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**An Algorithm for Computing
Significance Levels Using the
Kolmogorov-Smirnov Statistic
and Valid for both Large
and Small Samples**

S. E. Kurtz
D. E. Fields

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Health and Safety Research Division

AN ALGORITHM FOR COMPUTING SIGNIFICANCE LEVELS USING THE KOLMOGOROV-SMIRNOV STATISTIC AND VALID FOR BOTH LARGE AND SMALL SAMPLES

ORNL-5819

DESA 001506

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ABSTRACT

The KSTEST code presented here is designed to perform the Kolmogorov-Smirnov one-sample test. The code may be used as a stand-alone program or the principal subroutines may be excerpted and used to service other programs.

The Kolmogorov-Smirnov one-sample test is a nonparametric goodness-of-fit test. A number of codes to perform this test are in existence, but they suffer from the inability to provide meaningful results in the case of small sample sizes (number of values ≤ 80). The KSTEST code overcomes this inadequacy by using two distinct algorithms. If the sample size is greater than 80, an asymptotic series developed by Smirnov is evaluated. If the sample size is 80 or less, a table of values generated by Birnbaum is referenced. Valid results can be obtained from KSTEST when the sample contains from 3 to 300 data points.

The program was developed on a Digital Equipment Corporation PDP-10 computer using the FORTRAN-10 language. The code size is approximately 450 card images and the typical CPU execution time is 0.19 s.

1. INTRODUCTION

The Kolmogorov-Smirnov (K-S) one sample, two-sided test is a useful, yet infrequently used, statistical test. It is a nonparametric goodness-of-fit test; that is, the test is a measure of how well two probability distribution functions match one another, independent of the statistical distributions of the data being compared. There are several possible applications of the Kolmogorov-Smirnov test, but the most common application, which will be discussed in this paper, is the comparison of an empirical probability distribution of a set of data with an assumed theoretical probability distribution. The K-S test has the advantage over the most common goodness-of-fit test, the χ^2 test, in that the K-S test treats the data individually rather than in groups. It may be considered to be, in general, the more powerful of the two tests (Siegel, 1956).

The K-S test may have applications wherever a goodness-of-fit test is required. It has, for example, been applied in the computer code TERPED/P (Glandon and Fields, 1982), a code which determines whether a set of data may be considered to be distributed normally or log-normally. Such an application can be very useful for the evaluation and assessment of environmental impacts and the output of mathematical models (Fields, 1981).

The original release of the TERPED/P code used IMSL* subroutines for the evaluation of the K-S test. This approach had the drawback that the IMSL routines give valid results only when $N \geq 80$, where N is the number of data points. NAG** subroutines and SAS*** are somewhat superior in this regard, having lower limits of $N=35$ and $N=50$, respectively. The methodology we have developed is valid for $N > 3$ data points.

* International Mathematical and Statistical Library, Inc., Houston, Texas.

** Numerical Algorithms Group, Downers Grove, Illinois.

*** Proprietary software distributed by SAS Institute, Inc., Raleigh, North Carolina.

This work is an outgrowth of an attempt to increase the usefulness of the TERPED/P code by overcoming the $N \geq 80$ constraint imposed by the IMSL subroutines.

The KSTEST code which we have developed is a stand-alone program package for the performance of the Kolmogorov-Smirnov one-sample test. The program has been constructed so that the user may invoke the code directly to perform the K-S test or may excerpt the principle subroutines for use in other programs. These subroutines have already been applied as a section of the TERPED computer code (Kurtz and Fields, in press). The TERPED code may be used to examine the statistical distribution of a data set and plot the data set.

2. KOLMOGOROV-SMIRNOV ONE SAMPLE TEST

The K-S one sample, two-sided test is a measure of the goodness of fit between the cumulative probability distribution of a set of data and an assumed theoretical cumulative probability distribution. The K-S statistic, D_N , on which the test is based, is the maximum difference between corresponding terms of the two distributions. The statistic is independent of the distributions themselves, being instead a function of N , the size of the data sample. Associated with the statistic and the value of N there is a unique probability level. This value is the probability that the difference D_N is due to random chance rather than a genuine difference between the distributions. Hence, the number (one minus the probability) is the significance level, α , at which one may say the two distributions are the same.

Consider a random variable X with the continuous distribution function $F(x)$ defined as the probability that X is less than or equal to some value x . That is,

$$F(x) = P(X \leq x).$$

Given that $X_1, X_2, \dots, X_N$ is a sample of X of size N such that

$$X_1 \leq X_2 \leq \dots \leq X_N,$$

the empirical probability distribution function $F_N(x)$ is usually defined by

$$F_N(x) = \begin{cases} 0 & x < X_1 \\ i/N & X_i \leq x \leq X_{i+1} \quad i=1, 2, \dots, N-1. \\ 1 & X_N \leq x \end{cases}$$

This cumulative distribution is a step function of height $1/N$ with N steps. When the distribution under consideration is normal, its cumulative probability is more accurately given by $F_N(x) = (i-3/8)/(N + 1/4)$ in the range $X_i \leq x \leq X_{i+1}$, $i=1, 2, \dots, N-1$ and 0 or 1 elsewhere (Kimball,

1960). We have used this definition for the particular case of a normal distribution.

The Kolmogorov-Smirnov statistic is now defined as

$$D_N = \max |F(x) - F_N(x)| \quad x = X_1, X_2, \dots, X_N.$$

In his original work Kolmogorov found that the distribution of D_N was dependent on N but independent of $F(x)$. He also derived formulas for determining the probability associated with the value of D_N (Kolmogorov, 1933).

In the general case, for a sample of any size the probability is given in a form involving a recursive definition. If c is some integer less than N , where N is the sample size, the probability associated with a particular value of D_N is given by

$$P(D_N < \frac{c}{N}) = \frac{N!}{N^N} e^N R_{0,N}(c).$$

$R_{i,k}(c)$ is defined for i any integer, k any positive integer, and c any positive integer less than or equal to N , and is given by

$$R_{0,0}(c) = 1, \quad R_{i,0}(c) = 0 \quad i \neq 0$$

$$R_{i,k}(c) = 0 \quad i \geq c$$

$$R_{i,k+1}(c) = \frac{1}{e} \sum_{s=0}^{2c-1} R_{i+1-s,k}(c) \frac{1}{s!} \quad i \leq c-1.$$

A table of values of $P(D_N < c/n)$ for $N=1(1)100$ and $c = \min(N, 15)$ has been computed by Birnbaum and has been used as a part of the computational algorithm (Birnbaum, 1952).

The recursive definition above tends to become quite cumbersome even for small values of N . To avoid the computational problems inherent in that method, Kolmogorov derived an asymptotic relation for

the probability in the limit as N approaches infinity. This relation is expressed as

$$L(z) = \lim_{N \rightarrow \infty} P(D_N < \frac{z}{N}) = 1 - 2 \sum_{k=1}^{\infty} (-1)^{k-1} e^{-2k^2 z^2}$$

where $z = D_N \sqrt{N}$. An equivalent form is found to be

$$L(z) = \frac{\sqrt{2\pi}}{z} \sum_{k=1}^{\infty} \exp(-(2k-1)^2 \pi^2 / 8z^2)$$

which is more useful for small z . Both of these series have the advantage of very rapid convergence - three terms are usually sufficient. The condition that N approaches infinity, however, makes them unsatisfactory for small values of N with the cutoff point frequently being placed around $N=80$. $L(z)$ has been tabulated from $z=0.28$ to $z=3.00$ (Smirnov, 1948).

The first step in the computation of probability levels is to obtain the statistic D_N . When the continuous cumulative probability of the theoretical distribution is plotted together with the step function of the empirical distribution, the role of D_N becomes graphically clear (Fig. 1).

The endpoints of each line segment are determined by adjacent values of X , the data, and the step size is $1/N$. The difference between the two distributions are simply the distances from the ends of the line segments to the curve itself. The maximum difference, D_N , is the maximum distance. Clearly two values of the step function must be examined for each data point since the step function suffers both a left-handed and a right-handed discontinuity. Procedurally the approach is to calculate the ordinate of the theoretical distribution at each data point then determine the difference between this ordinate and the ordinates of the two steps of the empirical distribution, one on either side.

Since the differences are calculated at corresponding points of the two distributions, care must be taken in the evaluation of tied data points to insure that the correct ordinates are used. Tied data values may be thought of as line segments of width zero, appearing graphically as steps of size m/N where m is the number of times the value is

Figure 1. This figure shows the determination of the statistic D_N from the difference between corresponding data and fitted values.

repeated. For example, in Fig. 1 the value 0.2 occurs twice and appears as a step of size $2/10$. In the case of multiple tied data points, the right- and left-handed differences need not be calculated for each repeated values - once for each side is sufficient. However, since the cumulative probability of the empirical distribution continues to increase while the data values remain constant, the left-handed difference must be computed using the ordinate of the first tied value while the right-handed difference must be computed using the ordinate of the final tied value. The ordinate of the theoretical distribution is the same in either case.

After the statistic is found, the associated probability may be determined. In the case of small N , $N \leq 80$, the probability may be found from Kolmogorov's original formulation involving the recursive definition: D_N is characterized as c/N where the smallest positive integer value for c is chosen such that $D_N < c/N$. Application of the recursive definition for $R_{i,k}$ with parameter c then provides the value of $R_{0,N}(c)$ which may be substituted into Kolmogorov's equation to obtain the probability.

If the value of N is large, $N > 80$, it may be desirable to use one of the asymptotic relations. This is done by forming the weighted statistic $z = D_N \sqrt{N}$ and selecting the appropriate series based on the value of z . The summation will give the probability, $L(z)$.

Additional methodologies for calculating the value of the K-S statistics and its associated probability have been developed and a number of them are summarized by Pomeranz (Pomeranz, 1973).

3. KSTEST ALGORITHM

For computational purposes the algorithm used by the KSTEST code varies slightly from the procedure outlined in the preceding section.

For the case of $N \leq 80$ the table of probabilities generated by Birnbaum has been used to avoid the recursive calculation of the values of $R_{i,k}(c)$. At even moderate values of N , avoidance of the recursive calculation is almost essential since the number of terms generated by the recursive definition increases as N^3 . By using the tabulated values of the probabilities, we have avoided using Kolmogorov's equation at all and, hence, need not concern ourselves with the computation of $R_{0,N}(c)$.

Birnbaum's table gives probabilities for D_N parameterized by various values of c and N . Our method is to compute the statistic and then find integer values c_1 and c_2 such that

$$\frac{c_1}{N} \leq D_N \leq \frac{c_2}{N}.$$

Probabilities corresponding to these values of c_1/N and c_2/N are then obtained from the table and adjusted by linear interpolation. Probabilities in the table are given to five decimal places and error analysis of the computational methods used in its generation indicates that all five digits are significant (Birnbaum, 1952).

For the case of $N > 80$, the alternative approach of the asymptotic series is used. The weighted statistic, $z = D_N \sqrt{N}$, is formed and based on its size the appropriate form of the series is selected. $z=0.80$ is used as the transition value between the two series.

It is of interest to consider why $N=80$ was chosen as the transition between the Birnbaum method and the asymptotic series approach. Although Birnbaum's table goes to $N=100$, it was decided to use only the portion for which $N \leq 80$. This value was chosen for two reasons. First, whereas the values in the table are accurate to five places, the table itself is incomplete. To maintain five digit accuracy for all characterizations of D_N would have required values of c ranging from 1 to approximately $4\sqrt{N}$. This implies an upper value of $c=35$ for $N=80$ and $c=40$ for $N=100$. Birnbaum's table stops at $c=15$, however. The

discrepancy between $c=15$ and $c=35$ does cause some error at $N=80$ and below, but this was not deemed worthy of concern - the worst case relative error being approximately 0.62%. The discrepancy between $c=15$ and $c=40$ in the region of $80 < N \leq 100$ was found to be more significant, however. Worst case relative errors ranged from 0.66% to 1.98%, so this section of the table was omitted.

The second justification for the asymptotic series for $N > 80$ is that both methods - interpolation of the table and summation of the series - give comparable results in the immediate vicinity of $N=80$. So long as D_N is not characterized by a c value close to the upper limits of the table, the tabular method gives more accurate results.

For all but the smallest values of c , the asymptotic method gives good agreement even for $80 < N \leq 100$. However, in the case of small c , resulting from a small value for D_N , the asymptotic method can give poor results, a weakness shared by the IMSL routine for performing the K-S test. Relative errors can become as large as several hundred percent. Little benefit can be gained by extending the use of the table to $N=100$ since the poor performance of the asymptotic method at small c is not significantly better in the vicinity of $N=100$ than in the vicinity of $N=80$.

In the region of $80 < N < 100$ the user may wish to consult Birnbaum's table if $D_N \leq 4/N$.

4. PROGRAM STRUCTURE

The subprogram structure of the KSTEST code is shown in Fig. 2. The code consists of a main program and seven subprograms which perform the actual computations of the statistical parameters. A FORTRAN-10 listing of the code is given in Appendix A.

The Main Program structures flow of control, accepts data and control information from the user and outputs the computed values. Subroutine SISORT performs a straight-insertion sort of the data values, arranging them into ascending order. SANDM calculates the sample mean and standard deviation of the data set. Subroutine NKS calculates the K-S statistic after calling DSNORM which computes the normalized cumulative probability of the theoretical distribution. DSNORM is the probability distribution function generation subroutine which may be designated by the user and which gives the form of the theoretical cumulative probability distribution. In this case we have assumed a normal distribution and hence have added the NAG routine ERF which is called by DSNORM and calculates values of the error function.

Figure 2. The KSTEST computer code consists of a MAIN program and seven subprograms.

5. DISCUSSION OF CODE USAGE

KSTEST is an interactive code; a sample run showing the computer-user dialogue is reproduced in Appendix C.

Upon invoking the code, the user first specifies the method of data entry - by hand or from a data file. If file input is specified, a file number is requested. Regardless of the method of input, the number of values to be read is requested, followed by a query on the arrangement of the data. A negative response, indicating that the data are not ordered, will cause subroutine SISORT to be invoked. Once data input is complete, computations are begun and control does not return to the user until their completion, whereupon it is possible to select a listing of the ordered data. Finally, values of the calculated parameters are printed and the program is terminated or restarted at the user's discretion.

KSTEST performs a number of checks on user-supplied values and select calculated values and prints appropriate warnings. All control information submitted by the user is checked for correct form. If an input value is not in the correct form, the program prints a warning and awaits a valid response.

The program also checks the size of the data set. A warning is printed if there are more than 300 points. If hand input was specified, the program then repeats its request for the number of data points. If file input was specified with more than 300 points to be read, the same message is printed and a new response is awaited. If all points in the file are to be read, the program attempts to read 300 values. If the full 300 are read, a warning is issued and execution proceeds.

The 300 value limit may be modified by changing four lines in the main program. These statements are the data array DIMENSION statement and the three logical IF statements which check the size of the sample.

The KSTEST code is a stand-alone program package for the execution of the Kolmogorov-Smirnov one sample test. Care has been taken in the development of the code so that the principle subroutines are readily withdrawn for use in existing programs.

The subroutines which perform the actual computations for the K-S test are NKS and DSNORM, along with the real function ERF, NLE80 and NGT80. NKS is the calling subprogram for all of the other subroutines.

To excerpt these subroutines for use in an existing program, it will be necessary for the calling program (which will call NKS) to define the following items: the sorted data array, the length of this array, the variables to be used for the statistic D_N , and the significance level α . In addition, the sample mean and standard deviation have been passed to DSNORM through COMMON. The prospective user will need to pass these values as subroutine arguments for NKS and DSNORM or adjust COMMON storage appropriately.

The subroutine SANDM for calculating the mean and standard deviation and SISORT for ordering the data have not been included among the routines essential to the test. Many programs which might use the K-S test may already have provisions for these tasks. If desired, both SANDM and SISORT may be carried over intact with the other routines. Some discretion should be used regarding the SKSORT subroutine, however. SISORT arranges the sample data into ascending order by means of a straight insertion sort. This sorting algorithm is not highly efficient. If large data sets are to be evaluated, the user may wish to adopt a different sorting algorithm.

When the core of principal subroutines is used to service another program, the theoretical probability distribution subroutine DSNORM must be specified in the user's main program by means of an EXTERNAL statement. Further flexibility is available to the user by modifying DSNORM to calculate some distribution function other than normal. Several such possibilities and the requisite coding changes are given in Appendix D. If the user selects a distribution other than normal, some slight modifications should also be made to subroutine NKS. NKS calculates the cumulative probabilities of the empirical distribution based on the assumption that it is a normal distribution. Because of this assumption, mentioned in Sect. 2, NKS uses the modified formula for the cumulative probability due to Kimball. If the user selects a nonnormal distribution, two lines of code will need to be changed in the NKS subprogram. These changes are also outlined in Appendix D.

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APPENDIX A. FORTRAN-10 LISTING OF KSTEST CODE

The KSTEST code executes on the DEC PDP-10 computer. The only utility required is the NAG real function ERF for the evaluation of the error function.


```

C
C CODE KSTEST
C ORIGINAL VERSION RELEASED 8/83 BY S.E. KURTZ AND D.E. FIELDS
C FOR FURTHER DOCUMENTATION CCNSULT S.E. KURTZ AND D.E. FIELDS,
C AN ALGORITHM FOR COMPUTING SIGNIFICANCE LEVELS USING THE
C KOLMOGOROV-SMIRNOV STATISTIC AND VALID FOR BOTH LARGE AND SMALL
C SAMPLES, ORNL-5819, OAK RIDGE NATICNAL LABORATORY.
COMMON XMEAN,STDEDEV
EXTERNAL DSNCRM
DATA H,F/'HAND','FILE'//,YES,XNC/'YES','NO'//,R,E/'R','E'/
WRITE(5,1000)
10 WRITE(5,1100)
READ(5,1200) A
IF(A.NE.F) GOTO 20
WRITE(5,1300)
READ(5,1400) NIN
GOTO 40
20 IF(A.EQ.H) GOTO 30
WRITE(5,1500)
GOTO 10
30 NIN=1000
40 WRITE(5,1600)
WRITE(5,1700)
READ(5,1800) N
IF(N.LE.300) GOTO 50
WRITE(5,1900)
GOTO 40
50 WRITE(5,2000)
READ(5,1200) A
IF(A.EQ.XNO) GOTO 60
IF(A.EQ.YES) GOTO 60
WRITE(5,1500)
GOTO 50
60 IF(NIN.NE.1000) GOTO 70
WRITE(5,2100)
READ(5,2200) (X(I),I=1,N)
GOTO 110

```

```

70  IF(N.EQ.0)GOTO 81
    READ(NIN,2200) (X(I),I=1,N)
    GOTO 100
80  DO 90 I=1,300
    READ(NIN,2200,END=100) X(I)
90  N=I
100  REWIND NIN
110  WRITE(5,2300)
C    BEGIN DATA ANALYSIS
    IF(A.EQ.XNO)CALL SISOPT(X,N)
    CALL SANDY(X,N)
    CALL NKS(DSNORM,X,N,DN,ALPHA)
    WRITE(5,2400)
120  WRITE(5,2500) N
    WRITE(5,2600)
    READ(5,1200) A
    IF(A.EQ.YES)GOTO 130
    IF(A.EQ.XNO)GOTO 150
    WRITE(5,1500)
    GOTO 120
130  WRITE(5,2700)
    WRITE(5,2800)
    DO 140 I=1,N
140  WRITE(5,2900) I,X(I)
150  WRITE(5,3000) N
    WRITE(5,3100) XMEAN
    WRITE(5,3200) STDDEV
    WRITE(5,3300) DN
    WRITE(5,3400) ALPHA
160  WRITE(5,3500)
    READ(5,3600) A
    IF(A.EQ.F)GOTO 10
    IF(A.EQ.E)GOTO 170
    WRITE(5,1500)
    GOTO 160

```

```

170  WRITE(5,3700)
    STOP
1800  FORMAT(2X,'CODE KSTEST'//)
1900  FORMAT(2X,'ENTER FORM OF DATA INPUT: HAND INPUT OR FILE? (4 CHAR)')
2000  FORMAT(A4)
2100  FORMAT(2X,'ENTER INPUT FILE NUMBER')
2200  FORMAT(I2)
2300  FORMAT(2X,'INVALID INPUT; TRY AGAIN')
2400  FORMAT(2X,'ENTER NUMBER OF DATA POINTS TO BE USED')
2500  FORMAT(2X,' (IF YOU SPECIFIED FILE INPUT, 0 WILL CAUSE ALL POINTS
    'TO BE USED)')
2600  FORMAT(I3)
2700  FORMAT(2X,'NUMBER OF POINTS CANNOT EXCEED 300')
2800  FORMAT(2X,'ARE DATA ORDERED? (YES,NO)')
2900  FORMAT(2X,'ENTER DATA, ONE VALUE PER LINE')
3000  FORMAT(F12.4)
3100  FORMAT(2X,'DATA INPUT COMPLETED; ANALYSIS STARTED')
3200  FORMAT(2X,'ANALYSIS COMPLETED')
3300  FORMAT(2X,'THERE ARE ',I3,' DATA POINTS')
3400  FORMAT(2X,'WOULD YOU LIKE THEM PRINTED? (YES,NO)')
3500  FORMAT(2X,'RANK ORDERED DATA ARE:')
3600  FORMAT(2X,' I',8X,'VALUE')
3700  FORMAT(1X,I3,3X,E10.4)
3800  FORMAT(//2X,'SAMPLE SIZE',13X,'N= ',I3)
3900  FORMAT(2X,'MEAN',20X,'M= ',E9.4)
4000  FORMAT(2X,'STANDARD DEVIATION',6X,'SD= ',E9.4)
4100  FORMAT(2X,'K-S STATISTIC',11X,'DN= ',F7.4)
4200  FORMAT(2X,'LEVEL OF SIGNIFICANCE',3X,'ALPHA= ',F7.4)
4300  FORMAT(//2X,'ENTER R FOR RESTART, E FOR EXIT')
4400  FORMAT(A1)
4500  FORMAT(/2X,'END KSTEST')
    END

```

```

SUBROUTINE NGTR0 (STAT, PFCB2, N)
  XN=N
  Z=STAT*SQRT (XN)
  ZSQ=Z*Z
  C      SELECT FORMULA FOR AND COMPUTE PROBABILITY,
  C      BASED ON RANGE OF STATISTIC
  C      0.22 < Z
  IF (Z.GT.0.22) GOTO 10
  PFCB2=0.
  GOTO 40
  C      0.22 < Z < 0.80
  10 IF (Z.GT.0.80) GOTO 20
  PFCB2=(2.506628/Z)*EXP (-1.233700/ZSQ)
  GOTO 40
  20 IF (Z.GT.3.15) GOTO 30
  C      0.80 < Z < 3.15
  PFCB2=1.-2.*(EXP (-2.*ZSQ)-EXP (-8.*ZSQ)+EXP (-18.*ZSQ))
  GOTO 40
  C      3.15 < Z
  30 PFCB2=1.
  40 RETURN
END

```

```

SUBROUTINE SISORT(X,N)
DIMENSION X(N)
DO 5 I=2,N
  XT=X(I)
  L=I-1
C      COMPARE ADJACENT DATA ELEMENTS
1  IF(XT.LT.X(L).AND.L.GE.1)GOTO 3
C      GO TO NEXT DATA ELEMENT, NO INSERTION
  M=L+1
  X(M)=XT
GOTO 5
C      INSERT DATA ELEMENT AND DECREMENT
C      THEN RETURN TO COMPARE WITH NEXT LOWER ELEMENT
3  M=M-1
  X(M)=X(L)
  L=L-1
GOTO 1
5  CONTINUE
RETURN
END

```

```

SUBROUTINE SAND*(X,N)
COMMON XMEAN,STDDEV
DIMENSION X(N)
XN=N
SUM=0.
DO 10 I=1,N
10  SUM=SUM+X(I)
  XMEAN=SUM/XN
  SUM=0.
DO 20 I=1,N
20  SUM=SUM+(XMEAN-X(I))**2
  STDDEV=SQRT(SUM/(XN-1.))
RETURN
END

```

```

SUBROUTINE NKS (PDF,X,N,DN,ALPHA)
DIMENSION X(N)
C      INITIALIZE VARIABLES
EDF1=0.
EDF2=0.
DN=0.
XN=N
XN=XN+.25
10  I=1
20  J=I
C      CHECK FOR TIED DATA VALUES
30  IF(X(I).LT.X(I+1))GOTO 40
    I=I+1
    IF(I.EQ.N)GOTO 40
    GOTO 30
40  XI=I
    XJ=J
C      COMPUTE CUMULATIVE PROBABILITIES FROM ABOVE AND BELOW
EDF1=(XJ-1.375)/XN
EDF2=(XI-.375)/XN
CALL PDF(X(I),F)
C      COMPUTE FOR NEW MAXIMUM DIFFERENCE
DN=AMAX1(DN,ABS(F-EDF1),ABS(F-EDF2))
C      INCREMENT COUNTERS AND PICK FCINT OF LOOP REENTRY
I=I+1
IF(I.LT.N)GOTO 20
J=I
IF(I.EQ.N)GOTO 40
C      CHOOSE AND CALL PROBABILITY SUBROUTINE
IF(N.GT.80)GOTO 50
CALL NLF90(DN,PROB2,N)
GOTO 60
50  CALL NGT90(DN,PROB2,N)
60  ALPHA=1.-PROB2
    RETURN
END

```

	SUBROUTINE DSNCPM(X,F)		
	COMMON XMEAN,STDDEV		
	X1=(X-XMEAN)/STDDEV		
	P=0.5*(1.-ERF(-X1*0.7071068))		
	RETURN		
	END		
	REAL FUNCTION EFF(X)	ERF	0
C16	DOUBLE PRECISION FUNCTION EPFC(X,IFAIL)	ERF	5
C	MARK 4 RELEASE NAG COPYRIGHT 1974.	ERF	10
C	EDITED BY JOYCE CLARKE OXFORD OEG NUCLEAR PHYSICS 11TH SEP 1976	ERF	15
C	FORTRAN MACFC VERSION FDIA24.TEC	ERF	20
C	MARK 4.5 REVISED	ERF	25
C	EPFC(X)	ERF	30
	INTEGER NCF, IFAIL, J	ERF	35
	REAL XLC, XHI, ZERO, ONE, TWO, THREE, TWELVE, HALF, X, X2, BJP2,	ERF	40
	1 BJP1, BJ, SUM	ERF	45
C		ERF	50
C07	REAL C(11)	ERF	55
C07	DATA NCF/11/,XLC,XHI/-4.0,8.0/	ERF	60
C07	A,C(1),C(2),C(3),C(4),C(5),C(6),C(7),C(8),C(9),C(10),C(11)	ERF	65
C07	A/6.597255E-1,-4.536615E-1,1.603583E-1,-4.49081E-2,9.6414E-3	ERF	70
C07	A,-1.4425E-3,1.010E-4,1.21E-5,-3.6E-6,1.0E-7,1.0E-7/	ERF	75
C		ERF	80
	REAL C(15)	ERF	85
	DATA NCF/16/,XLO,XHI/-6.0,12.0/,C(1),C(2),C(3),C(4),C(5),	ERF	90
1	C(6),C(7),C(8),C(9),C(10),C(11),C(12),C(13),C(14),C(15)	ERF	95
1	,C(16)/6.5972554951E-1,-4.5366152054E-1,1.6039834082E-1,-	ERF	100
1	4.490812115E-2,9.64142865E-3,-1.44250808E-3,1.0095437E-4,	ERF	105
1	1.208502E-5,-3.62542E-6,7.187E-8,8.914E-8,-7.42E-9,-2.39E-9,	EPF	110
1	3.1E-10,3.E-11,-1.E-11/	ERF	115
C		ERF	120
C14	REAL C(21)	ERF	125
C14	DATA NCF/21/,XLO,XHI/-6.0,14.0/	ERF	130
C14	A,C(1),C(2),C(3),C(4),C(5),C(6),C(7),C(8),C(9),C(10),C(11),C(12)	ERF	135
C14	A,C(13),C(14),C(15),C(16),C(17),C(18),C(19),C(20),C(21)	ERF	140
C14	A/6.5972554950607E-1,-4.5366152053780E-1,1.6039834082156E-1	ERF	145

C14	A, -4.497812115112E-2, 9.64142865070E-3, -1.44250807774E-3	ERF	150
C14	A, 1.0099436735E-4, 1.208502453E-5, -3.62542060E-6, 7.186811E-8	ERF	155
C14	A, 8.914236E-8, -7.42157E-9, -2.38934E-9, 3.0622E-10, 7.789E-11	ERF	160
C14	A, -1.797E-11, -3.09E-12, 3.5E-13, 1.4E-13, -1.0E-14, -1.0E-14/	ERF	165
C		ERF	170
	DATA ZERO, ONE, TWO, THREE, TWELVE, HALF /0.0, 1.0, 2.0, 3.0, 12.0,	ERF	175
	1 0.5/	ERF	180
C		ERF	185
	IF (X.LE.XLC) GO TO 30	ERF	190
	IF (X.GE.XHI) GO TO 40	ERF	195
	X2 = TWO - TWELVE/(ABS(X) + THREE)	ERF	200
C		ERF	205
C	SUMMATION	ERF	210
	BJP2 = ZERO	ERF	215
	BJP1 = C(NCF)	ERF	220
	J = NCF - 1	ERF	225
10	BJ = X2*BJP1 - BJP2 + C(J)	ERF	230
	IF (J.EQ.1) GO TO 20	ERF	235
	BJP2 = BJP1	ERF	240
	BJP1 = BJ	ERF	245
	J = J - 1	ERF	250
	GO TO 10	ERF	255
20	SUM = HALF*(BJ-BJP2)*EXP(-X*X)	ERF	260
	IF (X.LT.ZERO) GO TO 50	ERF	265
	ERFC = SUM	ERF	270
	GO TO 60	ERF	275
30	ERFC = TWO	ERF	280
	GO TO 60	ERF	285
40	ERFC = ZERO	ERF	290
	GO TO 60	ERF	295
50	ERFC = TWO - SUM	ERF	300
60	EXP=1.-ERFC	ERF	305
	RETURN	ERF	310
	END	ERF	315


```

SUBROUTINE NLE8C (STAT, FPCP2, N)
  DIMENSION XKS (15, 80)
  DATA XKS/ 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000, 1.00000,
& .50000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000,
& .22222, .92593, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000,
& .09375, .81250, .99219, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000,
& .03840, .69120, .96992, .99936, 1.00000, 1.00000, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000,
& .01543, .57656, .93441, .99623, .99996, 1.00000, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000,
& .00612, .47446, .88937, .98911, .99960, 1.00000, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000,
& .00240, .38659, .83842, .97741, .99849, .99996, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000,
& .00094, .31261, .78442, .96121, .99615, .99982, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& 1.00000,
& .00036, .25128, .72946, .94101, .99222, .99943, .99998, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& .00014, .20100, .67502, .91747, .98648, .99865, .99993, 1.00000,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& .00005, .16014, .62209, .89126, .97885, .99732, .99979, .99999,
& 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000,
& .00002, .12715, .57136, .86304, .96935, .99530, .99953, .99997,

```

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8	.00000	.07950	.47795	.80275	.94517	.98882	.99837
8	.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8	.00000	.06265	.43564	.77158	.93081	.98425	.99736
8	.99997	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8	.00000	.04927	.39637	.74019	.91517	.97875	.99598
8	.99994	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8	.00000	.03869	.35991	.70887	.89844	.97235	.99419
8	.99999	.99999	1.00000	1.00000	1.00000	1.00000	1.00000
8	.00000	.03033	.32636	.67784	.88079	.96506	.99195
8	.99980	.99998	1.00000	1.00000	1.00000	1.00000	1.00000
8	.00000	.02374	.29553	.64728	.86237	.95693	.98924
8	.99968	.99996	1.00000	1.00000	1.00000	1.00000	1.00000
8	.00000	.01857	.26729	.61733	.84335	.94802	.98605
8	.99949	.99997	.99999	1.00000	1.00000	1.00000	1.00000
8	.00000	.01450	.24147	.58811	.82386	.93837	.98236
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8	.99890	.99982	.99998	1.00000	1.00000	1.00000	1.00000
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8	.99725	.99943	.99990	.99999	1.00000	1.00000	1.00000
8	.00000	.00416	.14334	.45517	.72309	.88128	.95661
8	.99645	.99921	.99985	.99998	1.00000	1.00000	1.00000
8	.00000	.00323	.12885	.43145	.70288	.86851	.95010
8	.99551	.99894	.99979	.99997	1.00000	1.00000	1.00000
8	.00000	.00251	.11575	.40870	.68280	.85541	.94318
8	.99441	.99861	.99971	.99995	.99999	1.00000	1.00000
8	.00000	.00195	.10392	.38693	.66290	.84203	.93588
8	.99315	.99821	.99960	.99992	.99999	1.00000	1.00000
8	.00000	.00151	.09325	.36612	.64323	.82843	.92822

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8	.99912,	.99717,	.99931,	.99985,	.99997,	1.00000,	1.00000,
8	.99994,	.99991,	.99947,	.99929,	.99947,	.99969,	.99912,
8	.99834,	.99652,	.99911,	.99980,	.99996,	1.00000,	1.00000,
8	.99996,	.99971,	.99717,	.99923,	.99950,	.99963,	.99932,
8	.99838,	.99578,	.99886,	.99973,	.99994,	.99999,	1.00000,
8	.99991,	.99954,	.99916,	.99925,	.99944,	.99929,	.99947,
8	.99823,	.99494,	.99857,	.99965,	.99992,	.99999,	1.00000,
8	.99991,	.99942,	.99936,	.99970,	.99934,	.99931,	.99938,
8	.99819,	.99399,	.99824,	.99954,	.99990,	.99998,	1.00000,
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8	.99613,	.99596,	.99504,	.99842,	.99955,	.99988,	.99997,
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8	.99990,	.99905,	.99916,	.99911,	.99988,	.99958,	.99915,
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8	.99529,	.99833,	.99923,	.99942,	.99919,	.99977,	.99994,
8	.99990,	.99903,	.99915,	.99922,	.99908,	.99912,	.99983,
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8	.92161,	.96304,	.98392,	.99356,	.99762,	.99919,	.99975,
8	.92161,	.96304,	.98392,	.99356,	.99762,	.99919,	.99975,
8	.91662,	.96011,	.98237,	.99280,	.99729,	.99906,	.99970,
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8	.91152,	.95708,	.98073,	.99200,	.99693,	.99891,	.99964,
8	.91152,	.95708,	.98073,	.99200,	.99693,	.99891,	.99964,
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8	.90632,	.95393,	.97900,	.99113,	.99654,	.99875,	.99958,
8	.90102,	.95069,	.97721,	.99022,	.99611,	.99857,	.99951,
8	.90102,	.95069,	.97721,	.99022,	.99611,	.99857,	.99951,
8	.89562,	.94734,	.97531,	.98924,	.99565,	.99837,	.99943,
8	.89562,	.94734,	.97531,	.98924,	.99565,	.99837,	.99943,
8	.89013,	.94390,	.97334,	.98821,	.99515,	.99815,	.99934,
8	.89013,	.94390,	.97334,	.98821,	.99515,	.99815,	.99934,
8	.88455,	.94036,	.97129,	.98712,	.99462,	.99791,	.99925,
8	.88455,	.94036,	.97129,	.98712,	.99462,	.99791,	.99925,
8	.87889,	.93674,	.96916,	.98598,	.99406,	.99765,	.99914,
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8	.87316,	.93302,	.96695,	.98477,	.99345,	.99737,	.99902,
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8	.86736,	.92921,	.96466,	.98351,	.99281,	.99707,	.99889,
8	.86736,	.92921,	.96466,	.98351,	.99281,	.99707,	.99889,
8	.86150,	.92533,	.96231,	.98218,	.99212,	.99674,	.99874,
8	.86150,	.92533,	.96231,	.98218,	.99212,	.99674,	.99874,
8	.85557,	.92136,	.95986,	.98080,	.99140,	.99639,	.99858,
8	.85557,	.92136,	.95986,	.98080,	.99140,	.99639,	.99858,
8	.84958,	.91731,	.95735,	.97936,	.99063,	.99602,	.99841,
8	.84958,	.91731,	.95735,	.97936,	.99063,	.99602,	.99841,
8	.84355,	.91320,	.95476,	.97786,	.98983,	.99562,	.99823,
8	.84355,	.91320,	.95476,	.97786,	.98983,	.99562,	.99823,
8	.83746,	.90901,	.95211,	.97630,	.98898,	.99519,	.99803,
8	.83746,	.90901,	.95211,	.97630,	.98898,	.99519,	.99803,

8	.77000	.00000	.00137	.73828	.17061	.36736	.56217	.71919
8	.83133	.90475	.94938	.97469	.98809	.99474	.99781	
8	.00000	.00000	.00122	.03589	.16406	.35819	.55280	.71115
8	.82516	.90042	.94659	.97201	.98716	.99425	.99758	
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8	.00000	.00000	.00096	.03155	.15165	.34043	.53437	.69510
8	.81271	.89159	.94080	.96950	.98518	.99321	.99707	
8	.00000	.00000	.00086	.02958	.14578	.33183	.52531	.68712
8	.80644	.88719	.93781	.96765	.98412	.99264	.99678	
8	.00000	.00000	.00076	.02772	.14013	.32342	.51635	.67916
8	.80014	.88253	.93476	.96576	.98302	.99204	.99648	
8	.00000	.00000	.00068	.02598	.13468	.31519	.50750	.67123
8	.79382	.87792	.93165	.96380	.98187	.99142	.99616	
8	.00000	.00000	.00060	.02435	.12943	.30714	.49875	.66333
8	.78748	.87326	.92848	.96180	.98065	.99076	.99582	
8	.00000	.00000	.00053	.02282	.12438	.29928	.49011	.65546
8	.78112	.86856	.92525	.95974	.97946	.99008	.99546	
8	.00000	.00000	.00047	.02138	.11951	.29159	.48158	.64764
8	.77475	.86381	.92197	.95762	.97819	.98936	.99508	
8	.00000	.00000	.00042	.02003	.11482	.28407	.47316	.63985
8	.76836	.85902	.91864	.95546	.97687	.98861	.99468	
8	.00000	.00000	.00037	.01877	.11031	.27672	.46485	.63211
8	.76197	.85419	.91525	.95324	.97552	.98783	.99426	
8	.00000	.00000	.00033	.01758	.10597	.26955	.45664	.62441
8	.75557	.84932	.91182	.95098	.97412	.98702	.99382	

DIFS=.0

XN=N

C FIND VALUES ON EACH SIDE OF STAT, EXPRESSIBLE AS C/N,

C WHERE C IS AN INTEGER

DC 10 I=0,N

XI=I

DIF=XI/XN

IF(STAT.LT.DIFS)GOTO 5

IF(STAT.IT.DIF)GOTO 21

8	.99172,	.99773,	.99946,	.99989,	.99998,	1.00000,	1.00000,
8	.00000,	.00117,	.00363,	.00624,	.00882,	.01143,	.01402,
8	.99012,	.99717,	.99931,	.99985,	.99997,	1.00000,	1.00000,
8	.00000,	.00091,	.00182,	.00273,	.00364,	.00455,	.00546,
8	.98934,	.99652,	.99911,	.99980,	.99996,	1.00000,	1.00000,
8	.00000,	.00071,	.00142,	.00213,	.00284,	.00355,	.00426,
8	.98638,	.99578,	.99886,	.99973,	.99994,	.99999,	1.00000,
8	.00000,	.00054,	.00108,	.00162,	.00216,	.00270,	.00324,
8	.98423,	.99494,	.99857,	.99965,	.99992,	.99999,	1.00000,

8	.00000,	.00000,	.00137,	.00382,	.00661,	.00936,	.01217,	.01499,
8	.83133,	.90475,	.94938,	.97469,	.98809,	.99474,	.99781,	.99911,
8	.00000,	.00000,	.00122,	.00358,	.00640,	.00919,	.01195,	.01468,
8	.82516,	.90042,	.94659,	.97201,	.98716,	.99425,	.99758,	.99900,
8	.00000,	.00000,	.00109,	.00336,	.00617,	.00892,	.01163,	.01431,
8	.81895,	.89604,	.94373,	.97128,	.98619,	.99374,	.99733,	.99891,
8	.00000,	.00000,	.00096,	.00315,	.00596,	.00871,	.01142,	.01409,
8	.81271,	.89159,	.94080,	.96950,	.98518,	.99321,	.99707,	.99881,
8	.00000,	.00000,	.00086,	.00295,	.00578,	.00853,	.01124,	.01391,
8	.80644,	.88700,	.93781,	.96765,	.98412,	.99264,	.99678,	.99861,
8	.00000,	.00000,	.00076,	.00272,	.00555,	.00830,	.01101,	.01368,
8	.80014,	.88253,	.93476,	.96576,	.98302,	.99204,	.99648,	.99841,
8	.00000,	.00000,	.00068,	.00259,	.00542,	.00817,	.01088,	.01355,
8	.79382,	.87792,	.93165,	.96380,	.98187,	.99142,	.99616,	.99819,
8	.00000,	.00000,	.00060,	.00243,	.00526,	.00801,	.01072,	.01339,
8	.78748,	.87326,	.92848,	.96180,	.98065,	.99076,	.99582,	.99805,
8	.00000,	.00000,	.00053,	.00228,	.00511,	.00786,	.01057,	.01324,
8	.78112,	.86856,	.92525,	.95974,	.97946,	.99008,	.99546,	.99791,
8	.00000,	.00000,	.00047,	.00213,	.00496,	.00771,	.01042,	.01309,
8	.77475,	.86381,	.92197,	.95762,	.97819,	.98936,	.99508,	.99781,
8	.00000,	.00000,	.00042,	.00203,	.00486,	.00761,	.01032,	.01299,
8	.76836,	.85902,	.91864,	.95546,	.97687,	.98861,	.99468,	.99761,
8	.00000,	.00000,	.00037,	.00187,	.00470,	.00745,	.01016,	.01283,
8	.76197,	.85419,	.91525,	.95324,	.97552,	.98783,	.99426,	.99731,
8	.00000,	.00000,	.00033,	.00178,	.00461,	.00736,	.01007,	.01274,
8	.75557,	.84932,	.91182,	.95098,	.97412,	.98702,	.99382,	.99711,

DIFS=1.0

XN=N

C FIND VALUES ON EACH SIDE OF STAT, EXPRESSIBLE AS C/N,

C WHERE C IS AN INTEGER

DO 10 I=0,N

XI=I

DIF=XI/XN

IF(STAT.LT.DIFS)GOTO 5

IF(STAT.LT.DIF)GOTO 21

```

5      DIFS=DIF
10     ISAVE=I
20     ISAVE1=ISAVE+1
C      ACCESS PROBABILITIES FROM BLOCK DATA
      IF (ISAVE.EQ.0) GOTO 30
      PLOW=XKS (ISAVE,N)
      PHIGH=XKS (ISAVE1,N)
      GOTO 40
30     PLOW=0.
      PHIGH=XKS (ISAVE1,N)
C      INTERPOLATE PROBABILITIES
40     FPAC=(STAT-DIFS)/(DIF-DIFS)
      PFCB2=FLOW+(PHIGH-FLW)*FPAC
      RETURN
      END

```


APPENDIX B. INPUT DATA USED IN SAMPLE RUN OF APPENDIX C

The data set provided here is that used in the sample run of Appendix C.

.000270
.000460
.000410
.000092
.002100
.003900
.003000
.000770
.000024
.000480
.000180
.000970
.000074
.000430
.000130
.001100
.000780
.000210
.000290
.000990
.000078

APPENDIX C. SAMPLE RUN OF KSTEST CODE

The sample run of the KSTEST code which is produced here uses file input of the data in Appendix B. The SISORT sorting routine is selected and printing of the data file is suppressed.

This run is made in interactive mode on a DEC PDP-10 computer system.

```

EX KSTEST
LINK:  Loading
[LNKXCT KSTEST execution]

CODE KSTEST

ENTER FORM OF DATA INPUT:HAND INPUT OR FILE? (4 CHAR)
FILE
ENTER INPUT FILE NUMBER
43
ENTER NUMBER OF DATA POINTS TO BE USED
(IF YOU SPECIFIED FILE INPUT, 0 WILL CAUSE ALL POINTS
  TO BE USED)
0
ARE DATA ORDERED? (YES,NO)
NO
DATA INPUT COMPLETED; ANALYSIS STARTED
ANALYSIS COMPLETED

THERE ARE 21 DATA POINTS
WOULD YOU LIKE THEM PRINTED? (YES,NO)
YES
RANK ORDERED DATA ARE:

  I      VALUE
  1      .2400E-04
  2      .7800E-04
  3      .9200E-04
  4      .9400E-04
  5      .1300E-03
  6      .1800E-03
  7      .2100E-03
  8      .2700E-03
  9      .2900E-03
 10      .4100E-03
 11      .4300E-03
 12      .4600E-03
 13      .4800E-03
 14      .7700E-03

```

15	.7800E-03
16	.9700E-03
17	.9900E-03
18	.1100E-02
19	.2100E-02
20	.3000E-02
21	.3900E-02

SAMPLE SIZE	N= 21
MEAN	M= .7980E-03
STANDARD DEVIATION	SD= .1016E-02
K-S STATISTIC	DN= 0.2408
LEVEL OF SIGNIFICANCE	ALPHA= 0.1508

ENTER R FOR RESTART, E FOR EXIT
E

END KSTEST
STOP

END OF EXECUTION
CPU TIME: 0.19 ELAPSED TIME: 44.87
EXIT

APPENDIX D. NONNORMAL THEORETICAL DISTRIBUTION FUNCTIONS

It may be desirable in some cases to have a theoretical probability distribution other than the normal. The major modification to the code which is required for the use of other distributions is the replacement of the probability distribution function subroutine DSNORM. Several common distributions along with the corresponding DSNORM code, due to IMSL, are given below. In each case the lines of coding should be preceded by a SUBROUTINE DSNORM (X,F) statement and followed by RETURN and END statements. Note that distributions 2 and 3 include these final RETURN's as numbered statements. Additional information can be found in IMSL documentation for the MDSMR subroutine.

1. Cauchy with median H,
first quartile at H-Q,
Q $\neq$ 0
$$F = \text{ATAN}((X-H)/Q) * 0.3183099 + 0.5$$
2. Uniform in [A,B]
where B>A
$$\text{If } (X.GT.A) \text{ GOTO } 5$$

$$F=0.0$$

$$\text{GOTO } 15$$

$$5 \text{ If } (X.LT.B) \text{ GOTO } 10$$

$$F=1.0$$

$$\text{GOTO } 15$$

$$10 F=(X-A)/(B-A)$$

$$15 \text{ RETURN}$$
3. Exponential with mean A
$$\text{If } (X.GT.0) \text{ GOTO } 5$$

$$F=0.0$$

$$\text{GOTO } 10$$

$$5 F=1.0 - \text{EXP}(-X/A)$$

$$10 \text{ RETURN}$$

In addition to these changes which must be made to DSNORM to allow for nonnormal distributions, three lines of code must be changed in subroutine NKS as well.

These changes are:

delete $XN = XN + 0.25$

change $EDF1 = (XJ - 1.375)/XN$
and
 $EDF2 = (XI - 0.375)/XN$

to $EDF1 = (XJ - 1.00)/XN$
and
 $EDF2 = XI/XN$

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