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PROTON SPECTRA FROM 800 MEV PROTONS
ON SELECTED NUCLEI

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Robert L. Stearns

667 3000

(Vassar College

Poughkeepsie, N.Y. 12601

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I. INTRODUCTION

Ever since the work of Rutherford, who established the very existence of the nucleus by studying the scattering of α -particles from thin gold foils, the scattering of various particles by nuclei has been used as a tool in nuclear structure studies.

The nature of the probe--whether hadronic, leptonic, or electromagnetic in character--and its size or wavelength--largely determines the nature of the information gained. In the present experiment, for example, a hadronic probe, the proton, is used. At low energies the proton deBroglie wavelength is large compared to nuclear dimensions. Hence a study of the inclusive $(p,p')^+$ reaction can yield information on the collective excitation of the nucleus, in which many nucleons participate in a coherent manner. As we raise the energy of the proton probe to say, 800 MeV, as in the present experiment, the wavelength falls to about 0.8 F, which is of the order of magnitude of the nucleon separation in nuclear matter.

The elementary aspects of nuclear reactions at high energies were first pointed out by Serber.¹ At high energies high momentum transfers to the nucleus occur and the cross section is no longer dominated by coherent effects, but is more affected by the individual nucleon-nucleon interactions. We would thus expect that nucleon-nucleus collision at large energies can be treated as a series of nucleon-nucleon collisions;² in many cases only one such collision takes place and the interacting nucleons may leave the nucleus without further collisions. Such a collision is called "quasi-elastic" and the process has been previously studied in a number of experiments.³ If the scattering process is sufficiently inelastic, mesons can be produced. In the spirit of the two-body interaction philosophy outlined

+ In this paper the description (p,p') refers to any reaction involving the emission of a proton.

above, we might expect that this deep inelastic process should be dominated by the (3,3) pion-nucleon resonance.⁴ This resonance can also be described as the formation of a " Δ " isobar.

We can illustrate these aspects of the nucleon-nucleus scattering process by reference to Fig. 1, which is a typical cross section curve based on the data of the present experiment, for the case of ^{12}C (p,p') at a laboratory scattering angle of 11° . In the elastic and near elastic region (A) one sees the ground state and various excited states of the recoiling ^{12}C system. These states are not of primary interest in our experiment and will not be further discussed here. In the momentum region corresponding to the free nucleon-nucleon scattering kinematics (B), one sees a broad peak which evidently related to nucleon-nucleon scattering in the nuclear interior. The width of the peak can be attributed to the internal momentum distribution, or Fermi motion, of the nucleons.

At still lower momenta for the outgoing proton, one arrives at a region (C) where meson production is energetically possible. In this region the nucleon-nucleon cross section is dominated by the Δ isobar, and we therefore expect to see the effect of that resonance in the proton spectra from the nucleus.

It should be evident from these qualitative considerations that inelastic proton scattering can excite a variety of nuclear processes and this offers a technique for probing gross nuclear features. In the present work a variety of targets have been subjected to 800 MeV protons and the spectra of emerging protons have been examined. The angle and momentum differential cross sections have been determined. The systematic behavior of the gross features of these spectra with target mass and scattering angle are described and the

quasi-elastic region of the spectra are interpreted within the framework of a distorted-wave impulse approximation (DWIA) calculation.

A number of previous experiments on the inclusive proton reactions are described in the literature, although none cover as broad a range of targets and momenta as those described here. They may be found in Refs. 5-11. Much of the DWIA theory has been developed to interpret the more restrictive (p,2p) reaction as first investigated⁺ by Tyren, Maris and Hillman.¹² In that analysis the restriction is made that both struck and incoming nucleons leave the nuclear volume and are detected in coincidence. While the coincidence restriction is capable of yielding more definite information about nuclear structure, such as shell effects, the inclusive reaction, or singles measurements, can cover a broader range of possible excitation processes. To present the systematic behavior of the inclusive reaction is the goal of the present work.

II. DESCRIPTION OF EXPERIMENT

The measurements were performed at the Los Alamos Clinton P. Anderson Meson Physics Facility (LAMPF) using the High Resolution Proton Spectrometer (HRS).¹³ HRS is a QDD momentum-loss spectrometer with a horizontal scattering plane and a vertical dispersion plane. The 800 MeV proton beam (nominal value-- actually 795 MeV) was derived from stripping the accelerated H⁻-ion beam, with typical beam currents of 0.25 to 10 nanoamperes on the target.

HRS operated at an effective resolution, $\Delta p/p$, of 2×10^{-4} , a momentum bite of $\pm 1.1\%$, and a solid angle of ~ 2 msr. The resolution was limited by the presence of mylar vacuum windows at the scattering chamber exit and the entrance aperture of the spectrometer. However, for the purposes of this

+ In Ref. 10, the first (p,2p) experimental measurement was attributed to O. Chamberlin, but this work apparently remains unpublished.

experiment, the resolution was quite adequate.

Detection was accomplished by a set of 5 scintillation detectors and 4 multiwire proportional counters with delay line readouts. These detectors were mounted near the spectrometer focal surface as indicated in Fig. 2. The specifications for these detectors may be found in Table I. The multiwire counters had spatial resolution (σ) of about 0.3 mm and were used to determine particle trajectories in the region of the focal surface. Pulse height and time of flight information was derived from the scintillators and used for particle identification, particularly against the large deuteron component present in the spectrometer. Veto counters, V_1 and V_2 , to define precisely the spectrometer acceptance and to discriminate against pole-face scattering were arranged as shown in Fig. 2.

Targets of ${}^6\text{Li}$, Li , ${}^9\text{Be}^+$, ${}^{12}\text{C}$, ${}^{27}\text{Al}$, ${}^{40}\text{Ca}$, ${}^{51}\text{V}$, ${}^{90}\text{Zr}$, and Pb were run at angles of 5° , 7° , 9° , 11° , 13° , 15° , 20° , 25° , and 30° . Only the data for angles larger than 9° have been processed. Calibration targets of deuterated polyethylene and polyethylene were run at all angles. For cross section normalizations, we made use of the differential cross sections measured by Willard et al.¹⁴ Table II lists in detail the target parameters and the angle and momentum ranges covered. Not all targets were investigated in the momentum region corresponding to delta isobar production, although the quasi-elastic region was investigated for all.

The targets, which range typically from 10 to 50 mg/cm^2 were 4.5×10 cm in size and were mounted on a remotely actuated target wheel located in the scattering chamber.

+ For the case of ${}^9\text{Be}$ only a limited momentum range was covered. The primary goal of these runs was to identify α -like nuclear clusters by looking for peaks in the excitation energy spectrum, corresponding kinematically to scattering from particles of mass $A=4$. This topic will be covered in a separate publication.

Several beam monitors were simultaneously available. Two Ar-CO₂ ion chambers at 50 mm pressure were located inside the scattering chamber 1 meter behind the target, and a third was placed about 10 meters downstream. The ion chambers saturated at the higher currents used. To cover the higher current region a secondary emission monitor (SEM) with a gain of 0.4 was placed 4 meters downstream of the target. In addition a pair of scintillator counter telescopes viewed the beam-target interaction region at 45 and 120 degrees. These were useful for monitoring beam steering and phase space. An additional monitor which was used occasionally was a thin CH₂ foil in the beam upstream of the scattering chamber and viewed by two counter telescopes to the left and right of the beam. The redundant beam monitors were very useful for readily identifying malfunctioning instruments and bad beam conditions. The configuration of monitors within and near the scattering chamber is drawn in Fig. 3.

The HRS spectrometer is used with the horizontal plane containing the principal component of scattering angle and a vertical plane containing the bending or dispersion direction. The data from events detected by the multiwire counters were used to define the particle trajectories and compute their intersections with the spectrometer focal surface, from which both momentum difference $\delta = \frac{P_o - P}{P}$ and scattering angle θ could be determined.

The momentum region was covered by a series of runs with $\delta = \pm 1.1\%$ from the elastic region to well below the quasi-elastic. An angular range of $\pm 1\%$ was covered by the focal plane. At each angle calibration runs were taken with polyethylene and deuterated polyethylene foils at momenta near the (p,p) and (p,d) elastic scattering points. To record the data, we used the standard HRS data acquisition system based on the PDP-11/45.

In general, event rates in the order of 100 per sec were recorded on magnetic tape, with about 85000 events for each spectrometer momentum and angle setting. About 1500 runs were taken in all, 3 runs per 2400 foot reel of tape. Later the tapes were scanned to produce a matrix of 461 x 20 channels as a function of δ and θ respectively on the focal plane. An example of the data, binned in increments of $\delta = \frac{P_o - P}{P}$ of 7×10^{-4} and $\Delta\theta = 0.09375^\circ$ is shown in Fig. 4. This figure shows a portion of the focal plane binned in the above size channels for a deuterated polythene scatterer. The shaded portion indicates the region of the (δ, θ) scattering plane containing protons scattered from deuterium. Spectra were produced from data similar to those of Fig. 4; for example, a momentum spectrum could be produced from a cut holding θ constant, or alternatively a "missing mass" or excitation energy spectrum could be produced by integrating along the trajectory defined in the (δ, θ) plane by a 2-body relativistic kinematics relationship. Separate runs corresponding to different momentum (δ) regions were joined together and the trajectories integrated across the boundary region.

The double-differential cross sections $d^2\sigma/d\Omega dp$ determined in this experiment are averaged over the spectrometer angular acceptance of 2° . In principle this procedure would lead to an additional broadening of the quasi-elastic peak, since the peak position follows the kinematics for (p, p) scattering, which defines a relationship between scattering angle and momentum. However the kinematic variation of (p, p) scattering over $\pm 1^\circ$ centered at 15° is 22.6 MeV/c and at 30° is 36.2 MeV/c. The natural line width due to Fermi motion is ~ 100 MeV/c. Thus no significant distortion of the data is introduced by this averaging. The inherently high resolution of the HRS is not needed for the smoothly varying cross sections characteristic of quasi-elastic

scattering or delta isobar production. The high resolution is however essential in identifying and correcting for the usually troublesome problem of hydrogen contamination of the target surfaces. For the ${}^6\text{Li}$, ${}^{12}\text{C}$, ${}^{40}\text{Ca}$, and Pb samples, some surface hydrogen contamination, probably from water, is evident. Figure 5 shows a plot of the quasi-elastic region in ${}^{12}\text{C}$. The hydrogen contamination shows up as a pronounced narrow peak superimposed on the smoothly-varying ${}^{12}\text{C}$ cross section. The sizes of the peak in the various runs indicate contamination at a level of approximately 1000 atomic ppm. The resulting incremental cross section is readily identified and subtracted. All of our targets were surveyed in this manner and corrected as necessary.

III. SURVEY OF RESULTS

The results of this experiment are summarized in the form of tables of the double differential cross section $d^2\sigma/d\Omega dp$, mb per steradian per MeV/c, for the nine targets of this experiment, in steps of about 10 MeV/c. For visualization of the momentum distribution, the 11° , 20° , and 30° laboratory angle spectra are also shown in Figs. 6-8.

In addition, Fig. 9 shows the region of the Δ -isobar production, which was examined in detail for hydrogen deuterium, ${}^6\text{Li}$, ${}^{12}\text{C}$, ${}^{27}\text{Al}$, and ${}^{90}\text{Zr}$ at 15° . For the elemental targets Li and Pb, the elemental cross sections are shown. The hydrogen and deuterium targets, in the form of polyethylene and deuterated polyethylene have been corrected for carbon content.

Inspection of Figs. 6-9 indicate the general features observed for the (p,p') reaction. The broad quasielastic peak located near the momentum corresponding to free p,p scattering is clearly evident. For the 15° data of Fig. 9 we see also the smaller peak associated with pion production through the 3,3 resonance corresponding to Δ -isobar formation. A distinct minimum

separates the two peaks. We see a tendency, in these figures, for the isobar peak to be less pronounced as the mass of the target increases. This tendency may be ascribed to the relatively more important role played by multiple scattering processes as the number of target nucleons increases. Harp et al.¹⁵ have obtained reasonable fits for the emitted proton spectra using an intranuclear cascade model^{16,17} in which the Δ -isobars are allowed to interact with other nucleons.

The area under the quasi-elastic peak may be more or less directly related to the number of effective nucleons participating in the quasi-free scattering process. The integral under the peak is fairly well defined for angles of 13° and larger and is shown in Fig. 10 as a function of target mass. The cross sections form straight lines on a log-log plot and thus the cross section is well-represented by a functional dependence of the form $\sigma = \sigma_0(A)^n$ where n is approximately 0.4. Also shown on the plot is the (p,p) cross section on hydrogen, plotted for $A=1$. This point lies quite close to the straight line. This dependence is close to what may be expected for a purely peripheral scattering process, since the projected annular area of the nucleus would vary as $A^{1/3}$. Consequently the quasi-elastic scattering process is apparently a surface phenomenon.

In general the statistical accuracy of the data, binned in segments of 10 MeV/c for the outgoing proton, is between 1 and 2% over the region of the quasi-free peak. The contributions of systematic errors in this experiment are much larger and much more difficult to evaluate. We have attempted to do so on the basis of reproducibility from run to run.

The significant systematic errors arise from several distinct sources:

1. Beam monitoring: An intercomparison of the several beam monitors shows discrepancies attributable to changes in beam quality (phase space distribution) and beam intensity variations. The redundant monitoring employed, and the repetition of supposedly identical runs allow us to assign an error based on observed variations. This amounts to a standard deviation of 2%, after exclusion of obviously malfunctioning monitors in each run.

2. Target errors: These arise from target non-uniformities, errors in thickness determinations, target backing correction, and correction for impurities, such as water. In some cases, especially the hydrogen and deuteron targets, a correction was applied for the carbon present in the polythene material of which the target is composed. Target thickness errors range from 0.3% to about 2% in the worst cases.

3. Corrections for efficiency across the focal plane: A correction was applied to the data to account for the change in effective aperture, or efficiency from one edge of the focal plane to the other. This correction was measured for ^{12}C elastic scattering. It is very nearly linear over the focal plane, and amounted to $\pm 8\%$, referred to the center. We allow a 1% error component due to this effect.

4. Non-linearity corrections: The non-linearity of the delay line readout multi-wire proportional counters was inferred from a set of calibration chambers with accurately known wire spacings. The non-linearity correction was fitted to a second degree polynomial; the maximum deviation from linearity amounts to $\pm 10\%$ over the focal plane. The error introduced by this correction was considered less than 0.1% and thus negligible.

5. Momentum errors: These arise in setting the spectrometer magnetic fields, when piecing together a spectrum comprised of the many individual runs, each reflecting the narrow momentum bite of the spectrometer. We estimate the resulting momentum uncertainty as about 1 MeV/c, which has a negligible influence on cross sections.

6. Reference standard errors: The authors ascribe an error of between 2.3% and 5% to the (p,p) standard data of Ref. 14.

IV. DWIA ANALYSIS

The analysis of the quasi-elastic scattering data presented here has been performed by means of the distorted wave impulse approximation (DWIA). The DWIA is extensively reviewed in Ref. 3. In the present application, the expression for the differential cross section associated with two observed outgoing nucleons must be integrated over all final momenta of the unobserved struck nucleon. This integration has been performed by Kroll and Wall.¹⁸ With the aid of several approximations, they derive the following expression for the differential cross section corresponding to scattering of the incident proton into solid angle $d\Omega_1$ and energy interval dE_1 :

$$\frac{d^2\sigma}{d\Omega_1 dE_1} = \frac{1}{(\hbar c)^2} \sum_{\alpha=1}^A \frac{4\pi k_1}{k_0^x} \int dk_{A-1} k_{A-1} \frac{m_0^2 + E_0 E_\alpha - p_0 p_\alpha \cos(x_1 k_\alpha) \cos(x_1 k_0)}{E_\alpha} \quad (1)$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{p-N}^{CM} = \frac{\sum_{m_\ell} |g_{n j \ell m_\ell}(\vec{k}_\alpha)|^2}{2\ell + 1}$$

* In the present analysis the energy interval dE_1 has been replaced by the momentum interval dp_1 .

In Eq (1), $\vec{x} = \vec{k}_0 - \vec{k}_1$ and

$$g_{nj\ell m_\ell}(\vec{k}_\alpha) = \frac{1}{(2\pi)^{3/2}} \int d\vec{r}_\alpha e^{-i\vec{k}_\alpha \cdot \vec{r}_\alpha} D_2^*(\vec{r}_\alpha) D_1^*(\vec{r}_\alpha) D_0(\vec{r}_\alpha) \frac{u_{n\ell j}(r_\alpha)}{r_\alpha} Y_{\ell m}(\theta_\alpha, \phi_\alpha) \quad (2)$$

The subscripts 0, α , 1 and 2 refer to the incident proton, the struck nucleon, the observed proton and the unobserved nucleon respectively. The residual recoiling nucleus is labeled "A-1". The differential cross section $\left(\frac{d\sigma}{d\Omega}\right)_{p-N}^{CM}$ refers to the free p-N cross section in the CM system corresponding to the p-N collision specified by the laboratory momenta $\vec{k}_0$, $\vec{k}_\alpha$, $\vec{k}_1$ and $\vec{k}_2$. (In Ref. 18, the p-N differential cross section is taken to be the p-p differential cross section.) The D's are the distortion factors while the u's are the optical model bound state wave functions. Further details concerning the notation in Eqs. (1) and (2) can be found in Ref. 18. In these equations $g(\vec{k})$ is called the "distorted momentum distribution." It differs from the actual distribution, obtained from a Fourier transform of the bound state wave function, by the presence of the distortion factors D_i . These serve to modify the momentum of the target nucleon in the nucleon-nucleon collision to account for the effect of multiple collisions on the incoming and outgoing projectile nucleon (refraction and absorption).

The bound state wave functions $u_{n\ell j}(r_\alpha)$ of the struck nucleons have been calculated with the A-Three optical model program¹⁹ using parameters found in the literature.²⁰⁻²² Binding energies not available in the literature were obtained by searching in A-Three using predetermined well-depths. To simplify the calculations proton optical model wave functions were used for all target nucleons (whether neutrons or protons) in filled shells, while for the unfilled shells, neutron wave functions were generally used. It is emphasized, however, that in the calculation

of Eq. 1, the proper distinction between struck neutrons and protons has been made in evaluating the elementary differential cross section $(d\sigma/d\Omega)_{p-N}$, as has already been mentioned above.

The distortion factors $D_i(\vec{r})$ ($i = 0, 1, 2$) are evaluated using a distorted high-energy optical model square-well potential, $V_i + i W_i$, as discussed in Ref. 18. Solving a one-dimensional Schroedinger equation with this potential and with a WKB approximation, one obtains²

$$D_i(\vec{r}) = \exp \{ [-iE_i / (hc)^2 k_i] [V_i + i W_i] \Delta S \} \quad i = 0, 1 \quad (3)$$

where ΔS is the path length of the particle i in the nuclear medium. Since, in the present experiment, particle 2 is unobserved, we have, following the argument in Ref. 18, set $V_2 = W_2 = 0$. Moreover, it has been found that the calculated cross sections are very insensitive to the values of V_0 and V_1 , which have therefore been set to equal to 0.

As might be expected, the calculated cross sections are found to be very sensitive to W_0 and W_1 , since these lead to attenuation of the incoming and outgoing waves. We have chosen to let $W_0 = W_1$ and to regard W_0 as a fitting parameter. More precisely, we have determined W_0 by fitting the calculated peak value of the spectrum at $\theta = 13^\circ$ to the peak of the 13° experimental data. This value of W_0 is then used for the calculation of the spectra at the other angles (20° and 30°). The values of W_0 so obtained are listed in Table IX.

From Eq. (3) it can be seen that the mean free path ℓ for absorption of protons in the nucleus can be related to the value of W_0 by means of

$$\ell = (\hbar c)^2 k / 2E |W_0| \quad (4)$$

If we take $|W_0| = 24$ MeV, which is the average value of $|W_0|$ as determined from Table I and, $E = 1738$ MeV, $k = 7.415$ MeV/c fm^{-1} , we find $\ell = 3.46$ fm. On the other hand, the value of ℓ can also be found from the relation

$$\ell = \frac{1}{n\sigma_a} \quad (5)$$

where n , the density of nucleons, is roughly 0.14 fm^{-3} and σ_a , the cross section for removal of the incident proton from the quasi-elastic peak, i.e., the cross section for meson production, is roughly 1.7 fm^2 .¹⁶ We find therefore

$$\ell = 4.20 \text{ fm},$$

which is in fair agreement with the value obtained in (4).

We conclude that these values for W_0 , shown in Table XII, are in qualitative agreement with what is expected from general physical arguments.

A computer code to evaluate Eqs. (1) and (2) had been developed by Kroll,²³ and applied by him to ^{12}C and ^{40}Ca quasi-elastic scattering. We have substantially modified this code in the following ways.

1. An empirical energy- and isospin-dependent elementary proton-nucleon differential cross section $\left(\frac{d\sigma}{d\Omega}\right)_{\text{p-N}}^{\text{CM}}$ has been inserted. The elementary proton-nucleon differential cross section is parameterized following the analysis and data contained in the review article of Alkhazov et al.²⁴ The cross section contains contributions from both central (C) and spin-orbit (S) amplitudes:

$$\frac{d\sigma}{d\Omega} = |f^{\text{C}}(q)|^2 + |f^{\text{S}}(q)|^2, \quad (6)$$

where q is the momentum transfer and a cross term has been eliminated by averaging over spins. The central amplitude is written as:

$$f^C(q) = f^C(0) e^{-\beta^2 q^2/2}, \quad (7)$$

$f^C(0)$ being the forward scattering amplitude and β^2 the "shape parameter". To find the imaginary part of $f^C(0)$ the optical theorem is used, i.e., since $f^S(0) = 0$,

$$\text{Im}(f^C(0)) = \frac{k\sigma_{\text{total}}}{4\pi}, \quad (8)$$

where σ_{tot} is the total cross section at momentum k . The total cross section is found from a parameterization due to Harp.²⁵ Combining (7) and (8) we get

$$f^C(q) = \left(\frac{k\sigma_{\text{tot}}}{4\pi}\right) (\epsilon + i) \exp[-\beta^2 q^2/2] \quad (9)$$

where ϵ is the ratio of the real to the imaginary part of the $f^C(q)$. The spin-orbit amplitude is parameterized by analogy with (9) :

$$f^S(q) = \left(\frac{k\sigma_{\text{tot}}}{4\pi}\right) (\epsilon_S + i) q \exp[-\beta_S^2 q^2/2]. \quad (10)$$

Eqs. (6), (9) and (10) are now used to calculate the differential cross section. The values of the parameters in these expressions are as follows.

In the case of the p-p cross section, we have, from the data of Alkhazov et al., the empirical relation

$$\epsilon_{pp} = .7505 - 4.72 \times 10^{-4} p_{OL} \quad (11)$$

where p_{OL} is the laboratory momentum of the incident particle expressed in MeV/c.

For β_{pp}^2 we find

$$\beta_{pp}^2 = 11.16 (\log_{10} p_{OL} - 2.751) \quad (12)$$

For the p-n cross section we find

$$\epsilon_{pn} = .7184 - 7.184 \times 10^{-4} p_{OL} \quad (13)$$

and

$$\beta_{pn}^2 = 9.439 (\log_{10} p_{OL} - 26.157) \quad (14)$$

For both p-n and p-p

$$\gamma = 0.16 \text{ fm}, \epsilon_S = -0.1, \beta_S^2 = .66 \text{ fm}^2.$$

2. The algorithm for the calculation of the distortion factors $D_i(\vec{r})$ has been altered to take the distortion into account when the struck nucleon is outside the nominal nuclear radius. Let R denote the radius of the target nucleus as used in the bound state calculation, and let r, θ, ϕ denote the spherical coordinates of the struck nucleon, the polar axis being along the direction of the incident beam.

If $r \leq R$, Kroll's algorithm is followed. If $r > R$, whereas Kroll's algorithm would yield no distortion, the new version of the code first compares the impact parameter, $r \sin \theta$, to R . If $r \sin \theta \geq R$, there is no distortion, but if $r \sin \theta < R$, the total path length is assumed to be the average chord length through a sphere of radius R , i.e. $\frac{4R}{3}$, with $\frac{2R}{3}$ traversed before collision and $2R/3$ traversed after collision. These path lengths are then inserted into the expressions for the distortion factors as given by Eq. (3).

3. The code has been extended to include nuclei heavier than ^{12}C .

4. The need for a Fourier transform of the bound state wave function as input to the code has been eliminated.

The above modifications have substantially improved the agreement between the data and predictions of the program. Calculations with the modified DWIA code have been performed for scattering angles of 13° , 20° , and 30° and for all the target nuclei used in the experiment have been compared with the data. Examples of these fits are discussed in the next section.

V. CONCLUSIONS AND SUMMARY

We illustrate the application of the DWIA analysis for several of the targets and the scattering angles measured. Some typical fits are shown in Figs. 11-13. These fits display two features of the DWIA analysis which seem to be characteristic for all the targets considered.

1. The DWIA analysis tends to produce a quasi-elastic peak which is narrower than the data, especially at laboratory angles larger than 20° .

2. The DWIA analysis predicts a peak momentum which is consistently below that of the data. In fact the centroids of the observed nuclear quasi-free peaks are (within the experimental uncertainty, typically ± 1 MeV/c) identical to that for the free ${}^1\text{H}(p,p){}^1\text{H}$ elastic scattering process. The shift predicted by the DWIA results simply from the binding of the nucleon within the nuclear well. The ${}^{51}\text{V}$ and ${}^{12}\text{C}$ fits are obtained by setting $W_0=W_1=24$ to 26 MeV, and $V_0=V_1=0$. The values for $W_{0,1}$ were obtained by adjusting the calculation to produce the observed peak cross section at 13° . For most nuclides investigated reasonable fits for values near $W=24$ MeV can be obtained, in the sense that the cross sections near the quasi-elastic peak are reproduced.

The experimental data also tend to show structure at the positions expected for nucleon scattering from mass 2 or mass 4 clusters. These structures are most pronounced, however, for ${}^6\text{Li}$ and ${}^9\text{Be}$ and are the subject of a continuing investigation which will be reported elsewhere.

While the problem of matching width and position persists at all angles, there are systematic aspects to the discrepancy which can be illustrated by several examples. In Fig. 12 are shown DWIA fits for ${}^{12}\text{C}$ at 13° and for ${}^{90}\text{Zr}$ at 30° . For the 13° ${}^{12}\text{C}$ data, the fit is excellent provided one translates the DWIA curve by 20 MeV/c from what is shown in the figure.

At 30° , as shown for ^{90}Zr , the peak position is apparently given well by DWIA; here, however the width of this peak attributed to quasi-free scattering is significantly narrower than the observed distribution. The width deficiency is even more emphatic for a heavier nucleus such as ^{208}Pb .

These general trends are observable for all nuclides, and are possibly due to the gross approximation inherent in our use of the Kroll and Wall version of the DWIA calculation. One essential feature of that calculation is the treatment of the struck nucleon: it is assumed to escape from the nucleus, and its binding energy is included in the kinematic calculations. At small scattering angles, however, the momentum transferred to the struck nucleon is small enough that its escape from the nucleus is far from probable. The WKB approximation of Eq. (3) is not valid for very small values of k_i . We have avoided this problem by not distorting the struck or unobserved particle. A proper quantum mechanical treatment, however, must include this particle, and we conclude that the shift in peak position produced by the calculation is attributable to our neglect of the distortion for this particle. Thus at small scattering angles ($<20^\circ$) in the laboratory frame a significant component of the scattering cross section in the neighborhood of the quasi-elastic peak corresponds to coherent inelastic scattering events to highly excited states in the residual target nucleus. A phenomenological way to treat the shift is to introduce an "effective" binding energy $E'_b < E_b$, into the calculation. The first order effect of such an E'_b is just to shift the calculated peak position. Agreement in position may be obtained with an effective binding energy of a few MeV. At laboratory scattering angles larger than 15° , the escape of the struck nucleon is virtually certain and the DWIA calculation is a good representation of the data.

At angles larger than 20° , and particularly for heavy nuclei, the observed proton will have a significant probability for undergoing several collisions, so that a single scattering at an angle θ may be simulated by a number of small

angle scatterings. The DWIA calculation, which is a single scattering approximation, can fail badly. Abul-Magd and Guardiola²⁶ have shown that double scattering effects are small at small angles, but are increasingly important at angles corresponding to large momentum transfers. The quasi-elastic peak is therefore considerably spread by the effect of these multiple scatterings. Only near 20° is the straightforward DWIA calculation capable of accurately reproducing the data.

At this angle the imaginary part of the optical model potential used in DWIA distortion factors yields a value for the nuclear mean free path for absorption which is satisfactory agreement with the nucleon density and the pion production cross sections.

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Figure Captions

- Fig. 1: The smoothed double-differential cross section curve for protons from ^{12}C at a laboratory scattering angle of 11° . The quasi-elastic and delta isobar peaks can be seen at momenta near 1410 and 1020 MeV/c respectively. The elastic scattering and scattering to discrete states of ^{12}C are not shown.
- Fig. 2: The detector configuration for this experiment. The counters labeled "DL" are multi-wire proportional counters with delay line readout. The counters labeled "V" are veto scintillators to define the spectrometer aperture. The scintillators labeled "S" are used for particle identification through time-of-flight and energy loss measurements. The angular and momentum apertures in the bending plane are shown.
- Fig. 3: The arrangement of beam monitors near the target wheel in the scattering chamber. The incident beam, shown at "A", interacts the target wheel "B" and traverses ion chambers "C" (2 chambers) and "D". The scattered beam "E" leaves the scattering chamber at "F" and enters the spectrometer at "G".
- Fig. 4: A portion of the data across the focal plane and binned as indicated in the text. The dotted lines highlight the trajectory obtained from elastic scattering of protons off deuterium. All the data of this experiment were analyzed by first binning the digital output of the delay line chambers in the δ (bending) and θ planes, and performing calculations on the binned data.

- Fig. 5: A portion of the outgoing momentum region for ^{12}C at 15° . Superimposed on the broad quasi-free peak is a narrow peak corresponding to elastic scattering from a hydrogen impurity.
- Figs. 6-8: Proton spectra for eight targets over the quasi-free region at laboratory scattering angles of 11° , 20° , and 30° respectively.
- Fig. 9: Proton spectra for the δ isobar and quasielastic regions at a laboratory scattering angle of 15° .
- Fig. 10: The integrated cross section under the quasielastic peak as a function of mass number. Data from successive angles are marked alternatively 0 and X. Hydrogen elastic scattering is plotted at $A=1$.
- Fig. 11a: DWIA fit to ^{12}C at 20° for an absorptive potential of 26 MeV.
- Fig. 11b: DWIA fit to ^{51}V at 20° for an absorptive potential of 24 MeV.
- Fig. 12: DWIA fit to ^{12}C at 13° for an absorptive potential of 26 MeV.
- Fig. 13: DWIA fit to ^{90}Zr at 30° for an absorptive potential of 20.5 MeV.

Table Captions

1. Dimensions for the multiwire proportional counters and the scintillation detectors used in this experiment.
2. A summary of the target parameters and the momentum ranges covered in this experiment.
- 3.-8. Smoothed cross sections in units of millibarns/steradian/MeV/c for angles of 11, 13, 15, 20, 25, 30°, respectively. These cross sections were obtained by smoothing point by point data obtained in this work. In some cases the interpolation has been performed between non-overlapping data sets at different momenta.
9. The values for W_0 , the absorptive potential, required to fit the peak values for the quasi-elastic region in the DWIA analysis of the data.

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TABLE I
HRS DETECTOR SPECIFICATIONS

V1	12.70 x 17.78 x 0.3175 cm	Pilot B
V2	12.70 x 17.78 x 0.3175 cm	Pilot B
S1	76.2 x 8.89 x 0.6350 cm	Pilot B
S2	77.47 x 10.795 x 0.6350 cm	Pilot F
S3	106.68 x 13.97 x 0.9525 cm	NE110
S4	106.68 x 13.97 x 0.9525 cm	NE110
DL1:	X, 58 cm, 1 mm spacings (cathode)	
	Y, 11 cm, 3 mm spacings (anode)	
*DL2:	X, 58 cm, 4 mm spacings (anode)	
DL3:	X, 58 cm, 1 mm spacings (cathode)	
	Y, 11 cm, 3 mm spacings (anode)	
*DL4:	Same as DL2	

* These chambers are calibration chambers.

TABLE II

Target	Thickness mg/cm ²	Angles°	Momentum Range MeV/c
⁶ Li*	48.81	11	1450 → 880
	48.81	13	1452 → 1230
	48.81	15	1460 → 725
	48.81	20	1440 → 1070
	48.81	25	1450 → 947
	48.81	30°	1400 → 907
Li*	54.378	11	1450 → 880
	54.378	13	1452 → 1230
	54.378	15	1460 → 1180
	54.378	20	1440 → 1070
	54.378	25	1450 → 946
	54.378	30°	1402 → 905
¹² C	40.83	11	1450 → 880
	40.83	13	1452 → 1230
	40.83	15	1460 → 725
	40.83	20	1440 → 1070
	40.83	25	1450 → 946
	40.83	30°	1438 → 907
²⁷ Al	57.58	11	1450 → 880
	57.58	13	1452 → 1230
	57.58	15	1460 → 725
	57.58	20	1440 → 1070
	57.58	25	1452 → 946
	57.58	30°	1401 → 906
⁴⁰ Ca	48.44	11	1450 → 1230
	48.44	13	1452 → 1230
	48.44	15	1460 → 725
	48.44	20	1455 → 1070
	48.44	25	1447 → 1066
	48.44	30°	1452 → 907
⁵¹ V	18.92	11	1450 → 880
	18.92	13	1452 → 1230
	18.92	15	1460 → 1160
	18.92	20	1455 → 1070
	18.92	25	1447 → 1035
	18.92	30	1452 → 907
⁹⁰ Zr	49.82	11	1450 → 1230
	49.82	13	1452 → 1230
	49.82	15	1460 → 725
	49.82	20	1455 → 1070
	49.82	25	1426 → 1035
	49.82	30	1452 → 906

TABLE II. (Cont'd.)

Target	Thickness mg/cm ²	Angles°	Momentum Range MeV/c
²⁰⁸ Pb	54.907	11	1450 → 1230
	54.907	13	1452 → 1230
	54.907	15	1460 → 725
	54.907	20	1428 → 1070
	54.907	25	1447 → 1034
	54.907	30	1452 → 907
¹ ₁ H	43.78 (CH ₂)	15°	1355 → 730
² ₁ H	19.6 (CO ₂)	15°	1300 → 730
⁹ Be	46.2	15°	1454 → 1377
	46.2	20	1443 → 1317
	46.2	25	1414 → 1300
	46.2	30	1428 → 1140

TABLE III

11°

τ P (MeV/c)	${}^6\text{Li}$	Li	${}^{12}\text{C}$	${}^{27}\text{Al}$	${}^{40}\text{Ca}$	${}^{51}\text{V}$	${}^{90}\text{Zr}$	${}^{208}\text{Pb}$
880	.137	.162	.222	.360		.600		
900	.165	.163	.235	.363		.600		
920	.180	.171	.241	.378		.604		
940	.188	.179	.247	.390		.608		
960	.190	.182	.253	.395		.612		
980	.190	.185	.259	.399		.616		
1000	.188	.188	.265	.400		.620		
1020	.184	.190	.270	.405		.624		
1040	.190	.192	.275	.400		.628		
1060	.190	.188	.272	.395		.632		
1080	.180	.185	.265	.390		.635		
1100	.168	.177	.250	.380		.625		
1120	.153	.155	.228	.350		.580		
1140	.138	.140	.206	.328		.524		
1160	.119	.122	.184	.306		.467		
1180	.100	.105	.162	.283		.414		
1200	.083	.087	.140	.260		.370		
1220	.065	.074	.125	.239	.22	.340	.350	.42
1240	.060	.68	.110	.218	.22	.320	.320	.44
1260	.062	.072	.115	.204	.22	.330	.340	.47
1280	.065	.082	.150	.190	.26	.380	.380	.48
1300	.080	.112	.180	.220	.330	.480	.500	.62
1320	.125	.14	.260	.360	.48	.620	.690	.83
1340	.195	.23	.37	.630	.81	.900	1.010	1.200
1360	.385	.540	.62	.970	1.22	1.420	1.55	1.75
1380	.700	.735	1.041	1.200	1.64	1.900	2.0	2.30
1400	1.025	1.065	1.252	1.660	2.05	2.28	2.500	3.11
1410	1.165	1.197	1.415	1.710	2.20	2.52	2.820	3.40
1415	1.212	1.198	1.430	1.770	2.21	2.57		3.46
1420	1.130	1.130	1.423	1.730	2.18			
1425	1.060	1.100	1.390	1.740	2.17			
1430	1.020	1.325	1.060	1.860				
1435	1.22	1.425	.940	1.960				
	$\pm 6.5^a$	± 6.5	± 6.2	± 5.9	± 6.2	± 5.9	± 5.9	± 6.2

a. Approximate systematic cross section errors in %.

TABLE IV

13°

P (MeV/c)	⁶ Li	Li	¹² C	²⁷ Al	⁴⁰ Ca	⁵¹ V	⁹⁰ Zr	²⁰⁸ Pb
1215	.045	.050	.097	.155	.19	.200	.290	.34
1225	.050	.055	.100	.160	.21	.215	.300	.36
1235	.055	.060	.110	.175	.22	.230	.310	.37
1245	.060	.068	.122	.178	.23	.250	.340	.40
1255	.065	.075	.136	.202	.24	.270	.360	.42
1265	.070	.085	.150	.225	.28	.300	.400	.46
1275	.090	.100	.170	.255	.30	.335	.450	.53
1285	.110	.122	.205	.300	.36	.385	.530	.60
1295	.140	.150	.250	.355	.44	.455	.620	.69
1305	.160	.188	.305	.420	.520	.535	.720	.81
1315	.210	.242	.379	.517	.650	.640	.840	.97
1325	.280	.320	.467	.636	.750	.760	.970	1.11
1335	.360	.410	.557	.750	.870	.880	1.110	1.28
1345	.445	.505	.645	.860	1.01	.995	1.255	1.48
1355	.550	.622	.729	.966	1.16	1.105	1.430	1.72
1365	.700	.775	.830	1.080	1.31	1.210	1.630	2.01
1375	.835	.882	.912	1.155	1.41	1.335	1.780	2.23
1380	.870	.910	.940	1.175	1.44	1.385	1.850	2.31
1385	.872	.912	.950	1.190	1.48	1.422	1.900	2.37
1390	.875	.910	.952	1.20	1.52	1.455	1.940	2.40
1395	.865	.908	.955	1.20	1.52	1.478	1.970	2.43
1400	.855	.890	.955	1.21	1.54	1.490	1.990	2.44
1405	.840	.855	.952	1.21	1.55	1.510	2.000	2.44
1410	.820	.815	.950	1.20	1.55	1.515	1.990	2.45
1415	.765	.790	.930	1.19	1.52	1.480	1.980	2.40
1420	.700	.780	.840	1.17	1.44	1.442	1.960	2.36
1425	.685	.780	.780	1.14	1.33	1.390	1.940	2.34
1430	.700	.790	.750	1.115		1.480	1.910	2.36
1435	1.00	.810	.775	1.090		1.580	1.880	
1440	1.665	1.610	2.04	1.050				
	±6.5 ^a	±6.5	±6.2	±5.9	±6.2	±5.9	±5.9	±6.2

a. Approximate systematic cross section errors in %.

TABLE V

15°

P (MeV/c)	⁶ Li	Li	¹² C	²⁷ Al	⁴⁰ Ca	⁵¹ V	⁹⁰ Zr	Pb	² H	¹ H
725			.141	.219	.294		.459	.605	.0365	.0337
730	.086		.137	.22	.2935				.0370	.0340
790	.090		.135	.221	.293		.438	.569	.0477	.038
850	.097		.146	.226	.304		.430	.568	.0565	.0417
900	.102		.148	.227	.312		.435	.570	.0610	.0451
940	.106		.154	.228	.310		.430	.565	.0640	.0471
1000	.105		.150	.237	.305		.433	.550	.630	.0478
1050	.096		.149	.229	.295		.401	.502	.0570	.0430
1090	.088		.138	.218	.280		.360	.462	.0485	.0367
1120	.078		.129	.190	.263		.360	.432	.0420	.030
1160	.067	.067	.112	.181	.244	.274	.360	.434	.0280	.017
1200	.057	.066	.108	.180	.250	.310	.356	.436	.0185	
1220	.062	.073	.115	.192	.259	.365	.368	.470	.0168	
1240	.070	.092	.143	.235	.295	.400	.400	.505	.0160	
1260	.102	.130	.207	.297	.337	.455	.500	.630	.0162	
1280	.155	.188	.278	.410	.514	.57	.670	.780	.0220	
1300	.234	.291	.411	.560	.710	.72	.890	1.05	.0300	
1310	.300	.359	.458	.647	.817	.80	1.02	1.21		
1320	.382	.441	.532	.713	.925	.89	1.140	1.36		
1330	.455	.506	.600	.777	.960	.95	1.270	1.50		
1340	.542	.624	.678	.856	1.106	1.05	1.370	1.71		
1350	.616	.619	.695	.870	1.12	1.10	1.47	1.81		
1360	.670	.700	.719	.915	1.255	1.16	1.59	2.01		
1365	.708	.745	.747	.930	1.26	1.18	1.59	2.08		
1370	.710	.721	.748	.920	1.262	1.19	1.590	2.16		
1375	.692	.714	.736	.918	1.245	1.20	1.58	2.12		
1380	.690	.680	.710	.925	1.23	1.20	1.57	2.08		
1385	.682	.685	.690	.930	1.22	1.20	1.55	2.06		
1390	.650	.661	.660	.900	1.21	1.19	1.52	2.05		
1395	.610	.577	.635	.880	1.12	1.17	1.500	1.97		
1400	.550	.539	.584	.860	1.05	1.14	1.460	1.89		
1405	.510	.500	.547	.830	.995	1.10	1.420	1.85		
1410	.511	.441	.530	.820	.986	1.05	1.380	1.82		
1415	.460	.447	.497	.830	.869	1.015	1.34	1.77		
1420	.408	.400	.437	.840	.853	.97	1.30	1.73		
1425	.353	.403	.360	.800	.848	.889	1.26	1.68		
1430	.359	.356	.390	.785	.844	.85	1.23	1.604		
1435	.418	.310	.43	.837	.797	.810	1.21	1.545		
1440	.617	.890	1.06	.949	.750	.762	1.20	1.46		
	±6.5 ^a	±6.5	±6.2	±5.9	±6.2	±5.9	±5.9	±6.2	±6.6	±6.6

a. Approximate systematic cross section errors in %.

TABLE VI

20°

P (MeV/c)	⁶ Li	Li	¹² C	²⁷ Al	⁴⁰ Ca	⁵¹ V	⁹⁰ Zr	Pb
1070	.0575	.0600	.103	.1675	.205	.2100	.2950	.360
1080	.0560	.0625	.105	.1700	.207	.2225	.2975	.365
1090	.0550	.0600	.108	.1725	.208	.2250	.2988	.360
1100	.0565	.0613	.110	.1750	.210	.2300	.3000	.368
1110	.0580	.0625	.113	.1775	.211	.2320	.3050	.380
1120	.0563	.0638	.115	.1825	.214	.2400	.3100	.390
1130	.0610	.0663	.121	.1900	.220	.2450	.3150	.400
1140	.0615	.0688	.124	.1975	.230	.2550	.3250	.410
1150	.0688	.0738	.129	.2100	.240	.2625	.3375	.420
1160	.0713	.0800	.138	.2175	.255	.2700	.3500	.428
1170	.0825	.0900	.150	.2375	.270	.2850	.3725	.438
1180	.0938	.1050	.170	.2525	.290	.3200	.4000	.460
1190	.1063	.1200	.190	.2850	.310	.3275	.4250	.488
1200	.1213	.1400	.208	.3050	.330	.3525	.4550	.523
1210	.1463	.1625	.228	.3325	.355	.3800	.4925	.565
1220	.1700	.1900	.254	.3600	.400	.4100	.5250	.620
1230	.1888	.2075	.285	.3850	.430	.4400	.5500	.675
1240	.2200	.2413	.313	.4125	.470	.4725	.6100	.743
1250	.2525	.2800	.338	.4400	.513	.5025	.6588	.798
1260	.2875	.3175	.355	.4600	.550	.5300	.7050	.860
1270	.3238	.3400	.380	.4875	.590	.5625	.7575	.930
1280	.3500	.3650	.390	.5150	.615	.5950	.8150	.990
1290	.3700	.3800	.398	.5300	.640	.6175	.8550	1.045
1300	.3838	.3825	.403	.5450	.645	.6250	.8700	1.080
1310	.3788	.3750	.403	.5538	.650	.6275	.8750	1.105
1320	.3513	.3550	.395	.5500	.638	.6200	.8300	1.108
1330	.3250	.3300	.385	.5400	.610	.5975	.8355	1.090
1340	.2700	.3000	.345	.5100	.575	.5563	.8038	1.053
1350	.2225	.2525	.308	.4700	.535	.5475	.7550	1.005
1360	.1850	.2050	.265	.4288	.485	.4825	.7025	.994
1370	.1463	.1625	.228	.3850	.425	.4375	.6275	.830
1380	.1150	.1150	.191	.3350	.368	.4100	.5600	.798
1390	.0800	.0850	.148	.2750	.300	.3250	.4950	.695
1400	.0538	.0625	.120	.2250	.260	.2975	.4400	.605
1410	.0438		.108	.1900	.215	.2775	.3925	.550
1420			.073	.1550	.170	.1975	.3200	.440
1430				.1300	.140	.1650	.1325	.380
1440				.1050		.1275		
1450								
	±6.2 ^a	±6.2	±5.8	±5.5	±5.8	±5.5	±5.5	±5.8

a. Approximate systematic cross section errors in %.

TABLE VII

25°

P (MeV/c)	⁶ Li	Li	¹² C	²⁷ Al	⁴⁰ Ca	⁵¹ V	⁹⁰ Zr	Pb
970	.0455	.051	.092	.147				
980	.0450	.052	.092	.147				
990	.0445	.052	.091	.148				
1000	.0440	.053	.091	.149				
1010	.0455	.053	.091	.151				
1020	.0460	.054	.092	.154				
1030	.0470	.054	.093	.156				.340
1040	.0480	.056	.096	.161		.211	.278	.341
1050	.0500	.057	.099	.165		.216	.284	.343
1060	.0520	.060	.103	.169		.220	.285	.345
1070	.0540	.065	.107	.174		.224	.289	.348
1080	.0590	.072	.115	.182	.208	.229	.298	.353
1090	.0650	.077	.124	.189	.228	.234	.305	.360
1100	.0730	.083	.129	.195	.231	.239	.315	.368
1110	.0810	.091	.135	.202	.240	.245	.328	.380
1120	.0910	.100	.143	.212	.252	.256	.340	.390
1130	.1020	.113	.153	.224	.268	.266	.355	.408
1140	.1110	.123	.162	.234	.278	.277	.370	.428
1150	.1220	.134	.170	.244	.290	.289	.385	.448
1160	.1340	.146	.177	.254	.303	.300	.405	.470
1170	.1480	.158	.186	.264	.315	.312	.423	.495
1180	.1610	.168	.197	.276	.330	.324	.443	.528
1190	.1710	.176	.202	.284	.341	.335	.458	.558
1200	.1780	.181	.204	.290	.354	.344	.473	.595
1210	.1840	.186	.210	.294	.364	.352	.483	.620
1220	.1870	.186	.216	.296	.368	.358	.493	.638
1230	.1880	.184	.218	.296	.374	.363	.498	.651
1240	.1835	.178	.214	.294	.364	.372	.496	.654
1250	.1730	.170	.206	.288	.360	.361	.484	.653
1260	.158	.159	.196	.278	.344	.353	.465	.645
1270	.140	.143	.184	.268	.320	.342	.449	.630
1280	.120	.119	.175	.254	.310	.329	.451	.600
1290	.103	.110	.162	.237	.294	.309	.450	.575
1300	.091	.094	.144	.230	.272	.291	.410	.553
1310	.077	.075	.130	.209	.253	.276	.358	.523
1320	.065	.065	.115	.186	.220	.240	.323	.488
1330	.053	.054	.100	.160	.202	.225	.310	.430
1340	.044	.045	.087	.136	.170	.190	.283	.385
1350	.037	.038	.074	.118	.152	.171	.249	.340
1360	.031	.034	.064	.102	.142	.161	.223	.295
1370	.025	.028	.054	.091	.120	.133	.193	.263
1380	.021	.025	.051	.084	.094	.118	.158	.230
1390	.020	.022	.045	.065	.076	.104	.125	.205
1400	.019	.019	.037	.052	.068	.083	.108	.184
	±4.8 ^a	±4.8	±4.3	±3.9	±4.3	±3.9	±3.9	±4.3

a. Approximate systematic cross section errors in %.

TABLE VIII

30°

P (MeV/c)	⁶ Li	Li	¹² C	²⁷ Al	⁴⁰ Ca	⁵¹ V	⁹⁰ Zr	²⁰⁸ Pb
900	.0355	.0379	.076		.161	.168	.251	.326
910	.0365	.0382	.078		.162	.168	.252	.324
920	.0375	.0388	.079	.130	.162	.169	.252	.322
930	.0379	.0396	.080	.131	.163	.169	.253	.321
940	.0385	.0402	.082	.131	.164	.169	.253	.320
950	.0391	.0414	.083	.133	.166	.170	.254	.317
960	.0400	.0426	.084	.133	.167	.170	.255	.318
970	.0412	.0447	.085	.134	.168	.172	.256	.318
980	.0426	.0471	.088	.134	.169	.174	.258	.321
990	.0440	.0497	.090	.135	.170	.176	.260	.323
1000	.0459	.0532	.091	.136	.173	.179	.261	.324
1010	.0473	.0560	.092	.137	.178	.180	.263	.327
1020	.0495	.0592	.095	.140	.181	.182	.264	.331
1030	.0512	.0631	.097	.144	.186	.185	.265	.337
1040	.0542	.0661	.099	.147	.189	.188	.267	.341
1050	.0592	.0702	.100	.149	.193	.191	.270	.346
1060	.0669	.0743	.103	.154	.199	.192	.272	.350
1070	.0737	.0781	.105	.156	.204	.195	.276	.355
1080	.0791	.0817	.108	.160	.207	.199	.282	.367
1090	.0828	.0850	.110	.162	.211	.201	.286	.374
1100	.0858	.0876	.111	.164	.214	.205	.291	.380
1110	.0885	.0897	.114	.167	.218	.207	.297	.388
1120	.0909	.0909	.115	.169	.219	.208	.300	.394
1130	.0922	.0909	.116	.173	.220	.207	.303	.407
1140	.0928	.0903	.117	.174	.218	.205	.303	.414
1150	.0917	.0882	.118	.174	.213	.204	.303	.421
1160	.0844	.0850	.117	.173	.209	.201	.302	.421
1170	.0773	.0808	.115	.169	.206	.199	.299	.420
1180	.0698	.0758	.110	.164	.203	.195	.294	.409
1190	.0627	.0704	.103	.160	.199	.193	.289	.405
1200	.0560	.0651	.098	.151	.190	.188	.284	.398
1210	.0509	.0592	.091	.143	.183	.183	.277	.399
1220	.0468	.0539	.088	.136	.176	.177	.269	.379
1230	.0430	.0483	.080	.128	.166	.169	.254	.359
1240	.0391	.0426	.076	.121	.157	.161	.244	.334
1250	.0361	.0385	.071	.112	.147	.153	.232	.329
1260	.0331	.0408	.065	.105	.136	.143	.218	.320
1270	.0305	.0320	.059	.097	.125	.135	.206	.301
1280	.0284	.0288	.053	.091	.111	.121	.174	.253
1290	.0246	.0258	.049	.083	.102	.109	.155	.238
1300	.0222	.0234	.043	.074	.088	.101	.148	.211
1310	.0195	.0207	.038	.066	.080	.088	.134	.185
1320	.0160	.0187	.033	.058	.071	.080	.114	.173
1330	.0134	.0157	.028	.048	.061	.070	.103	.140
1340	.0110	.0128	.025	.043	.051	.056	.088	.125
1350	.0086	.0110	.022	.036	.043	.047	.076	.112
1360	.0075	.0098	.019	.030	.038	.040	.064	.097
1370	.0080	.0080	.017	.024	.033	.034	.052	.084
1380	.0057	.0053	.012	.018	.021	.027	.043	.058
1390	.0033	.0027	.008	.014	.016	.019	.036	.050
1400	.0021	.0024	.006	.009	.013	.015	.027	.038
	±5.4 ^a	±5.4	±5.0	±4.6	±5.0	±4.6	±4.6	±5.0

a. Approximate systematic cross sections errors in %.

TABLE IX.

<u>Target</u>	<u>$-W_0$ (MeV)</u>
${}^6\text{Li}$	17
${}^7\text{Li}$	10
${}^{12}\text{C}$	26
${}^{27}\text{Al}$	27
${}^{40}\text{Ca}$	26
${}^{51}\text{V}$	24
${}^{90}\text{Zr}$	21
${}^{208}\text{Pb}$	37

Fig. 1

HRS DETECTOR CONFIGURATION (BENDING PLANE)

Fig. 2

Fig. 3

99	89	85	89	87	80	73	85	51	80	74	53	47	56	56	72	60	52	56	209	503	45	17	24	12	13	12	23	16
98	105	83	88	85	77	89	77	73	62	70	70	65	84	61	67	40	32	58	566	207	28	26	22	11	15	20	30	24
104	79	90	76	64	76	80	70	63	48	57	71	46	65	62	50	32	33	186	457	37	25	36	28	17	18	24	34	21
89	81	82	70	64	75	67	69	79	59	54	67	56	53	46	48	36	68	490	164	23	21	20	28	18	13	16	22	19
107	102	74	75	74	82	68	73	87	66	45	70	53	72	57	36	31	202	478	42	26	19	13	13	8	15	25	27	21
85	76	92	61	66	59	60	62	64	47	52	45	59	62	41	28	79	435	145	20	36	28	21	14	19	21	17	19	27
81	74	70	79	63	53	60	58	65	56	59	55	52	66	33	29	229	371	43	33	21	18	20	18	16	8	26	19	17
72	59	67	74	61	59	63	68	52	50	52	63	56	37	31	86	433	153	40	28	24	19	14	12	15	12	24	18	11
68	60	65	59	63	63	68	63	49	44	64	54	32	48	41	239	328	27	26	24	24	19	12	17	13	10	20	27	15
63	67	65	46	49	62	51	64	48	57	49	32	52	39	106	432	119	26	23	20	20	13	16	8	15	13	18	19	9
61	53	65	61	60	59	63	60	50	49	52	39	32	48	309	301	45	24	23	18	20	17	19	15	20	12	21	20	26
64	65	60	57	60	54	40	49	53	47	46	30	38	113	381	88	26	32	21	15	16	24	18	10	11	21	18	14	15
59	65	60	55	67	50	54	52	45	53	42	36	56	283	233	26	21	28	21	26	26	22	16	21	6	10	22	22	14
70	52	57	48	59	45	46	42	42	37	42	28	153	325	65	15	21	17	19	19	11	16	21	17	10	17	20	13	16
66	62	60	46	31	67	47	39	48	41	38	65	285	177	21	20	21	18	27	21	20	22	13	15	7	18	17	20	15
58	63	49	42	46	51	47	33	36	18	35	183	312	57	18	22	19	21	15	12	17	15	17	8	6	20	13	12	16
52	43	61	61	40	48	32	47	32	27	98	289	123	19	28	25	16	17	13	11	13	17	17	14	7	25	17	13	16
47	44	43	41	35	32	55	31	27	65	227	217	42	14	13	26	17	18	13	16	15	16	19	8	10	15	17	15	14
50	42	41	42	42	28	34	29	32	126	303	88	21	20	23	20	17	14	16	28	14	16	12	9	11	17	8	10	19
49	40	42	34	34	28	41	28	75	254	179	25	22	19	19	21	18	19	18	18	12	10	12	9	3	21	17	7	10

8 →

↑ 9

Fig. 4

Fig. 5

Fig. 6

Fig. 8

Fig. 9

FIG. 10

Fig. 11a

Fig. 11b

Fig. 12

Fig. 13