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A Model Approach for Simulating the Thermo-
dynamic Behavior of the MFEF Cryogenic
Cooling Systems -- A Status Report

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Abstract

A numerical model for calculating the thermodynamic behavior of the MFTF-B cryogenic cooling system is described. Nine component types are discussed with governing equations given. The algorithm for solving the coupled set of algebraic and ordinary differential equations is described. The model approach is demonstrated on a simplified coolant loop problem. Completion of the model and its application to the MFTF-B cryogenic cooling system has not been possible due to lack of funding.

Nomenclature

a	- speed of sound
A	- flow cross-sectional area
A_s	- surface area for convection purposes
A_s'	- surface area for radiation transport purposes
C	- specific heat of solid components, average specific heat in helium
C_p	- specific heat of fluid at constant pressure
C_v	- specific heat of fluid at constant volume
f	- Darcy friction factor, fraction of maximum flow rate
F	- gray body shape factor
h	- specific enthalpy
h	- average or mean convection coefficient
k	- ratio of fluid specific heats
k_j	- unit normal to element flow surfaces
K	- lumped pressure loss coefficient, orifice factor, valve loss coefficient
L	- element length for frictional loss considerations
$\dot{m}$	- mass flow rate
M	- mass
P	- pressure
Q	- heat flux
r_c	- critical pressure ratio
t	- time
T	- fluid temperature, wall temperature vector
T_w	- component bulk wall temperature

Nomenclature (continued)

- T_{∞} - ambient temperature for radiation purposes
- V - bulk velocity ($= \dot{m}/\rho A$)
- z - algebraic dependent variables (independent variables, fluid model)
- B - specified enthalpy change in auxiliary heat exchanger
- Δ - denotes difference
- ϵ - heat exchanger effectiveness, flow area or contraction ratio
- η_s - isentropic efficiency
- ρ - fluid density
- σ - Stefan-Boltzmann constant

Subscripts

- 1,3 - denotes element flow entrance
- 2,4 - denotes element flow exit
- c - at the vena-contracta
- h - by convective heat transfer
- r - by radiative heat transfer
- T - at the throat

I. Introduction

To provide cryogenic cooling to the superconducting magnets and cryopanel in the MFTF-B experiment, LLNL has contracted with CVI Incorporated, Columbus, Ohio, for the design, fabrication and installation of an 8000 watt helium cryogenic cooling system (see schematic, Fig. 1). In mid-1982, at the request of the MFTF program, we undertook the task of developing a computer model to simulate the thermodynamic behavior of the proposed cooling system during the mode of operation where the magnets are being cooled and filled with liquid helium (LHe). A long program plan development period ended with submission of a proposal [1] for developing a numerical model to simulate the behavior of the 8000 watt cryogenic cooling system during the cool-down mode of operation. In June 1983, support for this thermodynamic modeling effort was terminated by the MFTF staff.

This report documents the status of the model development at the time of its termination. Nine component types were identified in the early stages of the work. A brief description of current thinking on the model for each type of component is provided. It must be emphasized that in most instances these models are not complete since termination of funding prevented complete implementation during which simulation could have identified model inadequacies.

All models evolve around the concept of a "quasi-steady" analysis. This is a common method in the engineering sciences where only the dominant time derivative terms in the governing equations are retained. The resulting system of equations is a coupled set of algebraic equations and ordinary

differential equations. As will be subsequently shown in detail, the fluid behavior is governed by steady-state models and the thermal behavior of massive solid components is governed by a lumped mass ordinary differential equation approximation. It is the coupling between these two model types that generates the "quasi-steady" or "quasi-dynamic" nature of the model approach. The use of steady-state approximations for the fluid behavior is justified by the observations that the sonic wave transit time through the system is very short compared to the multi-day cool-down time and that the cool-down phenomena is relatively free of sudden, pronounced transients.

Each model for the fluid behavior in a component consists of three governing equations of algebraic form for mass conservation, momentum conservation and energy conservation. In many instances the equation identified as momentum conservation is really a pressure loss equation, which does not contain momentum-type terms. It is, however, a correct and necessary equation. Each model for a lumped-mass solid component consists of one ordinary differential equation.

The equation set was designed to be solved using a computer software package developed by A. Hindmarsh. The package is comprised of a collection of routines obtained from other operating codes for solving a system of coupled non-linear algebraic and ordinary differential equations using a Newton-type iteration and sparse matrix manipulation concepts [2]. A brief description of this software package is provided in a subsequent section.

The equations for fluid behavior are written in terms of the independent variables mass flux ($\dot{m}$), pressure (P), and specific enthalpy (h). In many of the equations the fluid density (ρ) appears. This is treated as a dependent

variable since it is obtained from the equation-of-state using the pressure and enthalpy.

II. Model Descriptions

Lines

Our definition of lines (see Fig. 2) includes any sequential set of pipe segments of constant area and the elbows that join these segments. The pressure loss due to friction in the pipes is modeled using the Darcy friction factor. The pressure loss in the elbows and other fittings is accounted for in a lumped-loss coefficient. The governing conservation equations for the fluid behavior are:

$$\text{mass: } \dot{m}_2 - \dot{m}_1 = 0$$

$$\text{momentum: } \frac{A_1^2}{2\dot{m}_1} (P_1 - P_2) - \left(\frac{fL}{160} + \frac{K}{160} \right) (\rho_1 + \rho_2) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)^2 - \left(\frac{1}{\rho_2} - \frac{1}{\rho_1} \right) = 0 \quad (1)$$

$$\text{energy: } h_1 + \frac{1}{2} \left(\frac{\dot{m}_1}{\rho_1 A_1} \right)^2 + \frac{\dot{Q}}{\dot{m}_1} - h_2 - \frac{1}{2} \left(\frac{\dot{m}_2}{\rho_2 A_2} \right)^2 = 0$$

In the energy conservation equation, $\dot{Q}$ is the convected heat gain from the wall of the pipe to the helium and is defined as

$$\dot{Q} = h A_s \left(T_w - \frac{T_1 + T_2}{2} \right) \quad (2)$$

Note that the average fluid temperature has been used to represent the bulk temperature for convective heat transfer purposes.

The wall is treated as a lumped-mass body. Most of the lines are vacuum jacketed and nitrogen shielded as shown in Fig. 2. The equation governing the thermal behavior of this component is

$$\begin{aligned} MC \frac{dT_w}{dt} &= -\dot{Q}_h - \dot{Q}_r \\ &= -h A_s \left(T_w - \frac{T_1 + T_2}{2} \right) - \sigma A_s' F (T_w^4 - T_\infty^4) \end{aligned} \quad (3)$$

This model accounts for convective heat transfer from the wall to the helium and radiative heat transfer from the wall forming the nitrogen barrier to the lumped mass body.

Heat Exchangers

There are two basic types of heat exchangers in the cooling system:

1) two-sided boiling liquid nitrogen heat exchangers, and 2) multi-sided heat exchangers where as many as four different fluid streams participate in the heat exchange process. Each type will now be discussed.

Two-Sided Boiling Liquid Nitrogen Heat Exchangers

An example of this type is the auxiliary skid heat exchanger used in the initial stages of cool-down. This is a boiling type heat exchanger (see Fig. 3) where the flow of liquid nitrogen is regulated to control the temperature decrease in the helium stream. The inlet and outlet areas for the helium stream are assumed to be the same. The pressure loss is accounted for in a lumped friction loss parameter. The governing conservation equations are:

$$\text{mass: } \dot{m}_2 - \dot{m}_1 = 0$$

$$\text{momentum: } \frac{A_1^2}{\dot{m}_1^2} (P_1 - P_2) - \frac{K}{16} (\rho_1 + \rho_2) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)^2 - \left(\frac{1}{\rho_2} - \frac{1}{\rho_1} \right) = 0 \quad (4)$$

$$\text{energy: } h_2 - h_1 - \beta = 0$$

The constant β represents the enthalpy change associated with the specific temperature drop.

Multi-Sided Heat Exchangers

The heat exchangers in the cold box have at least three different fluid streams participating in the heat exchange process. These heat exchangers are designed with multiple passages for each fluid stream stacked such that in effect the fluid streams see each other. For simplicity, we think of these as a set of parallel two-sided heat exchangers and branch the fluid streams before the heat exchanger and merge the branches of each stream after the heat exchanger. For a three stream heat exchanger there are three such two-sided heat exchangers as depicted in Fig. 4. The theory for a two-sided heat exchanger is well understood [3]. The governing conservation equations are:

$$\text{mass: } \dot{m}_2 - \dot{m}_1 = 0$$

$$\dot{m}_4 - \dot{m}_3 = 0$$

$$\text{momentum: } \frac{A_1^2}{\dot{m}_1^2} (P_1 - P_2) - \frac{K}{16} (\rho_1 + \rho_2) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)^2 - \left(\frac{1}{\rho_2} - \frac{1}{\rho_1} \right) = 0$$

$$\frac{A_4^2}{\dot{m}_4^2} (P_4 - P_3) - \frac{K}{16} (\rho_3 + \rho_4) \left(\frac{1}{\rho_3} + \frac{1}{\rho_4} \right)^2 - \left(\frac{1}{\rho_3} - \frac{1}{\rho_4} \right) = 0 \quad (5)$$

$$\text{energy: } T_2 - T_1 - \varepsilon (T_3 - T_1) = 0$$

$$\dot{m}_1 C_{12} (T_2 - T_1) - \dot{m}_3 C_{34} (T_4 - T_3) = 0$$

The above energy equations are identically correct for a constant property fluid. For our system, however, where the thermal properties of helium are very temperature sensitive, Wu [4] has suggested that a mean effective specific heat be used in place of the actual specific heat, where these are defined as

$$C_{12} = \frac{h_2 - h_1}{T_2 - T_1}$$

$$C_{34} = \frac{h_4 - h_3}{T_4 - T_3}$$
(6)

In addition, it is generally necessary to divide each heat exchanger section into several (as many as ten) serial sections to minimize the property variation across each section. The above equations are applied at each section.

Magnets

Each magnet is modeled as several sections with an equivalent cross-sectional area, friction factor, and $\frac{L}{D}$ in calculations for momentum change. All flow entering is assumed to exit. Heat transfer from the magnet metal to the helium flowing through it, is based on a heat transfer constant, $\bar{h}$, and a temperature difference ΔT .

The three steady-flow conservation equations are:

$$\text{mass: } \dot{m}_2 - \dot{m}_1 = 0$$

$$\text{momentum: } A \Delta P - f \frac{L}{D} A \rho \frac{V^2}{2} - \dot{m}_1 (V_2 - V_1) = 0 \quad (7)$$

$$\text{energy: } \dot{m}_1 \left(h_1 + \frac{V_1^2}{2} \right) + \bar{h} A \Delta T - \dot{m}_1 \left(h_2 + \frac{V_2^2}{2} \right) = 0$$

The primary variables used in our calculations are enthalpy, h , mass flow rate, m , and pressure P . The above equations written in these variables are:

$$\dot{m}_1 - \dot{m}_2 = 0$$

$$(P_1 - P_2) \frac{A^2}{\dot{m}_1^2} - \frac{f}{16} \frac{L}{D} (\rho_1 + \rho_2) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)^2 - \left(\frac{1}{\rho_2} - \frac{1}{\rho_1} \right) = 0 \quad (8)$$

$$h_2 - h_1 - \frac{\bar{h}}{\dot{m}_1} \left(T_w - \frac{T_1 + T_2}{2} \right) + \frac{1}{2} \left[\left(\frac{\dot{m}_2}{\rho_2 A} \right)^2 - \left(\frac{\dot{m}_1}{\rho_1 A} \right)^2 \right] = 0$$

The density is calculated from the equation-of-state package.

The heat transfer calculations are based on transient heat transfer from a slab of metal with an appropriate heat transfer surface area, A , mass, and heat transfer coefficient, $\bar{h}$, and temperature difference ΔT . The governing equation is

$$MC \frac{dT}{dt} = \bar{h} A \Delta T \quad (9a)$$

using the primary variables this is rewritten as:

$$MC \frac{dT}{dt} = \bar{h} A \left(\frac{T_1 + T_2}{2} - T_w \right) \quad (9b)$$

where T_w represents the slab temperature and $\frac{T_1 + T_2}{2}$ represents the average helium temperature.

Surge Tank

The functions of the surge tank are to fill the system with gaseous helium (GHe) initially and to serve as a mass source or sink to maintain the desired suction pressure of the first-stage compressors. Flow from the surge tank should always be choked due to the large pressure differential. Therefore, assuming ideal gas behavior,

$$\dot{m}_s = \rho_s a_s A_v = p_s A_v \sqrt{\gamma / RT_s} \quad , \quad (10)$$

where the subscript s refers to the surge tank and A_v is the effective orifice area of the valve between the surge tank and the line to the compressors. The tank is modeled as the stem in a Tee junction, where the downstream arm is at a known pressure (i.e., the required suction pressure (0.84 atm) plus pipe friction losses). The effective area in the mass flow rate Eq. (10) will be varied to maintain the prescribed pressure. The calculated mass flow rate (averaged over the timestep) will be compared with the maximum flow rate through the valve. If exceeded, a warning message will be written in the code.

Compressors and Associated Equipment

According to CVI, the mass flow rates through the compressors are controlled such that the suction and discharge pressures are maintained at the values prescribed by the appropriate operating scenario. Modeling, therefore, consists of specifying pressures and discharge temperature.

While the compression, of course, increases the temperature of the gas, each compressor has an oil separator and a water-gas heat exchanger downstream. The flow rate and temperature of the water is controlled such that the net rise in GHE temperature is no more than about one degree. Rather than model this in detail, it is appropriate to treat the entire compressor subsystem as an element in which there is no temperature increase. If criteria were available, it might be worthwhile making checks to ensure that such things as maximum heat transfer rates for the heat exchangers were not being exceeded. The oil separators have a maximum pressure drop of 250 pa (0.036 psi) and maximum flow rates of 1000 g/s which could be checked by the code and warning messages printed if appropriate.

Turbo-Expanders

The turbo-expanders could be modeled in great detail, but for our purposes they can be modeled quite simply. Mass is conserved. The definition of isentropic efficiency (also called adiabatic efficiency), η_s , provides a relationship [5,6] between the actual downstream enthalpy, h_2 , and the pressures:

$$h_2 - h_1 - \eta_s \left(\frac{k}{k-1} \right) (P_1/\rho_1) \left[(P_2/P_1)^{1-(1/k)} - 1 \right] = 0 \quad (11)$$

Here $k = c_p/c_v$ ($= \gamma$ for ideal gas). Pressures are related by $(P_1/\rho_1^k) = (P_2/\rho_2^k)$.

If mass flow exceeds the maximum ($\dot{m}_{\max} = KP_1/\sqrt{T_1}$, where K is an orifice factor) upstream pressure will build up. At about 10^5 Pa (1 atm) overpressure, valves will open and the turbines will cut out. This condition should be flagged by the code.

Heat leaks to the turbines are traditionally handled by adding enthalpy in the downstream pipe segment.

A complication is presented by turbo-expander #2. This has an electrical heater upstream which is used to add heat to the flow if necessary to prevent formation of LHe in the turbo-expander. (This is not a problem for turbo-expander #3 as it can tolerate liquid.) The criterion is that the temperature downstream of turbine #2 not fall below 8°K. A check should be put in for this condition and provision made for addition of the appropriate amount of heat upstream and iteration of the solution within the time-step.

Valves

There are three functional types of valves in the system: open/closed, control, and Joule-Thompson. The first type does not require modeling when closed as they are accounted for by the use of different configurations (see Section V). When opened they are treated like a fully-open control valve. For the other types, we need to know the pressure losses and heat leaks, masses, etc.

In general, to calculate the pressure drop across a valve, we need an empirical factor, such as the valve flow coefficient, C_v . This depends on the valve stem position (i.e., its degree of openness). The maximum flow rate, $\dot{m}_{\max}$, through a valve occurs when the flow is choked with the valve fully opened. (The maximum flow rate is one of the input parameters for the valves.) The fraction of the maximum that the actual flow represents is $f = \dot{m}/\dot{m}_{\max}$. The manufacturers' catalogs provide the relationship between the

flow coefficient for a partially opened valve (C_v) and that for a fully opened one (C_v^o). For a linear valve plug, $C_v = fC_v^o$. The pressure drop (in English units) is [7,8]

$$P_2 - (P_1/2) \left[1 + \sqrt{1 - (4F/P_1^2)} \right] = 0. \quad (12)$$

for $P_2/P_1 \geq r_c = .487$ for helium. (Here P_1, P_2 are the upstream and downstream pressures, $F = 7428 \dot{m}^2 T_1 / C_v^2$, $\dot{m}$ = mass flow rate (lbm/s), T_1 = upstream temperature (R).) The flow through the orifice is traditionally taken as isenthalpic. Any heat leak to the valve is added downstream.

Joule-Thompson valves are modeled as sonic orifices flowing single-phase supercritical gas. (They are individually designed with special trims and flow coefficients.) Consequently, there is no connection between the upstream and downstream pressures, save the requirement that $P_2/P_1 \leq r_c$. However, the downstream pressure and temperature are specified (the J-T valve position is set to give this). Upstream the pressure is related to the mass flow rate by $\dot{m} = KP_1/\sqrt{T_1}$, where K is a calculated factor supplied by the valve manufacturer.

Once the mass flow rate and downstream pressure and temperature are known, the quality and then the downstream enthalpy can be calculated.

Multi-Pipe Junctions

In the cooling system network there are several junctions where a helium stream is divided into two or more streams or two or more streams are merged into one stream. At the present time we neglect any pressure losses or energy losses/gains associated with this type of junction. The governing conservation equations are:

$$\text{mass: } \sum \dot{m}_i \cdot k_i = 0$$

$$\text{momentum: } \sum \frac{\dot{m}_i}{\rho_i A_i} (\dot{m}_i \cdot k_i) = 0 \quad (13)$$

$$\text{energy: } \sum h_i (\dot{m}_i \cdot k_i) = 0$$

It is presently not known if the above equation for momentum conservation is the appropriate equation to apply or if it is better to apply several equations that simply equate pressures. Similarly it may be more correct to apply several equations that equate enthalpy than to apply the energy equation as written. To ascertain the better approach, it would be necessary to have an operational computer code.

Sudden Expansions and Contractions

The piping network contains several sudden area expansions or contractions. These are modeled in the manner used in the RELAP5 computer code [9]. The governing conservation equations are:

$$\text{mass: } \dot{m}_2 - \dot{m}_1 = 0$$

$$\text{momentum: } p_1 - p_2 - \frac{1}{2} \rho_2 \left(1 - \frac{\epsilon}{\epsilon_c \epsilon_T}\right)^2 \left(\frac{\dot{m}_2}{\rho_2 A_2}\right)^2 = 0 \quad (14)$$

$$\text{energy: } h_1 - h_2 = 0$$

The factors ϵ , ϵ_c and ϵ_T are contraction ratios defined as:

$$\epsilon_c = \frac{A_c}{A_T}, \quad \epsilon_T = \frac{A_T}{A_1}, \quad \epsilon = \frac{A_2}{A_1} \quad (15)$$

The factor ϵ_c is empirical since the area at the vena-contracta is a function of the flow properties. For a pure sudden expansion, $\epsilon_T = \epsilon_c = 1$ and $\epsilon > 1$. For a pure sudden contraction, $\epsilon_T = \epsilon < 1$ and $\epsilon_c < 1$.

In the energy equation we have assumed no energy gain from the surroundings. This is easily modified should it be determined at a later date that there is significant energy gain at these pipe junctions.

III. Equation-of-State

The subroutine EOS drives the various NBS Helium subroutines and functions. Currently the calling subroutine provides one of nine pairs of input thermodynamic parameters and an index parameter. EOS converts the units, iterates if necessary, and returns the desired quantity in S. I. units.

Several points must be emphasized about the current package. It was input from listings supplied by NBS-Boulder and FERMILAB. (FERMILAB supplied some non-standard NBS routines which are faster but slightly less accurate than the standard routines.) The numerical coefficients should be checked and some representative curves generated for comparison with published data. The iteration scheme suggested in the EOS subroutine for getting enthalpy or temperature given density and pressure should be checked out. Also, not all of the NBS routines are being used. For example, there are two which calculate the fraction of liquid in a two-phase flow; these may be useful as the load models are refined.

IV. Summary of EDAS: Explicit Differential -
Algebraic System Solver

The differential-algebraic system (DAS), consisting of equations such as Eq. 1 may be written generally as:

$$g(t, T, z) = 0 \quad (16)$$

where z and g are vectors of length M and t is time, $t \geq t_0$. The vector z represents the variables for pressure, mass flow rate, and enthalpy.

Similarly, the DAS, representing heat balances for cryogenic system components, consisting of equations such as Eq. 3 may be generally written as:

$$\dot{T} = f(t, T, z) \quad (17)$$

where T and f are vectors of length N . We are given $T_0 = T(t_0)$ corresponding to the initial time $t = t_0$. Assume that the equation $g(t, T, z) = 0$ always has a unique solution for z , given t and T . Then under very mild conditions on f (that f is not too wildly behaved), this initial value problem has a unique solution

$$T = T(t), \quad z = z(t).$$

To generate a numerical solution, we use a simple explicit Euler scheme, coupled with an iterative method of solving $g = 0$ for z . Given a suitable time increment Δt , the overall scheme is as follows:

1. Set $n = 0$.
2. Solve $g(t_n, T_n, z_n) = 0$ for z_n .
3. Set $T_{n+1} = T_n + \Delta t f(t_n, T_n, z_n)$. (18)
4. Set $t_{n+1} = t_n + \Delta t$.
5. Set $n = n + 1$.
6. If t_n has reached output time, stop; otherwise go to 2.

Aside from the details of step 2 above, the only change made to this algorithm is to alter Δt slightly at step 3 if necessary to hit a desired output time. Continuing the solution after an output time amounts to starting at step 2.

The solution for z_n in step 2 uses a form of Newton's method, generalized to M variables. For this we need values of the $M \times M$ Jacobian matrix.

$$J(t, T, z) = \partial g(t, T, z) / \partial z \quad (19)$$

The method is iterative, and generates successive approximations $z_n(k)$ (the iterates) to z_n ($k = 0, 1, \dots$), based on a linear approximation to g as a function of z . The basic iteration formula for Newton's method is, therefore,

$$g(t_n, T_n, z_{n(k)}) + J(t_n, T_n, z_{n(k)}) \{z_{n(k+1)} - z_{n(k)}\} = 0, \quad (20)$$

which must be solved for $z_{n(k+1)}$. This is a linear system, and it is assumed that the matrix J involved is always nonsingular. However, it is expensive to reevaluate J at every iteration, and a good tradeoff is to evaluate J only periodically and use the matrix J currently available in (20) even though it may be somewhat inaccurate. The result is a modified Newton algorithm:

1. Compute a guess $z_{n(0)}$ to z_n . Set $k = 0$.
2. Decide whether to replace the current J (if any exists) with a new value, and if so compute J .
3. Solve $J \Delta z_k = -g(t_n, T_n, z_{n(k)})$
and set $z_{n(k+1)} = z_{n(k)} + \Delta z_k$. (21)

4. If Δz_k is sufficiently small, stop and accept $z_{n(k+1)}$ as z_n . Otherwise, set $k \leftarrow k+1$ and go to 2. (In any case, stop if k reaches 5.)

The initial guess $z_{n(0)}$ is gotten by extrapolation from data at t_{n-1} and t_{n-2} , when $n \geq 2$, i.e.,

$$z_{n(0)} = z_{n-1} + \frac{t_n - t_{n-1}}{t_{n-1} - t_{n-2}} (z_{n-1} - z_{n-2}) \quad (22)$$

For $n = 0$, it is assumed that an initial guess $z_{0(0)}$ to z_0 is supplied.

For $n = 1$, we use

$$z_{1(0)} = z_0 \quad .$$

The Jacobian J in step 2 of (21) is generally computed by a difference quotient approximation,

$$J_{ij} = \frac{\partial g_j}{\partial z_j} \equiv \frac{g_j(t, T, z + d_j e_j) - g_j(t, T, z)}{d_j} \quad , \quad (23)$$

where e_j is the j^{th} unit vector and d_j is a small increment. If available, a closed-form routine for J can be used instead.

The solution for Δt_k in step 3 of (21) uses the LU method, whereby J is first factored as the product of a lower-triangular and an upper-triangular matrix. The factors are kept until a reevaluation of J is called for, and the linear system is solved by back-substitution operations. The low cost of the latter, compared to that of the evaluation of factorization of J , is the reason that the modified Newton method is more efficient than the true Newton method.

The stopping criterion in step 4 of (21) uses tolerances that are supplied. A scalar relative tolerance RTOL and an absolute tolerance ATOL that is a scalar or an array are supplied, and the basic criterion is

$$||\Delta z_k|| = \frac{1}{M} \sum_{i=1}^M (\Delta z_k^i / EWT^i)^2^{1/2} \leq 1, \quad (23)$$

where

$$EWT^i = RTOL \cdot |z^i| + ATOL^i$$

(superscripts i denoting components). The criterion is modified to $C ||\Delta z_k|| \leq 1$, where C is an estimate of the convergence rate constant, i.e., the limiting value of $||\Delta z_{k+1}|| / ||\Delta z_k||$, formed by means of such ratios. The use of C tends to save nearly one iteration per step.

The decision to reevaluate J is made if convergence cannot be achieved with an out-of-date J , or if 20 time steps have been taken since the last J evaluation.

For small problems, it is feasible to treat J as a full (dense) $M \times M$ matrix. EDAS uses full linear system solvers (LU and backsolve routines). But for large problems, the use of sparsity in J is essential. A sparse version, called SEDAS, also exists. It generates the sparsity structure of J and exploits this in the evaluation of J , the factorization of J , and in the back-solution operation.

V. Input/Output Organization

Generation of files to input the characteristics of the many system components will be a tedious job, but once done should be amenable to minor variations in a simple fashion. The procedure envisioned is as follows. For the operating scenario of interest, identify the appropriate system configurations. A "configuration" is a schematic which includes all

components participating in the process of interest. The configuration would change, for example, if some previously closed valves were opened and others closed according to criteria contained in the operation scenario. For each configuration there would be a corresponding input file to be read by subroutine START. The disadvantage of requiring perhaps three configurations is mitigated by not having to check at every timestep which components have fluid flowing. Moreover, there would be a large amount of overlap between configurations, so generating additional input files from one basic configuration should not be an enormous task.

Each element type has an associated parameter matrix. These matrices have one row for each element of the type and a column for each parameter. An element numbering convention specifies a range of element numbers for each of the 11 element types. These are the "external" element numbers. (To save space, the code assigns internal element numbers.) Initial fluid properties (mass flow rate, pressure, enthalpy) are read in for each node. Equation of state calls are made to get density, temperature, and entropy for each node. The initial temperatures of all the component masses are also read in.

All the data read in is output for reference. At subsequent selected timesteps, the fluid parameters are output only for pre-selected nodes (called "process points"). Similarly, the temperatures of components (e.g., lines, loads) -- "process elements" -- are output. All the results could be output at every timestep if desired, but this should not generally be necessary nor useful.

The information stored in the output file can then be plotted using a post-processor (PSTCRY). Currently this can overlay results for up to five

nodes. For each such set, the user can plot all six fluid variables vs. time or element temperature vs. time. With a small amount of effort the program could be modified to allow plotting of one variable against another.

Currently PSTCRY only does plotting. It would, of course, be possible to modify it to do many other things with the results.

VJ. Simplified Coolant Loop Model

A simplified loop of the helium cryogenic system was modeled. Figure 5 shows the loop consisting of a heat exchanger, valves, magnet and connecting piping. The compressor is modeled as specified boundary conditions.

Calculations were made for four timesteps of 100 sec each. Figure 6 shows the computer code results for each of the eight nodes at times of 100, 200, 300, and 400 seconds. Slab temperatures for the five slabs modeled are given by $x(i)$, $i = 1, 5$ in the output.

VII. Summary

A modeling approach for simulating the thermodynamic behavior of the 8,000 watt helium cryogenic cooling system has been described. The computer program incorporating this model philosophy is nearly complete. However, lack of funding prevents completion of this effort.

The solution approach has been demonstrated on a small test problem. The techniques for solving the coupled system of non-linear algebraic equations and ordinary differential equations have been shown to work efficiently, thus providing a framework for evolving a computer algorithm that could simulate the multi-day cool-down process in several minutes of computer time.

VIII. References

1. Communication from S. Sutton to W. Sterbentz, 5 April 1982.
2. A. C. Hindmarsh, "ODEPACK, A Systemized Collection of ODE Solvers," UCRL-88007, August 1982.
3. F. Kreith, Principles of Heat Transfer, International Textbook Company, Scranton, 1966.
4. K. C. Wu, "Calculation of Off-Design Performance for Helium Refrigerators," Brookhaven National Laboratory, Isabelle Project Technical Note No. 20, 14 July 1980.
5. R. Barron, Cryogenic Systems, McGraw-Hill, New York, 1966.
6. G. J. Van Wylen, Thermodynamics, John Wiley, New York, 1959.
7. CVI, Inc., "Cryogenic Standard Products".
8. J. L. Lyons, Lyons' Valve Designers Handbook, Van Nostrand Reinhold, New York, 1982.
9. V. H. Ransom, et al., "RELAP5/MOD1 Code Manual, Volume 1: System Models and Numerical Methods (Draft)," EG&G Idaho, NUREG/CR-1826, EGG-2070, November 1980.

Figure 1. Representative Schematic of the MFTF-B Cryogenic Cooling System.

(a)

(b)

Figure 2. Schematic of the line geometry. a) representation of a multi-layered pipe. b) representation of the heat flow at the inner pipe wall.

Figure 3. Schematic of the boiling nitrogen heat exchangers.

Figure 4. Representation of the partitioning used for multi-sided heat exchangers.

- Notes: Initial slab temperatures = 250°K
 Node numbers given by circled numbers, i.e. ①
 Boundary conditions used are -
 pressure at ① = 1.013×10^6 Pa
 pressure at ⑧ = 5.065×10^4 Pa
 enthalpy at ① = 1.576×10^6 J/kg
 Density at ① = 1.625 kg/m^3

Figure 5. Schematic of the simplified coolant loop.

at t = 1.0000e+02 edas returned istate = 2
 x(1) = 2.7574e+02 2.3605e+02 2.2736e+02 2.247e+02 2.1155e+02

flow rate	enthalpy	pressure	density
1.09930e+00	1.57600e+05	1.01300e+06	1.62517e+00
1.09930e+00	1.58420e+05	1.00921e+06	1.63803e+00
1.09930e+00	1.14000e+05	1.21106e+06	1.23762e+01
1.09930e+00	1.14277e+05	9.26145e+05	2.05401e+00
1.09930e+00	1.14250e+05	9.17178e+05	2.02243e+00
1.09930e+00	1.14236e+05	8.00046e+05	1.91571e+00
1.09930e+00	9.88231e+05	2.95522e+05	7.56455e+01
1.09930e+00	6.08421e+05	5.05500e+04	1.58429e+01

at t = 2.0000e+02 edas returned istate = 1
 x(1) = 2.8814e+02 2.3416e+02 2.3064e+02 2.1107e+02 1.8567e+02

flow rate	enthalpy	pressure	density
1.09721e+00	1.57600e+05	1.01300e+06	1.62517e+00
1.09721e+00	1.55573e+05	1.00902e+06	1.63803e+00
1.09721e+00	1.16165e+05	1.00587e+06	2.19575e+01
1.09721e+00	1.16000e+05	9.27289e+05	2.02255e+00
1.09721e+00	1.16011e+05	9.13323e+05	1.98531e+00
1.09721e+00	1.15572e+05	8.13199e+05	1.78002e+00
1.09721e+00	9.83405e+05	2.66270e+05	7.21113e+01
1.09721e+00	7.48505e+05	5.06800e+04	1.71064e+01

at t = 3.0000e+02 edas returned istate = 2
 x(1) = 2.9428e+02 2.2726e+02 2.2742e+02 2.0351e+02 1.7025e+02

flow rate	enthalpy	pressure	density
1.09325e+00	1.57600e+05	1.01300e+06	1.62517e+00
1.09325e+00	1.56613e+05	1.00598e+06	1.62407e+00
1.09325e+00	1.17200e+05	1.00880e+06	2.17614e+00
1.09325e+00	1.16855e+05	9.27418e+05	2.00561e+00
1.09325e+00	1.16852e+05	9.13054e+05	1.97549e+00
1.09325e+00	1.16233e+05	8.16780e+05	1.78104e+00
1.09325e+00	8.98331e+05	2.46797e+05	6.94515e+01
1.09325e+00	7.09661e+05	5.06500e+04	1.80425e+01

at t = 4.0000e+02 edas returned istate = 2
 x(1) = 2.9721e+02 2.2579e+02 2.2558e+02 1.9894e+02 1.6962e+02

flow rate	enthalpy	pressure	density
1.08937e+00	1.57600e+05	1.01300e+06	1.62517e+00
1.08937e+00	1.57117e+05	1.00598e+06	1.61886e+00
1.08937e+00	1.17712e+05	1.00978e+06	2.16670e+00
1.08937e+00	1.17271e+05	9.27656e+05	2.00045e+00
1.08937e+00	1.17273e+05	9.13551e+05	1.96537e+00
1.08937e+00	1.16560e+05	8.26025e+05	1.77673e+00
1.08937e+00	8.75210e+05	2.24121e+05	6.76227e+01
1.08937e+00	6.84140e+05	5.05500e+04	1.87144e+01

Figure 6. Output from the simplified coolant loop model giving state point information at the eight nodal points.