ANALYSIS OF PHOTOVOLTAIC TOTAL ENERGY SYSTEMS FOR SINGLE FAMILY RESIDENTIAL APPLICATIONS

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FOR SINGLE FAMILY RESIDENTIAL APPLICATIONS

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FOREWORD

This report documents the results of one of several analytical investigations conducted under Contract No. EY 77-C-03-1101, Project Agreement 8, "Photovoltaic Mission Analysis", for the U.S. Department of Energy, Division of Distributed Solar Technology. The analyses described here were carried out as part of Task 2: Photovoltaic Total Energy Systems.

The work was accomplished by the Energy Systems Directorate of the Energy and Resources Division of The Aerospace Corporation. Dr. Harold Macomber of the Silicon Technology Programs Branch is the DOE Program Manager for this contract, and Dr. M. B. Watson, Principal Director of the Energy Systems Directorate, is the Principal Investigator. Dr. S. L. Leonard, Director, Photovoltaic Projects office, has provided the day-to-day program management.

This document was prepared by Dr. V. Chobotov of the Orbital and Environmental Analysis Office and Mr. B. Siegel of the Energy and Resources Division.

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2) The solar panel areas, storage levels, and fraction of the demand provided by photovoltaic total energy systems at minimum annual energy cost all decrease as array cost increases with respect to backup power cost. This suggests an approach where small systems are initially installed, and then, as the cost of system components is reduced or the cost of backup electricity increases, additional solar capability is added.

3) Systems with separate thermal and photovoltaic panels usually provide energy at lower total annual costs than systems with combined panels. Except at very low array costs ($\$100/\text{kW}_{\text{pk}}$), the savings in structural support and installation costs available with combined panels is not capable of off-setting the reduced thermal and electrical efficiencies of such panels in comparison to separate-panel systems. Separate-panel systems may become even more attractive if low-cost evacuated-tube collectors are developed. Such collectors could provide higher efficiencies and operating temperatures than the flat panels assumed in this report. Future studies may also wish to consider the use of concentrating collectors in evaluating the relative desirability of separate and combined panel systems.

4) All-photovoltaic total energy systems are preferable to either separate-panel or combined-panel systems in cold climates for array costs of $\$500/\text{kW}_{\text{pk}}$ or less. This result is due primarily to the poor efficiency of thermal panels in cold climates. (In this report, the installed cost of solar thermal panels was assumed to be $\$50/\text{m}^2$ which is the solar-thermal program cost objective for the mid-1980s. If more expensive solar thermal panels are assumed, all-photovoltaic systems will be preferable to separate or combined-panel systems at array costs greater than $\$500/\text{kW}_{\text{pk}}$.)

The performance and economics of separate or combined panel systems may be improved by operating at panel temperatures that are sufficient for space heating and using utility power to provide the incremental energy to meet the higher temperatures needed for domestic hot water.

5) All-photovoltaic total energy systems are preferable to either separate or combined-panel systems where both array costs are low and backup power costs are high. At low array cost ($\$100\text{--}\$500/\text{kW}_{\text{pk}}$), the cost of electrically generating thermal energy with air-cooled photovoltaic arrays is less than the cost of generating thermal energy with a solar thermal panel.

The advantages of all-electric systems would likely have been even greater if the analysis had considered installation of such systems in homes with electric heat pumps.

It would appear that if photovoltaic array cost objectives are achieved ($\$500/\text{kW}_{\text{pk}}$ in 1986), all-photovoltaic total energy systems would be more economic than combined or separate-panel systems in the post-1986 time period. However, in the near term, where array costs are high and backup power costs are low, separate or combined-panel systems could provide lower annual energy costs than all-photovoltaic systems. Future studies should consider how this transition to all-photovoltaic systems might occur and how it might be affected by widespread use of such options as heat pumps, absorption air conditioning, and utility purchase of excess electrical energy.

6) For array costs of $\$500/\text{kW}_{\text{pk}}$ or less, the photovoltaic system that results in the minimum total annual cost of service (including the cost of backup energy) meets 40-90% of the total annual demand in Phoenix. In Madison, the

minimum-annual-cost system meets only 20-40% of this demand with solar energy. The poor system performance in Madison is due primarily to the large night time space heating demand.

7) Selling excess electrical energy to the utility is primarily attractive at low array costs. The relative desirability of selling electrical energy, in comparison to such alternatives as additional on-site storage, is determined primarily by the cost of storage, by utility charges for backup energy, and by utility prices for purchased energy. It is recommended that future studies concentrate on providing a better definition of such charges and prices and, in particular, consider the effects of time-of-day pricing, increasing solar penetration, and off-peak utility charging of storage.

8) The breakeven array cost can exceed $\$1000/\text{kW}_{\text{pk}}$ in Phoenix and $\$500/\text{kW}_{\text{pk}}$ in Madison, without sale of excess electrical energy, for levelized backup power costs in excess of 7¢/kWh -- provided that utility backup power rates for photovoltaic total energy system owners are the same as utility electricity rates for other residential customers. (In this case non-solar users partially subsidize the cost of additional generating facilities and transmission and distribution equipment required to provide backup energy for solar users). Under these assumptions, if photovoltaic array cost objectives are achieved ($\$500/\text{kW}_{\text{pk}}$ in 1986), and the cost of electricity continues to increase, photovoltaic total energy systems would likely be competitive with utility power for residential applications in the mid 1980s in Phoenix and the early-1990s in Madison, without sale of excess electrical energy.

In general, it appears that the relationship established between the homeowner and the local utility in regard to such issues as the cost

of utility electricity, the ability of a home with photovoltaic systems to sell electricity to the utility, and the price the utility will pay for such electricity will be major factors in determining the characteristics, required costs, and market penetration of residential photovoltaic total energy systems. This report and related studies (References 1-7) are providing preliminary information on these issues, but as noted above, further work is clearly needed. In particular, a detailed study is needed of the legal, institutional, and technical issues involved in the interface between the utility and the homeowner in order to define and evaluate potentially attractive system ownership and operational arrangements.

The following sections of this report present a discussion of the analysis procedures used, the analysis results, and conclusions.

II. ANALYSIS APPROACH

The performance of the system concepts under consideration and the resulting cost of energy has been determined through an analysis procedure which is outlined in Figure 1. The purpose of this analysis is to define, for each system concept, the minimum levelized annual cost to the homeowner (including charges for backup energy) for the energy required to satisfy the thermal and electrical demand of a residence. The minimum levelized annual cost is then used to compare and draw conclusions about the relative desirability of the system concepts for various cost and economic assumptions.

The analysis procedure shown in Figure 1 consists of several steps:

- A) Definition of the residential thermal and electrical loads
- B) Use of load and insolation information, along with an appropriate simulation model, to compute the hourly performance, over a typical year, of the electrical and thermal parts of each photovoltaic total energy system (PTES). The simulation provides information, as a function of panel area and storage capacity, on the amount of backup energy that must be purchased as well as the amount of electrical energy that might be available for sale to the utility.
- C) The use of simulation results in combination with economic and cost assumptions to determine the minimum levelized annual cost of service associated with each system concept. A mathematical optimization procedure is used to determine the minimum levelized annual cost.

Each of these steps is discussed in greater detail in the following sections of this report.

III. ELECTRICAL AND THERMAL LOADS

The hourly electrical demand representations used in the simulation computations of PTES performance in a single-family residence were synthesized by adding together a standard baseload profile and a temperature-dependent air conditioning load. Similarly, the thermal demand representations were synthesized by adding a temperature-dependent space heating load to a standard profile corresponding to the thermal demand for domestic hot water. The thermal and electrical baseload profiles and the functional relationships between the temperature-dependent loads and the ambient temperature were all based on measurements representing average values for a number of homes. Because of the diversity factor in such measurements, the resulting demand representations (especially the electric demand) are undoubtedly considerably smoother than would be actual measured profiles for a single residence. The loss of realism in the simulation as a result of this smoothing effect is believed to be small, however. The principal difference is the absence of the large "spikes" in electric demand that occasionally occur in real demand profiles. It is probably not unreasonable to postulate that such spikes are accommodated by backup power from the utility grid and that they represent a negligible fraction of the annual average amount of energy consumed.

The assumed load profile for baseload electric power (lights, appliances, dryer, range) is shown in Figure 2 and is based on national averages for new construction (Reference 8). It is assumed that baseload electric power requirements are not affected by geographic location.

The required electric power for air conditioning as a function of ambient temperature is shown in Figure 3. This curve is based on information collected on a number of homes in the Phoenix area during a summer day in 1973 (Reference 9). The information in Figure 3, when

Figure 2
Baseload Electric Power Requirements:
1600 ft² Residence

Figure 3
Power Requirement for Residential Air Conditioning

combined with hourly values for the ambient temperature, permits calculation of the hourly use of electricity for air conditioning.

The assumed thermal load profile for domestic hot water heating is shown in Figure 4. The load profile and maximum values are based on national averages (Reference 10) and correspond to the requirements for a family of four. It is assumed that the domestic hot water demand is not a function of location.

The required thermal power for space heating as a function of ambient temperature is shown in Figure 5. The information in this figure is based on a survey of homes in the Philadelphia area using resistance heating (Reference 11). The hourly use of energy for space heating is computed in the same manner as is the air conditioning demand.

The total estimated annual thermal and electric requirements for typical homes in Phoenix and Madison are shown below in Table 1.

Table 1

Annual Residential Energy Demand (kWh)

	Phoenix	Madison
o Electrical Demand	16,550	10,160
- Baseload	(9745)	(9745)
- Air Conditioning	(6805)	(415)
o Thermal Demand	12,300	29,750
- Hot Water	(5585)	(5585)
- Space Heating	(6715)	(24,615)
o Total Demand	28,850	39,910

Figure 4
Thermal Energy Requirements for Domestic Hot Water:
Single Family Residence, Four Persons

Figure 5
Power Requirement for Residential Space Heating

IV. SYSTEM SIMULATION

In the system simulation procedure used in this study, all of the energy flows in the system were calculated on an hour-by-hour basis with the aid of a computer program modelling the PTES system concept under investigation. Required inputs to the computation include hourly values for the energy demand, the ambient temperature, and the insolation. The demand representations used in these simulations were discussed in the previous section. The hourly values for insolation and ambient temperature were based on historical values, obtained during a particular reference year at each of the two locations considered (1964 for Phoenix, 1960 for Madison), and were provided in the form of a magnetic tape record. The reference years for the insolation data were chosen on the basis that the annual averages of the daily insolation for those particular years agreed closely with the 24-year averages. Computations were carried out for a full year of simulated operation and yielded values for the total amount of solar energy received, stored, dumped (wasted), or delivered to the load. The simulation models used for the three PTES concepts are illustrated schematically in Fig. 6 and are discussed below.

A. System Concepts

1. All-Photovoltaic System

The all-photovoltaic system consists of a flat-plate air cooled photovoltaic array with an array efficiency of 13.5% at 20°C, a battery for electrical energy storage, and a power conditioner to convert the DC electricity generated by the array to AC electricity. The electric energy generated by the array is delivered to the load via the power conditioner, or if the load is satisfied, is stored for delivery to the load at a later time, or, if the load is satisfied and the battery is fully charged, is dumped to the atmosphere through a resistive heating element. If excess electrical energy is sold to the utility, all or part

Figure 6

System Concept Simulation

of the energy that would have been dumped is assumed fed into the utility grid after power conditioning. If the electrical energy provided by the array and storage is insufficient to meet load requirements, electrical energy is provided by the utility. The load for this system is the total thermal and electrical demand, which is estimated to be 28,850 kWh per year for the Phoenix residence and 39,910 kWh per year for the Madison residence.

The DC electric power generated by the all-photovoltaic system is computed as follows:

$$P_{Ge} = E_g \eta_c PF A_e \quad (1)$$

where

E_g = Insolation reaching the array (taking into account collection efficiency)

η_c = Solar cell conversion efficiency

PF = Array packing factor (fraction of array covered by cells) (assumed to be .9)

A_e = Array area

and for silicon cells

$$\eta_c = .12 \left[1 - .005 (T_A - 70) \right] \quad (2)$$

where T_A = Average array temperature ($^{\circ}\text{C}$) (it is assumed that the average array and cell temperature are equal).

For passive, free convection cooling

$$T_A = T_{\text{ambient}} + \frac{E_g (\alpha - \eta_c PF)}{h_c (1 + F) + 2 h_r} \quad (3)$$

where

T_{ambient}	= Ambient temperature ($^{\circ}\text{C}$)
α	= Absorbtivity (assumed equal to .77)
h_c	= Convective heat transfer coefficient (assumed equal to $.017 \text{ kW/m}^2\text{-}^{\circ}\text{C}$)
h_r	= Radiative heat transfer coefficient (assumed equal to $.0085 \text{ kW/m}^2\text{-}^{\circ}\text{C}$)
F	= Ratio of cooling area to the array area (assumed equal to 1)

In addition, it is assumed in the simulation that 5% of the DC electricity generated by the array is dissipated as heat, that the power conditioning efficiency is 85% regardless of DC input level and that the battery storage efficiency is 70%.

2. Separate-Panel System

The separate panel system essentially consists of separate thermal and electrical systems. The thermal system includes a water cooled solar thermal panel operating at 70°C , a tank of water for thermal energy storage, and heat exchangers. A 70°C panel temperature is capable of providing domestic hot water at 60°C (140°F) as well as thermal energy for space heating. Thermal energy provided by the solar thermal panel is used for space heating through a water-to-air heat exchange, used for domestic hot water heating through a water-to-water heat exchange, stored in a hot water tank, or dumped to the atmosphere if the water in the tank is already at a specified maximum temperature. When solar insolation is not capable of maintaining the temperature of the water in the thermal storage tank at or above a prescribed minimum level, electrical resistance heating of the water in the storage tank is provided by utility power. The annual thermal demand for the Phoenix residence is 12,300 kWh and is 29,750 kWh for the Madison residence.

The thermal power that can be transmitted to the coolant (i.e., useable thermal power) by the solar thermal panel is

$$P_{Gth} = \left[E_g \alpha - U_o (T_A - T_{ambient}) \right] A_{th} \quad (4)$$

where A_{th} is the thermal panel area and U_o is the convective heat transfer coefficient between the thermal panel and the air. Note that T_A is the average panel temperature, which as noted above is assumed to be 70°C .

It is further assumed that:

- a. No thermal energy is available until the heat absorbed, $E_g \alpha$, is greater than the convective heat loss with $T_A = 70^\circ\text{C}$. A value of $0.007 \text{ kW/m}^2\text{-}^\circ\text{C}$ has been assumed for U_o (Ref. 13). The hourly ambient temperature is provided on the insolation tape.
- b. Thermal losses between the array and the load are zero. In an actual system there will be thermal losses from distribution pipes and in heat exchangers.
- c. The thermal storage efficiency is 80%.

The electrical system and its simulation is identical in characteristics and operation to the all-photovoltaic system except that the load includes only the electrical demand of the residence. In Phoenix the annual electrical demand was 16,550 kWh and in Madison it was 10,160 kWh.

3. Combined-Panel System

The combined panel system consists of a water-cooled photovoltaic array that operates at a panel temperature of 70°C and

which provides both thermal energy and DC electricity, and an air cooled photovoltaic array that also provides DC electricity. The thermal and electrical energy provided by the combined-panel system is handled in the same manner as in the separate-panel system. However, the thermal and electrical energy available are different than in the separate panel system. The thermal power that can be transmitted to the coolant in the combined-panel system is:

$$P_{Gth} = \left[E_g (\alpha - \eta_c PF) - U_o (T_A - T_{ambient}) \right] A_{th} \quad (5)$$

where A_{th} is the area of the combined photovoltaic/thermal panel and the term $\eta_c PF$ accounts for the fraction of insolation that is converted into electricity.

The D.C. electrical power provided by this system is

$$P_{Ge} = E_g \cdot PF \left[(\eta_c A)_e + (\eta_c A)_{th} \right] \quad (6)$$

where the subscripts e and th refer, respectively, to the air cooled panel and the photovoltaic/thermal panel. The value of η_c for both panels is computed using Equation (2). Note, however, that since the average temperature of the photovoltaic/thermal panel has been assumed to be 70°C, the electrical efficiency of this panel has a constant value of 12%

B. Simulation Results

Typical results from a computer simulation are shown in Figure 7 for the case of a Phoenix, Arizona residence served by a system incorporating combined thermal-electric panels. The figure shows the fraction of the annual demand for thermal energy that is provided by the panel as a function of panel area and thermal storage capacity. Curves of similar shape and trend are obtained when other locations are considered or when it is the delivery of electrical, rather than thermal

Figure 7
Fraction of Thermal Demand Provided by PTES,
Combined-Panel System, Phoenix

energy that is being computed. (In the case of electrical energy, the simulation also determines the amount of energy that is available for sale to the utility.) In general, as the panel area is increased with storage capacity held constant, a larger fraction of the thermal energy generated is simply wasted (as a result of mismatches between available energy and the demand) and the amount of thermal energy actually delivered to the load approaches a limit. Similarly, as the storage capacity is increased for fixed panel area, energy that would otherwise be wasted is instead stored for use at a later time; as a result a larger fraction of the thermal demand is met by solar energy and the required amount of backup energy is reduced.

Figures 7 and 8 show the fraction of the residential thermal demand that is met by solar energy from combined-panel and separate-panel systems installed on a home located in Phoenix. Figures 9 and 10 provide the same information for a Madison residence. These figures show that each system concept provides about the same fraction of the thermal demand for the same values of thermal panel area and storage capacity. As expected, the combined-panel system provides slightly poorer performance because some of the insolation received is converted to DC electricity and is not available for thermal uses. The poor thermal performance of the combined and separate-panel systems in Madison is due to a combination of several factors, including low insolation levels, low ambient temperatures (which reduce thermal panel efficiency by increasing the convective heat loss), and the large space heating demand, particularly at night.

Two methods of improving the computed thermal performance of these systems in Madison were considered. The first assumes that an improved thermal panel with lower convective heat loss coefficient is used. Figure 11 shows the thermal system performance for the separate-panel system with a panel convective heat loss coefficient of $.005 \text{ kW/m}^2 - ^\circ\text{C}$. A comparison of Figures 10 and 11 shows that an

Figure 8
Fraction of Thermal Demand Provided by PTES,
Separate-Panel System, Phoenix

Figure 9
Fraction of Thermal Demand Provided by PTES,
Combined-Panel System, Madison

Figure 10

Fraction of Thermal Demand Provided by PTES,
Separate-Panel System. Madison

Figure 11.
Fraction of Thermal Demand Provided by PTES,
Separate-Panel System, Improved Efficiency Panel, Madison

improved panel can significantly increase the fraction of thermal demand provided by PTES in Madison. For example, with a 40 m^2 panel and 25 kWh of thermal storage, the baseline panel provides only about 12.7% of the thermal demand (Figure 10), while the improved panel provides about 20% of the thermal demand (Figure 11). Whether selection of a more efficient and therefore more costly panel is economically desirable will be considered later in this report. The second method considered assumes that the thermal panel is operated at a lower average temperature, thereby also reducing the convective heat loss. Figure 12 shows the fraction of thermal demand provided by PTES in Madison for a 40°C average panel temperature. With careful design, a 40°C panel can provide thermal energy at about 35°C , which is sufficiently high for space heating but inadequate for domestic hot water. A comparison of Figures 12 and 10 shows that a significant increase in the fraction of thermal demand that is provided by solar energy can be achieved by reducing the panel temperature. This increase in thermal system performance must be balanced against the cost of purchasing additional electricity to raise the temperature of the hot water supply from 35° to 60°C . The economic desirability of operating the thermal panel at a lower temperature will be considered later in this report.

Figures 13 and 14 show the fraction of the electrical demand that is met in Phoenix by solar energy from the combined-panel and separate-panel systems as a function of array area and electrical storage capacity. Similar information is presented in Figures 15 and 16 for a Madison residence. Note that for the combined-panel system, the array area includes the photovoltaic part of the combined photovoltaic/thermal panel as well as any additional air cooled array. A comparison of the electrical performance of the combined-panel and separate-panel systems at equivalent array areas and electrical storage levels shows that, as expected, the combined-panel system has slightly poorer performance because part of its array is operating at an elevated temperature (70°C) in comparison to the totally air cooled photovoltaic array of

Figure 12

Fraction of Thermal Demand Provided by PTES,
Separate-Panel System, 40°C Panel Temperature, Madison

Figure 13
Fraction of Electrical Demand Provided by PTES,
Combined-Panel System, Phoenix

Figure 14
Fraction of Electrical Demand Provided by PTES,
Separate-Panel System, Phoenix

Figure 15
Fraction of Electrical Demand Provided by PTES,
Combined-Panel System, Madison

Figure 16
Fraction of Electrical Demand Provided by PTES,
Separate-Panel System, Madison

the separate-panel system. These figures also show that, at a specified array area and storage capacity, about the same fraction of the electrical demand is provided by a PTES in Phoenix as in Madison. For example, with a 60 m² array and 25 kWh electrical storage capacity the separate-panel system provides about 64% of the electrical demand in Phoenix (Figure 14) and 62% of the electrical demand in Madison. Note, however, that the average electrical demand in Phoenix is 16,550 kWh and only 10,160 kWh in Madison.

The fraction of the total thermal and electrical demand provided by the all-photovoltaic system in Phoenix and Madison is shown in Figures 17 and 18.

The simulation also provides an estimate of the amount of electrical energy that is generated by the PTES but cannot be delivered to the load because of mismatches between the load and the insolation and also cannot be stored. This surplus electrical energy is either wasted or can potentially be sold to the utility to provide revenue for the homeowner. Figure 19 shows the amount of surplus electrical energy available for each of the system concepts for a Phoenix home with no electrical storage.

Figure 17
Fraction of Total Demand Provided by PTES,
All-Photovoltaic System, Phoenix

Figure 18
Fraction of Total Demand Provided by PTES,
All-Photovoltaic System, Madison

Figure 19
Surplus Electrical Energy Available,
No Electrical Storage, Phoenix

V. THE MINIMUM LEVELIZED ANNUAL COST OF SERVICE

The levelized annual cost of service to the owners of a home equipped with a PTES is the constant annual amount of money which, if paid in each year of system life, will defray all the costs of acquiring the required thermal and electric energy. These costs include the annual cost of the capital equipment (i.e., the cost of amortizing the capital expenditure), the annual costs of operation and maintenance of the system, and the annual cost of backup energy from the utility as well as any revenue received from selling surplus electrical energy to the utility. In equation format,

$$\text{Levelized annual cost} = (\text{CRF}) C_c + C_{\text{O\&M}} + C_{\text{Bu}} - C_R \quad (7)$$

where CRF = Capital Recovery Factor
 C_c = Installed Capital Cost of the System Concept
 $C_{\text{O\&M}}$ = Levelized Annual Cost of System Operations and Maintenance
 C_{Bu} = Levelized Annual Cost of Backup Energy
 C_R = Levelized Annual Revenue from Sale of Surplus Electrical Energy

In this study it has been assumed that the levelized cost of operations and maintenance is a fixed percentage of the capital cost, that the levelized cost of backup power is the product of the amount of backup energy required and the cost of electricity (it is assumed that the cost of backup electricity is fixed, independent of the time of day or the amount of backup energy required), and that the revenue provided from the sale of surplus electrical energy is the product of the amount of energy sold and an assumed fixed price per kWh.

The calculation of the levelized annual cost of service as defined above is a straightforward process. By means of simulation computations (typically for a full year of operation) the annual amounts

of electric and thermal energy provided by the PTES are determined, as are the amounts of backup energy required, and the amounts of surplus electricity available for sale to the utility. An estimate of the installed capital cost of the system and assumed values for the capital recovery factor, the operation and maintenance factor, the levelized unit cost of backup power, and the levelized revenue per unit of energy sold to the utility are then used to compute the separate terms of Equation 7. The minimum value of the levelized annual cost and the panel areas and storage levels that provide this minimum are then defined through a mathematical optimization procedure. Before describing the optimization procedure and the results obtained, a discussion is presented of the reasons for the existence of a minimum in the annual cost and of the effect of changes in assumptions on this minimum cost.

A. DISCUSSION

The results of a typical calculation of the levelized annual cost of the thermal energy required by a single family home in Phoenix are presented in Figure 20. The upper curve presents this cost as a function of panel area, where the levelized backup power cost is 5¢/kWh and CRF = .09. (If the solar system is financed as part of the homeowner's mortgage, a CRF of .09 implies a mortgage interest rate of about 8%. Present mortgage rates are 9-10%.) Results are shown for a combined thermal-electric panel where the thermal elements (i.e. everything but the photovoltaic modules) were assumed to cost \$50/m² installed. The system was assumed to incorporate 50 kWh of thermal storage (at \$10.66/kWh). The figure shows that at a panel area of about 30 m² the levelized annual cost to the user is a minimum and equal to about \$400 per year (including the cost of backup energy from the utility grid). At panel areas smaller than the optimum, the incremental cost of the additional thermal energy delivered to the load resulting from incremental increases in panel area is less than the cost of backup utility power, while at larger panel areas the converse is true (i.e., as the panel area is increased, a larger fraction of the thermal energy

Figure 20
 Levelized Annual Cost of Thermal Energy, Phoenix

generated cannot be delivered to the load due to mismatches between the insolation and the load). By considering other thermal storage levels, the combination of panel area and storage level that provide the minimum levelized annual cost for thermal energy can be defined.

The horizontal line labeled "Annual Cost of Utility Power" is the levelized annual cost to the user if all the thermal energy required were purchased from the utility at 5¢/kWh (i.e. the levelized cost of utility and backup power are identical). The amount of thermal energy delivered to the load as a function of panel area is also displayed (right hand vertical scale), illustrating the relationship between thermal energy delivered by the panel and the levelized annual cost. Note that in general, the amount of solar thermal energy delivered at minimum levelized annual cost is significantly less than the total demand (in this case about 75% of the total), and that any significant increase in the amount of solar energy delivered to the load will result in a large increase in levelized annual cost.

The kind of results shown in Figure 20 are also obtained when electrical energy is being considered or when the combined cost of thermal and electrical energy is being calculated. The procedure can be used to approximately size any system using solar collectors in any application where the thermal and/or electrical demand is being provided by solar energy with utility or other conventional backup.

In general, (1) there is a panel area and storage capacity that will minimize the total annual cost of energy, (2) the amount of solar energy delivered at the minimum annual cost point is generally less than the annual demand, and (3) annual costs rise rapidly as the energy delivered to the load by a solar panel approaches the total annual demand.

The levelized annual cost of service obviously depends strongly on the system parameters and on the assumed costs of the major subsystems. The sensitivity of results to changes in some of these

assumptions is illustrated in Figure 21. The example shown to illustrate these effects is a calculation of the levelized annual cost of electric service to a single-family Phoenix residence that is equipped with passively air-cooled photovoltaic arrays. In this example, it was assumed that the levelized cost of utility and backup electricity is 5¢/kWh and that the capital cost of the storage capacity (present value cost over the life of the PTES, including the cost of any replacement batteries) is \$40/kWh. Figure 21 shows that for a given storage level, the array areas that provide minimum levelized annual cost decrease as array cost increases and also that, as array cost increases, smaller storage capacity becomes desirable. For example, if the array cost is \$100/kW_{pk}, a storage capacity of 25 kWh and an array area of about 90 m² provide minimum levelized annual cost, while at \$500/kW_{pk} array cost, zero storage and an array area of about 10 m² are optimum. This suggests an approach where, initially systems with small photovoltaic panels and no electrical storage are installed and then, as the cost of panels is reduced, more panel area and storage is added. This procedure would minimize homeowner annual costs if the levelized cost of backup power either remained constant or did not decrease more rapidly than the cost of photovoltaic panels.

If, as has been suggested (Reference 2), photovoltaic electric energy that would have been wasted can be used to generate revenue through sale to the utility, then it may be possible to reduce the minimum levelized annual cost. Of particular interest under such an approach are systems without electrical storage which, as noted previously, provide maximum excess electrical energy at a given array area. In addition, this approach would reduce system capital costs by eliminating the capital costs associated with electrical storage. The results of the analysis for this case (i.e., no electric storage, surplus electricity sold to the utility) are shown in Figure 22. In the graph, the levelized annual cost of electricity for the same Phoenix home is presented, as a function of array area, for array prices of \$100/kW_{pk} and \$500/kW_{pk}. At the \$100/kW_{pk} array price, the sale price of the

Figure 21
Effect of Changes in System Cost Assumptions, Phoenix

Figure 22
Effect of Sale of Surplus Electrical Energy to Utility,
Phoenix

surplus electricity greatly affects the levelized annual cost and optimum array area. If the utility is willing to pay 3¢/kWh for this energy, which happens to be greater than the cost of photovoltaic generation for the conditions assumed in this analysis, then no optimum array area occurs, and the homeowner should purchase and install as large an array as possible within the limits imposed by roof area. For comparison, the minimum levelized annual cost point for the situation where excess electrical energy is wasted (Point A in Figure 21) is also shown (again labeled Point A) in Figure 22. Since Point A corresponds to a case where the electrical storage level is 25 kWh, it appears that, at least in this example, a system with no electrical storage but able to sell excess electrical energy at about 2¢/kWh would have the same levelized annual cost as a system with storage (25 kWh at \$40/kWh) where the excess electrical energy is not sold.

Figure 22 also shows that the effect of the sale of excess energy to the utility is much less significant at an array price of \$500/kW_{pk}. This result is primarily due to two factors:

- 1) When the array price is \$500/kW_{pk}, the cost of photovoltaic electricity generation is large in comparison to the surplus energy sale prices shown (0-3¢/kWh).

- 2) As noted previously, at \$500/kW_{pk} array cost, the minimum energy cost is realized, in the case where there is no sale of surplus energy, when the array area is small and there is no electrical storage (Point B in Figure 21). There is thus no storage to be eliminated and no associated reduction in capital costs.

It appears that for a given surplus energy sale price and utility electricity cost, selling excess photovoltaic electricity to the utility is most attractive at low array costs and becomes less useful in reducing homeowner annual costs as the array cost increases. Stated

another way, it could be concluded that as the photovoltaic array price increases, larger surplus energy sale prices are required if such sales are to significantly reduce homeowner annual costs.

B. COMPUTATION PROCEDURE

The procedure that has been developed for locating the minimum value for the levelized cost of service under a given set of conditions has two parts: a) development of an analytic expression relating the levelized cost of service to the principal parameters of interest (solar panel areas and thermal and electrical storage capacities) and b) application of the Davidon computer optimization procedure to minimize the cost of service and determine the parameter values leading to the minimum. The method used in developing the analytic expression for cost of service is outlined below and described in greater detail in Appendix A. The Davidon optimization method is discussed in Appendix B.

The basic expression for the levelized cost of service is Equation 7:

$$\text{Levelized annual cost} = (\text{CRF}) C_c + C_{\text{O\&M}} + C_{\text{Bu}} - C_R \quad (7)$$

where

$$C_c = \text{Thermal system capital cost} + \text{electrical system capital cost}$$

$$\text{Thermal system capital cost} = C_1 A_{\text{th}} + C_2 P_{\text{ths}} + C_3 \quad (8)$$

$$C_1 = \text{Thermal panel cost per unit area } (\$/\text{m}^2)$$

$$C_2 = \text{Thermal storage cost } (\$/\text{kWh})$$

$$C_3 = \text{Balance of plant cost (pipes, pumps, heat exchangers, etc.) } (\$)$$

$$A_{\text{th}} = \text{Thermal panel area } (\text{m}^2)$$

$$P_{\text{ths}} = \text{Thermal storage capacity (kwh)}$$

$$\begin{aligned} \text{Electrical system capital cost} &= (1 + d_1) A + d_2 A \\ &+ d_3 P_{es} + d_4 + d_5 \end{aligned} \quad (9)$$

- d_1 = Array cost (\$/m²)
- d_2 = Air cooled array structural cost (\$/m²)
- d_3 = Electrical storage cost (\$/kWh)
- d_4 = Power conditioner cost (\$)
- d_5 = Wiring, junction box, cost (\$)
- A = Total photovoltaic array area
- = $A_{th} + A_e$ for combined-panel system
- = A_e for all-photovoltaic or separate-panel system

If it is assumed that the thermal system balance of plant cost and the power conditioning cost are either constant or a function of panel area or storage capacity, then the capital cost C_c has the functional dependence

$$C_c = \phi_1 (A_{th}, A_e, P_{ths}, P_{es}) \quad (10)$$

It is assumed that the levelized operations and maintenance cost, $C_{O\&M}$, is a fixed annual percentage of the capital cost.

The levelized cost of backup energy, C_{Bu} is

$$C_{Bu} = C_7 \left[(1 - R_e) P_e + (1 - R_{th}) P_{th} \right] \quad (11)$$

where

- C_7 = Levelized Cost per kWh of backup energy
- R_e = Fraction of the annual electrical demand provided by the PTES
- R_{th} = Fraction of the annual thermal demand provided by the PTES
- P_e = Annual electrical demand (kWh)
- P_{th} = Annual thermal demand (kWh)

The simulation results of Figures 7 to 18 provide both R_e and R_{th} as a function of panel area and storage capacity. An inspection of these figures shows that R_{th} and R_e can be expressed analytically to an acceptable degree of approximation, as

$$R_{th} = K_{th} \left(1 - e^{-A_{th}/a_{th}} \right) \quad (12)$$

$$R_e = K_e \left(1 - e^{-A_e/a_e} \right) \quad (13)$$

where K_{th} , K_e , a_{th} and a_e are functions only of panel area and storage capacity and are evaluated, as shown in Appendix A, to provide optimum fit to the curves in Figures 7-18.

It then follows that C_{Bu} can be expressed in the form

$$C_{Bu} = \phi_2 (A_{th}, A_e, P_{ths}, P_{es}) \quad (14)$$

In a similar manner (see Appendix A) it can be shown that

$$C_R = \phi_3 (A_{th}, A_e, P_{ths}, P_{es}). \quad (15)$$

Combining the above equations leads to an expression for the levelized annual cost of service associated with any system PTES concept as a function only of A_{th} , A_e , P_{ths} , P_{es} . The minimum value of this function, for assumed values of the cost constants, can, then be determined by means of the Davidon optimization procedure which is outlined in Appendix B and discussed in Refs. 14 and 15.

The results of an analysis using the above procedures are described in the next section.

C. RESULTS

1. Phoenix Residence

The methodology described in the preceding section, in conjunction with the simulation results described in Section IV. B, has been used to compute values for the minimum levelized annual cost of service to Phoenix homes equipped with each of the three PTES system concepts considered in this report. The baseline assumptions used in these computations are listed in Table 2 and the results are presented in Tables 3-5 and in Figs. 23-28, as a function of photovoltaic array cost for levelized backup energy costs of 5, 7, and 10¢/kWh and for capital recovery factors of .09 and .11. The tables also show the panel areas (A_{th} and A_e) and storage levels (P_{th} and P_{es}) that provide the minimum levelized annual cost, as well as the fraction of the thermal (R_{th}), electrical (R_e) and total demand (R) provided by the PTES at this cost. In these computations it was assumed that excess electrical energy is wasted.

In connection with Table 2 it should be noted that

a) The cost of the power conditioner, \$800, is assumed to be independent of array size or storage capacity and is obtained multiplying the estimated peak load, 8 kW, by the assumed unit cost, \$100/kW_{pk}.

b) The choice of the thermal panel temperature indicated in the table, 70°C, was based on the assumption that both domestic hot water and space heating loads would be served by solar energy from the PTES. For space heating alone, a lower temperature would be adequate. (See discussion in Section C.2.).

c) The indicated thermal panel heat loss coefficient is typical of single-glazed panels for which the installed cost

Table 2

Baseline Assumptions

<u>Symbol</u>	<u>Definition</u>	<u>Assumed Value</u>
C_1	Thermal Panel Cost	$\$50/m^2$
C_2	Thermal Storage Cost	$\$10.66/kWh$
C_3	Balance of Thermal System Cost	$\$1000$
C_4	Capital Recovery Factor	.09 or .11 (30 year System Life)
d_1	Photovoltaic Array Cost Per Unit Area = η_a ($\$/kW_{pk}$)	Variable
η_a	Photovoltaic Array Efficiency at $20^\circ C$	.135
$\$/kW_{pk}$	Photovoltaic Array Cost Per Peak Kilowatt	Variable
d_2	Air Cooled Photovoltaic Array Structural Cost	$\$10/m^2$
d_3	Electrical Storage Cost	$\$40/kWh$
d_4	Power Conditioner Cost	$\$800$
d_5	Wiring, Junction Boxes etc.	10% of sum of $d_3 P_{es} + d_4$
T_A	Thermal Panel Temperature	$70^\circ C$
U_o	Thermal Panel Heat Loss Coefficient	$.007 kW/m^2-^\circ C$
$C_{O\&M}$	Levelized Annual Operations and Maintenance Cost	1.5% of capital cost

Table 3

Optimum System Parameters

- o Combined-Panel System
- o Phoenix

C_p \$/kW _{pk}	C \$/year	A_{th} m ²	A_e m ²	P_{ths} kWh	P_{es} kWh	A m ²	R_{th}	R_e	R
A) $C_4 = .09$, $C_7 = \$.05/\text{kWh}$									
100	1127	27.9	67.7	34.6	26.3	95.6	.555	.84	.718
300	1369	27.9	34.8	34.6	20.9	62.7	.555	.63	.598
500	1471	22.5	0	31.0	0	22.5	.51	.22	.356
1000	1511	12.9	0	23.7	0	12.9	.4	.125	.242
B) $C_4 = .09$, $C_7 = \$.07/\text{kWh}$									
100	1282	36.2	79.0	41.0	32.5	115.2	.61	.91	.782
300	1552	36.2	43.8	41.0	27.6	80	.61	.775	.705
500	1788	36.2	25.0	41.0	24.3	61.2	.61	.62	.615
1000	2003	20.2	0	30.6	0	20.2	.48	.2	.319
C) $C_4 = .09$, $C_7 = \$.10/\text{kWh}$									
100	1463	46.5	91.9	47.7	38.8	138.4	.675	.95	.832
300	1767	46.5	52.4	47.7	33.5	98.9	.675	.86	.781
500	2016	46.5	32.2	47.7	30.5	78.7	.675	.77	.729
1000	2525	46.5	4.6	47.7	25.5	51.1	.675	.53	.592
D) $C_4 = .11$, $C_7 = \$.05/\text{kWh}$									
100	1280	24.1	61.9	31.3	22.8	86	.52	.76	.658
300	1508	24.1	28.3	31.3	15.5	52.4	.52	.525	.522
500	1555	18.2	0	27.0	0	18.2	.46	.175	.296
1000	1608	9.54	0	19.1	0	9.54	.38	.10	.219
E) $C_4 = .11$, $C_7 = \$.07/\text{kWh}$									
100	1427	31.7	73.1	37.7	29.4	105	.58	.88	.752
300	1744	31.7	39.3	37.7	24.4	71	.58	.70	.649
500	1974	31.7	20.6	37.7	20.1	52.3	.58	.53	.551
1000	2106	16.1	0	26.8	0	16.1	.44	.155	.276
F) $C_4 = .11$, $C_7 = \$.10/\text{kWh}$									
100	1618	41.2	85.4	44.4	35.7	127	.64	.945	.815
300	1985	41.2	48.2	44.4	30.7	89.4	.64	.82	.743
500	2263	41.2	28.9	44.4	27.7	70.1	.64	.71	.680
1000	2777	41.2	1.33	44.4	20.8	42.5	.64	.43	.519

C_4 = Capital recovery factor
 C_7 = Levelized backup power
 cost (\$/kWh)

Electrical Demand = 12,300 kWh/yr
 Thermal Demand = 16,550 kWh/yr

Table 4.

Optimum System Parameters

- o Separate-Panel System
- o Phoenix

C_p \$/year	C \$/year	C_e \$/year	C_{th} \$/year	A_{th} m^2	A_e m^2	P_{ths} kWh	P_{es} kWh	R_{th}	R_e	R
A) $C_4 = .09, C_7 = \$.05/\text{kWh}$										
100	1116	565	551	23	82.3	26.7	24.4	.520	.820	.692
300	1351	800	551	23	47.1	26.7	18.8	.520	.550	.537
500	1468	917	551	23	27.9	26.7	14.3	.520	.360	.422
B) $C_4 = .09, C_7 = \$.07/\text{kWh}$										
100	1312	650	662	29	105	31	30.5	.570	.870	.744
300	1542	882	662	29	65.2	31	24.6	.570	.720	.656
500	1745	1083	662	29	45.2	31	21.1	.570	.550	.558
1000	1912	1250	662	29	2.2	31	0	.570	.030	.260
C) $C_4 = .09, C_7 = \$.10/\text{kWh}$										
100	1508	694	814	36.2	134	35.5	36.8	.610	.950	.803
300	1808	994	814	36.2	86.5	35.5	30.1	.610	.820	.732
500	2018	1204	814	36.2	64.0	35.5	26.5	.610	.720	.674
1000	2477	1663	814	36.2	34.5	35.5	21.2	.610	.430	.507
D) $C_4 = .11, C_7 = \$.05/\text{kWh}$										
100	1267	670	597	20.1	71.7	24.4	21.1	.490	.728	.628
300	1480	883	597	20.1	37.8	24.4	15.1	.490	.460	.473
500	1529	932	597	20.1	6.29	24.4	0	.490	.081	.255
E) $C_4 = .11, C_7 = \$.07/\text{kWh}$										
100	1470	741	729	25.8	92.8	28.7	27.3	.530	.830	.702
300	1713	984	729	25.8	55.7	28.7	21.7	.530	.658	.603
500	1933	1204	729	25.8	36.4	28.7	18.0	.530	.450	.484
F) $C_4 = .11, C_7 = \$.10/\text{kWh}$										
100	1695	805	890	32.6	119	33.3	33.7	.580	.912	.770
300	2011	1121	890	32.6	75.8	33.3	27.4	.580	.775	.692
500	2228	1338	890	32.6	54.6	33.3	24.0	.580	.661	.626
1000	2701	1811	890	32.6	26.1	33.3	18.2	.580	.320	.430

C_4 = Capital Recovery Factor
 C_7 = Levelized Backup Power
 Cost (\$/kWh)

Electrical Demand = 12,300 kWh/year
 Thermal Demand = 16,550 kWh/year

Table 5

Optimum System Parameters

- o All-Photovoltaic System
- o Phoenix

C_p \$/kW _{pk}	C \$/year	A_e m ²	P_{es} kWh	R_e
A) $C_4 = .09, C_7 = \$.05/\text{kWh}$				
100	896	155	56.5	.900
300	1328	75.4	22.4	.498
500	1468	26.9	0	.200
B) $C_4 = .09, C_7 = \$.07/\text{kWh}$				
100	950	163	51.1	.900
300	1418	125	40	.770
500	1826	84.5	32.6	.561
1000	2081	13.2	0	.116
C) $C_4 = .09, C_7 = \$.10/\text{kWh}$				
100	1036	163	51.1	.900
300	1534	160	50	.880
500	1939	140	50	.840
1000	2739	47.1	19	.363
D) $C_4 = .11, C_7 = \$.05/\text{kWh}$				
100	1035	163	51.1	.900
300	1416	44.4	5.8	.315
500	1524	18.5	0	.145
E) $C_4 = .11, C_7 = \$.07/\text{kWh}$				
100	1092	163	51.1	.900
300	1559	125	40	.790
500	1955	46.2	12.3	.344
F) $C_4 = .11, C_7 = \$.10/\text{kWh}$				
100	1179	163	51.1	.900
300	1707	160	50	.880
500	2190	125	40	.780
1000	2877	28.6	8.74	.238

 C_4 = Capital Recovery Factor

Total Demand = 28,850 kWh/year

 C_7 = Levelized Backup Power Cost (\$/kWh)

Figure 23
Minimum Levelized Annual Cost of Service,
Combined-Panel System, Phoenix, CRF = .09

Figure 24
Minimum Levelized Annual Cost of Service,
Combined-Panel System, Phoenix, CRF = .11

Figure 25
Minimum Levelized Annual Cost of Service
Separate-Panel System, Phoenix, CRF = .09

Figure 26
Minimum Levelized Annual Cost of Service,
Separate-Panel System, Phoenix, CRF = .11

Figure 27
Minimum Levelized Annual Cost of Service,
All-Photovoltaic System, Phoenix, CRF = .09

Figure 28
Minimum Levelized Annual Cost of Service,
All-Photovoltaic System, Phoenix, CRF = .11

objective is $\$50/\text{m}^2$ (C_1). The present installed cost of such panels is $\$100\text{--}150/\text{m}^2$.

The results shown in Tables 3-5 and Figures 23-28 indicate that for a given backup energy cost, as photovoltaic array cost increases, the optimum photovoltaic panel area and electrical storage capacity both decrease. The fraction of the electrical and total demand provided by solar energy is also, of course, reduced. For example, in Table 5 (all photovoltaic system), for CRF of .09 and levelized backup energy cost of $7\text{¢}/\text{kWh}$, as the array cost increases from $\$100/\text{kW}_{\text{pk}}$ to $\$1000/\text{kW}_{\text{pk}}$, the optimum array area decreases from 163 m^2 to 13 m^2 , the optimum electrical storage capacity drops from over 50 kWh to 0 kWh, and the fraction of the demand provided by solar energy drops from .90 to .36.

Figure 29 shows the minimum levelized annual cost for each of the system concepts for a CRF of .09 and levelized backup energy costs of 5, 7, and $10\text{¢}/\text{kWh}$. It appears that at array costs of less than about $\$500/\text{kW}_{\text{pk}}$, the use of an all-photovoltaic system yields a lower annual cost of service than do either combined or separate-panel systems. This result is due primarily to the fact that at low array cost, the cost of thermal energy generation is lower with a photovoltaic panel than with a solar thermal panel. (The solar thermal panel cost is fixed at $\$50/\text{m}^2$; see Table 2.) However, as the array cost increases from $\$500/\text{kW}_{\text{pk}}$, the separate panel system becomes more economic. In addition, Figure 29 shows that at low array costs, the economic advantages of all-photovoltaic systems increase as the levelized cost of backup power increases. For example, at $\$100/\text{kW}_{\text{pk}}$ and backup power cost of $5\text{¢}/\text{kWh}$, the minimum levelized annual cost of service in the all-photovoltaic system case is about $\$900$, while for the combined or separate-panel systems this cost is about $\$1100$. If the backup power cost were $10\text{¢}/\text{kWh}$ at the same array cost, the minimum levelized annual cost in the all-photovoltaic system case would be about $\$1050$, but in the combined or separate-panel system cases, the annual service costs would increase to about $\$1500$. This

Figure 29

Comparison of Minimum Levelized Annual Cost of Service,
Phoenix, CRF = .09

result is again due to the low cost of energy generation by the all-photovoltaic system at low array costs.

It also appears that except at very low array costs, the higher thermal and electrical efficiencies of the separate-panel system more than compensate for the savings in structural and installation costs provided by the combined-panel system; and, as a result, lower annual costs are achieved with the separate-panel system than with the combined-panel system.

If the breakeven array cost is defined as the array cost for which the minimum levelized annual cost of service with a PTES is equal to the cost of purchasing all the required energy from the utility, then it can be seen from Figure 29 that combined and separate-panel systems can have breakeven array costs over \$1000/kW_{pk} if both the levelized backup and utility energy costs are 7¢/kWh or greater. Figures 30 and 31 show the breakeven array cost as a function of levelized backup energy cost for the case where the backup and utility energy costs are equal and the CRF value is .09 or .11. For levelized backup costs in excess of 5¢-6¢/kWh, the breakeven array cost is greater than \$500/kW_{pk}, and it appears that combined or separate-panel systems provide higher breakeven array costs than the all-photovoltaic system.

It is important to note that breakeven array cost is very sensitive to the assumed values of levelized backup power and utility rates. When backup power and utility rates are assumed to be equal as in the above discussion, non-solar users are, in effect, providing a subsidy to cover the cost of electrical generation, transmission, and distribution facilities that are required as backup for solar powered homes. If the utility charged only the owners of solar powered homes for the cost of these facilities, then the cost of backup power would be greater than the cost of utility power and the breakeven array cost would drop significantly. For example, in Figure 29, if the levelized backup and utility charges are both 7¢/kWh, the breakeven array cost for separate or

Figure 30
Effect of Levelized Backup Power Cost on Breakeven Array Cost,
Phoenix, CRF = .09

Figure 31

Effect of Levelized Backup Power Cost on Breakeven Array Cost
Phoenix, CRF = .11

combined-panel systems is in excess of \$1000/kW_{pk} and about \$800/kW_{pk} for the all-photovoltaic system (Point A). However, if the levelized backup cost is 7¢/kWh and the utility charge is 5¢/kWh, the breakeven array cost is about \$250/kW_{pk} (Point B) for the combined or separate-panel systems, and about \$350/kW_{pk} for the all-photovoltaic system (Point C)

The 1975 cost of residential electricity in Phoenix was about 3.35¢/kWh (Ref. 16). If the cost of utility power rises at an annual rate slightly in excess of 1% over the assumed annual inflation rate (5% per year), the levelized cost of this power would be about 7¢/kWh for a PTES that goes into operation in 1986 and has a 30 year lifetime. If it is assumed that the cost of backup and utility power are equal, then PTES would be competitive with utility power for new Phoenix residences by the mid-1980s if the array cost reduction objectives of the Photovoltaic Program are achieved (\$1000/kW_{pk} array in 1982 and \$500/kW_{pk} array by 1986).

Figure 32 shows the fraction of the total demand provided from solar energy by each of the system concepts when in the configuration that leads to minimum levelized annual cost of service. The results are presented as a function of array cost, for levelized backup energy costs of 5¢ and 10¢/kWh and CRF = .09. At low array costs, the PTES provides 70-90% of the total annual energy required for the residence. As expected this fraction drops sharply with increasing array cost when the levelized cost of backup energy is 5¢/kWh but more gradually when this cost is 10¢/kWh. Over the range of array costs that were investigated, the combined-panel system provides a slightly higher fraction of the total demand from solar energy than does the separate-panel system, although, as noted previously, the use of a combined-panel system leads to a slightly higher annual cost of service at a given array cost.

Breakdowns of the capital costs of representative examples of each of the three systems concepts are presented in Table 6. In each case the assumed array cost was \$500/kW_{pk} and the assumed cost of backup energy was 7¢/kWh, values that are believed likely to prevail in

Figure 32

Fraction of Total Demand Provided by PTES at Minimum Annual Cost,
Phoenix, CRF = .09

Table 6

Typical Residential PTES in Phoenix for the Mid-1980s

A) Combined-Panel System

<u>Thermal System</u>	<u>Size</u>	<u>Cost (1975 \$)</u>
Photovoltaic/Thermal Panel	36.2 m ²	1810
Thermal Storage	41 kWh	437
Balance of Plant		1000
Capital Cost		<u>\$3247</u>

Fraction of Thermal Demand Provided, 61%
 Backup Electrical Power Required, 4797 kWh
 Annual Cost (CRF = .09, Levelized
 Backup Power at \$.07/kWh,
 O&M = 1.5% of Capital Cost)

\$ 676

Electrical System

Air Cooled Photovoltaic Panel	25 m ²	4131
Photovoltaic/Thermal Panel	36.2 m ²	
Array Structure	25 m ²	250
Electrical Storage	24.3 kWh	972
Power Conditioning	8 kW _{pk}	800
Wiring/Miscellaneous		238
Capital Cost		<u>\$6391</u>

Fraction of Electrical Demand Provided, 62%
 Backup Electrical Power Required, 6289 kWh
 Annual Cost (CRF = .09, Levelized Backup
 Power at \$.07/kWh, O&M = 1.5% of Capital
 Cost

\$1111

Total Incremental Capital Cost for PTES \$9638

Total Annual Cost of Service to Homeowner
 with PTES \$1787

Total Annual Cost of Service to Homeowner
 with Utility Power (\$.07/kWh) \$2020

Total Annual Savings to Homeowner with PTES \$ 233

Table 6

Typical Residential PTES in Phoenix for the Mid-1980s (Cont'd)

B) Separate-Panel System

<u>Thermal System</u>	Size	Cost (1975 \$)
Thermal Panel	29 m ²	1450
Thermal Storage	31 kWh	330
Balance of Plant		1000
Capital Cost		<u>\$2780</u>

Fraction of Thermal Demand Provided, 57%
 Backup Electrical Power Required, 5289 kWh
 Annual Cost (CRF = .09, Levelized Backup
 Power at \$.07/kWh, O&M = 1.5% of Capital
 Cost)

\$ 662

Electrical System

Air Cooled Photovoltaic Panel	45.2 m ²	3051
Array Structure	45.2 m ²	452
Electrical Storage	21.1 kWh	844
Power Conditioning	8kW _{pk}	800
Wiring/Miscellaneous		209
Capital Cost		<u>\$5366</u>

Fraction of Electrical Demand Provided, 55%
 Backup Electrical Power Required, 7448 kWh
 Annual Cost (CRF = .09, Levelized Backup Power
 at \$.07/kWh, O&M = 1.5% of Capital Cost)

\$1084

Total Incremental Capital Cost for PTES

\$8146

Total Annual Cost of Service to Homeowner
with PTES

\$1746

Total Annual Cost of Service to Homeowner
with Utility Power (\$.07/kWh)

\$2020

Total Annual Savings to Homeowner with PTES

\$ 274

Table 6

Typical Residential PTES in Phoenix for the Mid-1980s (Cont'd)

C) All-Photovoltaic System

	<u>Size</u>	<u>Cost (1975 \$)</u>
Air Cooled Photovoltaic Panel	84.5 m ²	5704
Array Structure	84.5 m ²	845
Electrical Storage	32.6 kWh	1304
Power Conditioning	8 kW _{pk}	800
Wiring/Miscellaneous		295
Capital Cost		\$8948
Fraction of Total Demand Provided, 56.1%		
Backup Electrical Power Required, 12,665 kWh		
Annual Cost (CRF = .09, Levelized Backup Power at \$.07/kWh, O&M = 1.5% of Capital Cost)		
		\$1826
Total Annual Cost of Service to Homeowner with Utility Power (\$.07/kWh)		
		\$2020
Total Annual Savings to Homeowner with PTES		
		\$ 194

Phoenix in the mid-1980's. The array sizes and storage capacities of the systems represented in the Table are those that would lead to minimum levelized annual cost of service under the assumed economic (cost) conditions. For the cases illustrated, the annual cost of service is almost the same for each of the system concepts, and the annual savings to the homeowner amounts to 2-3% of the incremental capital cost of the PTES.

The effect of selling surplus electrical energy to the utility on the minimum levelized annual cost of service is shown for the separate-panel system in Table 7 and Figure 33 and for the all-photo-voltaic system in Table 8 and Figure 34. It was assumed, in developing these tables and figures, that the photovoltaic total energy system has no electrical storage and that any electrical energy not needed to satisfy residential loads is converted to AC electricity and sold to the utility at a specified price per kWh. This price is assumed to be constant, regardless of the time of day or amount of energy sold. To account for power conversion losses, 15% of the excess DC electrical energy is assumed not to be available for sale. In addition, since these systems have no electrical storage, it is assumed that the size (rating) of the power conditioner is a function of array area rather than being fixed at 8 kW_{pk} . Power conditioner cost is assumed to be $\$100/\text{kW}_{\text{pk}}$, regardless of size.

Levelized sale prices of 1, 3, and 5¢/kWh ($J = \$0.01, \$0.03, \text{ and } \$0.05/\text{kWh}$ in Tables 7 and 8) were considered. As can be seen in the tables, for the higher sale prices and low array costs, very large array areas are desirable. For some cases the sale price is greater than the electrical generation cost and therefore as large an array as might reasonably be installed on the roof is desirable. It is assumed that the maximum photovoltaic array area or thermal panel plus photovoltaic array area is 160 m^2 (i.e. the available roof area is about equal to the residence floor space of 1600 ft^2). This means that for the separate

Table 7

Optimum System Parameters

- o Separate-Panel System
- o Phoenix
- o No Electrical Storage
- o Sale of Surplus Electricity to Utility

J = \$.01/kWh				J = \$.03/kWh			J = \$.05/kWh		
C_p	A_e	C_e	R_e	A_e	C_e	R_e	A_e	C_e	R_e
\$/kW _{pk}	m ²	\$/year		m ²	\$/year		m ²	\$/year	
A) $C_4 = .09, C_7 = $.05/kWh$									
100	62.5	645.2	.424	120*	290*	.5*	120*	-110*	.5*
300	27.6	765	.273	107	650	.491	120*	230*	.5*
500	9.96	816	.122	24.2	796	.250	120*	571*	.5*
B) $C_4 = .09, C_7 = $.07/kWh$									
100	76.9	830	.455	120*	455*	.5*	120*	56*	.5*
300	42	991	.353	120*	795*	.5*	120*	396*	.5*
500	24.4	1083	.251	45	1021	.366	120*	736*	.5*
C) $C_4 = .09, C_7 = $.10/kWh$									
100	91.4	1095	.476	120*	703*	.5*	120*	304*	.5*
300	56.6	1297	.407	120*	1043*	.5*	120*	644*	.5*
500	38.9	1430	.338	63.6	1318	.427	120*	984*	.5*
1000	14.3	1609	.166	20.3	1586	.219	31.9	1544	.3
D) $C_4 = .11, C_7 = $.05/kWh$									
100	49.6	687	.384	120*	380*	.5*	120*	-20*	.5*
300	18.5	795	.205	49	741	.382	120*	385*	.5*
500	1.71	827	.023	7.1	824	.091	43.8	798	.361
E) $C_4 = .11, C_7 = $.07/kWh$									
100	64	883	.428	120*	545*	.5*	120*	146*	.5*
300	33	1039	.307	70	934	.442	120*	550*	.5*
500	16.1	1121	.183	28	1089	.276	83.3	986	.465
F) $C_4 = .11, C_7 = $.10/kWh$									
100	78.6	1158	.458	120*	793*	.5*	120*	394*	.5*
300	47.5	1363	.376	88.4	1202	.472	120*	798*	.5*
500	30.7	1495	.293	46.5	1420	.372	108	1243	.492
1000	6.95	1643	.089	10.2	1634	.125	15.93	1618	.182

*Photovoltaic array limited to 120 m² (non-optimum)

Photovoltaic array plus thermal panel area equals 150 to 160 m², which is about the maximum area of a roof mounted system on a 1600 ft² home.

Table 8

Optimum System Parameters

- o All-Photovoltaic System
- o Phoenix
- o No Electrical Storage
- o Sale of Surplus Electricity to Utility

J = \$.01/kWh				J = \$.03/kWh				J = \$.05/kWh	
C_p \$/year	A_e m^2	C \$/year	R_e	A_e m^2	C \$/year	R_e	A_e m^2	C \$/year	R_e
A) $C_4 = .09$, $C_7 = \$.05/\text{kWh}$									
100	95.7	1113	.414	160*	679*	.45*	160*	195	.45*
300	46.4	1303	.297	140	1119	.45	160*	.694*	.45*
500	22.5	1398	.180	41.6	1382	.278	160*	1102*	.45*
B) $C_4 = .09$, $C_7 = \$.07/\text{kWh}$									
100	115	1444	.436	160*	997*	.45*	160*	513*	.45*
300	65.8	1688	.358	120	1439	.441	160*	967*	.45*
500	41.9	1838	.279	74.3	1763	.378	160*	1420*	.45*
1000	9.2	2006	.084	9.57	2010	.087	10.7	2013	.096
C) $C_4 = .09$, $C_7 = \$.10/\text{kWh}$									
100	134.5	1924	.451	160*	1474*	.45*	160*	990*	.45*
300	85.2	2224	.399	120	1916	.441	160*	1444*	.45*
500	61.3	2429	.346	100.8	2279	.421	160*	1897*	.45*
1000	28.6	2734	.215	36.2	2714	.254	53.5	2669	.322
D) $C_4 = .11$, $C_7 = \$.05/\text{kWh}$									
100	77	1178	.383	160*	796*	.45*	160*	312*	.45*
300	34.1	1354	.244	88.4	1287	.404	160*	852*	.45*
500	11.7	1429	.104	15.2	1431	.130	120	1364	.441
E) $C_4 = .11$, $C_7 = \$.07/\text{kWh}$									
100	96.4	1523	.480	160*	1114*	.45*	160*	630*	.45*
300	53.5	1764	.322	121	1617	.441	160*	1170*	.45*
500	31.1	1904	.228	47.8	1872	.303	120	1683	.441
F) $C_4 = .11$, $C_7 = \$.10/\text{kWh}$									
100	115	2001	.436	160*	1591*	.45*	160*	1107*	.45*
300	72.3	2306	.373	147	2111	.45	160*	1647*	.45*
500	50	2510	.311	73.7	2418	.377	120	2160	.441
1000	18.4	2784	.153	21.7	2782	.174	28.1	2774	.213

*Photovoltaic array limited to 160 m^2 due to roof limits (non-optimum)

Figure 33

Minimum Levelized Annual Cost of Service with Sale of Surplus Electrical Energy, Separate-Panel System, Phoenix, Levelized Backup Energy Cost = \$.05/kWh, CRF = .09

Figure 34

Minimum Levelized Annual Cost of Service with Sale of Surplus Electrical Energy, All-Photovoltaic System, Phoenix, Levelized Backup Energy Cost = \$.05/kWh, CRF = .09

panel system, the maximum photovoltaic array area is about 120 m^2 , while the all-photovoltaic system can have an array as large as 160 m^2 .

As noted previously, Figures 33 and 34 show that sale of surplus electricity is most effective in reducing annual cost when array costs are low and becomes less effective as the array cost increases.

For comparison, the minimum levelized annual cost of service for systems with electrical storage, where excess electrical energy is dumped rather than sold, is also shown in Figures 33 and 34. For the separate panel system at \$100/kW array cost (Figure 33), systems with storage result in the same annual cost of service as do systems without storage where surplus electrical energy is sold for about 1.5¢/kWh. For the all-photovoltaic system (Figure 34), at \$100/kW_{pk} array cost, the annual costs are equal when the electrical energy sale price is about 2¢/kWh.

The desirability of selling excess electrical energy to the utility is therefore a function of a number of parameters, including array cost, the price the utility is willing to pay (which may vary during the day or over a year), and the cost of electrical storage.

2. Madison Residence

The minimum levelized annual cost of service for each of the three system concepts for a Madison home are shown as a function of array cost, capital recovery factor and the levelized cost of backup power in Tables 9-11 and Figures 35-40. The system capital cost assumptions are the same as for the Phoenix residence which are summarized in Table 2, and the results presented are based on the assumption that surplus electrical energy is wasted.

In general, the annual cost of service is much higher in Madison than in Phoenix, primarily because of the reduced insolation and the very large nighttime heating load.

Table 9

Optimum System Parameters

- o Combined-Panel System
- o Madison

C_p \$/kW _{pk}	C \$/year	A_{th} m ²	A_e m ²	A m ²	P_{ths} kWh	P_{es} kWh	R_{th}	R_e	R
A) $C_4 = .09, C_7 = \$.05/\text{kWh}$									
100	2093	19.9	51.8	71.7	25.4	20	.090	.701	.248
300	2173	13.7	0	13.7	20.9	0	.073	.129	.087
500	2200	5.48	0	5.48	10.9	0	.031	.052	.036
B) $C_4 = .09, C_7 = \$.07/\text{kWh}$									
100	2659	29.6	56.7	86.3	34.3	27.6	.126	.820	.303
300	2879	29.6	31.2	60.8	34.3	22.8	.126	.630	.254
500	2957	16.6	0	16.6	26.6	0	.078	.152	.097
C) $C_4 = .09, C_7 = \$.10/\text{kWh}$									
100	3491	40.5	60.7	101.2	42.8	34.2	.153	.864	.334
300	3737	40.5	34.9	75.4	42.8	30.6	.153	.751	.305
500	3929	40.5	20.5	61	42.8	27.8	.153	.642	.277
1000	4136	14.5	0	14.5	28.2	0	.070	.136	.087
D) $C_4 = .11, C_7 = \$.05/\text{kWh}$									
100	2173	14.9	46.2	61.1	19.9	13.5	.077	.600	.210
300	2224	7.7	0	7.7	12.6	0	.049	.076	.056
E) $C_4 = .11, C_7 = \$.07/\text{kWh}$									
100	2791	24.6	54.4	79	29.9	24	.112	.760	.277
300	2974	18.6	0.1	18.7	26.3	0	.088	.167	.108
500	3013	11.4	0	11.4	20.6	0	.064	.109	.075
F) $C_4 = .11, C_7 = \$.10/\text{kWh}$									
100	3606	35.1	58.8	93.4	38.7	31.1	.145	.860	.327
300	3871	35.1	33.5	68.6	38.7	27.3	.145	.730	.294
500	4092	35.1	18.5	53.6	38.7	23	.145	.590	.258
1000	4213	9.5	.1	9.6	22.1	0	.057	.092	.066

C_4 = Capital Recovery Factor
 C_7 = Levelized Backup Power Cost
 (\$/kWh)

Electrical Demand = 10,160 kWh/year

Thermal Demand = 29,750 kWh/year

Table 10

Optimum System Parameters

- o Separate-Panel System
- o Madison

C_p \$/kW _{pk}	C \$/year	A_{th} m ²	A_e m ²	P_{ths} kWh	P_{es} kWh	R_{th}	R_e	R
A) $C_4 = .09$, $C_7 = \$.05/\text{kWh}$								
100	2063	15.3	47.3	24.1	10.7	.0835	.558	.204
300	2148	15.3	16.9	24.1	3.68	.0835	.268	.130
B) $C_4 = .09$, $C_7 = \$.07/\text{kWh}$								
100	2674	23.2	67.1	29.5	16.3	.111	.687	.258
300	2810	23.2	33.5	29.5	9.83	.111	.466	.201
500	2889	23.2	16.7	29.5	5.10	.111	.276	.120
C) $C_4 = .09$, $C_7 = \$.10/\text{kWh}$								
100	3536	32.1	89.6	34.3	22.6	.132	.789	.299
300	3731	32.1	52.5	34.3	16.1	.132	.789	.256
500	3850	32.1	33.4	34.3	11.6	.132	.475	.219
1000	3991	32.1	5.0	34.3	0	.132	.0875	.121
D) $C_4 = .11$, $C_7 = \$.05/\text{kWh}$								
100	2147	10.9	37.4	20	7.79	.063	.475	.168
300	2232	10.9	8.04	20	0	.063	0	.047
E) $C_4 = .11$, $C_7 = \$.07/\text{kWh}$								
100	2783	19.1	56.6	26.9	13.4	.098	.625	.232
300	2912	19.1	24.9	26.9	6.76	.098	.373	.168
500	2966	19.1	6.56	26.9	0	.098	.110	.101
F) $C_4 = .11$, $C_7 = \$.10/\text{kWh}$								
100	3670	27.7	78.4	32	19.5	.122	.743	.280
300	3867	27.7	43	32	13	.122	.551	.231
500	3979	27.7	25	32	8.5	.122	.385	.189
1000	4076	27.7	1.97	32	0	.122	.037	.100

 C_4 = Capital Recovery Factor C_7 = Levelized Backup Power Cost (\$/kWh)

Electrical Demand = 10,160 kWh/yr

Thermal Demand = 29,750 kWh/year

Table 11

Optimum System Parameters

- o All-Photovoltaic System
- o Madison

C_p \$/kW _{pk}	C \$/year	A_e m ²	P_{es} kWh	R_e
A) $C_4 = .09, C_7 = \$.05/\text{kWh}$				
100	1761	150	25	.415
300	2023	51.8	8.1	.192
B) $C_4 = .09, C_7 = \$.07/\text{kWh}$				
100	2225*	160*	25*	.425*
300	2615	130	25	.39
500	2811	49.2	11	.19
C) $C_4 = .09, C_7 = \$.10/\text{kWh}$				
100	2844*	160*	40*	.46*
300	3300	150	35	.44
500	3672	100	35	.35
1000	4065	11.9	0	.05
D) $C_4 = .11, C_7 = \$.05/\text{kWh}$				
100	1872	120	25	.27
300	2076	23.3	0	.09
E) $C_4 = .11, C_7 = \$.07/\text{kWh}$				
100	2326	150	35	.44
300	2718	91.5	20.1	.317
F) $C_4 = .11, C_7 = \$.10/\text{kWh}$				
100	2975*	160*	40*	.46*
300	3494	130	35	.41
500	3839	90	23.8	.319

 C_4 = Capital Recovery Factor C_7 = Levelized Cost of Backup Power

Total Demand = 39,910 kWh/yr

* Roof-Limited

Figure 35
Minimum Levelized Annual Cost of Service,
Combined-Panel System, Madison, CRF = .09

Figure 36
Minimum Levelized Annual Cost of Service
Combined-Panel System, Madison, CRF = .11

Figure 37
Minimum Levelized Annual Cost of Service,
Separate-Panel System, Madison, CRF = .09

Figure 38
Minimum Levelized Annual Cost of Service
Separate-Panel System, Madison, CRF = .11

Figure 39
Minimum Levelized Annual Cost of Service,
All-Photovoltaic System, Madison, CRF = .09

Figure 40
Minimum Levelized Annual Cost of Service,
All-Photovoltaic System, Madison, CRF = .11

The system concepts are compared on the basis of minimum levelized annual cost in Figure 41. This figure shows that the all-photovoltaic system provides service at lower annual cost than either the separate or combined-panel systems for array costs of less than \$500/kW_{pk} and that this advantage increases with increasing cost of backup power. Figure 41 also shows that use of the combined-panel system leads to higher annual cost of service than does the separate-panel system.

Figures 42 and 43 show the effect of levelized backup power cost on breakeven array cost and Figure 44 shows the fraction of total demand provided at the minimum annual cost of service. These figures show that the all-photovoltaic system provides higher breakeven array costs than either the combined or separate-panel systems for backup power costs of less than 9-10¢/kWh (Figures 42 and 43) as well as meeting a higher fraction of the total demand with solar energy (Figure 44). The breakeven array cost for the all-photovoltaic system is about \$500/kW_{pk} when the cost of backup power is 7¢/kWh (Figure 41). If the cost of utility electricity rises at an annual rate slightly in excess of 1% over the assumed inflation rate (5%), the levelized cost of the electricity would be about 7¢/kWh for a PTES installed in Madison in the early 1990's that had a 30 year system life (1975 electricity cost was 2.65¢/kWh). It appears that if the DOE array cost reduction objectives are achieved, photovoltaic total energy systems will be competitive with utility power for new homes in the Madison area in the early 1990s, if the cost of backup electricity is the same as the cost of electricity for all residential customers (i.e., if non solar users partially subsidize the additional cost of generation and transmission required to support the photovoltaic systems).

The capital and annual costs for each of the system concepts is shown in Table 12 for a typical PTES that might be installed in a Madison residence in the mid-1990s. It is assumed that photovoltaic array costs are \$300/kW_{pk} (the array cost objective for 1990) and that levelized

Figure 41
Comparison of Minimum Levelized Annual Cost of Service,
Madison, CRF = .09

Figure 42

Effect of Levelized Backup Power Cost on Breakeven Array Cost,
Madison, CRF = .09

Figure 43
Effect of Levelized Backup Power Cost on Breakeven Array Cost,
Madison, CRF = .11

Figure 44

Fraction of Total Demand Provided by PTES at Minimum Annual Cost,
Madison, CRF = .09

Table 12

Typical Residential PTES in Madison for the Early 1990s

A) Combined-Panel System

<u>Thermal System</u>	Size	Cost (1975\$)
Photovoltaic/Thermal Panel	29.6 m ²	1480
Thermal Storage	34.3 kWh	366
Balance of Plant		1000
Capital Cost		\$2846
Fraction of Thermal Demand Provided, 12.6%		
Backup Electrical Power Required, 26,002 kWh		
Annual Cost (CRF = .09 Levelized Backup		
Power at \$.07/kWh, O&M = 1.5% Capital Cost)		
		\$2119
<u>Electric System</u>		
Air Cooled Photovoltaic Panel	31.2 m ²	2462
Photovoltaic/Thermal Panel	29.6 m ²	
Array Structure	31.2 m ²	312
Electrical Storage	22.8 kWh	912
Power Conditioning	8 kW _{pk}	800
Wiring/Miscellaneous		238
Capital Cost		\$4724
Fraction of Electrical Demand Provided, 63%,		
Backup Electrical Power Required, 3759 kWh		
Annual Cost (CRF = .09, Levelized Backup		
Power at \$.07/kWh, O&M = 1.5% Capital Cost)		
		\$759
Total Incremental Capital Cost for PTES		\$7570
Total Annual Cost of Service to Homeowner with PTES		\$2878
Total Annual Cost of Service to Homeowner with Utility Power (\$.07/kWh)		\$2794
Total Annual Savings (Loss) to Homeowner with PTES		\$ (84)

Table 12

Typical Residential PTES in Madison for the Early 1990s (Cont'd)

B) Separate-Panel System

<u>Thermal System</u>	Size	Cost (1975\$)
Thermal Panel	23.2 m ²	1160
Thermal Storage	29.5 kWh	314
Balance of Plant		1000
Capital Cost		\$2474

Fraction of Thermal Demand Provided, 11.1%
 Backup Electrical Power Required, 26,448 kWh
 Annual Cost (CRF = .09, Levelized Backup
 Power at \$.07/kWh, O&M = 1.5% Capital Cost)

\$2111

Electric System

Air Cooled Photovoltaic Panel	33.5 m ²	1357
Array Structure	33.5 m ²	335
Electrical Storage	9.8 kWh	392
Power Conditioning	8 kW _{pk}	800
Wiring/Miscellaneous		153
Capital Cost		\$3037

Fraction of Electrical Demand Provided, 46.6%
 Backup Electrical Power Required, 5425/kWh
 Annual Cost (CRF = .09, Levelized Backup
 Power at \$.07/kWh, O&M = 1.5% Capital Cost)

\$ 699

Total Incremental Capital Cost for PTES

\$5511

Total Annual Cost of Service to Homeowner
with PTES

\$2810

Total Annual Cost of Service to Homeowner
with Utility Power (\$.07/kWh)

\$2794

Total Annual Savings (Loss) to Homeowner with PTES

\$(16)

Table 12

Typical Residential PTES in Madison for the Early 1990s (Cont'd)

C) All-Photovoltaic System

	Size	Cost (1975\$)
Air Cooled Photovoltaic Panel	130 m ²	5265
Array Structure	130 m ²	1300
Electrical Storage	25 kWh	1000
Power Conditioning	8 kW _{pk}	800
Wiring/Miscellaneous		310
Capital Cost		<u>\$8675</u>
Fraction of Total Demand Provided, 39%		
Backup Electrical Power Required, 24,345 kWh		
Annual Cost (CRF = .09, Levelized Backup Power		
at \$.07/kWh, O&M = 1.5% of Capital Cost)		
		\$ 2615
Total Annual Cost of Service to Homeowner with		
Utility Power (\$.07/kWh)		
		\$ 2794
Total Annual Savings to Homeowner with PTES		
		\$ 179

backup and utility electricity costs are 7¢/kWh. Table 12 shows that under these conditions, the incremental capital cost of PTES can vary from \$5500-9000, and that while the all-photovoltaic system requires the largest investment, it can provide an annual savings, while the combined or separate-panel systems show a small loss in comparison to purchasing all of the energy required from the utility.

The greater relative attractiveness of the all-photovoltaic system in Madison is due primarily to the poor thermal performance of the combined and separate-panel systems in this location. Two approaches for improving the performance of the combined and separate-panel systems were evaluated by modifying the assumptions for the separate-panel system.

a) A thermal panel operating temperature of 40°C was assumed instead of 70°C. In this case, it was assumed that the additional energy required to increase the temperature of the domestic hot water to the required level (60°C) was purchased from the utility. In addition, the assumed cost of thermal storage per kWh was increased from \$10.66/kWh to \$33/kWh to account for the larger volume of water required per kWh of thermal energy stored. (A 25°C (77°F) minimum storage temperature was assumed, which appears barely adequate for space heating. With a 40°C panel, the maximum thermal storage temperature is estimated to be 35°C. Therefore, the energy stored is 10 Btu/lb. of water. With a 70°C panel, the maximum temperature is about 60°C and the energy stored is 35 Btu/lb. of water. Therefore, to store the same amount of energy with a 40°C panel requires 3.5 times the weight or volume of water needed with a 70°C panel.) A slightly lower cost per unit volume was used for the 40°C system to account for some savings resulting from increased tank size.

b) A thermal panel of improved efficiency was assumed ($U_o = .005 \text{ kW/m}^2\text{-}^\circ\text{C}$) without an increase in the panel cost.

Tables 13 and 14 show the minimum levelized annual cost of service for separate-panel systems with these modifications. Figures 45 and 46 compare the minimum levelized annual cost of service for the separate panel system with and without these changes for CRF of .09 and .11 and levelized backup power cost of 5¢/kWh. These figures and tables show that significant reductions in the levelized annual cost of service and improvement in the fraction of the demand provided by separate-panel PTES (see Figures 11 and 12) can be provided by these changes. (These improvements could also be achieved with combined-panel PTES). It appears that a reduction in thermal panel operating temperature is more effective in reducing annual costs than increasing panel efficiency, even if it is assumed that the more efficient panel is no more costly than the baseline panel.

Table 13

Optimum System Parameters

- o Separate-Panel System
- o Madison
- o $T_A = 40^\circ\text{C}$; $U_O = .007 \text{ kW/m}^2 \text{ } ^\circ\text{C}$

C_p \$/kW _{pk}	C \$/year	A_{th} m ²	A_e m ²	P_{ths} kWh	P_{es} kWh	R_{th}	R_e	R
A) $C_4 = .09$, $C_7 = \$.05/\text{kWh}$								
100	1919	30.6	47.2	43.2	10.7	.272	.558	.327
300	2004	30.6	16.9	43.2	3.68	.272	.268	.255
500	2028	30.6	3.7	43.2	0	.272	.068	.202
B) $C_4 = .09$, $C_7 = \$.07/\text{kWh}$								
100	2421	39.3	67.1	52.3	16.3	.312	.687	.387
300	2557	39.3	33.5	52.3	9.83	.312	.466	.331
500	2626	39.3	16.7	52.3	5.1	.312	.276	.283
C) $C_4 = .09$, $C_7 = \$.10/\text{kWh}$								
100	3082	49.6	89.6	61.8	22.6	.355	.789	.442
300	3277	49.6	52.5	61.8	16.1	.355	.621	.400
500	3396	49.6	33.4	61.8	11.6	.355	.475	.362
1000	3537	49.6	5.0	61.8	0	.355	.0875	.264
D) $C_4 = .11$, $C_7 = \$.05/\text{kWh}$								
100	2018	26.3	37.4	38.3	7.79	.252	.475	.292
300	2088	26.3	8.04	38.3	0	.252	.129	.205
E) $C_4 = .11$, $C_7 = \$.07/\text{kWh}$								
100	2549	34.7	56.6	47.6	13.4	.295	.625	.359
300	2679	34.7	24.9	47.6	6.76	.295	.272	.296
500	2732	34.7	6.56	47.6	0	.295	.11	.279
F) $C_4 = .11$, $C_7 = \$.10/\text{kWh}$								
100	3220	44.4	78.4	57.2	19.5	.349	.743	.428
300	3416	44.4	43	57.2	13	.349	.551	.378
500	3529	44.4	25	57.2	8.5	.349	.385	.336
1000	3626	44.4	1.97	57.2	0	.349	.374	.333

C_4 = Capital Recovery Factor
 C_7 = Levelized Backup Energy Cost
 (\$/kWh)

Electrical Demand = 10,160 kWh/yr
 Thermal Demand = 27,240 kWh/yr

Table 14

Optimum System Parameters

- o Separate-Panel System
- o Madison
- o $T_A = 70^\circ\text{C}$; $U_O = .005 \text{ kW/m}^2 \text{ } ^\circ\text{C}$

C_p \$/kW _{pk}	C \$/year	A_{th} m ²	A_e m ²	P_{ths} kWh	P_{es} kWh	R_{th}	R_e	R
A) $C_4 = .09$, $C_7 = \$.05/\text{kWh}$								
100	1987	25.3	47.3	28.6	10.7	.173	.558	.271
300	2072	25.3	16.9	28.6	3.68	.173	.268	.197
500	2096	25.3	3.74	28.6	0	.173	.068	.146
B) $C_4 = .09$, $C_7 = \$.07/\text{kWh}$								
100	2543	35.2	67.1	36.4	16.3	.208	.687	.329
300	2679	35.2	33.5	36.4	9.83	.208	.466	.274
500	2748	35.2	16.7	36.4	5.10	.208	.276	.225
C) $C_4 = .09$, $C_7 = \$.10/\text{kWh}$								
100	3314	46.7	89.6	44.3	22.6	.235	.789	.376
300	3509	46.7	52.5	44.3	16.1	.235	.621	.333
500	3628	46.7	33.4	44.3	11.6	.235	.475	.296
1000	3769	46.7	5.0	44.3	0	.235	.0875	.197
D) $C_4 = .11$, $C_7 = \$.05/\text{kWh}$								
100	2082	20.5	37.4	24.2	7.79	.151	.475	.233
300	2152	20.5	8.04	24.2	0	.151	.129	.145
E) $C_4 = .11$, $C_7 = \$.07/\text{kWh}$								
100	2664	30.0	56.6	32.4	13.4	.191	.625	.301
300	2793	30.0	24.9	32.4	6.76	.191	.372	.237
500	2847	30.0	6.56	32.4	0	.191	.110	.170
F) $C_4 = .11$, $C_7 = \$.10/\text{kWh}$								
100	3464	41.0	78.4	40.5	19.5	.223	.743	.355
300	3661	41.0	43	40.5	13	.223	.551	.306
500	3773	41.0	25	40.5	8.5	.223	.385	.264
1000	3870	41.0	1.97	40.5	0	.223	.037	.176

C_4 = Capital Recovery Factor
 C_7 = Levelized Backup Energy
 Cost (\$/kWh)

Electrical Demand = 10,160 kWh/yr
 Thermal Demand = 29,750 kWh/yr

Figure 45

Minimum Levelized Annual Cost of Service, Effect of Changes in Thermal Panel Characteristics, Madison, Levelized Backup Power Cost = \$.05/kWh, CRF = .09

Figure 46

Minimum Levelized Annual Cost of Service, Effect of Changes in Thermal Panel Characteristics, Madison, Levelized Backup Power Cost = \$.05/kWh, CRF = .11

VI. CONCLUSIONS

Procedures have been developed that permit comparison of alternative photovoltaic total energy systems for residential applications. These procedures provide the levelized annual cost of service as a function of collector area and energy storage level and can define the system configuration that provides minimum levelized annual cost of service for assumed values of the costs of equipment, operation, and backup power. In addition, the procedure can be used to evaluate the economic effects of selling surplus electrical energy to the utility and for other applications of solar energy. This procedure has been used to compare the annual cost of service of all-photovoltaic, separate-panel and combined-panel photovoltaic total energy systems for single-family homes in Phoenix and Madison.

In general, it appears that:

- 1) Systems providing energy at minimum total annual cost (including the cost backup energy) require significant amounts of backup energy and that annual costs rise rapidly as the energy delivered by a solar panel approaches the total annual demand.
- 2) Selling excess electrical energy to the utility is primarily attractive at low array costs.
- 3) Systems with separate thermal and photovoltaic panels usually provide lower annual costs of service than systems with combined panels. Separate-panel systems may become even more attractive if low cost, evacuated-tube thermal collectors are developed. Such collectors could provide higher efficiencies and operating temperatures than the flat plate collectors assumed in this report.

4) All-photovoltaic systems are preferable to either separate-panel or combined-panel systems in cold climates and in cases where both array costs are low and backup power costs are high. The advantages of all-photovoltaic systems would have been larger if the analysis had considered installation of such systems in homes with electric heat pumps.

5) The breakeven array cost can exceed \$1000/kW_{pk} in Phoenix and \$500/kW_{pk} in Madison if utility backup power rates for photovoltaic total energy system owners are the same as utility electricity rates for other residential customers. Under these assumptions, if photovoltaic array cost objectives are achieved (\$500/kW_{pk} in 1986), and the cost of electricity continues to increase, photovoltaic total energy systems would likely be competitive with utility power for residential applications in the mid 1980s in Phoenix and the early-1990s in Madison.

The analysis also indicates that additional study is needed of the interface between utilities and on-site photovoltaic total energy systems particularly in regard to the effects on utility costs and charges of solar penetration, time of day pricing, and the sale of surplus electrical energy.

APPENDIX A

EQUATIONS AND CONSTANTS

In the sections which follow, the analytical expressions for the levelized annual cost of service to the homeowner are derived and analyzed. Equations and constants are presented for the combined and separate panel systems and the all photovoltaic concept.

1. COMBINED PANEL SYSTEM

a. Thermal Subsystem

The cost function for the combined panel system consists of a thermal part (which also generates electrical energy) and an electrical (photovoltaic) part. The capital cost C_{thc} of the thermal part of the system can be expressed as

$$C_{thc} = C_1 A_{th} + C_2 P_{ths} + C_3 \quad (A-1)$$

where

C_1 = panel cost per unit area (e.g., \$50/m² thermal part only)

C_2 = thermal storage cost per kWh (e.g., \$10.66/kWh)

C_3 = balance of plant cost (e.g., \$1000.00)

A_{th} = thermal panel area (m²)

P_{ths} = thermal storage capacity (kWh)

Annual levelized cost of the thermal system is

$$\overline{CS}_{th} = C_{thc} \times CRF + O\&M \quad (A-2)$$

where

CRF = capital recovery factor

O&M = operating and maintenance cost per year

If it is assumed that the O&M cost is a fraction of C_{thc} then Equation (A-2) can be expressed as

$$\begin{aligned}\overline{CS}_{th} &= C_{thc} \times (C_4 + C_5) \\ &= C_6 C_{thc}\end{aligned}\tag{A-3}$$

where

$$C_4 = CRF \text{ (typically .09 or .11)}$$

$$C_5 = \text{fraction of } C_{thc} \text{ (typically .015)}$$

$$C_6 = C_4 + C_5$$

The levelized annual thermal backup cost is given by

$$\begin{aligned}C_{thb} &= C_7(1 - R_{th})P_{thl} \times 365 \\ &= C_8(1 - R_{th})P_{thl} \\ &= C_7P_{th}(1 - R_{th})\end{aligned}\tag{A-4}$$

where

$$C_7 = \text{levelized cost of backup energy \$/kWh}$$

$$C_8 = 365 C_7$$

$$P_{thl} = \text{total thermal load per day (kWh/day)}$$

$$P_{th} = 365 P_{thl} = \text{annual thermal load (kWh)}$$

$$R_{th} = \text{fraction of the thermal load (demand) provided by solar power}$$

The levelized annual cost of thermal energy is therefore

$$\begin{aligned}C_{th} &= \overline{CS}_{th} + C_{thb} \\ &= C_6 C_{thc} + C_7(1 - R_{th})P_{th} \\ &= C_6(C_1 A_{th} + C_2 P_{ths} + C_3) + C_7(1 - R_{th})P_{th}\end{aligned}\tag{A-5}$$

Equation (A-5) is a function of A_{th} , P_{ths} , P_{th} and R_{th} . The fraction of thermal load provided by solar power R_{th} as a function of A_{th} and P_{ths} can be determined by simulation of system performance. Examination of simulation data for R_{th} shows that it may be expressed as

$$R_{th} = K_{th} \left(1 - e^{-A_{th}/a_{th}} \right) \quad (A-6)$$

where

$$a_{th} = a_1 P_{ths} + a_2 P_{ths}^2 + a_3 \quad (A-7)$$

$$K_{th} = k_1 \left(1 - e^{-P_{ths}/p_{th}} \right) + k_2 \quad (A-8)$$

Equation (A-5) then becomes

$$C_{th} = C_9 A_{th} + C_{10} P_{ths} + C_{11} + C_7 F_{th} \left[1 - K_{th} \left(1 - e^{-A_{th}/a_{th}} \right) \right] \quad (A-9)$$

where

$$C_9 = C_1 C_6$$

$$C_{10} = C_2 C_6$$

$$C_{11} = C_3 C_6$$

The value of A_{th} which minimizes Eq. (A-9) can be found by differentiation with respect to A_{th} keeping P_{ths} constant. Thus,

$$\begin{aligned} \frac{\partial C_{th}}{\partial A_{th}} &= C_1 C_6 - C_7 P_{th} K_{th} \left(\frac{1}{a_{th}} e^{-A_{th}/a_{th}} \right) \\ &= 0 \end{aligned}$$

and

$$\left(A_{th} \right)_{opt} = a_{th} \ln \left(\frac{C_7 P_{th} K_{th}}{C_1 C_6 a_{th}} \right) \quad (A-10)$$

b. Electrical Subsystem

The levelized annual cost of the electrical energy can be expressed as

$$C_e = \overline{CS}_e + \text{backup energy cost} \quad (\text{A-11})$$

where

$$\overline{CS}_e = \text{CRF} \times C_{ec} + \overline{\text{O\&M}} \quad (\text{A-12})$$

and

$$\begin{aligned} C_{ec} &= (1 + d_1)A + d_2A_e + 1.1(d_3P_{es} + d_4) \\ &= \text{electrical system capital cost} \end{aligned} \quad (\text{A-13})$$

$$\begin{aligned} \overline{\text{O\&M}} &= 0.015 C_{ec} \quad (\text{A-14}) \\ &= \text{electrical system} \\ &\quad \text{operation and maintenance cost} \end{aligned}$$

Here

$$\begin{aligned} d_1 &= \text{array cost} \\ d_2 &= \text{support structure cost} \\ d_3 &= \text{electric storage cost} \\ d_4 &= \text{power conditioning cost} \end{aligned}$$

and

$$\begin{aligned} A_e &= \text{air cooled, photovoltaic panel area (m}^2\text{)} \\ A &= A_{th} + A_e = \text{total array area (m}^2\text{)} \\ P_{es} &= \text{electrical storage capacity (kWh)} \end{aligned}$$

Further reduction of Eq. (A-11) results in

$$C_e = g_1 A_{th} + g_2 A_e + g_3 P_{es} + g_4 + C_7(1 - R_e)P_e \quad (A-15)$$

where

$$g_1 = C_6(1 + d_1)$$

$$g_2 = C_6(1 + d_1 + d_2)$$

$$g_3 = 1.1 C_6 d_3$$

$$g_4 = 1.1 C_6 d_4$$

and

P_e = annual electrical power demand (load)(kWh)

The fraction of the electrical demand provided by solar energy in Eq. (A-15) can be expressed analytically as

$$R_e = K_1 \left[1 - e^{-e/a_e + K_2} \right] \quad (A-16)$$

where the parameters K_1 , K_2 , and a_e must be determined approximately by fitting the simulation data for the combined panel system (Figure 13).

R_e is thus an analytical function of the total array area A and the above parameters which can in turn be expressed analytically as functions of the electrical storage capacity P_{es} . The parameters K_1 , K_2 and a_e can be expressed as

$$K_1 = n_1(1 - e^{-P_{es}/p_1}) + n_2 \quad (A-17)$$

$$K_2 = n_3(1 - e^{-P_{es}/p_2}) + n_4 \quad (A-18)$$

$$a_e = a_4 P_{es} + a_5 P_{es}^2 + a_6 \quad (A-19)$$

The constants $n_1, n_2, n_3, n_4, p_1, p_2, a_4, a_5$, and a_6 are determined by fitting the analytic expression to the simulation results.

Differentiation of Eq. (A-15) with respect to A_e , with P_{es} assumed constant, yields an optimum electric array area $(A_e)_{opt}$ which minimizes cost. Thus

$$(A_e)_{opt} = a_e \ln \left(\frac{C_7 P_e K_1}{g_2 a_e} \right) \quad (A-20)$$

where A_{th} and A_e are in square meters. The value of (A_e) can be computed for a given A_{th} and P_{es} and substituted into Eq. (A-15). The optimum values for A_e and P_{es} which minimize C_e may thus be determined.

c. Combined Panel System (CP)

The levelized annual cost for the combined thermal and electrical system is equal to the sum of Eqs. (A-9) and (A-15), i. e.,

$$C = C_{th} + C_e \quad (A-21)$$

or

$$C = (C_1 C_6 + g_1) A_{th} + C_2 C_6 P_{ths} + C_7 P_{th} \left[1 - K_{th} \left(1 - e^{-A_{th}/a_{th}} \right) \right] \\ + g_3 P_{es} + g_2 A_e + (C_3 C_6 + g_4) + C_7 P_e \left[1 - K_1 \left(1 - e^{-A_e/a_e} \right) - K_2 \right] \quad (A-22)$$

This cost function is of the form

$$C = C_{12} A_{th} + C_{10} P_{ths} + C_7 P_{th} \left[1 - K_{th} \left(1 - e^{-A_{th}/a_{th}} \right) \right] \\ + f_2 A_e + f_3 P_{es} + C_{13} + P_e \left\{ C_7 + C_{14} \left[K_1 \left(1 - e^{-A_e/a_e} \right) + K_2 \right] \right\} \quad (A-23)$$

where:

$$C_{12} = g_1 + C_1 C_6$$

$$C_{10} = C_2 C_6$$

$$f_2 = g_2$$

$$f_3 = g_3$$

$$C_{13} = g_4 + C_3 C_6$$

$$C_{14} = -C_7$$

d. Minimum Annual Cost - Phoenix

The minimum annual levelized cost C and the associated optimum system parameters A_{th} , A_e , P_{ths} and P_{es} were determined numerically by the optimization of Eq. (A-23). The constants used in Eqs. (A-6) and (A-16) were:

$$k_1 = .7, \quad k_2 = .22, \quad P_{th} = 28$$

$$a_1 = .179, \quad a_2 = .00208, \quad a_3 = 5$$

$$n_1 = .57, \quad n_2 = .24, \quad p_1 = 22.2$$

$$n_3 = .18, \quad n_4 = .26, \quad p_2 = 77.8$$

$$a_4 = -.12, \quad a_5 = .0112, \quad a_6 = 36$$

The constants used for the baseline case ($C_7 = .05$, $C_4 = .09$) and the photovoltaic panel cost $C_p = \$100/\text{kW}_{pk}$ ($d_1 = \$13.5/\text{m}^2$) were:

$$C_7 = .05, \quad C_8 = 18.25, \quad C_{10} = 1.119$$

$$C_{12} = 6.773, \quad C_{13} = 197.4, \quad C_{14} = -.05$$

$$f_2 = 2.573, \quad f_3 = 4.62, \quad C_6 = .105$$

$$P_{thl} = 33.7 \text{ kWh/day}, \quad P_e = 16550 \text{ kWh/year}$$

$$P_{th} = 12300 \text{ kWh/year}$$

$$R = (R_{th} P_{th} + R_e P_e) / P \quad (A-24)$$

where $P = P_{th} + P_e$ represents the total annual energy demand for the residence.

e. Minimum Annual Cost - Madison

The combined thermal/electrical panel system results for Madison were obtained using

$$R_e = K_1 \left(1 - e^{-A_e / a_e} \right) + K_2 \quad (A-25)$$

and the Eqs. (A-17) to (A-19) where

$$\begin{aligned} n_1 &= .58, & n_2 &= .15, & p_1 &= 20.19 \\ n_3 &= .18, & n_4 &= .26, & p_2 &= 77.8 \\ a_4 &= .36, & a_5 &= .0096, & a_6 &= 34 \end{aligned}$$

The thermal system fraction R_{th} was defined as in the Phoenix case by the Eqs. (A-6) - (A-8) where

$$k_1 = .16, \quad k_2 = .054, \quad p_{th} = 26.4$$

and

$$a_1 = .252, \quad a_2 = 0, \quad a_3 = 13.4$$

For the baseline case ($C_7 = .05$, $C_4 = .09$, $C_p = \$100/\text{kW}_{\text{pk}}$), the constants used in Eq. (A-23) were:

$$\begin{aligned} C_7 &= \$0.05/\text{kWh} & C_{10} &= 1.12, & C_{12} &= 6.77 \\ f_2 &= 2.57, & f_3 &= 4.62, & C_{13} &= 197.4, & C_{14} &= -.05 \\ \text{and } P_e &= 10160/\text{kWh} \end{aligned}$$

2. SEPARATE PANEL SYSTEM

a. Cost Function

The equation for the levelized annual cost for the separate-panel system is similar to that for the combined-panel system except that no thermal area A_{th} term appears in the electrical part of the cost function and the R_{th} and R_e functions are different. The expression for the annual levelized cost may be written as

$$C = C_{\text{th}} + C_e \quad (\text{A-26})$$

where

$$C_{\text{th}} = h_1 A_{\text{th}} + h_2 P_{\text{ths}} + h_3 (1 - R_{\text{th}}) + C_{11} \quad (\text{A-27})$$

$$C_e = m_1 A_e + m_2 P_{\text{es}} + m_3 (1 - R_e) + g_4 \quad (\text{A-28})$$

and where

$$h_1 = C_9, \quad h_2 = C_{10}, \quad h_3 = C_8 P_{\text{thl}}$$

$$m_1 = g_2, \quad m_2 = g_3, \quad m_3 = C_7 P_e$$

Rewriting,

$$C = h_1 A_{th} + h_2 P_{ths} + h_3 (1 - R_{th}) + m_1 A_e + m_2 P_{es} + m_3 (1 - R_e) + m_4 \quad (A-29)$$

where

$$m_4 = C_{11} + g_4$$

Here,

$$\left. \begin{aligned} R_{th} &= K_{th} \left(1 - e^{-A_{th}/a_{th}} \right) \\ R_e &= K_e \left(1 - e^{-A_e/a_e} \right) \\ a_{th} &= a_1 P_{ths} + a_2 P_{ths}^2 + a_3 \\ a_e &= a_4 P_{es} + a_5 P_{es}^2 + a_6 \\ K_{th} &= k_1 \left(1 - e^{-P_{ths}/p_{th}} \right) + k_2 \\ K_e &= k_3 \left(1 - e^{-P_{es}/p_e} \right) + k_4 \end{aligned} \right\} \quad (A-30)$$

where $a_1, a_2, a_3, a_4, a_5, a_6, k_1, k_2, k_3, k_4, p_{th}$ and p_e can be determined from the simulation results.

Equations (A-27) and (A-28) may be optimized (minimized) with respect to A_{th}, P_{ths}, A_e and P_{es} separately or combined as in Eq. (A-29) for different values of the h and m constants.

b. Minimum Annual Cost - Phoenix

The minimum annual levelized cost C and the associated optimum system parameters A_{th}, A_e, P_{es}, R_e and R_{th} were determined numerically by optimizing Eq. (A-29). The following constants were used:

$$\begin{aligned}
k_1 &= .6, & k_2 &= .22, & p_{th} &= 17.37 \\
a_1 &= .207, & a_2 &= .00133, & a_3 &= 5.6 \\
a_4 &= -.171, & a_5 &= .0163, & a_6 &= 37 \\
k_3 &= .75, & k_4 &= .52, & p_e &= 32.4
\end{aligned}$$

The constant used for the baseline case ($C_7 = .05$, $C_4 = .09$, $C_6 = .105$) and the photovoltaic panel cost $C_p = \$100/\text{kW}_{pk}$ were:

$$\begin{aligned}
h_1 &= 5.25, & h_2 &= 1.12, & h_3 &= 615, & g_4 &= 92.4 \\
m_1 &= 2.573, & m_2 &= 4.62, & m_3 &= 827.5, & m_4 &= 197.4
\end{aligned}$$

c. Minimum Annual Cost - Madison

The optimization results for Madison were obtained using the following constants:

$$\begin{aligned}
k_1 &= .155, & k_2 &= .055, & p_{th} &= 25.61 \\
k_3 &= .675, & k_4 &= .35, & p_e &= 13.65 \\
a_1 &= -.105, & a_2 &= .00625, & a_3 &= 17.6 \\
a_4 &= 1.48, & a_5 &= -.0173, & a_6 &= 17.4
\end{aligned}$$

The baseline system constants were:

$$\begin{aligned}
C_4 &= .09, & C_7 &= .05, & h_1 &= 5.25 \\
h_2 &= 1.119, & h_3 &= 1487, & m_1 &= 2.57 \\
m_2 &= 4.62, & m_3 &= 508, & m_4 &= 197.4, & g_4 &= 92.4
\end{aligned}$$

d. Effect of Power Conditioning Cost - Phoenix
(No Electrical Storage)

In previous analyses the expression for C_e contained a constant power conditioning cost $d_4 = \$800$. If it is assumed that $d_4 = \$12 A_e$ for $d_4 \leq \$800$ (i.e., the array provides $.12 \text{ kW}_{pk}/\text{m}^2$ and the power conditioner costs $\$100/\text{kW}_{pk}$) and that the electrical storage capacity $P_{es} = 0$, then the cost of annual electrical power is

$$C_e = m_1 A_e + m_3 (1 - R_e) + m_4 \quad (\text{A-31})$$

where

$$m_1 = C_6 (1 + d_1 + d_2)$$

$$m_3 = C_7 P_e$$

$$m_4 = 880 C_6 = m_4' \text{ (Case A when } d_4 = \$800 \text{)}$$

$$= 13.2 C_6 A_e \text{ (Case B when } d_4 = \$12 A_e \text{)}$$

For Case A, $R_e = k_4 (1 - e^{-A_e/a_6})$ and therefore

$$C_e = m_1 A_e + m_3 \left[1 - k_4 (1 - e^{-A_e/a_6}) \right] + m_4 \quad (\text{A-32})$$

By differentiation of C_e with respect to A_e , the optimum panel area is found to be

$$(A_e)_{opt} = a_6 \ln \left(\frac{m_3 k_4}{m_1 a_6} \right) \quad (\text{A-33})$$

which substituted into Eq. (A-28) yields the minimum annual cost

$$(C_e)_{\min} = m_1 a_6 \left[1 + \ln \left(\frac{m_3 k_4}{m_1 a_6} \right) \right] + m_3 (1 - k_4) + m_4 \quad (A-34)$$

For Case B,

$$C_e = m_1' A_e + m_3 (1 - R_e) \quad (A-35)$$

where

$$m_1' = m_1 + 13.2 C_6$$

and

$$m_3 = C_7 P_e$$

$$R_e = k_4 \left(1 - e^{-A_e/a_6} \right)$$

as before.

e. Effect of Power Sale to Utility (No Electrical Storage) - Phoenix

The effect of selling power back to the utility can be examined by subtracting a term representing revenue or negative cost. The expression for the annual levelized cost [Eq. (A-35)] then becomes

$$C_e = m_1' A_e + m_3 (1 - R_e) + m_4 - C_R \quad (A-36)$$

where

$$C_R = 0.85 JE$$

Here the factor 0.85 accounts for the conversion loss of DC to AC power, J is a constant (\$0.01, \$0.03 or \$0.05 per kWh typically) representing the levelized price of energy sold to the utility, and E is the quantity of electrical energy sold. The latter is a performance simulation result and is shown in the text in Figure 19 for the combined, separate, and the all photovoltaic panel systems and no electrical storage capacity. E may be expressed analytically in the form

$$E = b \left(e^{\beta A_e} - 1 \right) + \gamma A_e \quad (A-37)$$

where b , β and γ are constants to be determined.

Rewriting Eq. (A-36) with $R_e = k_4 (1 - e^{-A_e/a_6})$

$$C_e = m_6 A_e + m_8 e^{-A_e/a_6} - m_7 e^{\beta A_e} + m_9 \quad (A-38)$$

where

$$m_6 = m_1 + 13.2 C_6 - 0.85 J \gamma$$

$$m_7 = 0.85 J b$$

$$m_8 = m_3 k_4$$

$$m_9 = m_3 (1 - k_4) + m_7$$

Differentiating Eq. (A-38) with respect to A_e and setting the result to zero yields

$$m_6 = m_7 \beta e^{\beta A_e} + \frac{m_8}{a_6} e^{-A_e/a_6} \quad (A-39)$$

Equation (A-39) can be readily solved for A_e , the optimum panel area which minimizes C_e , if $\beta = 1/a_6$. Then

$$\begin{aligned} A_e &= a_6 \ln \left(\frac{m_8 - m_7}{m_6 a_6} \right) \\ &= (A_e)_{\min} \end{aligned} \quad (A-40)$$

and

$$(C_e)_{\min} = m_6 \left[a_6 + (A_e)_{\min} \right] + m_9 \quad (A-41)$$

3. ALL-ELECTRIC (PHOTOVOLTAIC) PANEL SYSTEM

a. Minimum Annual Cost - Phoenix

The all photovoltaic panel system was evaluated for comparison with the combined and the separate panel systems using the equation

$$C = m_1 A_e + m_2 P_{es} + m_3 (1 - R_e) + m_4 \quad (A-42)$$

where the m_1 , m_2 , m_3 and m_4 constants and the A_e , P_{es} , R_e system parameters were as defined previously. The constants used for the fit of the R_e function were:

$$\begin{aligned} k_3 &= -.623, & k_4 &= .48, & p_e &= -58.8 \\ a_4 &= .422, & a_5 &= .0296, & a_6 &= 48 \end{aligned}$$

The baseline system constants (for $C_7 = .05$, $C_4 = .09$ and $C_p = \$100/\text{kW}_{pk}$) were:

$$m_1 = 2.573, \quad m_2 = 4.62, \quad m_3 = 1443, \quad m_4 = 92.4$$

b. Minimum Annual Cost - Madison

The all photovoltaic system was analyzed using the following constants:

$$\begin{array}{lll} k_3 = 1.2, & k_4 = .3, & p_e = 72.13 \\ a_4 = 2.7, & a_5 = 0, & a_6 = 65 \end{array}$$

The baseline constants were:

$$\begin{array}{lll} C_4 = .09, & C_7 = .05, & m_1 = 2.57 \\ m_2 = 4.62, & m_3 = 1996, & m_4 = 92.4 \end{array}$$

APPENDIX B

DAVIDON OPTIMIZATION METHOD

1. Theory^T

The mathematical basis of the Davidon method is based on the following arguments:

Let $\phi(\underline{x})$ be a cost function which is to be minimized by properly choosing the n state variables $\underline{x} = (x_1, x_2, \dots, x_n)$. The n -dimensional vector $\underline{g}(\underline{x})$ represents the gradient of ϕ , where $g_1 = \partial\phi/\partial x_1$. Let the (symmetrical) matrix A have second partial derivatives

$$a_{ij} = \frac{\partial^2 \phi}{\partial x_i \partial x_j}$$

Expanding $\phi(\underline{x})$ into the first two terms of a multi-dimensional Taylor's series about point $\underline{x}_i$ gives

$$\phi(\underline{x}) \cong \phi(\underline{x}_i) + \underline{g}^T(\underline{x}_i) (\underline{x} - \underline{x}_i) + \frac{1}{2} (\underline{x} - \underline{x}_i)^T A(\underline{x}_i) (\underline{x} - \underline{x}_i)$$

The partial derivative (gradient) of this expression with respect to $\underline{x}$ is

$$\left(\frac{\partial \phi}{\partial \underline{x}} \right)^T \equiv \underline{g}(\underline{x}) = \underline{g}(\underline{x}_i) + A \left[\underline{x} - \underline{x}_i \right]$$

At the minimum of the cost function

$$\underline{x} = \underline{x}^* \text{ and } \underline{g}(\underline{x}^*) = 0$$

Therefore,

$$\underline{g}(\underline{x}^*) = \underline{g}(\underline{x}_i) + A \left[\underline{x}^* - \underline{x}_i \right] = 0$$

^TFor a more complete description see References 14 and 15.

Solving for the required stepsize in the optimization procedure gives

$$(\underline{x}^* - \underline{x}_i) = -A^{-1} g(\underline{x}_i)$$

For a quadratic cost function ϕA^{-1} is a constant and this step will reach the minimum point of ϕ . For a nonquadratic cost function A^{-1} will vary and the minimum point must be iterated on by taking steps

$$\Delta \underline{x}_i = (\underline{x}_{i+1} - \underline{x}_i) = -A^{-1}(\underline{x}_i) g(\underline{x}_i)$$

The Newton-Raphson method attempts to minimize using these steps where A^{-1} is calculated analytically or numerically at each step. To avoid this time consuming computation Davidon (Reference 8) proposed to approximate A^{-1} by a matrix H . The matrix H is used like A^{-1} to generate search directions $\underline{S}_i$.

Thus,

$$\underline{S}_i = \frac{1}{C_i} (\underline{x}_{i+1} - \underline{x}_i) = -H_i g(\underline{x}_i)$$

where C_i is the distance moved in the $\underline{S}$ direction. A one-dimensional search is conducted along these directions to minimize ϕ and compute the corresponding C_i and $\underline{x}_{i+1}$ values. When a minimum is found a new "conjugate direction"[†] is evaluated at the new point and a new search conducted until convergence is achieved. If convergence is not achieved a new conjugate direction is evaluated. This procedure continues until convergence or a prescribed number of directions have been searched.

Davidon concluded that a recursive formula could represent the matrix A^{-1} or H from information acquired on previous one-dimensional searches. Such a formula requires only H_j , $\Delta \underline{g}_j$ and $\Delta \underline{g}_i$ obtained from the previous search and is given by

[†] $\underline{S}_i$ is conjugate to $\underline{S}_j$ if $\underline{S}_i^T \Delta \underline{S}_j = 0$ for all $i \neq j$.

$$H_{j+1} = H_j + \frac{\Delta \underline{x}_j \Delta \underline{x}_j^T}{\Delta \underline{x}_j^T \Delta \underline{g}_j} - \frac{H_j \Delta \underline{g}_j \Delta \underline{g}_j^T H_j}{\Delta \underline{g}_j^T H_j \Delta \underline{g}_j}$$

The steps $\Delta \underline{x}_i$ are along conjugate direction $\underline{S}_i$ and are eigenvectors of $H_{i+1} A$ with unit eigenvalue.[†]

Therefore, the optimal solution in the Davidon technique is written as

$$\underline{x}_{i+1} = \underline{x}_j + H_j \underline{g}(\underline{x}_j)$$

where $H_j \cong -A^{-1}$

2. Example

An example of a computer optimization run for a combined panel system at Madison is shown in Tables B-1 to B-3. The $\underline{x}$ -vector in Table B-1 is an initial guess of the desired parameters, the thermal and the electrical panel areas A_{th} , A_e and the thermal and the electrical storage capacities P_{ths} and P_{es} respectively. Table B-2 shows the baseline case input parameters including the $\underline{x}$ vector values, as given in Section 1-e of Appendix A. Table B-3 is the computer printout of the cost function C after each iteration and its final value of \$2102.2 (iteration 9). The values of the parameters A_e , A_{th} , P_{ths} , and P_{es} are shown for this final value at the bottom of Table B-3.

[†] $\Delta \underline{x}_i$ must satisfy $\left[H_{i+1} A \right] \Delta \underline{x}_i = \Delta \underline{x}_i$

Table B-1

***** CASE 1 COMBINED PANELS *****

Madison

DIMENSION OF X-VECTOR = 4
 MAXIMUM NUMBER OF ITERATIONS = 20
 MAXIMUM NUMBER OF STEP REDUCTIONS = 20
 MINIMUM FRACTIONAL REDUCTION IN F = .1000000E-13
 MINIMUM STEP SIZE = .1000000E-12
 MINIMUM GRADIENT MAGNITUDE = .1000000E-09
 INITIAL DIAGONAL ELEMENT OF H-MATRIX = .1000000E+01
 INITIAL VALUE OF LAMBDA = .1000000E+01
 LAMBDA REDUCTION PARAMETER = .5000000E+00
 LAMBDA MULTIPLICATION PARAMETER = .2000000E+01
 NUMBER OF PURE GRADIENT ITERATIONS = 0

NORMAL DAVIDON METRIC USED

MODIFIED GRADIENT USED

USES REFINO

***** ITERATION 1 *****

	$A_{th} (m^2)$	$A_e (m^2)$	$P_{ths} (kWh)$	$P_{es} (kWh)$
X-VECTOR =	.3000000E+02	.7500000E+02	.5000000E+02	.2500000E+02
FUNCTION =	.2154106E+04			
RTH =	.1300176E+00	RE =	.8125847E+00	
GR. ENT VECTOR =	.2379913E+01	.16156E+01	.1187873E+01	.2402997E+00

Table B-2. Input Data - Case 1 Combined Panels - Madison

\$DATA1

C7	=	.5E-01,
C8	=	.1825E+02,
C10	=	.112E+01,
C12	=	.677E+01,
C13	=	.1974E+03,
C14	=	-.5E-01,
F2	=	.257E+01,
F3	=	.462E+01,
K1	=	.16E+00,
K2	=	.54E-01,
PTH	=	.264E+02,
P1	=	.2019E+02,
P2	=	.778E+02,
A1	=	.252E+00,
A2	=	0.0,
A3	=	.134E+02,
A4	=	-.36E+00,
A5	=	.96E-02,
A6	=	.34E+02,
PE	=	.1016E+05,
PTHE	=	.8151E+02,
n1	=	.58E+00,
n2	=	.15E+00,
n3	=	.18E+00,
n4	=	.26E+00,
XI	=	.3E+02, .75E+02, .5E+02, .25E+02,
IPARM	=	1, 1, 1, 1,
XLL	=	0 . 0.0, 0.0, 0.0,

Table B-3. Output Data - Combined Panels - Madison

ITERATION	Cost	SUMMARY - CASE 1		STEP MAGNITUDE	GRADIENT MAGNITUDE
	FUNCTION C	FRACTIONAL REDUCTION			
1	.2154106E+04	0.		0.	.3117108E+01
2	.2122142E+04	.1483877E-01		.1821025E+02	.1753501E+01
3	.2103426E+04	.8819054E-02		.2174940E+02	.8034375E+00
4	.2102652E+04	.3678892E-03		.12777144E+01	.4494558E+00
5	.2102208E+04	.2113195E-03		.2104608E+01	.5626279E-01
6	.2102202E+04	.2376373E-05		.2436234E+00	.6179639E-01
7	.2102183E+04	.9033434E-05		.8834288E+00	.4122177E-01
8	.2102176E+04	.13423237E-05		.4076626E+00	.3982612E-03
9	.2102176E+04	.1192991E-09		.1788933E-02	.1403886E-05
		$A_{th} (m^2)$	$A_e (m^2)$	$P_{ths} (kWh)$	$P_{es} (kWh)$
FINAL VALUE OF X-VECTOR =		.1991847E+02	.5185252E+02	.2540301E+02	.2000812E+02
RTH =		9.6965795E-02	RE =	6.6050653E-01	

LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
A	Total Photovoltaic Array Area	m ²
A _e	Air Cooled Photovoltaic Array Area	m ²
A _{th}	Thermal Panel or Water-Cooled Photovoltaic Array Area	m ²
a ₁	Constant	m ² /kWh
a ₂	Constant	m ² /(kWh) ²
a ₃	Constant	m ²
a ₄	Constant	m ² /kWh
a ₅	Constant	m ² /(kWh) ²
a ₆	Constant	m ²
a _e	$a_4 P_{es} + a_5 P_{es}^2 + a_6$	m ²
a _{th}	$a_1 P_{ths} + a_2 P_{ths}^2 + a_3$	m ²
b	Constant	kWh
C	Total Levelized Annual Cost of Service	\$
C ₁	Thermal Panel Cost	\$/m ²
C ₂	Thermal Storage Cost	\$/kWh
C ₃	Thermal Balance of Plant Cost	\$
C ₄	Capital Recovery Factor (CRF)	--
C ₅	Annual O&M Cost as Fraction of Capital Cost	--
C ₆	$C_4 + C_5$	--
C ₇	Levelized Backup Power Cost	\$/kWh
C ₈	$365 C_7$	\$/kWh

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
C ₉	C ₁ C ₆	\$/m ²
C ₁₀	C ₂ C ₆	\$/kWh
C ₁₁	C ₃ C ₆	\$
C ₁₂	g ₁ + C ₁ C ₆	\$/m ²
C ₁₃	g ₄ + C ₃ C ₆	\$
C ₁₄	-C ₇	\$/kWh
C _{Bu} , C _{thb}	Levelized Annual Cost of Backup Energy or Power	\$
C _c	Installed Capital Cost of System Concept	\$
C _e	Levelized Annual Cost of Electric Energy	\$
C _{ec}	Installed Capital Cost of Electrical System	\$
C _{O&M}	Levelized Cost of System Operations and Maintenance	\$
C _p	Array Cost	\$/kW _{pk}
C _R	Levelized Annual Revenue from Sale of Surplus Electricity	\$
C _{th}	Levelized Annual Cost of Thermal Energy	\$
C _{thc}	Installed Capital Cost of Thermal System	\$
CRF	C ₄	--
CS _e	Levelized Annual cost of Electrical System	\$
CS _{th}	Levelized Annual Cost of Thermal System	\$
d ₁	Array Cost	\$/m ²
d ₂	Air Cooled Array Structural Cost	\$/m ²
d ₃	Electrical Storage Cost	\$/kWh
d ₄	Power Conditioner Cost	\$
d ₅	Wiring/Miscellaneous Cost	\$

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
E	Amount of Surplus Electricity Available for Sale	kWh
E_g	Insolation Falling on Panel or Array	kW/m ²
F	Ratio of Fin Cooling Area to Array Area	--
g_1	$C_6 (1 + d_1)$	\$/m ²
g_2	$C_6 (1 + d_1 + d_2) = f_2$	\$/m ²
g_3	$1.1 C_6 d_3 = f_3$	\$/kWh
g_4	$1.1 C_6 d_4$	\$
h_1	C_9	\$/m ²
h_2	C_{10}	\$/kWh
h_3	$C_8 P_{th}$	\$
h_c	Convective Heat Transfer Coefficient (Air Cooled Array)	kW/m ² - °C
h_r	Radiative Heat Transfer Coefficient (Air Cooled Array)	kW/m ² - °C
J	Levelized Sale Price of Surplus Electricity	\$/kWh
k_1	Constant	--
k_2	Constant	--
k_3	Constant	--
k_4	Constant	--
K_1	$n_1 \left[1 - e^{-P_{es}/P_1} \right] + n_2$	--

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
K_2	$n_3 \left[1 - e^{-P_{es}/P_2} \right] + n_4$	--
K_{th}	$k_1 \left[1 - e^{-P_{ths}/P_{th}} \right] + k_2$	--
K_e	$k_3 \left[1 - e^{-P_{es}/P_e} \right] + k_4$	--
m_1	g_2	\$/m ²
m_2	g_3	\$/kWh
m_3	$C_7 P_e$	\$
m_4	$C_{11} + g_4$	\$
m_6	$m_1 + 13.2 C_6 - .85 J Y$	\$/m ²
m_7	$.85 J b$	\$
m_8	$m_3 k_4$	\$
m_9	$m_3 (1 - k_4) + m_7$	\$
n_1	Constant	--
n_2	Constant	--
n_3	Constant	--
n_4	Constant	--
P_1	Constant	kWh
P_2	Constant	kWh
P_e	Constant	kWh
P_{th}	Constant	kWh
P	Total Annual Demand	kWh
P_e	Annual Electrical Demand	kWh
P_{es}	Electrical Storage Capacity	kWh

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
PF	Fraction of Array Covered by Cells (Packing Factor)	--
P_{Ge}	Array DC Power Generated	kW
P_{Gth}	Thermal Power Generated	kW
P_{th}	Annual Thermal Demand	kWh
P_{th}	Daily Thermal Demand	kW
P_{ths}	Thermal Storage Capacity	kWh
R	Fraction of Total Annual Demand Provided by PTES	--
R_e	Fraction of Electrical Demand Provided by PTES	--
R_{th}	Fraction of Thermal Demand Provided by PTES	--
T_A	Average Array or Panel Temperatures	$^{\circ}C$
$T_{Ambient}$	Ambient Air Temperature	$^{\circ}C$
U_o	Convective Heat Transfer Coefficient	$kW/m^2 - ^{\circ}C$
α	Panel or Array Absorbtivity	--
β	Constant	$1/m^2$
η_c	Solar Cell Conversion Efficiency	--
γ	Constant	$1/m^2$

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