency, the central cell alpha particle concentration is adjusted, as well as the g values of DT ions and alphas. For D-T ions, the trapping rate has the form:

$$I_{\text{trap,DT}} = 4.49 \times 10^{-33} H V_t \overline{n_{\text{pass,t}}^2} [1 + 0.0441 R_t]^{3.33} (1 + 2.5 C_{\alpha,t})$$

$$(x) \frac{g_{\text{DT}}^{3.33} \sqrt{2/A_{\text{DT}}}}{(g_{\text{DT}} - 1)^{2.33} T_c^{3/2}} \left(1 + 0.78 \frac{L_{\text{plug}}}{L_t} \right) \quad (32a)$$

where V_t is the volume of the transition region (both ends):

$$V_t = 2\pi r_c^2 \frac{B_{c,\text{vac}} \sqrt{1 - B_c}}{B_{t,\text{vac}} \sqrt{1 - B_t}} L_t \quad (32b)$$

L_t is the length of the transition plus anchor region, L_{plug} is the plug length, g_{DT} is defined by Eq. (3), and R_t is the mirror ratio between

choke coil field, $B_{\max}$, and transition region field B_t . The variable A_{DT} is the atomic mass of a composite D-T ion, taken to be 2.5. The original Futch-LoDestro formula was obtained using a code which solves for one trapped ion distribution function, and uses magnetic fields and potentials which are of the square well form. There has been a new code developed recently at LLNL which can treat several classes of trapped ions, as well as consider more general (and hence realistic) B and ϕ axial profiles. The H factor introduced in Eq. (32a) corrects for the effects of the B and ϕ profiles, as well as the treatment of multiple trapped ion distributions. Before the physics freeze in December 1982, preliminary estimates obtained from this multiregion code suggested $H \approx 2/3$, which has been adopted for the MARS baseline. More recent estimates⁸ using the same code after several bugs had been corrected have yielded values of H of around 1.5. The higher H factor will have an impact on the amount of central cell refueling necessary, the value of ϕ_e , and on the power drain on central cell ions due to radial loss. As far as power flow is concerned, using an H less than one tends to compensate for the fact that the average energy of an ion lost radially will be less than the $3/2 T_c$ we used. For the trapping rate of alpha particles, a phenomenological formula similar to the rate for D-T ions is used. To date, no multi-species trapping rate calculations have been performed. A multi-species, multi-region Fokker-Planck code would be necessary to do the alpha particle trapping rigorously. In the meantime, we use:

$$I_{\text{trap},\alpha} = 4.49 \times 10^{-33} H V_t \overline{n_{\text{pass},\alpha,t}^2} [1 + 0.0441 R_t]^{3.33} \quad (33)$$

$$(x) \frac{g_\alpha^{3.33}}{(g_{\alpha-1})^{2.33}} \frac{Z_\alpha^2}{T_{\text{eff}}^{3/2}} \sqrt{\frac{2}{A_\alpha}} \left[Z_\alpha^2 + \frac{g_\alpha}{g_{DT} C_{\alpha,t}} \right] \left(1 + 0.78 \frac{L_{\text{plug}}}{L_t} \right).$$

Here, Z_α is the charge on the alpha (2) and A_α is its mass (4). The effective temperature, T_{eff} is taken to be twice the ion temperature. This effect arises due to the fact that in the end cell region, the velocity space distribution of alphas and D-T ions are separated, the alphas having fallen down a deeper potential hill. Since the dominant contribution to small angle

scattering (which leads to trapping) of alphas comes from D-T ions, the scattering rate ($\sim 1/T^{3/2}$) should be evaluated with a "temperature" more representative of the relative velocity between the two distributions. The H factor is taken here to be 2/3, for the same reasons as the D-T case. Both equations for I_{trap} depends on g and, $C_{\alpha,t}$, and therefore consistency is achieved by adjusting these variables until Eq. (32a) equals the radial component of Eq. (21) for DT ions, and until Eq. (33) equals the sum of the first and third terms in Eq. (25) for alpha particles.

III.2.3 Central Cell Electron Energy Balance

Let us now consider the energy balance for the central cell electrons. We solve for the electron temperature, T_{ec} , through this equation. It can be written:

$$P_{\text{ecrh},a} + P_{\text{ecrh},b} - P_{\text{sync}} + P_{i \rightarrow e} + P_{e,\text{slosh}} - P_{\text{tran},e} + P_{\text{fus},e} = 0 \quad (34)$$

The terms with subscripts "ecrh" are ECRH powers which are delivered to the end cell warm and hot electrons and eventually appear, through scattering and drag, as a central cell power source. These terms will be described later in the section on each cell power balances. We subtract synchrotron power loss from the power sustaining the hot electrons, $P_{\text{ecrh},b}$. This is because the central cell electrons are assumed not to be able to absorb this radiation. The electron-ion equilibration term is given by Eq. (29a). The term $P_{e,\text{slosh}}$ is the energy source due to the introduction of cold electrons at point a' from the sloshing neutral beams. These electrons gain energy by falling down the potential hill of height $\phi_{a'}$. Therefore, this term can be written:

$$P_{e,\text{slosh}} = \phi_{a'} I_{\text{slosh}}, \quad (35)$$

with $I_{\text{slosh},e}$ the same term as given in Eq. (18). The term $P_{\text{tran},e}$ is the axial Pastukhov power loss. It is given by the second term in square brackets

of Eq. (21), times the energy lost per electron, $\phi_e + T_{ec}$. Finally, in Eq. (34), we have the power deposited into electrons from the fusion produced alpha particles. It has a form quite similar to Eq. (28a), except U_{gain} is replaced by $U_\alpha f_{\alpha,e}$ given by Eq. (28e).

III.2.4 Central Cell Ion Particle Balance

The central cell ion particle balance is solved for the cold refueling current, I_c , to be provided by some means of fueling. We are presently planning to inject frozen D-T pellets. This equation is given by:

$$I_c = I_{burn} + I_{tran,DT} + I_{neo,DT} \quad (36)$$

where I_{burn} is defined in Eq. (19), $I_{neo,DT}$ defined above Eq. (26a), and $I_{tran,DT}$ is the first term in Eq. (21).

III.2.5 Central Cell Auxiliary Equations

There are auxiliary equations needed to calculate parameters of the central cell species which will now be described. Since the value of central cell beta, β_c , is an input variable, and the ion temperature is solved for through Eq. (27), and the electron temperature, T_{ec} , is solved using Eq. (34), the D-T ion density, n_c , is computed consistent with the values of temperatures and β_c . One solves for n_c from the defining equation of β_c :

$$\beta_c = \frac{2 \mu_0}{B_{c,vac}^2} n_c [T_c (1 + C_\alpha) + T_{ec} (1 + 2 C_\alpha) + P_{\alpha,H}/n_c] \quad (37)$$

where $2 \mu_0 = 4 \times 10^{-16}$ in the code units, C_α is the central cell thermal alpha concentration, and $P_{\alpha,H}$ is the pressure due to hot alphas. It is given by:

$$P_{\alpha,H} = 8.3 \times 10^{10} \langle \sigma v \rangle_{DT} T_{ec}^{3/2} U_\alpha f_{\alpha,e} n_c \quad (38)$$

The central cell length, L_c , and fusion power, P_{fus} , are inputs to the code. The central cell hot plasma radius is computed using the equation:

$$r_c = \left[\frac{P_{fus}}{\frac{1}{4} n_c^2 \langle \sigma v \rangle_{DT} U_\alpha \pi L_c} \right]^{1/2} \quad (39a)$$

The hot plasma radius at any location in the end cell can be obtained by conservation of magnetic flux, assuming the magnetic field is predominantly axially directed. The local radius is given by:

$$r(z) = \sqrt{\frac{B_{c,vac} \sqrt{1 - \beta_c}}{B(z)}} r_c \quad (39b)$$

A value for the minimum first wall radius, r_{FW} , can be derived from (39a) having specified the minimum number of 3.5 MeV alpha gyro-radii, N , which must be between the wall and the fusion plasma:

$$r_{FW} = r_c + \frac{27 N}{B_{c,vac}} \quad (40)$$

For MARS, we take $N \approx 2$.

III.2.6 End Cell Species Power Balance

Now consider particle and power balance for the end cell electron and ion species. We will describe below the power balances for the warm and hot electrons and the sloshing ions and particle balances for the warm electrons and the sloshing ions. The particle balance for hot electrons is satisfied by moving the resonance zone for ECRH heating axially to create the proper particle source of cold electrons to sustain them. The cold electron density varies rapidly in the region inboard of the thermal barrier, so only small adjustments in resonance location will be necessary.

III.2.6.1 Hot Electron Power Balance

We first consider the power balance for the hot electrons. This equation describes the heating of these electrons by electron cyclotron resonance heating (ECRH) and their energy loss due to pitch angle scattering, drag with cold electrons, and synchrotron radiation losses. These terms of course balance in the steady state. This makes this balance equation translate to a requirement for the absorbed microwave power necessary to sustain the hot electrons. This requirement can be written:

$$P_{\text{ecrh,b}} = P_{\text{scatt}} + P_{\text{drag}} + P_{\text{sync}} \quad (41)$$

Way we are currently modeling the drag power loss, P_{drag} , in combination with the scattering power loss:

$$P_{\text{scatt} + \text{drag}} = f_{\text{drag}} \frac{q}{10^3} \frac{(1 - F_{\text{ec}})}{(n\tau)_{\text{scatt}}} G^2 \frac{1}{n_{\text{pass,b,DT}}^2} E_{\text{loss,e}} V_{\text{b,eff,e}} \quad (42)$$

where f_{drag} is currently taken to be 1.2 to account for drag losses. More detailed models developed recently⁹ suggest that the drag power is somewhat more than assumed here, approximately 40% higher. The scattering $(n\tau)$ value has been obtained from Fokker-Planck calculations, and is given by:

$$(n\tau)_{\text{scatt}} = 2.8 \times 10^8 E_{\text{eh}}^{3/2} \log_{10} (R_{\text{eff,eH}}) \quad (43a)$$

where E_{eh} is the hot electron average energy, and the effective mirror ratio, $R_{\text{eff,eH}}$, is:

$$R_{\text{eff,eH}} = \frac{B_{\text{ma}}}{B_{\text{b,pla}}} [1 + \phi_b/E_{\text{eh}}]^{-1} \quad (43b)$$

We compute the hot electron energy based on the plug beta deemed necessary for MHD stability of the entire configuration:

$$E_{\text{eh}} = \frac{(B_{\text{b,vac}}^2 \beta_b - P_{\text{other}})}{0.8 G^2 \mu_0 n_{\text{pass,b,DT}} (1 - F_{\text{ec}})} \quad (43c)$$

where P_{other} is the pressure at point "b" due to other species, i.e., sloshing ions, warm ions and cold electrons:

$$P_{\text{other}} = 2 \mu_0 n_{\text{pass,b,DT}} [g_{\text{DT}} T_c + G F_{\text{ec}} T_{\text{ec}} + E_{\text{inj,a}} (G-g) \times B_{\text{b,pla}}/B_{\text{a',pla}}] \quad (43d)$$

The average energy lost per hot electron $E_{\text{loss},e}$ is taken as $E_{\text{eh}}/2$ effective volume of the plug which hot electrons see is taken to be an effective length times the plasma cross sectional area evaluated at the cell midplane. The effective length here is taken to be one-third the mirror-to-mirror length. The one-third factor arises from the axial average of the hot electron power formula, accounting to some degree for field and density profiles. The formula for synchrotron radiation is based on a model neglecting reabsorption:

$$P_{\text{sync}} = 9.075 \times 10^{-20} B_{b,\text{vac}}^2 (1 - \beta_b) G \bar{n}_{\text{pass},b,DT} \left[\frac{\bar{E}_{\text{eh}}}{10^3} + \left(\frac{\bar{E}_{\text{eh}}}{10^3} \right)^2 \right] \quad (44)$$

(x) $V_{b,\text{eff},e}$ (MW)

where currently the same effective volume is used here as was used in the scattering and drag formula. This is not strictly correct, since the synchrotron power depends on n and B in a different way. The calculation of the effective lengths used in the present model can be improved upon. For all of the plug powers, where densities can vary appreciably axially, effective lengths need to be defined:

$$L_{\text{eff}} = \frac{1}{n_0^\beta B_0^{\alpha-1}} \int_{\text{plug}} \frac{dz}{B(z)} B^\alpha(z) n^\beta(z) \quad (45)$$

where the midplane density and field values are n_0 and B_0 respectively, and the powers α and β depend upon which power formula is being considered. For Eq. (42) for example, $\alpha = 0$ and $\beta = 2\nu$ where ν has been defined in Eq. (9). This effective length is then multiplied by the midplane cross sectional area.

III.2.6.2 Warm Electron Balance Equations

For the warm electrons, we solve both a particle and energy balance. The physics of the warm electrons differs significantly between MARS-like reactors and MFTF-B. This is because in MFTF, there are several sources of cold electrons which are introduced into the region where the warm electrons reside. These sources are from the high energy pump beam, the sloshing ion beam, and cold gas ionized as it penetrates the halo plasma. In MARS we use drift pumping instead of charge exchange pumping, which introduces no

cold electrons, the sloshing ions are injected on the inboard side of the plug so electrons produced from the injected neutrals can't reach point "a", and the halo plasma effectively shields the gas produced by the sloshing beam. With no ionization sources, the warm electron particle balance takes on the form of a "modified Boltzmann" relation^{4,10}:

$$\frac{n_{ec}(b)}{n_{ew}(a)} \sqrt{\frac{T_{ew}}{T_{ec}}} \exp(\delta\phi_a/T_{ew}) - 1 = 0 \quad (46)$$

where $n_{ec}(b)$ is the cold electron density at the bottom of the thermal barrier, $n_{ew}(a)$ is the warm electron density at the potential peak, and $\delta\phi_a$ is $\phi_c + \phi_b$. Equation (46) has been derived in the so-called "weak ECRH" limit where distribution functions are assumed Maxwellian. The form of (46) when distributions are non-Maxwellian is currently under investigation. For the power balance of these warm electrons, the dominant contributions to losses are (1) energy exchange with cold electrons, and (2) actual particle exchange between the cold and warm groups. This is in contrast to MFTF-B, where the largest power is that which is required to heat the cold electrons introduced. For our specific case, the ECRH power requirement at "a" can be written:

$$P_{ecrh,a} = 2 \frac{q}{10^3} \frac{n_{ew}(a)}{(n\tau)_{ew}} v_{b,eff,e} (T_{ew} - T_{ec}) \quad (47)$$

The factor of two accounts for power lost by both conduction and convection, since they are typically equal, and each term has approximately the same functional dependence. The confinement parameter $(n\tau)_{ew}$ is due to Cohen¹⁰:

$$(n\tau)_{ew} = 1.82 \times 10^8 T_{ew}^{3/2} [\exp(\delta\phi_a/T_{ew}) - 1] (1 + B_{b,pla}/B_{a,pla}) \quad (48)$$

The warm electron density, $n_{ew}(a)$, is obtained from Eq. (17).

There is work being done currently by Cohen¹¹ to develop new models for plug electron particle and energy balance. This was motivated by the desire to have the model distributions more accurately describe those generated by

ECRH simulations using Matsuda's multi-region Fokker-Planck code. The concept of three electron species residing in the plug (hot, warm, and cold) is convenient for visualization, but distributions characterized this way do not closely match Fokker-Planck distributions, nor are they continuous across velocity space boundaries. This latter condition is important for assuring that the ambipolar potential be a single-valued function throughout the barrier region. It must be realized, then, that the ECRH heating powers calculated for MARS are only estimates. Cohen's preliminary results for the particle balance of these plug electrons suggest that the density appears raised to the 2/3 power, rather than appearing in a logarithmic term, as implied by Eq. (46). This will allow the production of higher ion potential peaks for the same ECRH power at pt. "a". Cohen's power balance for the plug electrons has not been completed at this writing.

III.2.6.3 Sloshing Ion Balance Equations

The last plug balance equation is that which describes sloshing ions. These ions are injected perpendicular to the field at a location near the inboard mirror of the plug. The term "sloshing" refers to the fact that since they are injected off midplane, they tend to "slosh" back and forth between their injection point, "a'", and the ion confining potential peak, point "a". Distributions of this type trap cool ions in the density trough, and exhibit improved microstability properties over a distribution injected at the midplane. With the use of sloshing ions, it has been shown in TMX-U that end plug ion microstability can be achieved without a cold streaming plasma, which is a large power drain. The beam source at energy $E_{inj,a'}$ sustains the sloshing ion plasma against losses due to pitch angle scattering and drift pumping. The scaling of sloshing ion power has the parametric dependence of pitch angle scattering; the ratio of scattering loss to pumping losses is assumed constant. The sloshing ion power is given by:

$$P_{slosh} = \frac{q}{10^3} \frac{(G - g_{DT})^2}{(\pi\tau)_{slosh}} n_{pass,b,DT}^2 E_{inj,a'} v_{b,eff,i} \quad (49a)$$

Where the $(n\tau)$ expression was fit to Fokker-Planck calculations, based on injection at "a", incorporating both scattering and pumping:

$$(n\tau)_{\text{slush}} = 1.58 \times 10^{10} \sqrt{A_s} E_{\text{inj},a}^{3/2} \log_{10} (R_{\text{eff},s}) \quad (49b)$$

where $E_{\text{inj},a}$ is given by Eq. (11), and $R_{\text{eff},s}$ given by:

$$R_{\text{eff},s} = \frac{B_{ma}}{B_{a,v} \sqrt{1 - \beta_a}} \left[1 + \frac{(\phi_e + \phi_e)}{E_{\text{inj},a}} \right]^{-1} \quad (49c)$$

and A_s is the atomic mass in AMU of the injected ion.

The volume is given by the physical plug volume times the following factor:

$$\frac{L_{\text{eff}}}{L_b} = \frac{2.2}{(B_a/B_b)} \quad (49d)$$

obtained from Fokker-Planck calculations.

III.2.7 Anchor Hot Ion Balances

The last set of formulae required is that for the hot anchor ions, heated with ICRH. The input variables for this region are the centerline β value, the magnetic field profiles, and anchor length. The hot ion energy is taken to be the most restrictive of: (1) the adiabatic energy limit:

$$E_{\text{in}} < 5 \times 10^{-2} \frac{B_{\text{anc}}^2 L_{\parallel}^2 |\ln \cos^2 \theta_0|^2}{M \sin^4 \theta} \quad (50a)$$

where $L_{\parallel} = (2 B_0)^{1/2} / (\partial^2 B / \partial s^2)^{1/2}$ evaluated at the midplane and θ_0 is the pitch angle of a typical ion crossing the midplane, and (2) the requirement that the hot ions behave like an MHD fluid. This latter requirement translates to a constraint that the central cell diamagnetic drift frequency, ω_* must exceed the curvature drift frequency in the anchor, ω_k . These frequencies have the form:

$$\omega_* = \frac{T_c}{B_{c,vac} r_c \partial r_c} \quad (50b)$$

where δr_c is the density gradient scale length. For the curvature drift frequency,

$$\omega_K \approx \frac{E_{ih}}{B_{anc,vac} r_{anc} R_c(r_{anc})} \quad (50c)$$

Logan's original estimate suggested that the energy limit should be chosen on the basis of adiabaticity, with E_{ih} about 700 keV. Devoto, having done actual orbit calculations in the EFFI generated field, has more recently determined that the lower limit should be based upon the drift frequency requirement. He suggests the use of 500 keV ions as an upper limit. Knowing now E_{ih} and β_{anc} , we can compute the required midplane hot ion density, n_{iH} :

$$n_{iH} = 2.5 \times 10^{15} \beta_{anc} \frac{B_{anc,vac}^2}{0.8 E_{ih}}, \quad (50d)$$

ignoring the pressure contribution from warm ions and alpha particles. Before we can calculate a power requirement, we need to know the confinement of the hot ions. For mirror confinement,

$$(n\tau)_{mir} = 2.4 \times 10^{10} E_{ih}^{3/2} \log(B_{m,anc}/B_{anc,pla}) \quad (50e)$$

and the $(n\tau)$ due to drag on cold electrons

$$(n\tau)_{drag} = 10^{13} \frac{A_{anc} T_{ec}^{3/2}}{\ln \Lambda_{ei}} \quad (50f)$$

where $\ln \Lambda_{ei} \approx 18$, $B_{m,anc}$ ($B_{anc,pla}$) is the anchor mirror (midplane) field, and A_{anc} is the mass of the anchor ions. The total $(n\tau)_{anc}$ is the inverse sum of Eqs. (50e) and (50f). For an effective anchor volume, $V_{anc,eff}$, we take the flux-mapped cross-sectional area times an effective length assuming the hot density is not a function of z (Monte Carlo calculations have shown this) and parabolic (in z) magnetic field profiles.

Typically, the effective length is 30% shorter than the physical length. The anchor power is given by:

$$P_{anc} = \frac{q}{10^3} \frac{E_{i,loss}}{(n\tau)_{anc}} V_{anc,eff} \overline{n_{iH}^2} \quad (50g)$$

The loss energy is currently being taken as one-half E_{ih} , consistent with previous results of Fokker-Planck codes modeling beam-heated distributions.

IV. Other Constraints Implicit in Physics Modeling

In addition to the constraints imposed by plasma power balance, we satisfy several auxiliary constraints which we check external to TMRBAR.

IV.1 MHD Stable Plasma Pressures.

The first has to do with plasma beta values in the central cell, transition region, anchor and plug regions. Using the MHD stability code TEBASCO, and the EFFI generated axial magnetic field profile, a set of beta values for the reactor are calculated which are stable to interchange and $m = 1$ ballooning modes. The beta values inputted to TMRBAR and shown on Table 1 are results of such an auxiliary calculation.

IV.2 Sloshing Ion adiabaticity.

The sloshing ions in the plug of MARS are of a sufficiently high energy that one must design the magnetics so that they conserve the magnetic moment invariant. Otherwise they will suffer a large enhancement in pitch angle scattering losses. The magnetic field profile, plug length, and plug midplane field value, as well as ion midplane energy determine whether the sloshing ions are adiabatic. TMRBAR has an approximate law to check for adiabaticity assuming a parabolic field shape, but this relationship has been found to be overly pessimistic and is no longer used. When a new plug design is proposed, a separate calculation of adiabaticity is usually performed.

IV.3 Stability to Trapped Particle Modes.

Stability to trapped particle modes is checked external to TMRBAR. A sample calculation is shown below.

Here we briefly demonstrate, using the methodology outlined in the Axicell proposal for MFTF¹³, that MARS is stable to electrostatic ballooning modes. The stability criterion derived in that document is:

$$\frac{\overline{g n_{\text{pass}}}}{n_c} > \frac{8 D}{\gamma} \frac{r_c^2}{L_{\text{tran}}(\text{cc}) (L_{\text{anc}} + L_b)} \frac{\overline{B}_{\text{anc}}}{B_c}, \quad (51)$$

where D is the ratio of total bad curvature drive (transition plus central cell) to central cell drive, and $L_{\text{tran}}(\text{cc})$ is the characteristic scale length for the rise in magnetic field at the ends of the central cell. The barred quantities denote averages over the good curvature regions of the anchor and plug. These averages require knowledge of the axial mode structure of the instability, and therefore can only be estimated at the present time. The other variables have been defined elsewhere in this report. For the axial average of the net passing density, we take $g n_{\text{pass}}$ in anchor and plug, weighted by the lengths of the respective regions, $g n_{\text{pass}} = 1.6 \times 10^{13} \text{ cm}^{-3}$. A representative value of D for reactors quoted in the axicell proposal is adopted here, $D \sim 6$. We take the average beta reduced field in the good curvature region to be around 2 T. Taking the field gradient scale length to be around 3.25 m, and using the MARS baseline parameters, we see that the stability criterion is satisfied.

V. History of Power Balance Code

The power balance code had its origin as a scaling code utilizing the equations Q which appeared in "Physics Basis for MFTF-B".⁴ In this early code, temperatures and potentials were input variables, not solved through particle and power balances, and quasineutrality, as is done in the current code. This version of the code was set up to model only the A-cell-type end cell configuration,⁴ with charge exchange pumping. The first major modification to the code was to solve for electron temperature through the electron energy balance, ion confining potential by using the ion energy balance, and central cell floating potential using global machine quasineutrality. During FY 1981, LLNL performed a comparative study of several end cell configurations.¹² The A-cell, modified cusp, with an EBT-type electron ring, and TARA-like axicell designs were considered. The interested reader is encouraged to consult Ref. 12 for more detail. The physics code was modified from its original A-cell configuration to model each of the end cells. With the help of this code, it was determined that the best end cell was the TARA axicell, based on a combination of physics credibility and performance.

In mid 1982, after the TARA axicell had been chosen, Berk and Rosenbluth predicted a destructive, rapidly growing mode present in tandem mirrors, akin to the trapped particle modes present in tokamaks. Refinements to the theory¹³ showed that end plugs stable to these trapped particle modes could be designed, but it would be necessary to move the thermal barrier from the axicell (as in TARA) to the good-curvature yin-yang "anchor". The code was then changed to model this configuration.

The change introduced a number of difficulties not present in the old design. The problem of designing a magnet set which had good MHD equilibrium and stability properties in spite of considerable plasma pressure in the bad-curvature transition region was particularly challenging. To keep the density in the transition region as low as possible, the trapped density needs to be pumped. The original scheme called for charge exchange pumping, where a neutral ion injected in the passing region of velocity space charge exchanges with a trapped ion, which subsequently is lost as a neutral particle. The charge exchange pumping power was found to be quite large for the new design, over 100 MW.

This motivated research into developing a passive means of pumping trapped particles. Baldwin¹⁴ proposed using electric fields induced by oscillating currents produced in external coils, to create radial drifts of trapped particles. This is called "drift pumping". The method has since been refined,¹⁵ so that it is selective to trapped particles by inducing transport only in those particles with a range of bounce frequencies, the so-called "bounce resonance" drift pumping. It is this most recent scheme which is incorporated into the current physics code.

It should be noted that during the entire development of the code, formulas for central cell and end cell balances were modified to reflect our latest thinking, as well as results from other codes. This is one of the reasons why the equations documented here are not the same as those presented in Ref. 4. It is anticipated that further changes to the model will be made, as soon as better formulas for power requirements become available; this is a continuing process.

VI. Improvement of the Model and Code

It is anticipated that as improved formulae come into existence, the models embodied in the TMRBAR code will evolve. There are a number of areas where improved models are most likely to come. One is in the confinement and

power loss of the super thermal end cell species: electrons from Cohen¹¹ and Shearer⁹ and sloshing ions from additional Fokker-Planck calculations. Another area where model improvement is likely to come is in the calculation of trapping rates of ions in potential and magnetic wells, that is computing more realistic "H" factors as in Eq. (32a). Another model improvement would be to allow for startup modeling by making TMRBAR time-dependent. There is currently no plan to convert TMRBAR, but there is a project underway to develop a single time-dependent code¹⁶ which will model startup and steady-state operation of TMX-U, MFTF-B, and MARS. This is expected to be a long-term project.

A possible near term project is to improve the exportability of the code. In its current state, the code must be run in an interactive mode, and is not particularly easy to run by the uninitiated user. By perhaps changing the role of some of the variables (change from input to output and vice versa) or by changing some of the solution techniques, the code might be able to obtain solutions more easily. This is an area of pursuit which deserves further study.

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