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**Space-Charge Limit for an
Alternating-Phase-Focusing Structure**

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SPACE-CHARGE LIMIT FOR AN ALTERNATING-PHASE-FOCUSING STRUCTURE

by

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ABSTRACT

In a report titled "Space-Charge Limit for a Drift-Tube Linac from the Stability Point of View of Longitudinal Periodicity," the author showed that the stability condition for the longitudinal periodicity gives one of the upper limits and sometimes a lower limit for a drift-tube linac. The same method can be extended to an alternating-phase-focusing (APF) structure to determine the optimum parameters and also the current limit. Using the space-charge expression of Vlasov, the current limit for the Pion Generator for Medical Irradiation (PIGMI) APF structure is found.

I. INTRODUCTION

Some publications¹⁻⁶ have described the alternating-phase-focusing (APF) structure but they have discussed few practical applications of this device. However, B. P. Murin⁷ did report that an APF structure was used to accelerate 2 mA of protons from 50 to 575 keV.

Thus, the APF structure received little attention until D. A. Swenson of the Los Alamos Scientific Laboratory (LASL) adapted the low-energy structure for the Pion Generator for Medical Irradiation (PIGMI) project.^{8,9} A PIGMI model will accelerate the proton beam to a final 650-MeV energy with an average 100- μ A current for pion cancer therapy.

The purpose of the PIGMI project is to minimize the size of a pion-producing linear accelerator so that it can be installed beneath a hospital parking lot. To achieve this goal many innovations must be made in the technology for a conventional proton linear accelerator as used for high-energy physics research. For example, if the linac injection energy is

reduced to two or three hundred kilovolts, then the preinjector can be handled easily.

A paper by E. McMillan¹⁰ gives a general proof that phase stability and first-order focusing, which neglect the changes in velocity and in direction during one repeat length, are incompatible. However, both oscillations may be stabilized by an appropriate phase selection.¹¹

To avoid using focusing grids or foils to overcome the radial defocusing during acceleration, J. Blewett¹² proposed using successive magnetic fields of alternating transverse gradients. This method is essentially the development of the strong focusing theory.¹³ Blewett's paper discusses exclusively radial focusing for a constant synchronous phase linac.

In another paper¹⁴ I pointed out that it is possible to find one of the space-charge limits for an Alvarez linac through the extension of the strong focusing theory to longitudinal phase space.

This paper describes the application of this formulation to an APF structure.

II. TRANSFORM MATRIX OF PHASE SPACE

The longitudinal oscillation of a particle from the synchronous phase can be described by the following equations,

$$\frac{d}{dz} \left[\gamma_s^3 \beta_s^3 \frac{d}{dz} (\psi - \psi_s) \right] = - \frac{2\pi}{\lambda} \cdot \frac{e}{m_0 c^2} \left[E_z(r, z, t) - E_z(r, z, t_s) \right] , \quad (1)$$

or

$$\frac{d}{d\ell} \left[\gamma_s^3 \beta_s^2 \frac{d}{d\ell} (\psi - \psi_s) \right] = \frac{2\pi e \lambda \beta_s}{m_0 c^2} \left[E_z(r, z, t) - E_z(r, z, t_s) \right] , \quad (2)$$

where γ is the particle energy, β is the particle velocity, ψ is the particle phase, λ is the wavelength, and E is the electric field. The subscript s denotes the synchronous particle. Because it is convenient from a periodical

point of view, the independent variable $d\ell$ defines the number of rf cycles the synchronous particle needs to traverse the distance dz ,

$$dz = v_s dt_s = \beta_s \lambda d\ell, \quad (3)$$

where v is the particle velocity. The second term in brackets in Eq. (2) is the change in the rf electric field. For standing wave operation, this change is

$$E_z^{\text{rf}}(r, z, t) = E_{0z}(r, z) \cos \omega t, \quad (4)$$

plus the electric field caused by the space charge. Using the expression by Vlasov, (15) we get

$$E_z(r, z, t) - E_z(r, z, t_s) = - \left[E_{0z}(r, z) - E_0 T S \right] \sin \psi_s (\psi - \psi_s), \quad (5)$$

where

$$S = \frac{3\eta\lambda I M_z}{4\pi a_x a_y a_z T \sin \psi_s} \cdot \frac{1}{(\psi - \psi_s)_{\max}}, \quad (6)$$

$$M_u = \frac{a_x a_y a_z}{2} \int_0^\infty \frac{dS}{(a_u^2 + S) \sqrt{(a_x^2 + S)(a_y^2 + S)(a_z^2 + S)}}, \quad (7)$$

$\eta_0 = \sqrt{\mu_0/\epsilon_0} = 120 \pi \Omega$, a_u ($u = x, y, z$) is the half axis of the ellipsoid, I is the current, E_0 is the average field of the cell, and T is the transit time factor. Usually we approximate $E_{0z}(r, z)$ by $E_0 T$, so that Eq. (8) can be used to determine the longitudinal oscillation throughout the accelerator,

$$\frac{d}{d\ell} \left[\gamma_s^3 \beta_s^2 \frac{d}{d\ell} (\psi - \psi_s) \right] = \frac{2\pi e \lambda \beta_s E_0 T}{m_0 c^2} (1 - S) \sin \psi_s (\psi - \psi_s). \quad (8)$$

Sometimes it is convenient to divide Eq. (8) into two parts to represent a gap and a drift tube for an Alvarez structure¹⁴ or into several parts for an APF structure in which a superperiod consists of several gaps with different synchronous phases and drift tubes. The superperiod indicates the structural period, not the rf period. The drift-tube length is ℓ and the number of gaps is n . If we use the average field E_m in the first gap of a superperiod, the equation can be rewritten as follows.

$$\frac{d^2 X}{d\ell^2} = -n(\ell)X, \quad (9)$$

and

$$n(\ell) = (G - S')F, \quad (10)$$

where

$$F = - \frac{2\pi e \lambda E_m}{m_0 c^2 \gamma_s^3 \beta_s} \sin \psi_s, \quad (11)$$

$$S' = \frac{E_0 T}{E_m} S, \quad (12)$$

and

$$X = \psi - \psi_s. \quad (13)$$

The average field in a element divided by E_m is G . The approximation is made that γ_s and β_s are constant in Eq. (8).

The solution can be expressed by

$$X(\ell) = \begin{bmatrix} X(\ell) \\ X'(\ell) \end{bmatrix} = M(\ell|\ell_0) X(\ell_0) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} X(\ell_0) \\ X'(\ell_0) \end{bmatrix}, \quad (14)$$

where $X(\ell_0)$ is the initial value and $M(\ell|\ell_0)$ is a transformation matrix. Corresponding to the value of $n(\)$, the matrix $M(\ell|\ell_0)$ is

$$M(\ell|\ell_0) = \begin{bmatrix} \cos \alpha & n^{-1/2} \sin \alpha \\ (-n)^{1/2} \sin \alpha & \cos \alpha \end{bmatrix} \quad \text{for } n > 0, \quad (15)$$

$$M(\ell|\ell_0) = \begin{bmatrix} 1 & \ell - \ell_0 \\ 0 & 1 \end{bmatrix} \quad \text{for } n = 0, \quad (16)$$

and

$$M(\ell|\ell_0) = \begin{bmatrix} \cosh \beta & (-n)^{-1/2} \sinh \beta \\ (-n)^{1/2} \sinh \beta & \cosh \beta \end{bmatrix} \quad \text{for } n < 0, \quad (17)$$

where α and β are defined by

$$\alpha = n^{1/2}(\ell - \ell_0) \quad (18)$$

and

$$\beta = (-n)^{1/2}(\ell - \ell_0). \quad (19)$$

Even though the periodicity of ion linacs is less than that of synchrotrons, we can assume that the periodicity for every superperiod of L rf cycles is

$$n(\ell + L) = n(\ell) \quad . \quad (20)$$

If the index has i different values for $n_0, n_1, \dots, n_{i-1}$ and an rf length equal to $\ell_1 - \ell_0, \ell_2 - \ell_1, \dots, \ell - \ell_{i-1}$ in a superperiod L , then the matrix is

$$M(\ell | \ell_0) = M(\ell | \ell_{i-1}) \dots M(\ell_2 | \ell_1) M(\ell_1 | \ell_0) \quad . \quad (21)$$

For a linac with a superperiod m , the matrix becomes

$$M(mL | 0) = [M(L | 0)]^m \quad , \quad (22)$$

from which the stability condition,

$$-1 < 1/2 \operatorname{Tr} M < 1 \quad , \quad (23)$$

is obtained for a large m . Also, it is possible to have a series of different elements $M_1, M_2, \dots, M_n$ throughout the linac, such that the maximum M_n is at most several hundreds. The stability condition requires that the multiplication,

$$\prod_{i=1}^n M_i \quad ,$$

remain bounded.

In this case, to achieve stable motion it is sufficient that $|1/2 \operatorname{Tr} M_i| < 1$ for every i .

III. APPLICATION TO AN APF STRUCTURE

Applying this method to an Alvarez linac,¹⁴ we found one of the space-charge limitations. To understand the APF structure, three different analyses were made. The parameters, listed in Table I, are taken mainly from

TABLE I
APF STRUCTURE PARAMETERS

<u>C</u>	<u>CL</u>	<u>GL</u>	<u>DL</u>	<u>WS</u>	<u>PHI</u>	<u>T</u>	<u>E</u>	<u>E_m</u>	<u>G</u>	<u>l</u>
0				0.250						
1	1.853	0.542	0.465	0.258	-78.60	0.341	6.089	7.099	1.	0.339
2	1.936	0.565	1.694	0.265	80.00	0.336	6.197	7.135	-1.012	1.059
3	1.231	0.362	1.049	0.280	64.00	0.477	5.827	9.452	-1.223	0.353
4	1.282	0.376	0.690	0.298	-62.00	0.491	6.091	10.197	1.297	0.656
5	2.082	0.601	1.125	0.309	-76.00	0.382	5.717	7.491	1.	0.226
6	2.110	0.607	1.838	0.320	78.00	0.387	6.479	8.716	-1.173	0.431
7	1.361	0.396	1.168	0.338	63.60	0.510	5.832	10.223	-1.260	0.235
8	1.417	0.411	0.763	0.359	-61.60	0.519	6.004	10.743	1.300	0.703
9	2.280	0.646	1.251	0.375	-74.00	0.421	6.047	8.986	1.	0.341
10	2.316	0.654	2.018	0.389	76.00	0.430	5.811	8.849	-0.994	1.042
11	1.512	0.435	1.307	0.412	63.20	0.544	6.202	11.727	-1.212	0.344
12	1.573	0.450	0.850	0.437	-61.20	0.557	5.923	11.532	1.170	0.662
13	2.509	0.696	1.400	0.459	-72.00	0.470	6.037	10.229	1.	0.224
14	2.555	0.705	2.228	0.479	74.00	0.478	5.941	10.292	-1.017	0.432
15	1.688	0.477	1.472	0.506	62.80	0.581	6.023	12.383	-1.132	0.233
16	1.754	0.492	0.953	0.536	-60.80	0.593	5.912	12.499	1.122	0.709
17	2.771	0.750	1.574	0.566	-70.00	0.519	6.099	11.695	1.	0.332
18	2.829	0.762	2.470	0.594	72.00	0.530	6.043	11.891	-1.029	1.037
19	1.891	0.521	1.666	0.626	62.40	0.621	5.882	13.257	-1.069	0.336
20	1.958	0.535	1.077	0.663	-60.40	0.630	6.073	14.001	1.108	0.672
21	3.073	0.823	1.772	0.701	-68.00	0.556	5.937	12.326	1.0	0.224
22	3.146	0.843	2.729	0.738	70.00	0.563	6.108	12.833	-1.055	0.437
23	2.114	0.567	1.875	0.777	62.00	0.655	5.999	14.651	-1.132	0.231
24	2.182	0.578	1.220	0.820	-60.00	0.663	5.945	14.879	1.127	0.720
25	3.412	0.916	1.991	0.868	-66.00	0.582	5.943	12.883	1.0	0.323
26	3.499	0.942	3.003	0.914	68.00	0.585	5.999	13.036	-1.027	1.034
27	2.356	0.609	2.114	0.960	61.60	0.684	6.002	15.881	-1.187	0.327
28	2.424	0.662	1.385	1.011	-59.60	0.692	6.008	16.203	1.187	0.683
			2.224							0.221
										0.442
										0.228
										0.730
										0.313
										1.031
										0.318
										0.695
										0.217
										0.449
										0.223
										0.739
										0.308
										1.023
										0.316
										0.703
										0.213
										0.457
										0.217
										0.746
										0.309
										1.012
										0.317
										0.713
										0.205
										0.467
										0.210
										0.750

the PIGMI design.⁽¹⁶⁾ In case 1, the superperiod consists of two gaps and two drift tubes, with $S' = 0$. The indices are

$$n() = \left\{ \begin{array}{ll} n_1 = F & \text{in } \ell_1 \\ n_2 = 0 & \text{in } \ell_2 \\ n_3 = GF & \text{in } \ell_3 \\ n_4 = 0 & \text{in } \ell_4 \end{array} \right\}, \quad (24)$$

and

$$G = \frac{E_{m3} \sin \psi_{s3}}{E_{m1} \sin \psi_{s1}}, \quad (25)$$

based on the assumption that

$$\ell_1 + \ell_2 + \ell_3 + \ell_4 = 2. \quad (26)$$

Figure 1 shows the calculated stability region for $\ell_1 = 0.35$, $\ell_2 = 1.05$, $\ell_3 = 0.35$, and $\ell_4 = 0.25$. The abscissa is F and the ordinate is $(-G)$. This calculation shows that the stable region increases if F is made stronger,

and that it is preferable to have the absolute value of the synchronous phase in the second gap be larger than that in the first gap. However, note that the practical value of F is in a very limited region.

In case 2, the superperiod consists of two gaps and two drift tubes, with $G = -1$ ($\psi_{s3} = -\psi_{s1}$). We assumed that the superperiod is 2, as in case 1 and in Eq. (26).

Fig. 1. Stability diagram for $[F - (-G)]$.

TABLE II

SYNCHRONOUS PHASE CHANGES DETERMINE THE DRIFT-TUBE LENGTHS

$\frac{l_{s1}}{l_1}$	$\frac{l_1}{l_2}$	$\frac{l_2}{l_3}$	$\frac{l_3}{l_4}$
-30°	0.3	0.867	0.533
-60°	0.3	1.033	0.367
-80°	0.3	1.144	0.256

The indices are

$$n(l) = \begin{cases} n_1 = (1 - S')F & \text{in } l_1 \\ n_2 = -S'F & \text{in } l_2 \\ n_3 = (-1 - S')F & \text{in } l_3 \\ n_4 = -S'F & \text{in } l_4 \end{cases}, \quad (27)$$

If we assume constant rf gap lengths, we can change the rf lengths in the drift tubes by our selection of the synchronous phase. Table II gives examples.

Fig. 2. Stable region of FODO structure.

Figure 2 shows the stable region of S' vs F . The upper limit of S' becomes a limitation on the space-charge effect, as discussed in Ref. 14. The figure shows that a smaller synchronous phase is preferable to increase the stability region. However, the phase needs to be optimized for field strength (Eq. 11). For PJGMI the injection energy is 250 keV, the frequency is 450 MHz, and the average electric field is 6 MV/m. Then F is 0.5794, 1.0036, or 1.1452 for $\psi_s = -30^\circ$, -60° , or -80° , respectively.

For $\psi_s = -80^\circ$, $a_x = a_y = 2.5 \times 10^{-3}$ m, $M_z = 1/3$, and the upper current limit becomes 77.4 mA, using Eqs. (6) and (12).

In case 3, the superperiod consists of four gaps and four drift tubes.

$$\sum_{i=1}^8 \ell_i = 4 \quad . \quad (28)$$

In the PIGMI APF structure the synchronous phase of the first gap decreases by 2° in every superperiod. Thus, the stability region should be located between the values of the first and the last superperiods. The stability region was calculated using the same method. Figure 3 shows that the lower current limit decreased to 30% of the FODO structure in Case 2, but it still has a higher F value than the practical one. However, by using the FODODOFO structure and a strong electric field, the upper S' limit increases remarkably.

In the first cell, with F equal to 1.341, S' is 0.062 and the current is 367 mA [$(\psi - \psi_s)_{\max}$ equals 2 times 78°], or 291.7 mA [$(\psi - \psi_s)_{\max}$ equals 2 times 62°]. In the last cell, with F equal to 1.192, S' is 0.039 and the current is 242 mA [$(\psi - \psi_s)_{\max}$ equals 2 times 66°] or 220 mA [$(\psi - \psi_s)_{\max}$ equals 2 times 60°].

IV. CONCLUSION

The transformation matrix in phase space is an effective tool to study a periodical structure, especially an APF linac structure. Compared to a cyclic

Fig. 3. Stable region for FODODOFO structure.

accelerator, the linac beam is so strongly bunched that the space-charge force can be viewed as a defocusing element under some conditions. If this effect is included in the above formalism, we can determine one of the upper current limits by calculating the space-charge stability region.

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