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Phase-Space Structure of Cold Dark Matter Halos*

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Abstract:

A galactic halo of cold dark matter particles has a sheet-like structure in phase-space. The energy and momentum spectra of such particles on earth has a set of peaks whose central values and intensities form a record of the formation of the Galaxy. Scattering of the dark matter particles by stars and globular clusters broadens the peaks but does not erase them entirely. The giant shells around some elliptical galaxies may be a manifestation of this structure.

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A considerable body of observational and theoretical evidence indicates[1] that galaxies are surrounded by halos of dark matter that contribute 90% or more of the total galactic mass and have a density distribution $\rho(r) \sim \frac{1}{r^2}$ at large r. If dark matter does indeed provide the dominant contribution to the energy density of the universe, its nature, along with its primordial spectrum of density perturbations, strongly influences the details of galaxy formation. In this connection, it appears that <u>cold</u> dark matter with a flat (Zel'dovich-Harrison) spectrum of primordial density perturbations and some "biasing" in the extent to which light traces matter, yields what is perhaps the best available scenario of galaxy formation. The focus of this paper is the phase-space structure of galactic halos in this scenario. Our original motivation was to ask what information can be extracted from a signal in a galactic halo axion detector[2] if such a signal is ever discovered. The modulation of the signal by the orbital and rotational motions of the Earth has already been discussed.[3] However, because the cavity detector of galactic halo axions has unusually high energy resolution, additional information may be encoded there.

At present, the axion and the WIMPS (weakly interacting massive particles) are the two main particle physics candidates for cold dark matter. At the time t_G when galaxy formation starts, both of these candidates exhibit very small velocity dispersion. For axions, the initial velocity dispersion $< v_a(t_G)^2 >^{1/2}$ is due to the lack of homogeneity of the axion field at time $t_1 \simeq 10^{-6}$ sec when the axion mass turns on at the start of the QCD phase transition.[4] In case there is no inflation after the phase transition in which the $U_{PQ}(1)$ symmetry of Peccei and Quinn becomes spontaneously broken, one has $v_a(t_G) \sim \frac{1}{t_1 m_a}(\frac{R_1}{R_G}) \sim 10^{-17}$ for an axion mass $m_a \sim 10^{-5} eV$ and the scale factor ratio $\frac{R_1}{R_G} \sim \frac{T_G}{T_1} \sim \frac{10^{-4} eV}{1 GeV} = 10^{-13}$. If there is inflation after the phase transition in which the $U_{PQ}(1)$ symmetry becomes spontaneously broken, the initial velocity dispersion of cosmic axions is smaller still. For WIMPS, the initial velocity dispersion is of order $v_W(t_G) \simeq (\frac{2T_D}{m})^{1/2} \frac{R_D}{R_G}$ where m is the WIMP mass and T_D the temperature at which its kinetic energy leaves thermal equilibrium. For $m \sim 50\ GeV$ and $T_D \sim 1\ MeV$, one has $v_W(t_G) \sim 10^{-12}$, which is also very small.

Thus in both the axion and WIMP dark matter scenarios, the initial phase distribution is a very thin sheet near $\vec{v} = 0$, spread nearly uniformly over $\vec{r}$-space. When a galaxy forms, the sheet folds itself up in phase space. This process is illustrated in Fig. I for the simplified case where all dark matter particles move on radial orbits and the evolution is spherically symmetric.[5] It is clear from Fig. I that the energy momentum spectrum of dark matter particles as measured at any position $\vec{r}$ has a series of peaks whose number slowly increases with time. The sheet "winds up", rotating most rapidly near $r = \dot{r} = 0$. As time advances, successive layers form and move slowly outward in phase space.

It is generally true that in the limit of zero initial velocity dispersion $[< v(t_G)^2 >= 0]$, the spectrum of cold dark matter particles is a finite (although possibly very large) number N of zero-width peaks. Indeed, consider the particles which intially, at time t_G, have position $\vec{r}_G$. They all have the same initial velocity $\vec{V}_G(\vec{r}_G)$ and all end up, at time t, at the same position $\vec{r} = \vec{r}(\vec{r}_G, \vec{V}_G(\vec{r}_G), t) \equiv \vec{r}(\vec{r}_G, t)$ with the same velocity $\vec{v} = \vec{v}(\vec{r}_G, t)$. On the other hand, the particles which are at $\vec{r}$ at time t come in general from many different initial locations $\vec{r}^{\,n}_G(\vec{r}, t)$, $n = 1...N < \infty$. As a result, they have a discrete set of velocities $\vec{v}^{\,n}(\vec{r}, t) = \vec{v}(\vec{r}^{\,n}_G(\vec{r}, t))$. Using Liouville's theorem, one can express the spatial density of particles at time t as the sum

$$\begin{aligned} \rho(\vec{r}, t) &= \sum_{n=1}^{N} \rho_n(\vec{r}, t) \\ &= \sum_{n=1}^{N} \rho(\vec{r}^{\,n}_G(\vec{r}, t)) \mid det \left(\frac{\partial \vec{v}_G}{\partial \vec{v}} - \frac{\partial \vec{V}_G}{\partial \vec{r}_G} \cdot \frac{\partial \vec{r}_G}{\partial \vec{v}} \right)^{(n)}_{fixed\ \vec{r}} \mid^{-1} . \end{aligned} \tag{1}$$

If cold dark matter particles are ever detected on earth, the locations $\vec{v}^{\,n}$ and intensities ρ_n of these peaks form a record of the formation of the Galaxy. Of course, whether this is of any practical use depends very much on how large N is. For sufficiently large N the peaks will appear to form a continuum, because of the finiteness of the experimental resolution.

Note that to have a sheet-like structure in phase space it is sufficient that the dark matter be collisionless, since the sheet structure follows directly from Liouville's theorem and the fact that trajectories cannot cross in phase-space. Galactic halos made of neutrinos, for example, also have a sheet structure in phase space, but the sheets are so thick as to almost "touch" one another.[6] WIMPS and axions are collisionless because their scattering cross-sections are very small. In the case of axions, the relaxation rate gets multiplied[7] by their huge average quantum state occupation number, of order $6.10^{25}(\frac{m_a}{10^{-5}eV})^4$, but this results in relaxation times which are still much longer than the age of the universe.

How large is N? Let us first consider the idealized case where all structure on scales less than the overall size of the galaxy is absent, where the gravitational potential is spherically symmetric, and where the orbits of the dark matter particles are purely radial. It is easily shown that in this case $\frac{N-1}{2}$ is the maximum number of times a particle that is here now can have passed us in the past, and that $N \simeq 200$. We next we investigate the dependence of N upon the unrealistic assumptions we have made and which are listed at the beginning of this paragraph.

The baryons carry angular momentum as evidenced by the rotating galactic disk. This angular momentum is thought to have been produced by the gravitational forces of nearby galaxies when our own started to form. One should expect the dark matter to have similar amounts of angular momentum and hence to move on non-radial orbits. Nonetheless, the total number of phase space sheets will be roughly of the same order of magnitude as our estimate for the case of purely radial orbits. This is because, assuming spherical symmetry of the gravitational potential, the particles oscillate only in the radial r coordinate and hence the phase space sheet only folds in that direction. Angular momentum makes the location of the folds depend on the angular coordinates (θ, φ) but does not change very much the number of folds. If anything, angular momentum will tend to decrease N because it restricts the range of radii over which dark matter orbits vary. (In the extreme limit of circular orbits, $N = 1$.)

The gravitational potential of the Galaxy is not exactly spherically symmetric since the baryons have excess density in the galactic disk. This does increase the number N of phase space sheets because it causes the dark matter particles to oscillate in the z-direction (perpendicular to the disk) in addition to their oscillations in the r-coordinate. Taking the disc column density $\sigma \simeq \frac{75 M_\odot}{pc^2}$, one finds that a particle, whose orbit is such that it always stayed within a distance of order 1 kpc from the disk, will typically have oscillated ~ 300 times in the z-direction since the time of galaxy formation, with a velocity of order $v_z \sim 10^{-4}$. Thus a peak comprised of dark matter particles which spent all their time close to the galactic disc will fragment in about 10^3 subpeaks spread in a cone of angle $\theta \sim \frac{v_z}{v} \sim 10^{-1}$ about the direction of the velocity of the original peak and over an energy interval of order $\frac{\Delta E}{E} \simeq \frac{v_z^2}{v^2} \simeq 10^{-2}$. (Here and everywhere in this paper, E stands for the kinetic energy of the dark matter particles.) However, this is the most a peak can fragment. Most peaks will be much less fragmented because they are comprised of dark matter particles which spent relatively little time near the galactic disk.

Next, we must evaluate our neglect of all structure on scales smaller than that of the Galaxy as a whole. There are two kinds: structure due to primordial density perturbations in the dark matter, and aggregates of baryons such as stars and globular clusters. The primordial density perturbations are thought to have a flat (Zel'dovich-Harrison type) spectrum with power on all scales. The rate of growth of the density perturbations is of order $\sqrt{G\rho}$ in the linear regime for all scales. However, once the Galaxy has started to form, the further growth of the small scale density perturbations is disrupted by the tidal forces of the Galaxy as a whole. The question is: while the phase space sheet folds on the scale of the Galaxy as a whole, do individual sheets similarly fold on smaller scales, producing folds and subfolds on all scales over which the spectrum of primordial density perturbations had power? The answer is no because the rate of growth of density perturbations on a sheet is controlled by the density ρ_{sheet} of that sheet by itself. (The sheets move with respect to one another, so that growth on one sheet is not amplified by concurrent growth on the other sheets). The condition for the tidal forces of the Galaxy as a whole to disrupt the growth of density perturbations on a sheet is $\rho_{galaxy} > \rho_{sheet}$. This condition is satisfied as soon as the Galaxy has started to form.

Finally, we consider the effect of stars and globular clusters. The gravitational field of each star produces a fold in each phase space sheet. Thus the number of sheets increases exponentially with the number of stars in the Galaxy. Nonetheless, we will see that the phase space structure does not get completely washed out, because the dark matter particles get scattered mainly in the forward direction. Consider a sheet of dark matter particles which reach us from a particular direction in the absence of any stars. The first star along the way of the dark matter particles will split the original beam into two subbeams, the second star will split the two subbeams into four, and so on. The collective effect of successive scatterings by stars is to diffuse the beam over a cone of opening angle $\Delta\theta$ which results from a random walk:

$$(\Delta\theta)^2 = \int_{t_G}^{t_0} dt \int_{b_{min}}^{b_{max}} \frac{4G^2M_*^2}{b^2v^4} n_* v \; 2\pi b \; db$$
$$\simeq 1.6 \; 10^{-6} \left(\frac{t_0}{10^{10} \; years}\right)\left(\frac{n_*}{(pc)^{-3}}\right)\left(\frac{10^{-3}}{v}\right)^3\left(\frac{M_*}{M_\odot}\right)^2 , \quad (2)$$

where we have for simplicty assumed a single population of stars of mass M_* and density n_*, b is the impact parameter and $(\delta\theta)^2 = (\frac{2GM_*}{bv^2})^2$ is the scattering angle squared due to a single collision. The integral over b yields $\ell n(\frac{b_{max}}{b_{min}}) \simeq 8$ if we take $b_{max} \simeq 3.10^{18}\ cm$ (when the particle is more than $n_*^{-1/3} \sim 1\ pc$ away from the star, it feels the gravitational pull of other stars as well) and $b_{min} \simeq 10^{15} cm$ (collisions with $b < 10^{15} cm$ are negligibly rare). In the worst possible case of a beam of dark matter particles that reaches us after having traveled continually through regions with a star density of $1 M_\odot/(pc)^3$, one has $\Delta\theta \simeq 10^{-3}$. The contribution from scattering by globular clusters, again for dark matter particles that have spent most of their past in the dense regions ($r \lesssim 7\ kpc$) of the Galaxy, is roughly of the same order of magnitude, as Eq. (2) implies for $n_{gl.cl.} \simeq \frac{10^{-1}}{(kpc)^3}$ and $M_{gl.cl.} \simeq 10^5 M_\odot$.

Because the stars and globular clusters are moving themselves, the dark matter particles can gain or lose energy in the collisions. The kinetic energy change in a collision is $\frac{\delta E}{E} = 0(\delta\theta^2)$ and the resulting dispersion in the energies of dark matter particles in a given peak is of order $\Delta E \simeq 10^{-6} E$. However, a more important contribution to the energy dispersion follow from the fact that the dark matter particles in a given peak, because of their collisions with stars and globular clusters, came from different initial locations.
Eq. (2) implies that the initial locations of particles in the same peak are spread over a region of size

$$\ell \simeq v t_0 \Delta\theta \simeq 3kpc(\frac{n_*}{(pc)^{-3}})^{1/2} \frac{M_*}{M_\odot}. \tag{3}$$

The resulting dispersion in final energies is of order $\frac{\Delta E}{E} \simeq \frac{\ell}{L}$ where $L \sim 100\ kpc$ is the initial size of the Galaxy. Therefore, energy peaks constituted of particles that have continually been traveling in regions with star density $n_* \simeq \frac{1}{(pc)^3}$ since the formation of the Galaxy most likely do get washed out since $\frac{\Delta E}{E} \simeq 0.03$ in that case. But many peaks will be constituted of particles which have spent relatively little time in the star rich parts of the Galaxy, because their orbits have a large inclination with respect to the galactic plane or because their angular momentum keeps them away from the central parts of the Galaxy. A typical peak may have experienced an average star density $< n_* > \simeq \frac{10^{-2}}{(pc)^3}$ which implies $\Delta\theta \simeq 10^{-4}$ and $\frac{\Delta E}{E} \simeq 10^{-3}$. Such peaks are unlikely to be washed out. Also, the peak corresponding to particles which are falling onto the Galaxy for the first time or those corresponding to particles which have oscillated only a small number of times in the r-coordinate will certainly not be washed out.

The present cavity detectors[2] of galactic halo axions can readily measure the energy spectrum with a resolution of order $\Delta E \simeq 10^{-5} E \simeq 10^{-11} m_a$. The only obvious limitation on the resolution arises from the instability of the quartz local oscillator with which the microwave signal leaving the cavity is compared. If the quartz local oscillator ($\frac{\Delta f}{f} \simeq 10^{-11}$) is replaced by an atomic clock ($\frac{\Delta f}{f} \simeq 10^{-14}$), the energy resolution could be increased to $\frac{\Delta E}{E} \simeq 10^{-8}$. The intrinsic width in energy of a single unfragmented peak is of order

$$\frac{\Delta f}{f} = \frac{\Delta E}{m_a} \simeq v \; \Delta v_a(t_G) \left[\frac{1}{N} \frac{\rho_{halo}(t_0)}{\rho_a(t_G)} \right]^{1/3} \tag{4}$$

where the last factor, which accounts for the change in velocity dispersion of a single sheet from the onset of galaxy formation till the present, follows from Liouville's theorem. Using $v \simeq 10^{-3}$, $\Delta v_a(t_G) \simeq 10^{-17}$ and $\left[\frac{1}{N}\frac{\rho_{halo}(t_0)}{\rho_a(t_G)}\right]^{1/3} \simeq 10$, one obtains $\frac{\Delta f}{f} \simeq 10^{-19}$, which is smaller than the frequency dispersion of any known clocks including the millisecond pulsar observations.[8] The positions of the energy peaks shift in time not only due to the earth's rotation and its motion around the sun and the gravity of nearby celestial bodies, but also due to the motion of the sun around the galactic center ($\frac{1}{f}\frac{df}{dt} \simeq \frac{10^{-14}}{year}$) and the slow evolution of the phase-space sheets in the rest frame of the Galaxy ($\frac{1}{f}\frac{df}{dt} \simeq \frac{10^{-16}}{year}$). Each energy peak is affected differently by the various phenomena. Finally, the velocities $\vec{v}_n$ of the individual peaks can be measured directly by building two or more cavity axion detectors placed at some distance from one another and cross-correlating their signals.

Direct detection of galactic halo WIMPs may be achieved by looking for the recoil of nuclei which have scattered a WIMP. The nuclear recoil may be put into evidence by low temperature calorimetry, by ionization detection or by the detection of ballistic phonons.[9] Because the recoil energy depends on the scattering angle as well as the incident WIMP energy, it will be difficult to see peaks in the energy distribution. However, the ballistic phonons may show the anisotropy associated with discrete $\vec{v}_n$ values.

Finally we would like to suggest that the phase space structure of galactic halo dark matter may already have revealed itself indirectly through its gravitational effects. Because of its distribution on sheets in phase space, the dark matter is not distributed smoothly in physical space. There is excess density at the places in physical space where the phase space sheets fold back. As Fig. I illustrates, the fold locations are closed surfaces which move outward slowly, i.e. on a time scale of order the age of the galaxy, as opposed to the free infall (or dynamical evolution) time scale. The gravitational potential is somewhat deeper at the fold locations and hence the stars spend somewhat more time there. One expects

$$\rho_*(\vec{r}) = \rho_*^{sm}(\vec{r}) e^{-\frac{1}{\beta^2}(V(\vec{r}) - V^{sm}(\vec{r}))} \tag{5}$$

where $\rho_*(\vec{r})$ is the density of stars, $V(\vec{r})$ is the gravitational potential, $\rho_*^{sm}(\vec{r})$ and $V^{sm}(\vec{r})$ are those same quantities if the phase space structure of the dark matter particles were smoothed out, and β is a measure of the velocity dispersion in the galaxy. Observations of elliptical galaxies, using a special technique to enhance low contrast features, have revealed that approximately 10% of them are surrounded by giant shells,[10] from a few kpc up to $\sim$ 100 kpc in size. Surface photometry shows these shells to be sudden drops in the surface brightness superimposed on the steady outward decrease in the galaxy's surface brightness.[11] A number of different interpretations of this phenomenon have been given. The most cited one is that the shells form when the elliptical galaxy "eats" a low-mass companion.[12] But the interpretation we wish to suggest here is that the shells are merely regions where there is a small overdensity of stars of the parent galaxy in response – in the manner of Eq. (5) – to folds in the dark matter phase-space sheets of the halo of that galaxy. Several features of the observations support this interpretation. First, the fact that elliptical galaxies with giant shells are rather common; in particular, they appear to be much more common than elliptical galaxies which are about to eat up a low mass companion. Second, the fact that the shells are ripples on the surface brightness of the parent galaxy rather than separate features. More precisely, the shells roughly follow the same $r^{1/4}$ luminosity distribution as that which describes the galaxy itself.[11] This is what one would expect on the basis of Eq. (5). Third, the size of the effect, namely that the shells are an increase of a few percent in the surface brightness of the underlying galaxy, is also consistent with Eq. (5). Fourth, in most cases, the shells have the same color[10,11] as the underlying galaxy implying that the stars which make up the shells are of the same type as those of the parent galaxy.

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5. Note however that we do not believe the assumption of radial orbits and spherical symmetry to be valid. It is used in Fig. I only for purposes of illustration. Indeed, in

computer simulations and in an analytical treatment, we found that these assumptions lead to a halo density profile $\rho(r)$ which falls off much more sharply than $\frac{1}{r^2}$. It appears that cold dark matter particles need to acquire angular momentum – possibly due to the tidal forces of nearby galaxies – to produce a halo with flat rotation curve.

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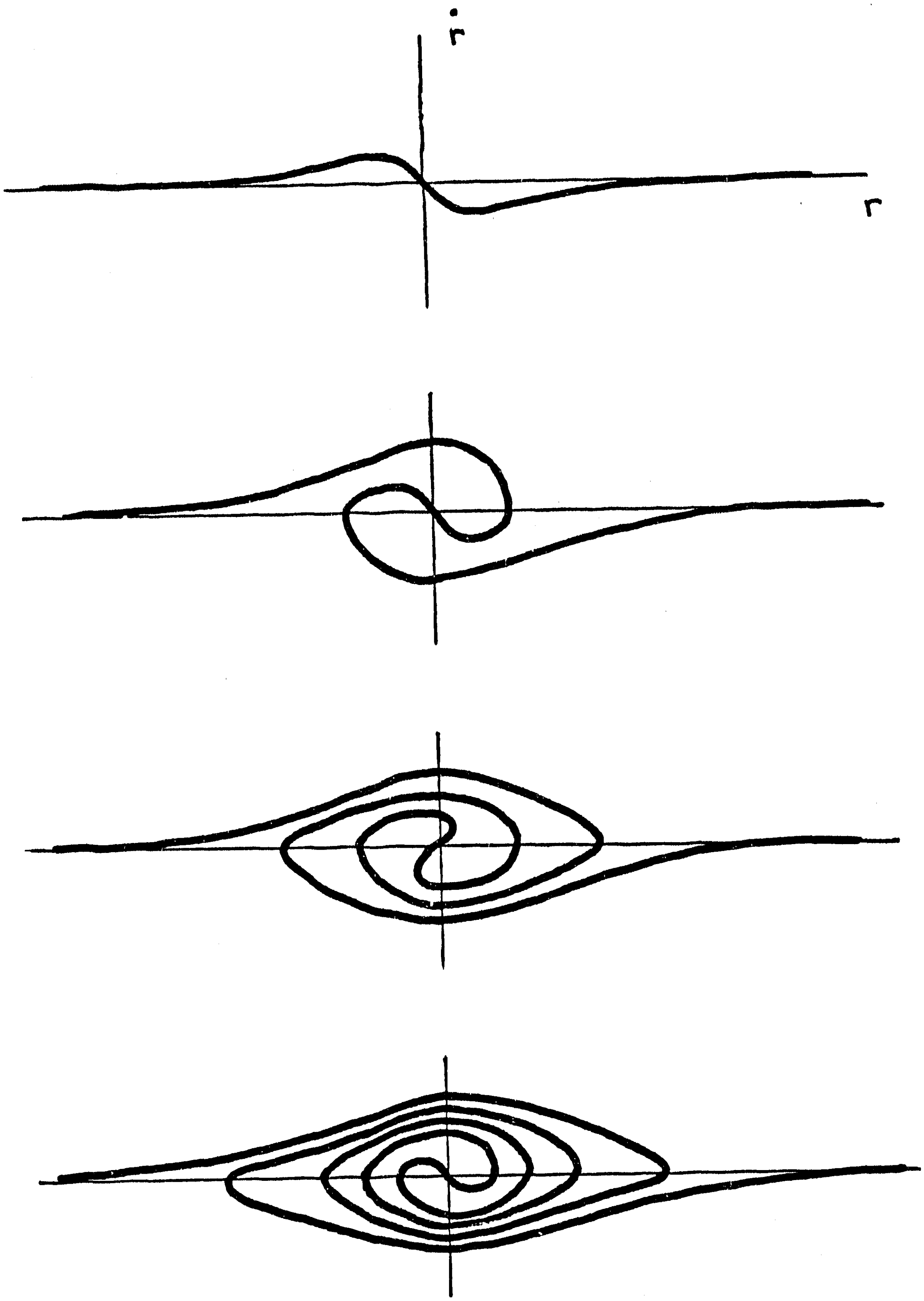
Figure Caption:

Time evolution of the phase-space distribution of cold dark matter particles in a galactic halo in the case of spherical symmetry and purely radial orbits.[5]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