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Dielectric Response in Guiding Center Plasma

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The dielectric function for guiding center plasma is derived from the direct-interaction approximation. For the special case of thermal equilibrium, the result agrees with, although is more detailed than, an earlier calculation of Taylor. An explicit formula for the collision operator Σ' is given. The calculation illustrates several important features of renormalized turbulence theory: cancellation between the so-called diffusion and polarization parts of Σ' , and the role of the renormalization in providing the proper description of adiabatic response.

I. INTRODUCTION

The dielectric response function $\epsilon(k, \omega)$ plays a central role in the turbulence theory of many plasma instabilities. For example, the growth rate γ_k is determined both in linear, weak turbulence, and renormalized weak turbulence theory as

$$\gamma_k = - \frac{1}{2\pi} \frac{\partial}{\partial \omega} \text{Im} \epsilon(k, \omega) \Big|_{\omega=\omega(k)} \quad (1)$$

where

$$\epsilon_r[k, \omega(k)] = 0 \quad (2)$$

(The subscripts "r" and "i" denote, respectively, the real and imaginary parts.) While the form of ϵ is well understood both in linear and weak turbulence theory,¹ the renormalized dielectric has until recently been obtained by more or less well-motivated but asystematic approximations.² In the well-known resonance-broadening theory of E×B turbulence,³ for example, the positive infinitesimal of linear theory is replaced by $k_{\perp}^2 D$, where D is a test particle diffusion coefficient. This approximation does not conserve energy,⁴ a defect which can be severe in many circumstances. The failure can be understood physically in terms of test wave arguments. Let us write $k \equiv (k_{\perp}, \omega_k)$. In resonance-broadening theory, a test wave k is selected and treated as a dynamical system subject to the random influences of all modes $\{q \neq k\}$ but not affecting those modes in any way. This means that the modes $\{q \neq k\}$ are treated as a fixed, statistically specified bath which can absorb an infinite amount of energy from the test wave. Thus, each test wave is predicted to damp at a rate $k_{\perp}^2 D$.

which in this approximation is also the rate $\Sigma^{(d)}$ of test particle diffusion ("d" denotes diffusion). Because all waves cannot damp in steady state, this approximation violates energy conservation among the isolated set of all modes $\{q\}$ and predicts incorrect nonlinear growth rates.

Energy conservation can be recovered only by restoring the correlation between each wave and its bath. This gives rise to two effects.⁵⁻⁷ First, the background distribution function is renormalized:

$$f_{\perp} = \langle f \rangle + \delta \bar{f}, \quad (3)$$

where $\delta \bar{f}$ is a nonlinear correction. In weak turbulence theory, this term is responsible for the well-known symmetry of the matrix elements. We have noted elsewhere⁸ that in renormalized theory $\delta \bar{f}$ is necessary to ensure that the adiabatic response is handled correctly; we shall return to this point. Furthermore, the presence of $\delta \bar{f}$ decouples the wave growth rate γ_k from the test particle scattering rate Σ_k' . (The nonlinear dispersion relation is not obtained from the linear one by $\epsilon(k, \omega) = \epsilon^{(0)}(k, \omega + ik_{\perp}^2 D)$.] Individual waves can either grow or decay, although total energy is conserved between waves and particles. Second, the particles polarize the medium, which adjusts to reduce the influence of the medium on the particles. The total particle scattering rate becomes

$$\Sigma_k' = \Sigma_k^{(d)} + \Sigma_k^{(p)} < \Sigma_k^{(d)} \quad (4)$$

("p" denotes polarization). The polarization effect is also well-known in weak turbulence theory, where it appears as the

so-called nonlinear scattering, or induced scattering from the shielding clouds. (The diffusion terms generate Compton scattering.) A polarization effect also appears in $\delta \bar{f}$:

$$\delta \bar{f}_k = \delta \bar{f}_k^{(d)} + \delta \bar{f}_k^{(p)} . \quad (5)$$

While some recent renormalized theories^{4,8,9} have included all or part of $\delta \bar{f}^{(d)}$, virtually all theories have omitted the polarization terms, either for technical reasons or because, in many cases, they did not arise naturally in the formalism. In general, this neglect is expected to be a poor approximation. For example, in unmagnetized weak turbulence theory the nonlinear scattering terms cancel the Compton scattering terms¹⁰ to $O(k_{\perp}^2)$, where $\lambda_D = (T/4\pi n e^2)^{1/2}$ is the Debye length. Though this cancellation is incomplete in the finite gyroradius case,¹¹ the polarization terms remain nevertheless both qualitatively and quantitatively important.¹² These considerations motivate a careful study of the polarization terms in renormalized theory. As a practical application in which these terms are likely to be important and certainly have not been shown to be small, we mention the theory of drift waves in sheared geometry, in which the nonlinear growth rates are extremely sensitive to the form of the nonlinear terms.

We have recently discussed the systematology of renormalized turbulence theory^{6,7} and written an explicit expression for $\delta \bar{f}$ in the direct-interaction approximation.^{5-7,13} Though the formalism is structurally simple, the details at the level of the Vlasov or drift-kinetic description are extremely complicated and seem to require extensive numerical analysis. It is desirable to find

a simple model problem in which the theory can be worked out explicitly. Such a model is provided by the guiding center plasma.

The guiding center plasma model was introduced in ref. 14; it consists of charged rods moving perpendicularly to a straight magnetic field under the $\mathbf{E} \times \mathbf{B}$ interaction. The charge density of this system obeys the Liouville equation

$$\frac{\partial \tilde{\rho}}{\partial t} + \tilde{\mathbf{V}}_E \cdot \nabla \tilde{\rho} = 0, \quad (6)$$

where

$$\tilde{\mathbf{V}}_E = (c/B) \tilde{\mathbf{E}} \times \hat{n}, \quad (7)$$

$$\tilde{\mathbf{E}} = -\nabla \tilde{\phi}, \quad (8)$$

$$\nabla^2 \tilde{\phi} = -4\pi \tilde{\rho}. \quad (9)$$

The superscript tilde denotes a random variable. We assume that the system is statistically stationary. This is automatic in thermal equilibrium, where the ensemble would be the Gibbs distribution describing spatial positions of the rods. Otherwise, forcing and dissipative terms must be added to Eq. (6) to maintain a steady state; we do not do this here. Now Taylor¹⁵ has found an approximate form for the thermal equilibrium dielectric associated with Eq. (6) by using the Kubo formalism. Simple manipulation of Eq. (20) of Ref. 15 yields Taylor's result in the form

$$\epsilon = 1 + \left(\frac{k_D^2}{k^2} \right) \left(1 + \frac{i\omega}{-i\omega + \left(\frac{k^2}{k^2 + k_D^2} \right) k^2 D} \right), \quad (10)$$

where $k_D = \lambda_D^{-1}$. This is to be compared with the result of (a modified) resonance-broadening theory¹⁶:

$$rht \sim 1 + \left(\frac{k_D^2}{k^2} \right) \left(1 + \frac{i\omega}{-i\omega + k^2 D} \right) . \quad (11)$$

Clearly, the difference between (10) and (11) lies in the form of the resonance function g , which describes the "nonadiabatic" response. We can write in both cases

$$= 1 + \left(\frac{k_D^2}{k^2} \right) (1 + i\omega g) , \quad (12)$$

$$g = (-i\omega + k^2 D)^{-1} , \quad (13)$$

where

$$g = \begin{cases} \left(\frac{k_D^2}{k^2 + k_D^2} \right) k^2 D & \text{(Taylor),} \\ k^2 D & \text{(resonance-broadening theory).} \end{cases} \quad (14)$$

We note that Taylor's "collision" term Σ' is reduced at long wavelengths from that of resonance-broadening theory by a factor of $(k/k_D)^2$. This is very reminiscent of the cancellation to $O(k^2 \lambda_D^2)$ in weak turbulence theory between Compton and nonlinear scattering. In fact, by direct computation beginning with the form of Σ given in Ref. 6, we will show that the analogy is maintained in renormalized theory: at least in the guiding center plasma, polarization terms significantly reduce, by $O(k^2 \lambda_D^2)$, the size of the diffusion terms. The calculation is more general than Taylor's, as it introduces the special properties of thermal equilibrium at

a later stage in the argument and allows for nonlocal transport effects (wavenumber dependence of D). Another desirable feature of the present formalism is that it is couched in a language quite close to that of conventional weak turbulence theory. In this language, certain intermediate terms, such as particle propagators, have significance in their own right; by exhibiting these, we clarify the structure of the theory. Nevertheless, our calculation upholds the general form of Taylor's result.

II. DIELECTRIC FUNCTION in the DIRECT-INTERACTION APPROXIMATION

Structurally, we have shown that the dielectric can be written in the "problem-independent" form⁷

$$\epsilon = 1 - \phi_k g_k i k \cdot \partial \bar{f}_k. \quad (15)$$

Here f is the Liouville function for the problem, ϕ is a potential operator which inverts Poisson's equation for the electrostatic potential in terms of f , g is the particle propagator, and ∂ is a known differential operator related to the form of the non-linear interaction. (In Vlasov plasma, for example, $\partial = (q/m) \partial / \partial v$.) We further write

$$g^{-1} = \left(g^{(0)} \right)^{-1} + \Sigma', \quad (16)$$

$$\bar{f} = \langle f \rangle + \delta \bar{f}, \quad (17)$$

where $g^{(0)}$ is the linear propagator, $\langle \dots \rangle$ denotes ensemble average, and Σ' and $\delta \bar{f}$ are the renormalizing factors, explicitly proportional to the spectral intensity. Equation (15) can be written

$$\epsilon = 1 + \chi, \quad (18)$$

where the internal susceptibility χ is defined as

$$\chi = \frac{\delta \phi_1}{\delta \phi_0}. \quad (19)$$

Here ϕ and ϕ_1 are the total and induced potential responses of the medium correct to first order in an external source potential ϕ_0 ; δ denotes a functional derivative. This is the form most closely related to linear theory; indeed, Eq. (15) reduces to linear theory when $\bar{f}$ and $\delta \bar{f}$ are omitted and when ϕ is replaced by the zeroth order distribution. An alternative, more fundamental form from which Eqs. (15) and (19) are, in fact, derived is

$$\epsilon = (1 - \chi_e)^{-1}, \quad (20)$$

where the external susceptibility χ_e is found from

$$\chi_e = \frac{\delta \phi_1}{\delta \phi_0}. \quad (21)$$

In terms of the total response function^{7,13}

$$R = g \left[1 + ik \cdot \delta \bar{f} \frac{\partial g}{\partial t} \right], \quad (22)$$

one has

$$\epsilon = (1 + \phi_k R_k ik \cdot \delta \bar{f})^{-1}. \quad (23)$$

Taylor discussed χ_e , the quantity easiest to define operationally, so in effect gave a theory for R . Fluid dynamicists work exclusively with R . This is the natural choice for strong turbulence, where "particles" and fields are tightly coupled. Introduction of the particle propagator g and the representations (16) and (22) is

appropriate only for weak turbulence. Nevertheless, although the guiding center approximation describes strong turbulence, we shall initially study the form (15) in order to exhibit as clearly as possible the parallels with earlier work on renormalized plasma turbulence. However, we will be led naturally to study the total response $\bar{F}$.

To describe the guiding center model, the following transcriptions can be made from the general formalism^{6,7}:

$$f = \dots, \quad (24a)$$

$$\epsilon_k = \frac{4\pi}{k^2}, \quad (24b)$$

$$\bar{\omega} = (c/B)\bar{n} \times \bar{V}. \quad (24c)$$

In the direct-interaction approximation, we have explicit expressions^{6,7} for $\bar{F}$ and $\Delta\bar{F}$:

$$\bar{F}_k^{(d)}(1, \bar{I}) = -\sum_q \epsilon \cdot \partial_1 g_{k-q}(1; 2) I_q \cdot \partial_2 \bar{F}_{k-q}(2; \bar{I}), \quad (25a)$$

$$\begin{aligned} \bar{F}_k^{(p)}(1, \bar{I}) = & -\sum_q \epsilon^{-1} [(k-q) \cdot \partial_1 \phi_{k-q}(\bar{2}) g_{k-q}(\bar{2}; 2) \cdot \partial_2 \bar{F}_{k-q}(1; \bar{I}) \\ & + q \cdot \partial_1 g_{k-q}(1; \bar{1}) i(k-q) \cdot \partial_1 \bar{F}_{k-q}(\bar{1}) \\ & \times \phi_{k-q}(\bar{2}) g_{k-q}(\bar{2}; 2) I_q \cdot \partial_2 \bar{F}_{k-q}(2; \bar{I})]; \end{aligned} \quad (25b)$$

$$ik \cdot \partial \delta \bar{F}_k^{(d)}(1) = -\sum_q \epsilon \cdot \partial_1 g_{k-q}(1; 2) k \cdot \partial_2 \langle \delta \bar{F}_{k-q}(2) \rangle_q, \quad (26a)$$

$$\begin{aligned} ik \cdot \partial \delta \bar{F}_k^{(p)}(1) = & -\sum_q \epsilon^{-1} [(k-q) \cdot \partial_1 \phi_{k-q}(\bar{2}) g_{k-q}(\bar{2}; 2) k \cdot \partial_2 \langle \delta \bar{F}_{k-q}(1) \rangle_q \\ & + q \cdot \partial_1 g_{k-q}(1; \bar{1}) i(k-q) \cdot \partial_1 \bar{F}_{k-q}(\bar{1}) \\ & \times \phi_{k-q}(\bar{2}) g_{k-q}(\bar{2}; 2) k \cdot \partial_2 \langle \delta \bar{F}_{k-q}(2) \rangle_q]; \end{aligned} \quad (26b)$$

Here we have merely transcribed Eqs. (23) and (24) of Ref. 6, using the relation (8). The numerical arguments in Eqs. (25) and (26) represent species and velocity dependence in the general case. For the guiding center plasma, there is no such dependence, but the notation does serve to indicate which functions are being acted on by the operator. Because in the present case the potential operator is purely multiplicative, the expressions are much simpler than in the more realistic models which permit parallel motion. Indeed, taking note of Eqs. (24), using

$$\Delta_k = \frac{k^2}{4\pi} \Delta_k, \quad (27)$$

and defining

$$M_{k,q} = (c/B)^2 |\mathbf{n} \cdot (\mathbf{k} \times \mathbf{q})|^2, \quad (28)$$

we find explicitly

$$\bar{\epsilon}_k^{(d)} = \sum_q M_{k,q} g_{k-q} I_q, \quad (29a)$$

$$\begin{aligned} \bar{\epsilon}_k^{(p)} = & - \sum_q \epsilon_{k-q}^{-1} \left[M_{k,q} \phi_{k-q} g_{k-q} I_q \left(\frac{q^2}{4\pi} \right) \right. \\ & \left. - g_{k-q} i(\mathbf{k}-\mathbf{q}) \cdot \partial \bar{\epsilon}_{k-q} \phi_{k-q} g_{k-q} I_q \right]; \end{aligned} \quad (29b)$$

$$ik \cdot \partial \bar{\epsilon}_k^{(d)} = \sum_q M_{k,q} g_{k-q} I_q \left(\frac{q^2}{4\pi} \right), \quad (30a)$$

$$\begin{aligned} ik \cdot \partial \bar{\epsilon}_k^{(p)} = & - \sum_q \epsilon_{k-q}^{-1} M_{k,q} \left[\phi_{k-q} g_{k-q} I_q \left(\frac{q^2}{4\pi} \right)^2 \right. \\ & \left. - g_{k-q} i(\mathbf{k}-\mathbf{q}) \cdot \partial \bar{\epsilon}_{k-q} \phi_{k-q} g_{k-q} I_q \left(\frac{q^2}{4\pi} \right) \right]. \end{aligned} \quad (30b)$$

Notice that we have *not* employed the coherent approximation discussed in Ref. 6; this approximation is not correct for strong

turbulence. If we now use the form of the dielectric given in Eq. (15), we observe that the diffusion and polarization terms combine:

$$\gamma_k^* = \gamma_k^{(d)} + \gamma_k^{(p)} = \gamma_{k,q}^* \quad , \quad (31)$$

$$\gamma_{k,q}^* = N_{k,q} g_{k-q} \frac{1}{q} \left(\frac{|k-q|^2 - q^2}{|k-q|^2} \right)_{k-q} ; \quad (32)$$

$$ik \cdot \Delta \bar{f} = ik \cdot \Delta \left\{ \bar{f}^{(d)} + \bar{f}^{(p)} \right\} = \frac{1}{4\pi} \sum_q q^2 \gamma_{k,q}^* \quad . \quad (33)$$

Notice in particular the factor $|k-q|^2 - q^2$ in Eq. (32); the q^2 arises from the polarization terms. This factor vanishes as $k \rightarrow 0$, a feature which has the important effect of raising the order of Δ by a factor $(k\lambda_D)^2$ relative to its order when only the diffusion terms are retained. We will demonstrate this explicitly in thermal equilibrium.

Let us define

$$4\pi ik \cdot \Delta \bar{f}_k = -k^2 \gamma_k'' \quad (34)$$

and

$$\Sigma_k = \Sigma_k' + \Sigma_k'' \quad . \quad (35)$$

Formula (15) then becomes

$$\begin{aligned} \epsilon &= 1 - \left(\frac{4\pi}{k^2} \right) \left(\frac{-\frac{1}{4\pi} k^2 \gamma_k''}{-i\omega + \Sigma_k'} \right) \\ &= \frac{-i\omega + \Sigma_k}{-i\omega + \Sigma_k'} \quad , \end{aligned} \quad (36)$$

so the dielectric vanishes for

$$\omega = -i\Sigma_k \quad . \quad (37)$$

We will be able to show that, in the long wavelength limit of thermal equilibrium, $\chi_k = k^2 D_k$, where D_k depends weakly on k , so Eq. (37) becomes

$$\epsilon_k = -ik^2 D_k. \quad (38)$$

This is one of Taylor's results.¹⁵ In plasma physics, result (38) would be called the "convective cell eigenfrequency."¹⁷

The result (36) can be interpreted in terms of the response function R . Representation (22) satisfies the identity⁶

$$iR = \epsilon^{-1} g, \quad (39)$$

Because in the guiding center plasma ϵ is purely multiplicative, i can be cancelled from both sides of Eq. (39):

$$R = g/\epsilon. \quad (40)$$

Conventionally, this would be interpreted as the shielding relation between the particle propagator and the total response function. Such an interpretation implies that one more or less knows ϵ , say from linear theory. Here, however, the fundamental variable is g or R , knowledge of either of which determines ϵ . It is therefore more useful to write

$$\epsilon = g/R \quad (41)$$

as the determining relation for ϵ . This is just Eq. (36), with

$$R = (-i\omega + \Sigma)^{-1}. \quad (42)$$

Thus, the zeros of ϵ and the system response to an external

perturbation are determined by the total response function. This is hardly surprising. Furthermore, notice that the form of Eqs. (32) and (33) is such that only the combination $q/\omega = R$ appears. This underscores our earlier observation that q has little direct relevance for a theory of strong interactions.

Combining results (32) and (33) according to Eqs. (35), (34), and (40), we find a formula for the mass operator Σ :

$$\Sigma_k = \sum_q M_{k,q} R_{k-q} I_q \left(1 - \frac{q^2}{|k-q|^2}\right) \left(1 - \frac{q^2}{k^2}\right). \quad (43)$$

The response function obeys the nonlinear equation (42).

Transforming this back into the time domain, we have

$$\frac{\partial R_k(\tau)}{\partial \tau} + \int_0^\tau d\bar{\tau} \Sigma_k(\bar{\tau}) R_k(\tau - \bar{\tau}) = \delta(\tau), \quad (44)$$

where

$$\begin{aligned} \Sigma_k(\tau) &= (c/B)^2 \int \frac{dp dq}{(2\pi)^2} \delta(k-p-q) |k+q|^2 R_p(\tau) I_q(\tau) \\ &\quad \times \left(1 - \frac{q^2}{p^2}\right) \left(1 - \frac{q^2}{k^2}\right) \\ &= k^2 \left[\frac{1}{(2\pi)^2} \left(\frac{c}{B}\right)^2 \left(\int_0^1 dp \int_{1-p}^{1+p} dq + \int_1^\infty dp \int_{p-1}^{p+1} dq \right) \right. \\ &\quad \left. \times q^2 |\sin \alpha| (p^2 - q^2) (1 - q^2) k^4 R_{kp}(\tau) I_{kq}(\tau) \right]. \quad (45) \end{aligned}$$

Here $p \equiv |p|$, we have normalized both p and q to k , and $\sin \alpha \equiv p \cdot q / pq$ with $p+q=k$. Formula (45) is valid for both equilibrium and nonequilibrium states. It agrees with Eqs. (4.1) and (4.2) of Ref. 18. In general, the direct-interaction approximation

provides a second independent equation for $I_k(\cdot)$; however, we do not write this here.

Following Taylor, let us now consider thermal equilibrium explicitly. A form of the fluctuation-dissipation theorem¹⁹ allows us to write

$$I_k(\tau) = R_k(\tau) I_k(0) \quad (\tau > 0) \quad (46)$$

with the thermal spectrum¹⁴

$$I_k(0) = \frac{4\pi T}{k^2(1+k_D^2)} \quad (47)$$

Now direct solution of the cubically nonlinear equation (44) requires numerical analysis. Although this is quite tractable, we do not discuss it here because it is peripheral to our central purpose, which is to elucidate the structure of the renormalized theory. Analytically, we may follow Taylor and many others and try to understand the qualitative behavior of (44) by assuming

$$R_k(\tau) = H(\tau) \exp(-k^2 D_k \tau) \quad (48)$$

and then attempting to determine the wavenumber dependence of the damping coefficient D_k . Since (48) is not correct for all times, it is appropriate to find an equation for D_k by integrating Eq. (44) from 0_- to ∞ . One finds

$$D_k = \frac{1}{\pi} \left(\frac{c}{B} \right)^2 T (1+k_D^2 \lambda_D^2) \int_{\Delta} dp dq |\sin \alpha| \times \frac{(p^2 - q^2)^2}{(p^2 D_{kp} + q^2 D_{kq}) (1+k_D^2 \lambda_D^2 p^2) (1+k_D^2 \lambda_D^2 q^2)} \quad (49)$$

where Δ denotes the integration domain written explicitly in Eq. (45). In arriving at (49), we symmetrized the integrand in p and q .

The function D_k defined by the solution of the nonlinear integral equation (49) has qualitatively different behavior in the two regimes $k \gg k_D$ and $k \ll k_D$. It vanishes as $k \rightarrow \infty$. To examine the long-wavelength behavior, let it be assumed temporarily that the limit as $k \rightarrow 0$ of D_k approaches a constant, as would be true if transport was Markovian. To examine the convergence of the resulting integral, it is convenient to change variables from (p, q) to (p, λ) , where $q = 1 + \lambda p$ for $p \leq 1$ and $q = p + 1$ for $p > 1$. The geometric factor $\sin \alpha$ can be expressed in terms of p and λ through the law of cosines. One then finds that the integral converges near $p = 0$, but diverges logarithmically as $p \rightarrow \infty$. This implies that D_k diverges logarithmically as $k \rightarrow 0$. The gross k dependence may be estimated by noting that in the regime $p \gg 1$, which dominates the integral (49), we have $q \sim p$ and that the spectral factors $(1 + k^2 \lambda^2 p^2)$ provide an effective cutoff to the p integral at $p \sim \omega k_D / k$, where ω is a k -independent constant of order unity. Noting that $\sin \alpha \sim 1/p$ and that $p^2 - q^2 \sim -2\lambda p$, we find

$$\begin{aligned} D_k &\sim \frac{1}{\pi} \left(\frac{c}{B} \right)^2 T \int_1^{\omega k_D / k} \frac{dp}{p D_{kp}} \int_{-1}^1 d\lambda \lambda^2 \\ &= b \left(\frac{c}{B} \right)^2 T \int_k^{\omega k_D} \frac{dp}{p D_p}, \end{aligned} \quad (50)$$

where b is another numerical factor of order unity. An equation of the form (50) has been previously derived by Krommes and Oberman¹⁷ in the course of their work on convective cells. It

can be solved by differentiation with respect to k , followed by rearrangement and integration; the final result is

$$D_k = D_0 \ln^2(ak_D/k) \quad (k < k_D) \quad , \quad (51)$$

where

$$D_0 = (b/2)^{-1} (c/B) \sqrt{\pi} \quad (52)$$

is proportional to the Taylor-McNamara Bohm-like diffusion coefficient.¹⁴ That D_k diverges as $k \rightarrow 0$ implies that the interactions are nonlocal in space, a well-known result.¹⁷ Needless to say, one should not take the detailed form of (51) very seriously. In particular, more detailed analysis shows that logarithmic, non-local behavior manifests itself in time as well as in space. Fortunately, the detailed form of $\chi_k(\tau)$ is not critical to the principal thrust of our arguments.

Similar manipulations show that

$$\chi_k''' = k^2 D_k'' \quad , \quad (53)$$

where D_k'' is determined from the right-hand side of Eq. (49) with the factor of $k^2 \lambda_D^2$ in the numerator omitted. Thus, defining

$$D_k' \equiv D_k - D_k'' \quad , \quad (54)$$

we find that

$$D_k' = \left(\frac{k^2}{k^2 + k_D^2} \right) D_k \quad , \quad (55)$$

$$D_k'' = \left(\frac{k_D^2}{k^2 + k_D^2} \right) D_k \quad , \quad (56)$$

so

$$\begin{aligned}
 &= 1 + \frac{\gamma''}{-i\omega + \gamma'} \\
 &= 1 + \left(\frac{k_D^2}{k^2} \right) \left(\frac{\left(\frac{k^2}{k^2 + k_D^2} \right) k^2 D_k}{-i\omega + \left(\frac{k^2}{k^2 + k_D^2} \right) k^2 D_k} \right). \quad (57)
 \end{aligned}$$

Simple manipulation reduces Eq. (57) to the form of Taylor's result (10), the sole difference being the wavenumber dependence of D_k .

III. SUMMARY and DISCUSSION

We have studied the dielectric function for the guiding center plasma in the direct-interaction approximation. In the special case of thermal equilibrium, the form of our result agrees completely with that of Taylor. The direct-interaction approximation predicts in addition an explicit formula for the broadening factor D_k , including the wavenumber dependence. In contradiction to most theories which postulate a wavenumber-independent diffusion coefficient, the proper result is logarithmically divergent at long wavelengths, indicating that a Markovian description is inappropriate. The significance of such behavior has been emphasized previously.¹⁷

Whereas in simple resonance-broadening theory the particle propagator g was broadened by the factor $k^2 D$, the analyses of Taylor and of us predict that this factor should be reduced by the amount $k^2/(k^2 + k_D^2)$, a reduction important at long wavelengths. We showed that this reduction arises from a cancellation between

the so-called diffusion and polarization parts of the turbulent collision operator $\mathcal{L}'$. Though we have demonstrated this cancellation explicitly only for our special model and only for thermal equilibrium, a reasonable assertion, supported by experience with weak turbulence theory, is that the polarization terms are important much more generally. Recent practical applications of resonance-broadening theory have ignored these terms.

The present analysis illustrates another important point which has been already suggested elsewhere⁸: the $\mathcal{L}'$ term is necessary for a proper treatment of the adiabatic response. Indeed, recalling that the numerator of Eq. (57) arises from $\mathcal{L}'$ and comparing Eq. (57) with Eq. (10), one observes in the numerator of Eq. (10) a complete cancellation between $\mathcal{L}'$ and $\mathcal{L}'$. In the framework of resonance-broadening theory, results of the form of Eqs. (10) or (11) are sometimes derived by integrating by parts^{9,16,20} prior to employing the statistical ansatz. However, we again caution that we have demonstrated the cancellation to be complete only for the special case of the thermal equilibrium guiding center plasma. Preliminary calculations suggest that the cancellation is not complete for the drift-kinetic equation at finite gyroradius, and that the correction terms may be qualitatively and quantitatively important, particularly in sheared geometries.

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