Golder Associates

CONSULTING MINING AND GEOTECHNICAL ENGINEERS

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VOLUME II - APPENDICES
REPORT TO
LAWRENCE LIVERMORE LABORATORY
ON THE
DERIVATION OF PARAMETERS NECESSARY
FOR THE EVALUATION OF PERFORMANCE OF SITES
FOR DEEP GEOLOGICAL REPOSITORIES
WITH PARTICULAR REFERENCE TO BEDDED SALT
LIVERMORE, CALIFORNIA

PREPARED FOR THE UNIVERSITY OF CALIFORNIA
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APPENDIX I

Specific Rock Mechanics Tests

I-1.0 SPECIFIC ROCK MECHANICS TESTS

The following sections describe in detail test methods referred to in Section 3 of the main text. It is considered that suitable reference material for the methods and techniques described is not readily available, but the length of the descriptions prevents inclusion of these sections in the main text.

I-1.0 POINT LOAD TEST

I-1.1 Introduction

The point load test has been developed as a means of rock strength testing in the field. It may be employed to test drill core diametrically or axially or to test irregular "lump" samples (Figure I-1). A strength index $Is(50)$, may be derived for use as a rock strength classification (Broch and Franklin 1972; Bieniawski 1974).

I-1.2 Point Load Tester

The point load testing frame (Figure I-2) normally consists of a hydraulic ram that applies load to the sample placed between two conical platens. The load (P) is increased steadily until the sample fails in tension. In certain cases a gauge is provided with a dial that is calibrated to read load directly. More commonly, however, a pressure gauge records the hydraulic pressure applied to the ram. If this is the case, the load applied to the platens must be calculated by multiplying the indicated pressure by the effective ram area or, preferably, by a valid calibration factor.

I-1.3 Procedure

Before using the point load tester the pressure gauge should be checked for accuracy and the platen tips should be checked for alignment.

I-1.3.1 Sample Selection

1) In core testing, a minimum of ten tests should be run for each rock unit. At least twice that number of tests should be carried out for irregular lump samples.

TYPES OF POINT LOAD TEST

Figure I - 1

DIAMETRAL
 $L \geq 1.4 D$

AXIAL
 $D = (1.1 \pm 0.5) d$

IRREGULAR
 $1.4 d > D > d$

POINT LOAD STRENGTH APPARATUS

Figure I - 2

ii) Guidelines for sample size:

- Rock core tested diametrically - test sample should have a length:diameter ratio $>1.5:1.0$
- Rock core axial test - test sample should have a length:diameter ratio of $1.1:1 \pm 0.05$
- Lump sample - test sample should have a typical diameter of approximately 50 mm and a ratio of longest to shortest diameter between 1.0:1 and 1.4:1.

I-1.3.2 Test Procedure

i) After the sample is placed between the platens the pump is operated to bring the platens into contact with the rock and the distance (D) between platens is recorded from the attached scale. The load, P, is increased at a continuous and moderate rate until sample failure. P at time of failure is recorded.

For a valid test result, the sample should fail along a clean, tensile fracture connecting one platen tip to the other.

For softer samples, where the platens will penetrate the sample to some degree before failure, it may be necessary to check D during the test and obtain its value immediately prior to failure.

ii) For standard classification, a "typical" moisture content should be measured for the test samples. Natural, air-dry samples give strengths 10 to 20 percent higher than those of saturated rock.

iii) Anisotropic rock, such as bedded rock, should be tested perpendicular and parallel to planes of weakness. The ratio of the two strengths is a measure of the degree of anisotropy.

I-1.4 Calculations and Recording of Results

i) The point load strength, I_s , is calculated accordingly to the formulae:

$$\text{For Diametral Tests} \quad I_s = \frac{P}{D^2}$$

$$\text{For Axial or "Lump" Tests} \quad I_s = \frac{P}{Dd}$$

where: P = the load required to split the sample,
 D = the distance between the platens, and
 d = the lateral dimension of the sample.

ii) The point load strength index ($I_s[50]$ or $I_s[1.97]$) for a 50 mm diameter core for each sample is estimated using the formulae:

$$I_s(50) = K I_s \quad \text{or} \quad I_s(1.97) = K I_s$$

$$\text{where:} \quad K = \frac{35}{85-D} \quad \text{where:} \quad K = \frac{1.38}{3.35-D}$$

and D is measured in mm and D is measured in inches

iii) To estimate the compressive strength, an approximate conversion of point load test results to uniaxial compression strength is:

$$\text{Uniaxial Compressive Strength} = 24 \times I_s(50)$$

This equation must be verified or modified prior to its use in this context, by comparison with uniaxial compressive strength values obtained directly through uniaxial or unconfined compression tests.

iv) If required, and if the samples have been tested in directions perpendicular and parallel to the weakness planes, a measure of the effects of such plans may be calculated as a strength anisotropy index, I_a .

This is calculated according to the formula:

$$I_a = \frac{Is(50), \text{ core tested in the strongest direction}}{Is(50), \text{ core tested in the weakest direction}}$$

I-2.0 BRAZILIAN TEST

I-2.1 Introduction

The Brazilian test is essentially a line-compressive loading test designed to produce indirect tensile failure of a cylindrical rock specimen. This type of failure is possible due to the approximately uniform tensile stress distribution induced by the line loading. It was first proposed by Carneiro (1947) and independently proposed by Akazawa (1953).

I-2.2 Test Principles and Procedure

If a disc of rock (see Figure I-3a) is subjected to compressive load at the two opposite points across a diameter, the stresses induced at the center of the disc specimen will be:

$$\text{Maximum principal stress: } \sigma_1 = \frac{6F}{\pi Dt} \quad [I-1]$$

$$\text{Minimum principal stress: } \sigma_3 = -\frac{2F}{\pi Dt} \quad [I-2]$$

where: F = the applied load,

D & t = the disc diameter and thickness, respectively.

The compressive stress, σ_1 , that develops across the diametral plane actually varies from above value, $\sigma_1 = \frac{6F}{\pi Dt}$, at the center to $\sigma_1 = \infty$ at the loading contact points. This compressive stress distribution induces high shear stresses and crushing at the loading points.

To minimize these effects, the loading is now applied in a line rather than point fashion, on a cylindrical specimen of length, L (Figure I-3b), with the line loading applied along AB and CD. In an attempt to diminish local crushing, Hondros (1959) distributed the line load over the area of a strip of width $D/12$ (see Figure I-3c). This procedure leaves the uniform tensile stress across the diametral plane unaffected, while its value is now expressed as:

BRAZILIAN TEST

Figure I-3

(a)

Stresses induced at the center of a disc specimen subjected to point loading across a diameter.

(b)

Indirect tensile test with line loading.

(c)

Indirect tensile test with load distributed on strips.

$$\sigma_3 = - \frac{2F}{\pi DL} \quad [I-3]$$

Although some horizontal compressive stresses still develop in the proximity of the loaded area, the distribution of loading helps ensure that failure should initiate in the interior of the specimen.

I-2.3 Calculation of Tensile Strength

If F_c is the applied line load at failure, the tensile strength, σ_t , is given by equation [I-3] as:

$$\sigma_t = - \frac{2 F_c}{\pi DL}$$

When above strip loading induces unduly high tangential stresses, an alternative approach (Paone and Bruce 1963) may be developed through empirical equations that take into account the local crushing action arising from line loading. However, this procedure is not yet widely accepted.

I-3.0 THERMAL CONDUCTIVITY MEASUREMENT

I-3.1 The Needle-Probe Versus the Divided-Bar

The major difficulties with the needle-probe are the very high initial temperature gradients close to the probe and the resulting effects in partially saturated materials (DeVries and Peck 1958), the high heat input required to get good temperature sensitivity, the gravitational instability of the sample, and the relatively low accuracy of the transient method. These must be offset against the ease of preparation of the sample, the high portability of the equipment involved, and the absolute nature of the measurement. To a large extent the absolute nature of the measurement is a misnomer, since calibration against a standard is necessary to decide whether heat losses or high contact resistances are causing erroneous results.

The divided-bar can be operated with a smaller temperature gradient across the sample, permits higher accuracy measurements, and can be made gravitationally stable by placing the hot plate at the top. However, it suffers from the problems of tedious sample preparation, its larger size, and the question of absolute conductivity standards.

I-3.2 Measurement Using the Divided-Bar

Of the several types of divided-bar which exist, the speediest uses constant temperature reservoirs at top and bottom. It is modified from a type described by Beck (1956) to eliminate the guard ring without increasing heat losses and thus simplify manufacture. Two cylindrical brass reservoirs are connected in parallel to, and maintained at two different but constant temperatures by, two Colson-type NB constant-

temperature baths circulating fluid through them. The hotter reservoir is placed at the top to avoid convection of moisture in the porous samples. Between the reservoirs are two stacks consisting of an 1/8 inch thick disc of Optosil-fused silica sandwiched and cemented between two brass discs and with a 1/16 inch disc of perspex cemented between the brass disc and the brass reservoir. This silica disc dampens out fluctuations of reservoir temperature with periods of 10 seconds or less. Holes drilled radially into the centers of the brass discs serve as wells for the temperature sensing devices, in this case Fenwall 1 percent glass probe thermistors with a resistance of 2 Kiloohms at 25°C. The complete assembly is mounted in a hydraulic jack to enable axial pressures to be maintained on the sample and thus reduce the contact resistance between the stack and the sample. In operation, the sample is inserted between the two stacks, the pressure is jacked up to between 10 and 30 psi, the assembly is allowed to reach a thermal steady state, and the temperature differences are compared across the fused silica discs of known conductivity and the sample disc whose conductivity is unknown. To avoid errors due to evaporation and/or sublimation of the sample moisture, the sample is wrapped in a double layer of "saran wrap" and, to avoid radial losses from the bar, the stack and sample are surrounded by a split styrofoam cylinder as insulation. A fifth thermistor taped to the outside of the insulation was used to monitor the ambient temperature of the bar.

I-4.0 SHEAR STRENGTH TESTING OF DISCONTINUITIES

I-4.1 Test Method

The steps involved in testing a sample in the portable shear machine are described below (see Figure 3.3-9 in Section 3.3.8):

i) A sample containing the discontinuity to be tested is trimmed to a size which will fit into the mould. The two halves are wired together in order to prevent movement along the discontinuity and the sample is then cast in plaster or concrete. Care must be taken to position the discontinuity accurately so that it lies in the shear plane of the machine. A bed of clean gravel, placed in the bottom of the first mould, is sometimes helpful in supporting the specimen during setting up and, provided that a wet mix is used, the gravel can be left in place so that it becomes part of the casting.

The sample is aligned in a mould and the lower half is cast in concrete, plaster, or similar material. When the lower half is set, the upper half of the mould is fitted and the entire mould with its sample is turned upside-down for the pouring of the second half of the casting.

ii) Once the castings have set, the mould is stripped and the specimen is transferred into the shear machine. The upper shear box is set in position, and a small normal load is applied in order to prevent movement of the specimen. The wires binding the two halves of the specimen together are then cut and the shear load cable is placed in position.

iii) The specimen is now ready for testing, and the normal load is increased to the value chosen for the test. This normal load is main-

tained constant while the shear load is increased. A note is kept of displacements during the application of the shear load.

iv) Once the peak strength has been exceeded, usually after a shear displacement of a few millimeters, the displacement is allowed to continue and it will be found that a lower shear load is required in order to sustain movement.

v) The machine illustrated is limited to a displacement of approximately 1 inch (2.5 cm) and, in order to determine the residual shear strength, a displacement in excess of this value is normally required. This can only be achieved if the normal load is released and the upper half of the specimen is moved back to its starting position. In another version of the machine, manufactured by Robertson Research Ltd., a second jack acting in opposition to the shear load jack has been added to allow for shear reversal under constant normal load.

Which of these systems is more representative of the shearing process in the rock mass is uncertain since, in one case, the detrital material is disturbed when the normal load is released while, in the other, the direction of rolling of the particles is reversed. It is possible that different rock structures behave differently under these conditions. The authors tend to prefer the single jack system, since they feel that it is important that the direction of shearing under loads should be kept constant.

vi) In this, as in most shear machines, the loads applied to the specimen are measured, and these have to be divided by the surface area of

the discontinuity surface in order to obtain normal and shear stresses. The initial area should be determined by direct measurement, and the reduction in surface area with displacement should be calculated.

vii) The shear strength of rock is not generally sensitive to the loading rate and no difficulty should arise if the loads are applied at a rate, which will permit measurements of loads and displacements to be carried out at regular intervals. A typical test would take between 15 and 30 minutes.

The specimen size which can be accommodated in the portable shear machine illustrated is limited to about 4 inches by 4 inches (10 cm by 10 cm) and this means that it is very difficult to test joints with surface roughness which is representative of the in situ conditions. Consequently, it is recommended that the use of this machine should be restricted to the measurement of the basic friction angle, ϕ . This can be done by testing sawn surfaces or by testing field samples and by subtracting the average roughness angle, i , measured on the specimen surface before testing, from the measured angle, $(\phi + i)$, as determined in the test.

I-5.0 CSIRO HOLLOW INCLUSION STRESS CELL

The following pages are taken from "Rock Instruments", Australia, technical publication on the use of the Commonwealth Scientific and Industrial Research Organization's (CSIRO) Hollow Inclusion Cell.

CSIRO HOLLOW INCLUSION STRESS CELL

The HI Stress Cell before grout extrusion

The Hollow Inclusion Cell provides a method of determining the three-dimensional stress state in rock, based on the overcoring principle. Extensive field use over recent years has proven it an accurate, reliable device with users reporting unsurpassed success rates.

The C.S.I.R.O. (Commonwealth Scientific and Industrial Research Organization) developed and patented* the Hollow Inclusion Cell after laboratory testing and field trials throughout Australia. It has subsequently been used for determining rock stresses in many underground operations, associated with mine design studies and in near-surface boreholes for geotectonic studies.

Construction

Basically, the HI Cell is a fully encapsulated array of 9 strain gauges, which is grouted in an EX diamond drill hole and monitored while being overcored. Strain response registered during this process is analysed to determine the principal stresses and their orientation. The HI Cell consists of three strain gauge rosettes, each of three gauges, precisely oriented at 120° along the circumference of an epoxy pipe. Full encapsulation and epoxy grouting means the instrument is unaffected by moisture. Each gauge is 10mm long, minimising the influence of rock grain size. Gauge orientation produces a statistically powerful result of high sensitivity when strain response is analysed to determine the stress tensor.

Installation

Grouting in the EX hole is by filling the epoxy tube with epoxy cement, then extruding the cement by displacement with a piston at target depth. A trip wire within the Cell registers completion of extrusion. Piston activation is either by forcing a protruding rod against the end of the borehole (depicted at left), or by manually pulling the piston into the epoxy shell. Multiple seals confine grout flow around the Cell. The strain gauges are thus fully bonded 1.5mm from the borehole wall; allowance for this geometry is made in the data reduction program.

Overcoring

Monitoring of strain response (and the trip-wire) is via a twelve-core cable from the HI Cell through the overcoring rods and out of the diamond drill through a modified water swivel. A 9-channel switchbox and quarter bridge strain indicator are connected to the cable and strain response is monitored during overcoring. When plotted graphically, this response is invaluable in indicating overcore conditions such as the onset of core cracking or inelastic response.

Advantages of the HI Cell

- Determines the complete stress tensor from one measurement.
- Multiple gauges provide inbuilt checks of results.
- Fully encapsulated strain gauges and leadwire connections.
- Simple grouting process.
- Overcoring monitored throughout drilling.
- Mechanical properties of actual rock overcore used in analysis.
- High success rate in field use.
- Result is accurate and reliable.

Technical Specifications

Strain Gauges: Gauge length: 10mm
 Gauge resistance: 120 ohms
 Gauge factor: 2.0
 Circuit: three-wire quarter bridge

Borehole Size: 38mm diameter (EX) for HI Cell
 150mm diameter for overcore (100mm diameter satisfactory in good coring conditions).

Operating Range: Temperature: 15°C – 45°C
 (Present field experience) Stress range: Up to 100MPa
 Accuracy: $\frac{\text{Standard Error of Stress}}{E} = 10\text{ppm}$

Accessories Available

1. **Installation Tool:** Incorporates a robust dual-caster trolley used to clean the EX borehole then place the HI Cell at target depth. It includes a mercury switch attachment for accurate orientation. The castors are adjustable to accommodate varying borehole diameter.

2. **Installation Rods:** Stainless steel rods with positive locking connections which attach to the installation tool. Standard rod length is 1.8m but may be varied on special order.

3. **Biaxial Pressure Chamber:** Used for determination of the deformation modulus and Poisson's Ratio of the overcore surrounding the Cell. It incorporates a steel pressure chamber and a neoprene sleeve into which the core slides very simply. The associated hydraulic hand pump and 3000psi (or metric equivalent) pressure gauge and hoses are also available. Specify core diameter when ordering.

4. **Data Reduction Program:** Analyses the stress tensor using the overcore strain data and biaxial pressurization results, giving the stress state in both principal form and oriented along coordinate axes. Associated statistical data accompanies the output. The program also accommodates rock properties derived from laboratory compression tests.

5. **Field Manual:** For assistance to field technicians conducting stress determination with the HI Cell. It covers site selection, equipment, installation, overcoring, biaxial testing and data input to the analysis program.

Ordering Requirements — HI Cell

1. **Read-out Cable:** Available shielded or unshielded at any length. At least 4m of cable should be allowed beyond the drill stem water swivel. Stock HI Cells are available with 15m of shielded cable.

2. **Piston Activation:** Push rod (standard)
 Pull wire (special order)

Special versions of the HI Cell are available on request e.g. with inbuilt dummy temperature compensation strain gauge.

Manufactured and Distributed by:

ROCK INSTRUMENTS

P.O. Box 246, East Caulfield, Melbourne, Victoria, 3146.
 AUSTRALIA Telephone: (03) 211 4882

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APPENDIX II

Drilling Investigation Techniques and Equipment

II-1.0 DRILLING INVESTIGATION TECHNIQUES AND EQUIPMENTII-1.1 Introduction

A variety of borehole techniques and drilling equipment may be used to gain further data useful in the engineering assessment of a particular site. They include methods of recovering the maximum amount of intact core, methods of absolute orientation of that core, and down-hole logging techniques as described below:

- Triple tube core barrel
- Craelius core orientator
- Christensen-Hugel orientation
- Sperry Sun Single Shot camera
- Tro-Pari Surveying Instrument.

II-2.0 DRILLING FOR STRUCTURAL PURPOSES

The requirements for structural drilling are somewhat more stringent than those for normal exploratory drilling. The aim is not only to recover a sample of the rock material but to recover a sample of the rock mass complete with discontinuities. This means that each piece of core must be recovered in its correct order.

It is convenient for summary logging and essential for detailed logging to lay out the core in a V-shaped guide rail. The core can be fitted together and depths marked at regular intervals. The time taken to do this will be amply repaid by speed of mapping.

Positive incentives for skill and motivation of the drillers are required and, for this reason, footage bonuses must be avoided.

The absolute cost of this type of drilling will be somewhat higher than that of exploration drilling. However, the cost must be viewed in the light of the additional information obtained.

II-2.1 Triple Tube Core Barrel

On all engineering/geotechnical drilling programs, maximum intact recovery of core is essential. It is often the soft or broken zones which will play a major role in stability problems, and it is these zones that are most difficult to recover. The Longyear Q-3 series triple tube wireline core barrel has been found, after extensive field proving, to be the most effective and reliable coring method.

Regular Q series wireline barrels can easily be converted to triple tube core barrels, and all necessary parts can be ordered in either 5- or 10- foot sizes. Instead of a simple double tube set up, in which the core is usually hammered from the core tube, a third tube is added, split

lengthwise, and nested in the inner tube. Its inside surface is plated with hard, low-friction chrome for smooth core entry. Coring with the triple-tube barrel proceeds in the usual manner, and the core laden inner tube assembly is retrieved through the drill string. The core lifter and spring case is removed and the split tube is hydraulically pumped from the inner tube. One-half of the split tube is carefully lifted off so the core can be inspected in a virtually undisturbed condition prior to transfer to the core box.

The combination of a step-type lifter case and face discharge diamond coring bit helps eliminate core erosion, particularly when drilling in soft conditions. A special brass piston with an O-ring seal is used to extrude the split tube, incorporating a pump-out group designed to utilize the rig's grout pump.

The split tubes are manufactured in paired sets, and matched numbers should always be used together. The walls of the tubes are fairly thin and, as they can easily be damaged through negligence, spare sets of splits should always be available and securely stored. Damaged splits are useful for carrying core which can then be transferred from split tube to core box with little or no disturbance.

II-2.2 Drilling Operation

Problems encountered in the drilling of the hole frequently tell more about the ground conditions than can be obtained from logging of the recovered core. Thus, a geotechnical logging method should contain the following information in addition to a record of geological features.

A summary of the drilling events, based on verbal conversation with the drillers, should be noted. This summary enables a reconstruction

of events that allows the engineer to see exactly what occurred in the drilling of the hole and also may aid in the interpretation of core condition. Items such as the use of bentonitic mud, the rods becoming stuck or hard to turn, requirement for replacement of core catcher or bit (including specific type) deviations in usual drilling techniques, etc., all add to this documentation and serve to produce a log that can be used in the interpretation of the ground conditions.

This information should be recorded on a driller's sheet. If and when zones of water loss are encountered, the specific depth should be noted and shown on the log; a general statement should be made if none are encountered. Driller's data sheets, with water levels, can be used on each core hole to provide additional information.

Rubblized zones within the hole that cave should be given special emphasis and noted on the log. If the caving ground is brought to the surface in the barrel, a description of it should also be noted (sand size, gravel size, etc.).

Areas that require grouting in order to continue coring should be noted, and the log should indicate the specific zones grouted, how much grout was used, and the hardness and description of the grout core.

II-3.0 CORE ORIENTATION

It is apparent that the dip and dip direction of discontinuities are most important in stability evaluations and in defining rock mass geometries. Consequently, however successful a drilling program has been in terms of core recovery, the most valuable information of all will have been lost if no effort has been made to orient the core.

II-3.1 Relative Orientation

One approach to this problem is to use inclined boreholes to check or to deduce the orientation of structural features. For example, if surface mapping suggests a strong concentration of planes dipping at 30 degrees in a dip direction of 130 degrees, a hole drilled in the direction of the normal to these planes (i.e., dipping at 60 degrees in a dip direction of 310 degrees) will intersect these planes at right angles, and the accuracy of the surface mapping prediction can be checked. This approach is useful for checking the dip and dip direction of critical planes.

Alternatively, if two or more non-parallel boreholes have been drilled in a rock mass in which there are recognizable marker horizons, the orientation and inclination of these horizons can be deduced using graphical techniques.

Where no recognizable marker horizons are present, as is frequently the case in metalliferous ore bodies, this technique is of little value.

II-3.2 Absolute Orientation

A second approach is to attempt to orient the core itself and, while the techniques available abound with practical difficulties which are

the despair of many drillers, these methods do provide some of the best results currently obtainable. In fact, the greatest possible service which could be rendered to the rock engineer by the manufacturers of drilling equipment would be the production of simple core orientation systems.

II-3.3 Craelius Core Orientator

One of the best core orientation devices is the Craelius Core Orientator, manufactured by Atlas Copco. It consists of a metal holder of the same diameter as the core which contains six movable pins. It is clamped in the front of the core barrel and lowered into the hole. Its orientation is fixed by the orientation of the rigid core-barrel or by a simple marker within the device. When the six pins come into contact with the end of the hole (where the core will usually have broken off or parted on an inclined surface) these pins take up the profile of the core stub. The pins are locked in place, and the device is released to move up the core barrel as drilling proceeds. When the core is removed, the core orientation tool is matched to the upper end of the core and the first piece of core is oriented with respect to the known orientation of the device at the time of the fixing of the pins. Provided that good core recovery has been achieved, it should then be possible to reconstruct the core, which is then oriented with respect to the first piece.

A technique currently being tested as a means of eliminating shortened drilling runs and drilling difficulties is to lower the craelius on the wireline overshot, take the impression of the core stub on the pins, and then remove the craelius prior to drilling the run. In this way, quick check can be made to ensure the device has operated correctly and the drilled run can then be of the standard 5 or 10 ft. This is most useful

for taking further orientations, since the drill rods may be broken at the same place each run without pulling the diamond bit back from the end of the hole too far.

II-3.4 Christensen-Hugel Orientation

An alternative method is used with the Christensen-Hugel core barrel. Three tungsten carbide points scribe the core continuously while coring. The reference marks are then oriented absolutely by a magnetic or gyroscopic survey instrument mounted in the core barrel. This technique is most successful when the ground being drilled is hard and non-magnetic.

II-4.0 HOLE SURVEYING

An integral part of any oriented core logging program is hole surveying. It is essential that the plunge and trend of the hole are known at enough intervals to interpolate the exact deviation at any depth. Golder Associates have developed a computer program which will handle this deviation interpolation. The most commonly used survey tools are the single- or multi-shot camera magnetic surveying instruments (Eastman and Sperry Sun are most commonly used) and the Tro-Pari Surveying Instrument.

II-4.1 Sperry Sun Single Shot

As an example of the photographic methods available, a description of the Sperry Sun single shot is given. This system can conveniently be used for hole surveying when the Craelius core orientator is being used for core orientation.

The single-shot is a magnetic survey instrument which is used to obtain records, on film, of the inclination and direction of inclination (or plunge and trend) at various depths in a borehole. The presence of steel will obviously affect the compass, and non-magnetic rods are used to hold the instrument below all casing and drill rods. The inclination unit is a form of inverted plum bob which is combined with the magnetic compass into a single unit available for a number of angles of plunge. The borehole orientation indicated by the compass-angle unit is recorded on a film disc when a preset timer illuminates two small lamps in the body of the instrument. Special loading and developing tanks permit handling of the photographic discs without the necessity of a darkroom.

It has been found from extensive field experimentation that hole surveys should be taken every 50 to 100 m and more frequently when the

deviation exceeds 1 degree in 30 m. The delay caused in taking the survey, usually about 30 minutes per shot, is easily compensated by the additional information available from a well-surveyed hole.

The single-shot camera must be retrieved and reloaded after every shot. This feature is the only major difference compared to the multi-shot camera which is preloaded with four film discs and preset at different time intervals to allow hole surveyings at several depths without removal of the instrument.

II-4.2 Tro-Pari Surveying Instrument

The Tro-Pari Borehole Surveying Instrument comprises a unit mounted in gimbals and is provided with a clockwork mechanism to clamp a compass to indicate trend and simultaneously to clamp the unit in its plumb position to indicate plunge.

By means of a timing ring suitably calibrated in 5 minute divisions, the unit may set to lock after a lapse of time sufficient to allow the placing of the instrument at the desired point in the drill hole where readings are to be taken. The maximum time lapse obtainable with the instrument designed for use in E- or A-size holes is 1 hour 30 minutes, which is considered to be ample time to lower to the greatest depths that which may be drilled with this size of equipment. A larger adaptor has been developed for larger diameter holes such as B and N sizes and a correspondingly greater time lapse has been provided for.

II-5.0 GEOTECHNICAL LOGGING

The objective of a geotechnical log is to record information from the drill hole that reflects the condition of the ground penetrated. This information comes both from the drilling process and the recovered core.

The data recorded should be relevant to the scope of the project; the logging detail and format for an underground pilot hole should reflect the problems of underground excavation and support and data obtained from holes drilled for initial pit slope investigations should reflect the larger scale of an open pit operation.

The information should also be directly useful in the interpretation of either the mechanical or hydrological properties of the rock mass. Geologic information (that which enables interpretation of the spatial variability of the several parameters) should be recorded and handled in the manner preferred by the responsible geologist(s). There is an overlap in that alteration, for instance, that will affect the strength of the rock materials. Probably the most important aspect of the entire geological/geotechnical investigation is the definition of the fault population, as faults frequently control the distribution of ore materials, water migration, and strength properties. Typically, geotechnical problem areas or failure situations in rock materials will directly relate to the occurrence of faults.

II-5.1 Core Logging

There is information obtainable from the drill core that is useful in defining mechanical and hydrologic parameters that is not normally recorded in the geologic logging. This information, which is most effi-

ciently obtained at the time the core is logged geologically, should be recorded on the logging form.

The thickness and characteristics of fault zones should be described with sampling of the thicker zones (>15 cm) for laboratory testing.

The presence of gypsum, anhydrite, calcite, chlorite, etc., as joint infillings and secondary mineralization should be noted as these influence the permeability of the rock mass and the frictional characteristics of the joint surfaces.

In addition to the above, particular requirements may suggest that such items as RQD, fracture frequency, angles to core axis, hardness, etc., should be recorded.

II-5.2 Core Photography

Colour photographs of drill core provide a permanent record of the core condition upon recovery and hence can be an invaluable aid to the interpretation of ground conditions and therefore to excavation design. The photographs which show detail that cannot be easily recorded on drill logs, can be filed for ready reference by both geologists and engineers.

A 28 mm, wide-angle lens and a high-quality camera (e.g., Cannon, Pentax, etc.) has proven to be most convenient when photographing core. A suitable frame can be constructed of dimensions to suit the size of the core boxes and the width to height ratio of the 35 mm transparency. A title block should be included to indicate project number and name, drill hole number, and footage interval. A meter scale should be shown, with an indication of "top of hole" direction. The core should be photographed dry.

Subdued daylight provides the best lighting for core photography and direct sunlight should be avoided. Flash or photofloods may be needed in adverse natural lighting conditions, in which case different film requirements must be observed with regard to flash or floodlamp type. Regardless of the lighting system used, consistency is of the utmost importance for interpretation of the core slides, as similar colours will be recorded differently in varying lighting conditions.

The following sections describe different mapping techniques and logging forms used for various types of project covered in the scope of this appendix. The parameters investigated are all described in detail in the main text. Codings used are free form and compatible with all Golder Associates data reduction programs. Listings of coding notations must be retained with all data listings to ensure unambiguous interpretation.

1) Oriented Core Logging

Where the core can be successfully oriented, valuable information can be obtained on the spatial relationships of the natural discontinuities in the rock. Because of the complexity of reducing the resulting orientation for the orientation of the drill hole, it is recommended that the oriented data is recorded in a format suitable for direct key punching. This will permit data reduction and subsequent analysis, usually by stereographic projection.

The Structural Mapping Coding Form used by Golder Associates is shown in the accompanying figure and is compatible with the Golder Computer program library. The format permits recording of details regarding the orientation and nature of the discontinuities. The data are recorded as follows:

- Job Number:
This is unique to the project.
- Data Unit:
Up to 7 characters to detail, for example, borehole number DD77803.
- Location:
Recorded to the nearest 0.1 m, and right-justified. (The decimal point is not recorded.)

All other columns are filled with details of parameters described in the main text.

ORIENTED CORE LOGGING FORMAT

STRUCTURAL MAPPING CODING FORM

[illegible]

EXAMPLE ONLY

ii) Summary Logging

Summary engineering logging can provide fairly detailed engineering data from drill core at the same time as geological logging is performed, without an appreciable decrease in the rate of logging. Performed on exploration and development core, this type of logging can provide much of the information necessary for preliminary slope design studies and can be used as a basis for planning detailed geotechnical programs.

The summary engineering data can be recorded on a structural summary log (see following figure).

The principal features to be recorded in summary logging are as follows:

- Discontinuity Orientations:
The angle between discontinuity or bedding planes and the core axis, recorded by fracture type.
- Rock Properties:
Strength (estimated according to Table 2.5-2)
Weathering (see Section 2.5.3.3)
Type (see Section 2.5.3.1)
- Gouge Zones, Broken Core, Core Loss Zones:
Depths concerned shaded with appropriate symbol.
Comments on nature of gouge.
Percent core loss shaded.
- Fracture Log:
Count number of natural fractures in each meter.

SUMMARY LOGGING FORMAT

SUMMARY STRUCTURAL LOG

PAGE ____ OF ____

MINE _____

SCALE: 1:100

LOGGED BY _____

CORE SIZE _____

HOLE _____

DATE _____

[illegible]

NOTE: Capacity 25 meters on standard Metric Sheet (A4)

iii) Mining Engineering Logging

A copy of the recommended engineering logging format follows. This format can be used either independently, or combined with an existing geological logging format.

Suggestions for recording data by columns follow. It is recommended that all histogram columns be shaded from the left. In this way, the poorer quality rock is highlighted by larger darkened areas.

In order, the columns are:

- Depth:
Each line usually represents a 2 m increment; this may be changed for particular projects. The beginning of each core run can be indicated in this column by a horizontal line and a figure showing the down-hole depth.
- Percent Core Loss:
The percent core loss in each run is plotted graphically for better definition of possible weak zones.
- Fractures:
Where detailed core orientation is being recorded, typical values only may be noted, with the detailed logging coded on a separate form. The fracture column is broken into three categories: type, infilling and inclination (the angle between the plane of the discontinuity and the core axis).

- Description and Remarks:
This column records the changes in lithology as observed from the core. General remarks are made about any significantly weak zones or other noticeable features.
- Graphic Log:
The local graphic symbols are used, or should be specified in advance.
- Breccia/Gouge and Broken Core:
Zones of broken core or fault material are recorded graphically.
- Hardness Code:
Core strength is recorded graphically, and the histogram is filled in from the left.
- Rock Quality Designation:
All fractures, natural and mechanical, are considered in the calculation, and core lengths are measured from the centre of the fractures along the core axis.
- Fracture Frequency:
The fracture frequency is usually calculated over 3 m increments and recorded as a histogram shaded from the left; the length increment may be changed. Fracture frequency counts and broken core/gouge zones are mutually exclusive records on the log.
- Blank Column:
The blank column is used for recording additional data, sample numbers, etc.
- Permeability Column:
This column can be used to record the daily fluid level recorded in the hole, measured permeabilities, and details of any piezometer installations.

MINING ENGINEERING LOGGING FORMAT

[illegible]

iv) Geological Engineering Logging

This logging technique is often performed immediately the core is recovered and, consequently, usually requires full-time rig supervision. It can be carried out consecutively with geological logging and oriented core logging.

Most of the attached logging form is self explanatory, the details of parameters recorded being included as the main text. Other information is detailed below.

- Type of Drilling:
Usually "rotary core" followed by the qualifying drilling medium "mudflush" or "waterflush".
- Reference Elevation:
Definition of the datum from which down-hole depths are measured, e.g., drill floor.
- Drilling Progress:
A line to indicate the end of run footage, with the down-hole depth recorded above the line. Data and shift (a.m. or p.m.) should also be recorded below the footage line at shift changes.
- Rate of Advance:
Time per unit length of hole drilled. Filled in as a histogram and shaded to highlight runs of slow drilling.
- RQD and Core Recovery:
Plotted graphically for better definition of possible weak zones. These are calculated for the core run interval shown.
- Depth:
The short lines usually represent 0.2 m increments; each 2 m line should be labelled. Down-hole depths at major rock type contacts should be included as accurately as possible, with a line drawn across the rest of the log.
- Reduced Level:
After the hole has been surveyed, this column can be completed with details of elevations, rather than down-hole depths, for major contacts.
- Water Level:
A line is drawn to indicate shift-change hole depth and a record is made of the fluid level in the hole, measured from the reference elevation.

- Test Results:
Details of sample location, sample type, Atterberg test results, penetration tests, etc., are included in this column.
- Bed/Fol:
Representative inclinations of bedding or foliation with respect to the core axis are recorded.
- Instrumentation and Legend:
Both columns are for graphic representation of data. The former is used to represent in-hole installations (e.g., piezometers, with details of grout, fill, cave, etc.). The latter is a geological legend and should comply with a standard local to the project or following AGI recommendations.
- Description:
The description follows the procedure outlined in "The Logging of Rock Cores for Engineering Purposes" (QJEG 1970), with major rock unit descriptions running across the whole column and minor details, with footages, indented 1 cm from the left.
- Remarks:
This space is used for indications of casing depths, hole surveys, zones of caving, etc.

GEOLOGICAL ENGINEERING LOGGING FORMAT

Type of drilling: ROTARY CORE DRILLING		Coordinates:		DRILLHOLE No. 77-302	
Rig: BOYLES 170 WIRELINE		Dip: 35°		Sheet 6 of 8	
Bit: IN DRUMS		Azimuth: 140°		Reference elevation: DRILL FLARE	

Drilling Program	Rate of Advance MIN/FT	R.Q.D.	Core Recovery %	Depth	Reduced Level	Water Level	Test Results	Bore/Fat.	Fracture Index	Weathering Index	Legend	Description
	3.6	0	50	200								201.5-206.0 poor recovery of mud, cemented conglomerate
	3.6	0	50	205								
	4.3	100	100	206.5-206.8								206.5-206.8 fresh, massive, w. some bedding lam., pale-ght. gray, mod. strong SILTSTONE
	4.3	100	100	210				10	0			206.8-214.3 mod. altered, massive, pale gray-pinkish, m-c, weak to mod. weak, v. silty SANDSTONE w. minor conglomerate. Some structure as low angle to core axis w. shearing. 206.8-207.8 v. weak zone
	5	100	100	214.5				-	0			213.0 some v. soft zones in sandstone showing shearing of clays & alteration
	6.0	100	100	215								214.3-232.0 REGOLITE composed of kaolinitized, hematized, sheared, weak, altered rock producing pink coloration in drilling mud. In places regolith appears as mottled siltstone, sandstone or conglomerate w. intense shearing.
	4	100	100	220				-	0			
	5.0	90	100	225				-	2.2			
	5.0	90	100	230				47 to 38	1.0			227.0-232.0 rock appears to be v. granular w. relic foliation & hematization throughout. Prob. represents alteration of basement.
46/7 PH	7	100	100	232.0				62 to 43	0.2			232.0-311.5 sl.-mod. altered, lt. reddish gray-grayish green, m-c grained mod. strong-strong KNEISS w. some pegmatite. Some hematization between 232.0 & 237.0.
	5.8	100	90	235								232.3-232.9 & 235.3-235.6 (pegmatites)
				240					0.8			237.0-242.0 some hematization & pegmatization

Contractor: RUMBLE		Logged by: PH		Remarks:	
Date started: 14 FEB 77		Checked by:		Golder Associates	
Date finished: 17 FEB 77		Date: 16 FEB 77			
				Scale:	

Appendix II - References

Geological Society Engineering Group Working Party Report, 1970. The logging of rock cores for engineering purposes. Quart. J. Eng. Geol., V. 3, No. 1, pp. 1-24.

APPENDIX III

Geophysical Surveying

III-1.0 GEOPHYSICAL SURVEYINGIII-1.0 Introduction

Geophysical surveying involves taking remote measurements, generally at the ground surface, but may also entail surveying in boreholes or in underground excavations. These physical measurements are indirect and yield information as to the contrasts detected within the survey area. For example, a sufficient density contrast enables the geophysicist to outline a massive sulphide deposit surrounded by less dense country rocks through the use of the gravity technique.

The direct information available through geological surveying and sampling enables the interpreter to expand on the geological information. This results in a more thorough understanding of the site geology. To be effective for this purpose, the data from the geophysical surveys must be expressed in terms of the geology. The reliability of the geological picture thus derived, which is greater for some techniques than others, depends on the quality of the data and the skill displayed in their interpretation.

The reliability of the geological interpretation concerning the use of geophysical measurements further depends on the accuracy of the method or methods employed. Surface techniques such as gravity, seismic and resistivity are generally less accurate than borehole techniques. This is due to the fact that borehole probes are used in close proximity to the formation rocks and also to the construction of borehole probes which provide high resolution detail of the physical property measured. This high

degree of resolution is sacrificed at the expense of penetration. Surface techniques, on the other hand, possess excellent penetration capability at the expense of resolution. Surface techniques are utilized, therefore, as a means for mapping regional or site geology. Borehole techniques, while extremely useful for geological mapping, provide more accurate definition of some physical properties such as bulk density, porosity, hydrogen content, fracturing and shear bulk, and Young's moduli plus Poisson's ratio. In addition, borehole techniques can be combined to more accurately define porosity and permeability.

The following tables summarize techniques used for the various phases of the site evaluation process for deep repositories in bedded salt.

TABLE III-1

GEOPHYSICAL SURVEYING RELATING TO BEDDED SALT DEPOSITS
REGIONAL TECHNIQUES

<u>Technique</u>	<u>Purpose</u>	<u>Comments</u>
Surface gravity surveying	To define regional stratigraphy and structure	Variations in the near surface component of the earth's gravity field are measured. The contoured Bouguer gravity map is interpreted with regard to lithology, faulting, folding, and bedrock topography (see Section 2.1).
Limited surface seismic reflection/seismic refraction surveying	To outline in detail stratigraphy and/or structure at selected locations	Seismic refraction surveying is generally accepted as accurate to within 10% of the true depth of a refractor. Topography of the refractor is obtained also. Seismic reflection surveying may provide multiple reflections which mirror the topography, thickness, and attitude of reflectors (see Sections 2.2 and 2.3).

TABLE III-2

GEOPHYSICAL SURVEYING RELATING TO BEDDED SALT DEPOSITS
SITE SPECIFIC, SURFACE TECHNIQUES

<u>Techniques</u>	<u>Purpose</u>	<u>Comments</u>
Resolution seismic reflection or seismic refraction surveying	To outline in greater detail the stratigraphy and/or structure at the site	Detailed seismic refraction surveying together with drillhole information will provide detailed geological information. If geological conditions are not favourable for refraction surveying, then detailed reflection surveying will be undertaken (see Sections 2.2 and 2.3).
Electrical resistivity surveying	To map the variations of resistivity over the area of interest	Lateral changes in earth resistivity may be caused by facies changes, layers thickening or thinning, groundwater effects, and structure (see Section 3).
Electrical resistivity sounding	To define the changes in earth resistivity with depth	Vertical changes in earth resistivity are caused by lithological changes and ground water (see Section 3).

TABLE III-3

GEOPHYSICAL SURVEYING RELATING TO BEDDED SALT DEPOSITS
SITE SPECIFIC, BOREHOLE TECHNIQUES

<u>Techniques</u>	<u>Purpose</u>	<u>Comments</u>
Low frequency pulse-radar electromagnetic surveying	Detection of non-salt anomalies laterally from the borehole such as water, anhydrite, sandstone, shale, or faulting	Discontinuities lateral to the borehole can be detected at distances greater than 300 m (see Section 3.2).
Seisviewer system	Determination of fractures and their orientation or highly disturbed zones in close proximity to the borehole	Seisviewer is an acoustical device. The amplitude of the signal reflected from the wall depends upon the acoustical impedance of the wall rock and associated physical properties of the wall (see Section 3.3).
3-D velocity system	Determination of fractures and their orientation together with the compressional and shear wave velocities; also lithological log	Combined with the gamma-gamma (density) log, provides values for in situ shear, bulk, and Young's moduli plus Poisson's ratio (see Section 3.4).
Gamma-Gamma density log	To determine true rock density in situ	Source sends active gamma radiation into the borehole wall. Returned gamma particles are a direct function of rock density (see Section 3.5).
Gamma Ray	Delineation of higher natural radiation lithological units such as shales and clay zones	Passive tool which measures the amount of natural gamma radiation emanating from rocks containing higher concentrations of montmorillonite (see Section 3.6).
Temperature log	To record the thermal conductivity in the wall rocks; also to outline aquifers and perhaps identify their relative permeabilities	Probe is of the thermister type. A useful long term monitoring tool (see Section 3.7).
Neutron log	To define porosity below the phreatic surface and moisture content above the phreatic surface	Essentially a groundwater tool. The method depends on hydrogen content of the host rock which captures neutrons from the source (see Section 3.8).

TABLE III-4

GEOPHYSICAL SURVEYING RELATING TO BEDDED SALT DEPOSITS
SITE SPECIFIC, CONSTRUCTION TECHNIQUES

<u>Techniques</u>	<u>Purpose</u>	<u>Comments</u>
Caliper log (hole diameter log)	To determine hole roughness which may reflect fractured zones. Also used to apply corrections to other logs	Tool essentially consists of four spring loaded mechanical arms (see Section 3.9).
Electrical resistivity logging	May provide an estimate of effective porosity and water quality in overburden granular aquifers; excellent for determining lithology and stratigraphic correlation	The response depends upon the nature of the pore spaces and the interstitial fluids (see Section 3.10).
Underground low frequency radar surveying	Detection of non-salt anomalies laterally from the underground opening such as water, anhydrite, sandstone, shale, faulting, or boreholes.	Given relatively clean salts, some of these anomalies can be detected up to 1,560 m from the workings (see Section 3.2).
Micro-seismic technique	To determine and monitor stress changes in the surrounding rock as a result of mining operations	See Section 3.11.

III-2.0 REGIONAL/AREA SURVEY

III-2.1 Gravity Method

Gravity surveying entails the measurement of the earth's gravitational attraction at points within the survey area relative to each other; small changes are recorded in the pull of gravity at stations relative to a base station. These variations in gravity observed at the ground surface depend upon the lateral changes in the density of earth materials. The value obtained at a station is the composite of contributions from all depths down to and including the Mohorovicic Discontinuity. The density of the earth materials is considered to be the wet bulk density. The gravimeter essentially measures the change in pull on a small mass supported by either a spring or mechanical arms.

The survey positioning should be on a rectangular basis, with survey positions at 0.5 to 1.0 km intervals. Due to instrument drift and the need for correcting the drift, base stations must be reoccupied within 1.0 to 1.5 hours from the time of initial observation. This entails the survey being either helicopter- or vehicle-supported.

The unit of measurement in gravity surveying is a unit of acceleration and is measured in gals ($1 \text{ gal} = 1 \text{ cm/sec}^2$). Due to the small variations in gravitational attraction generally encountered in a survey, the milligal is the more acceptable practical unit.

Several corrections need to be applied to the raw gravity data. An elevation correction is applied which requires a complete topographic mapping of the survey area. This may be done by EDM instrumentation. Next, the free air correction is applied to correct for changes in distance from the centre of the earth. A latitude correction is then applied. A further correction, the Bouguer correction, is applied to take into account the density of a slab above and below the survey datum.

The resultant contour map is called a Bouguer gravity map. This map is composed of a regional component and a near-surface component. The regional gradient is removed to isolate smaller, local anomalies which are of interest.

The most critical phase or the final phase of interpretation is to fit a geological explanation to the reduced gravity or Bouguer gravity map which accounts for the shape and nature of the gravity contours. Faults, for example, show up markedly on gravity maps as steep gradients.

III-2.2 Seismic Refraction

Refraction is often applied as a reconnaissance tool. Geophones are placed in a straight line along the ground surface, and their separations and elevations are recorded. Explosive charges are detonated at several locations along the refraction traverse and the travel times from shots to each geophone are recorded on analog or digital tape.

The speed or velocity at which the shock wave travels in the various earth media is controlled mainly by Young's modulus and, to a lesser extent, by the density of the medium.

Travel paths differ for refracted waves from those of reflected waves. The refracted ray, at the critical angle, travels along the denser medium and at some point is refracted back toward the ground surface. The explanation for this phenomenon requires familiarity with wave theory.

The environment amenable to refraction prospecting must have a density increase with depth. If a denser medium overlies a less dense medium, this gives rise to a velocity reversal, and the calculated depths to a refractor will have large errors. Similarly, beds which are too thin or which lack sufficient velocity contrast may not be detectable by the refraction method.

The arrival times and geophone separations and elevations form the raw data for interpretation. The interpretation is based either on ray path or wavefront techniques. Given a favourable geological environment, the calculated depths to a refractor are generally accepted as being within 10 percent of the true depth to a refractor. Additional information provided by the interpretation include dip, orientation, thickness, and topography of refractors.

III-2.3 Seismic Reflection

The seismic reflection technique may employ the same equipment as the refraction technique. The field procedure may differ somewhat, in that it is not necessary to have the large separations of the surface geophones or the rather large charge sizes that generally accompany refraction shooting.

The use of nearly vertical ray paths to map reflecting horizons is essentially the working basis of the reflection method of seismic prospecting. Unlike refraction, multiple reflections from various subsurface layers may be picked up from one location. The most common method of reflection shooting is to straddle the shot point with geophones on either side. The geophones again are laid out in a straight line.

Velocity information is not obtained by the seismic reflection technique. The most common methods of obtaining velocity data are by well shooting, continuous velocity logs, or drill hole information.

Perhaps the most complex aspect of reflection surveying is in the data processing. The analog or digital arrival information may require filtering and any one of a number of processes which can best be described by communications theory.

Seismic reflection profiling may be used to detect faults, stratigraphy, depositional history, pinchouts, and other erosional truncations, and facies changes.

The resolution of the method decreases with increasing depth. However, repetitive source methods such as Vibroseis and Mini-Sosie provide a means of obtaining high resolution in the depth range of interest.

III-3.0 SITE SPECIFIC

III-3.1 Electrical Resistivity

Surface resistivity techniques generally employ two current electrodes and two potential electrodes. Generally, the electrodes are laid out in a straight line. The outside electrodes introduce current into the earth, while the inside pair of electrodes measure the potential associated with the current. The quantity obtained for each reading is the apparent resistivity which is the voltage divided by the current times the geometric factor. The geometric factor depends on which electrode configuration is employed. For example, some of the more popular arrangements are the Wenner, Lee, Schlumberger, Pole Dipole, and Dipole Dipole.

The value of apparent resistivity in ohm-meters is equivalent to the true resistivity only when the latter is uniform throughout the subsurface. This generally includes only the first layer situation where the separation of the electrodes is small. If you employ a larger electrode separation, the apparent resistivity obtained is the sum of contributions from all layers down to the level of penetration achieved.

The apparent resistivity value obtained for any station depends on several factors. In most rock materials, the porosity and the chemical content of the water filling the pore spaces is more important in governing resistivity than the conductivity of the mineral grains of which the rock itself is composed. The salinity of the water in the pores is probably the most critical factor determining the resistivity. When porous rocks, particularly those with large concentrations of magnetite or graphite, lie above the water table at shallow depths or when they occur at such great depths that all pore spaces are closed by ambient pressure, the conduction through them takes place within the mineral grains themselves. Under these conditions, the resistivity of the rock will depend on the resistivity of

the grains. When the pores are saturated with fluids, it will be governed by the fluid resistivity as well.

The resistivity method is generally employed to find lateral variations in the stratigraphy (horizontal profiling) or vertical variations in the stratigraphy (vertical sounding). Additionally, faulting can be detected by horizontal profiling.

In reconnaissance horizontal resistivity profiling, the roving dipole and dipole dipole techniques are generally employed. The roving dipole technique is designed to facilitate rapid reconnaissance of large areas in order to select specific targets for detailed investigation. The advantages of this technique are that it is lower in cost per unit area, it is faster than other methods, and it does not require straight survey lines (which makes it especially useful in rugged terrain). The dipole-dipole technique has the advantage of providing a resistivity cross-section in line with the dipole-dipole configuration.

The interpretation of horizontal resistivity data consists of a qualitative assessment of the contoured resistivity values for constant depth of exploration. The lateral changes in resistivity values may indicate faulting, changes in thickness of stratigraphic units, or changes in basement topography.

In vertical electrical sounding the electrodes are laid out along a line and their separations are increased in a systematic manner. For deep electrical sounding the Schlumberger electrode configuration is utilized. The change in apparent resistivity with electrode spacing makes it possible to determine the variations of resistivity with depth. The interpretation of these data consists of separating out from the curve

of apparent resistivity with depth the thicknesses of the various stratigraphic units which contribute to the overall resistivity curve.

There are several techniques available to the interpreter. The most accurate is the comparison of field curves with published theoretical master curves. The accuracy of the results in electrical sounding depends mainly on the resistivity contrast existing between the various lithological types. The interpretation yields information mainly about the structure, stratigraphy, and phreatic surface.

III-3.2 Low Frequency Radar Surveying

At present, all successful ground probing pulse radars employ electromagnetic waves of relatively low frequencies, generally between 15 and 500 MHz. These waves are emitted in broad-band pulses of the shortest possible duration (2 to 70 nanosec) to provide good distance resolution and discrimination between reflectors at slightly different ranges. Average transmitted power levels are generally only a few watts.

The effectiveness of ground probing radar is determined both by radar system limitations and by propagation loss in the medium probed. Salt is an excellent transmitter of electromagnetic waves, with radar probing ranges to discontinuities in salt of up to 10 km. A radar system probing into salt will detect discontinuities in the rock material if a facies change is represented by a change in the electric permittivity (dielectric constant) of the rock or if the rock remains the same but is filled with water. Thus, the losses are greatly increased (by factors up to 12) by thorough wetting in many rocks.

Distances out to discontinuities are obtained by multiplying one-half of the reflection time by the velocity of radar in salt. In order to obtain salt velocity, tests may have to be run through known thicknesses such as salt pillars underground or between boreholes. Discontinuities such as water, anhydrite, sandstone, shale, faulting, or boreholes can be detected at large distances.

Continuous recording of the radar data can be accomplished underground by use of a video tape recorder (VTR). The use of a VTR has two distinct advantages: one is an audio channel which is convenient for recording position locations, the other, the ability to play back the whole recording to see if the data are satisfactory.

Like most geophysical methods, the accuracies of radar surveying are highest at short probing distances. Accuracies of better than 3 m in probing over 300 m have been obtained; other researchers indicate that at distances in excess of 30 m the inaccuracies to be expected are of the order of 0.6 to 3 m.

III-3.3 Seisviewer Logs

The acoustical Seisviewer logging device produces a photograph of the lithologic and stratigraphic features exposed on the face of an uncased borehole or the physical conditions inside cased boreholes.

The Seisviewer system, originally developed by Mobile Oil Company as the borehole televiewer, logs a continuous acoustic picture of a borehole wall as if the hole were split vertically at magnetic North and laid flat. Recording in the variable density mode displays any physical changes in the borehole wall as changes in picture intensity.

In open holes, fractures, vugs, washouts, and thin beds are clearly mapped. In casing inspection applications, the Seisviewer log reflects the exact size and shape of perforations, corrosion pits, and casing splits.

The down-hole unit comprises an electronics section and a motor driven section containing a piezoelectric transducer and a flux-gate magnetometer. The surface panel contains controls for operating the down-hole functions of the Seisviewer. Additional equipment required to operate the system includes a power supply, a monitor oscilloscope with polaroid camera attachment for "quick-look" record sampling, and a Birdwell 3-D Camera for continuous film recording.

In operation, a motor rotates the downhole scanner-transducer and flux-gate magnetometer about their common vertical axis at a uniform rate. The transducer emits a pulsed narrow-beam acoustic signal which travels through the borehole fluid striking the wall of the borehole. A portion of the signal is reflected and subsequently detected by the receiving transducer. The amount of energy reflected by the borehole wall is a function of the physical properties of that surface. In general, any irregularities reduce the amplitude of the reflected signal and thus the

intensity of the output of the surface recording equipment. The magnetometer senses the earth's magnetic field and produces a pulse at the instant the transducer is pointed toward magnetic north. Data gathered by the downhole unit are transmitted up-hole by wireline. Because the entire wall of the borehole is scanned, the resulting log picture is a direct and descriptive reproduction of the borehole wall oriented to magnetic north.

III-3.4 The 3-D Velocity or Variable Density Log

In recent years a considerable amount of research involved detailed studies of complete acoustical wave signals. A modified acoustic logging system called the "three-dimensional" or "3-D" velocity log was developed by the Birdwell Division in 1960.

The 3-D velocity logging system utilizes transmitting and receiving transducers placed a known distance from each other in the borehole. The transmitter generates pulses of 22 microseconds in width at a rate of 20 times per second and the receiving transducer detects any pressure waves reaching it in a borehole and converts these pressure waves to electric signals which are transmitted, along with a synchronized pulse, to the surface recording equipment. The total wave train, including pressure, shear, and boundary waves, can be displayed as intensity modulation, or variable density, by a specially designed camera with a 210 mm fibre optic face. This information, in addition to time and depth lines, is recorded on film which moves past the face of the fibre optic tube synchronized with the depth measuring device. The length of the sweep can be adjusted to display selected portions of the received wave trains ranging from 250 microseconds to 25 milliseconds.

Computer programs are applied to the raw data obtained from the 3-D velocity log and the density log to compute the corrected compressional and shear wave velocities; the shear, bulk, and Young's moduli; and Poisson's ratio. These elastic moduli, computed from theoretical relations developed for homogeneous, isotropic and elastic materials indicate good agreement with other sources of measurements.

In subsurface engineering problems associated with the science of rock mechanics, the 3-D velocity log can provide in situ measurements with

sufficient accuracy to be of practical field application. The in situ dynamic properties obtained from the 3-D velocity log have been employed to investigate rock mechanics problems involved with dam construction, tunnels and canals, and for foundation studies for buildings and nuclear power plants. The 3-D velocity log also is utilized to investigate the fracturing and rate of subsidence of the roof-rocks over caverns. Recently, some researchers utilized the 3-D log for bulk lithology evaluations in carbonates and evaporites.

III-3.5 The Density Log

The density log was introduced in 1950, but was not used widely in the mineral industries until the borehole compensated tool was introduced in 1964. This improved tool eliminates undesirable borehole and mud cake effects, provides the most accurate percent porosity in sand-shale sequences, and is one of the two most widely used tools for porosity and lithology evaluation.

Density logs are used for correlation, for measurement of bulk density, for accurate porosity evaluation, for lithology and mineral identification, and for formation fluid evaluation.

Gamma radiation is attenuated or scattered by all material through which it travels as a function of the electron density of the material. The gamma-gamma log is a record of changes in detected gamma radiation, derived from a source in the probe, caused by differences in the density of material in and adjacent to the hole. In some logging equipment, 10 to 35 millicuries of cobalt⁶⁰ is used as a gamma source attached below a sodium iodide detector. Both an axially symmetric and a decentralized side collimated tool are used. The axially symmetric tool employs Mallory 1000 metal shielding between the source and detector and is not positioned in the hole. The decentralized device employs two motor-driven arms to force the tool against the wall of the hole. Mallory 1000 metal is used as a shield around the source and detector so that most of the signal recorded is from the portion of the hole wall in contact with the tool.

The decentralized tool has the advantage of being much less affected by changes in drill bit size or position in the hole. It is, however, still affected by hole diameter changes occurring between the source and detector.

Gamma photons radiating from the source and are scattered and absorbed by fluid in the hole, casing, and formation. The relative percentage of photons absorbed and scattered and radius of investigation depend greatly on the type and size of the source, spacing between the source and detector, distance to the wall of the hole, and material penetrated. With the equipment and source described, Compton scattering is probably the most important process of photon energy degradation taking place. Therefore, gamma radiation absorbed is proportional to the electron density of the material penetrated, and the electron density is approximately proportional to the bulk density of most material encountered in logging. The count rate plotted on a gamma-gamma log is an exponential function of bulk density. The calibrated and corrected gamma-gamma log may be used for the determination of porosity by the application of the following equation:

$$\text{Porosity} = \frac{\text{grain density} - \text{bulk density (from log)}}{\text{grain density} - \text{fluid density}}$$

Grain density may be derived from laboratory analysis of samples or, in the case of quartz sandstone, a value of 2.6 gm/cc may be assumed. The fluid density in most water wells may be assumed to be 1 gm/cc. If a high content of dissolved solids is suspected, a laboratory analysis may be necessary. Errors in bulk density obtained by the gamma-gamma method may be as high as ± 0.04 gm/cc. A linear plot is obtained when bulk density is plotted against the logarithm of count rate.

In addition to measurement of bulk density or calculation of porosity, the gamma-gamma log may be used to locate casing or collars, including one string of casing outside another. It may also be used to locate the position of grout or hole enlargements through casing and detect the water level and significant changes in fluid density.

III-3.6 Natural Gamma Log

Natural gamma logs are continuous records of gamma radiation produced by radioisotopes naturally occurring in all rocks. Where borehole temperatures are less than 66° C, gamma logs usually are made with a thallium activated sodium iodide detector; at higher temperatures, the less sensitive Geiger tubes must be used. The gamma-emitting radioisotopes normally present are potassium 40 and the daughter products of uranium and thorium. Potassium 40 is probably the most common radioisotope in sedimentary rocks, and the ratio of potassium 40 to stable potassium is usually constant. The total potassium content of rocks is extremely variable, but the world wide average for argillaceous sediments is about 3 percent and for sandstones is about 1 percent. For this reason, it is generally true that the finer-grained detrital sediments in a section exhibit the maximum radiation on a gamma log, but exceptions are common. The identification of shaley or clay-rich sediments with the gamma log is important in distinguishing sediments with a high total porosity from those with a high effective porosity. Evaporates and chemical sediments are generally lower in gamma activity than detrital sediments, and mafic igneous rocks are usually lower than the felsic rocks.

When gamma rays emitted by the formations penetrate suitable detectors in boreholes, an electrical pulse is generated and sent to a surface recorder. The pulses are counted systematically and expressed on the logs in analog form in terms of gamma ray intensity on a standard API quantitative scale. Detectors are usually Geiger-Mueller or scintillation types employing photoelectric techniques.

The log is used for correlation of subsurface formations, for the evaluation of shaliness and formation permeability, for the quantitative

evaluation of potassium-bearing rocks, for lithology studies, and for the environment of deposition exploration.

The chief uses of the natural-gamma log in groundwater hydrology are stratigraphic correlation and the identification of lithological units. It is the simplest and least expensive of the radiation logs and permits identification of aquifers in either cased or open holes. The gamma sonde is also used for location of radioactive wastes disposed of in the ground, in radioactive tracer work, and in the evaluation of deposits of radioactive minerals.

III-3.7 Temperature Log

Logging equipment utilizes a single probe to make a record of fluid temperatures down the borehole. Temperature is measured with a calibrated thermistor inside a protective tube designed to force water through the resistivity electrode and past the thermistors as the probe is lowered. A small change in temperature causes a large change in electrical resistance, and this resistance is recorded at the surface recorder. The thermistor is mounted mechanically so that the effect of mounting and protected material around it has a minimum effect on the change in temperature sensed by the thermistor. The resistance changes are calibrated with a resistance decade box.

Computerized Differential Temperature Surveys were introduced a few years ago to amplify the small temperature anomalies between adjacent depth levels. Magnetic tape is employed, and the log represents a continuous analog plot of differential temperatures versus depth.

Differential logging systems are of two types. One system employs two thermistors spaced from one to a few meters apart along the axis of the drill hole (Bashman and Macune 1952). The differential curve is a record of changes in the temperature difference between these two thermistors. Another system employs one thermal sensor with a memory device to compare the temperature at a point in time with a preselected past time (Johns 1966).

The first system is subject to error due to disturbance of thermal conditions by the lower thermistor. The second system is subject to error caused by changes in logging speed. Both systems have a theoretical sensitivity of $.0005^{\circ}\text{C}$, much more than conventional gradient logging systems.

It is much easier to measure small differences in a factor on a differential than on an absolute scale. Gradient or absolute scales should be selected to cover the temperature ranges expected in the hole and thus reduce scale changing to a minimum. The differential settings are selected so that a reference gradient near the top of the hole is recorded at a zero baseline and slight increases or decreases in gradient are recorded as sharp deflections to the right or left. Most of these changes are not detectable on the gradient log.

The geothermal gradient is recorded on a temperature log of a well that is, in thermal equilibrium with the adjacent rocks where there is no vertical fluid circulation. This gradient is largely a function of the thermal conductivity of the rocks modified by water moving through the system. Temperature logs may be used to identify aquifers or perforated sections contributing water or gas to the well, to provide information on the source of water, as an aid in identifying rock types, and for calculating fluid viscosity and specific conductance from fluid-resistivity logs. Temperature logs made before and after pumping or injection may identify the aquifers and suggest the relative permeabilities. The sensitivity of the differential device suggest the use of water itself as a tracer. Temperature logs have been used to distinguish moving and stagnant water in a well and identify the source of recharge or injected waste water. Seasonal recharge may be reflected in cyclic temperature fluctuations, and it is possible to interpret the vertical movement of water in the unsaturated zone from very sensitive temperature logs (Stallman 1965).

III-3.8 Neutron Log

The various types of neutron logs include potentially the most useful techniques in borehole geophysics as applied to groundwater investigations. This is because most of the log response is caused by changes in hydrogen content in the environment; in water wells this is primarily due to changes in water content.

In neutron logging, a high-energy neutron source such as plutonium-beryllium, radium-beryllium, or americium-beryllium is attached to the lower end of the downhole tool to bombard the formations with neutrons. After emission from the source, the neutrons are slowed to thermal velocities by collisions with hydrogen atoms which have virtually the same size and mass. After the neutrons have been slowed to thermal velocities, they may be captured by other hydrogen atoms, which then emit gamma rays of capture. The gamma rays of capture are detected by one or both of the neutron-gamma ray detectors and telemetered to the surface as electrical pulses. The number of these pulses is logged as an indication of the hydrogen density in the pore space of formations surrounding the borehole. The long detector spacing provides greater depths of investigation outside the borehole, and the short detector spacing provides for investigation of the formation near the source.

The average energy lost by a neutron in a collision with a hydrogen nucleus is 63 percent as compared to 12 percent for the nucleus of an oxygen atom. Thus, the range of a neutron is largely a function of the hydrogen content of the material through which it passes. In the long spaced neutron probe used in well logging, the number of neutrons detected and recorded is reduced by increasing hydrogen content of the environment.

Above the zone of saturation, the calibrated and corrected neutron log provides a continuous measurement of the moisture content; in the zone of saturation it provides a record of porosity. Neutron logs are also used for geologic correlation, shaliness evaluation, bulk lithology evaluation, invasion and permeability studies, and porosity type evaluation.

III-3.9 Caliper Log

The caliper or hole diameter log is essential to the correct interpretation of other logs and has considerable use in providing information on hole construction, mudcake thickness, and lithology. Caliper logs measure the diameter, volume, and geometry of boreholes. Because most lithological geophysical logs are affected to some degree by hole diameter, the caliper should be included in any open-hole logging program.

Caliper devices are either one-arm, two-arm, three-arm, four-arm, or six-arm in construction. Birdwell-SSC developed huge six-arm caliper and density tools especially for use in large-diameter holes drilled by the Atomic Energy Commission in Nevada.

The tool is lowered to the bottom of the hole, the arms are opened by a motor and maintained against the wall of the hole by spring pressure. A continuous record of average hole diameter is made as the tool is raised. Accuracy and reproducibility is better than 0.64 cm of hole diameter. In practice, this tool is run with a gamma ray detector to provide precise depth correlation between the caliper log and other logs run in the same hole.

Because of the independent action of each of the arms, the diameters recorded are those of the circles described by the tips of the three sets of arms, regardless of their radial position in relation to the center of the tool. These diameters can be recorded with a choice of 1 inch (2.5 cm), 2 inch (5.1 cm), or 4 inch (10.2 cm) scales per horizontal inch (cm) of record. All scales are shown in their correct depth relationship.

When hole volumes are desired at the time of logging, a special surface panel may be used to obtain these data. The panel is capable of taking two input signals and providing an integrated output signal. One

input signal is the depth factor, which is generated by a selsyn system. The other signal is the radius squared of the average diameter as recorded with the six-arm caliper.

In certain localities, the caliper is used for subsurface formation correlations based on the differential "washing out" or caving of diverse formations. It has also been possible to detect casing collars, very small fractures, and vesicular tops of basalt flows.

III-3.10 Resistivity Logs

Theoretically, the resistivity values recorded on a log are a measurement of the resistance of a cube of material measuring 1 m along each edge, hence the units are ohm-metres² per m or simply ohm-m. Since most rocks consist of non-conductive particles, the nature of the pore spaces and interstitial fluids determines the character of the resistivity curve.

The specific resistivity curves of the borehole electrical log represent a measurement in which an electrical current is passed between two electrodes in the conductive fluid of a borehole and the potential difference between two adjacent potential electrodes is observed. This potential, after suitable corrections for borehole influences, is related through Ohm's law to the resistivity of the formations near the measuring electrodes.

Normal symmetrical curves are obtained by having a current electrode at the recording depth and the return current electrode at a distance at least 10 times the distance from the potential electrode to the nearest current electrode. Depth of investigation into formations adjacent to the borehole is controlled by the potential-current electrode spacing. The conventional electric log includes a short 16 inch (40.6 cm) spacing for shallow investigation and a longer 64 inch (162.6 cm) spacing for deeper investigation. Spacings have varied throughout the years and from one geologic province to another, depending upon the problems existent.

The lateral non-symmetrical curve is obtained by having two fairly closely spaced potential electrodes at an appreciable distance, usually 19 ft. (5.8 m) or more, from the nearest current electrode. The lateral depth

of investigation is approximately the potential-current electrode spacing. The Lateral curve obtains a resistivity reading of the rock usually unaffected by mud filtrate invasion, thus permitting a quantitative measurement of the per cent of water in the rock pore spaces. The Archie water saturation equation was originated in 1938 and is still the basis for approximately 70 percent of the formation evaluation methods for fluid saturation determinations. This equation is:

$$S_w = (R_w / \phi^m \times R_t)^{1/n}$$

S_w represents the fraction of water in the pore space; R_w is the formation water resistivity; ϕ is the fractional porosity; m is a cementation exponent obtained from core measurements and logging methods; and R_t is the true resistivity from a Lateral or other resistivity device.

The lateral curve also is useful for delineation of thin resistive formation breaks and for geological correlation in exploration. The lateral and normal resistivity curves largely have been replaced by today's more sophisticated focused current logs.

III-3.10.1 Specialized Resistivity Devices

1) Guard and Laterologs

The guard-type log employs the principle of a focused current device. The focusing effect of the guard electrodes is obtained by maintaining an equal potential parallel to the axis of the tool in the borehole adjacent to the instrument. The components of the guard instrument consist of two guard electrodes 64 inches (162.6 cm) in length, a measuring electrode 5-1/2 inches (13.9 cm) in length, and a return current electrode placed in the borehole from other electrodes.

The measuring electrode is connected electrically with the guard electrodes through an electronic measuring system. Any difference in potential between the measuring electrode and the guard electrodes indicates an unbalanced condition. The servo balance system will then change the current on the guard electrodes until a balanced condition exists.

With the guard electrodes maintaining an equal potential 64 inches (162.6 cm) above and below the measuring electrode, the constant current from the 5-1/2 inch (13.9 cm) measuring electrode is confined and flows laterally out into the formation in a disc-like pattern. Any change in resistivity of the formation will cause a change in potential adjacent to the measuring electrode. This change in potential is detected instantly, and the balance circuit changes the current to the guard electrodes. The changes in potential required to maintain a balanced condition are recorded as formation resistivities.

$$V = \rho KI$$

V = Voltage at measuring electrode

K = Constant related to volume measured

I = Fixed current

ρ = Resistivity

The guard and laterolog logs are used principally where the salinity of the drilling mud is the same as that of the formation water. The logs resolve the resistivity of relatively thin beds and, when corrected for invasion effects, provide a true resistivity that permits quantitative evaluation of formation water.

Laterologs are focused current devices employing a central current source electrode and two other current electrodes above and below, which emit a current of equal intensity to that of the current source electrode. This technique forces the current out laterally into a flat disc around the

borehole and increases the depth of investigation of the device. Additional current monitoring electrodes and an electronic monitoring system ensure that specific resistivity is measured accurately. Laterologs have a greater depth of investigation than guard logs, which they are gradually replacing in petroleum exploration.

Recently, a simultaneous dual laterolog system utilizing a short spacing for shallow investigation and a longer spacing for deep investigation was introduced. This system permits the measurement of depth of invasion of mud filtrate and improves the evaluation of true resistivity required for oil and gas evaluation.

ii) Induction Log

Induction logs of various types largely have replaced normal and lateral resistivity curves and guard and laterolog logs where borehole conditions are normal and where formation resistivities do not exceed 250 ohms. The deep induction logging device obtains about 50 percent of its signal from 130 inches (330.2 cm) and is the deepest lateral investigation device available for resistivity logging.

Induction logs can be recorded in non-conductive muds and in empty or gas-filled boreholes, whereas normal, lateral, guard and laterolog logs cannot be obtained under the same hole conditions.

Induction logging requires no direct contact with the mud or sidewall in order to determine the conductivity accurately. The formation surrounding the tool is subjected to focused high-frequency electro-magnetic wave energy from the transmitter coils. The magnetic field set up by this energy induces eddy currents in the formation which in turn have their own magnetic field that induces an electromotive force in the receiver coil.

If a constant transmitter current is maintained, the magnitude of the eddy currents will be proportional to the conductivity of the surrounding media. The detected signal is a measure of the conductivity which is inversely proportional to the resistivity. To obtain resistivity, a solid-state reciprocator network is incorporated in the surface panel. Thus, the resistivity value of the formation is recorded simultaneously by reciprocating the conductivity recorded over the interval.

The Induction instrument is unique in that the medium immediately surrounding the borehole has little effect on the conductivity recorded by the instrument. The conductivity, subsequently resistivity, recorded is that of a cylinder with a minimum radius of 10 inches (25.4 cm) and a maximum radius of approximately 250 inches (635 cm). Because the induction instrument is insensitive to high-resistivity materials in close proximity to the device, it is an ideal tool for looking beyond a high-resistivity cylinder into a larger low-resistivity cylinder. This condition exists many times in reservoirs invaded by fresh mud filtrate. Another advantage of the induction system is its ability to record thin high-conductivity beds bounded above and below by low-conductivity horizons.

The modern dual Induction log simultaneously records medium and deep investigation curves along with a modified short-spaced laterolog for the most complete investigation of the resistivity profile extending laterally from a borehole. Under many conditions this combination log provides in the absence of an accompanying porosity log values of R_{XO}/R_t which permits an evaluation of water saturation.

iii) The Microlog

The Microlog, utilizing micro-resistivity spacings, is used to log thin 1 inch (2.5 cm) to 2 inch (5.1 cm) formation breaks, to obtain porosity under ideal conditions, to locate fracture and vugular-type porosities, and for detailed correlation. The tool consists of a 1-1/2 inch (3.8 cm) lateral spacing and a 2 inch (5.1 cm) normal spacing, the electrodes being recessed on a hard rubber or inflatable pad that rubs the borehole. The electrode assemblage and pad are mounted on one arm of a spring- or motorized-arm caliper to insure good formation contact. The Microlog constituted the first fairly effective borehole logging method for obtaining rock porosity, but its accuracy is limited to porosities exceeding 20 percent.

iv) The Proximity Log

The proximity log was introduced to obtain more accurate resistivities of the rocks and flushed zones immediately behind thick mud cakes on the borehole walls. The Proximity logging device utilizes rectangular electrodes while the current beam is focused somewhat deeper than the Microlog device. This is essential for obtaining the resistivity of the flushed zone, which is necessary for the semi-quantitative evaluation of formation permeability and productivity.

III-3.11 Microseismic Technique

Microseismic techniques for monitoring mine structures are being accepted in the mining industry. The installation or use of automatic controlled rock burst monitoring (RBM) systems are now realities.

The purpose of an RBM system is to provide a means of obtaining continuous up-to-date information that can be used to evaluate changes in rock stress underground.

The primary function of an RBM is to detect and locate the source of each microseismic or seismic event and to determine the relative magnitudes of that event. The microseismic technique is based on the fact that as stresses increase, the number and magnitude of the rock noises generated increases markedly. The source location of each event allows one to monitor any number of structures or specific parts of a particular structure and in addition, provides a means of continuously monitoring seismic velocity. This latter feature is important, as seismic velocities increase and decrease with stress changes.

Components of an RBM system consist of geophones, preamplifiers, recorder, and computer.

Accuracy for the determination of arrival times is about ± 10 milliseconds, and the maximum deviation in the P-wave velocity due to inhomogeneities can be assumed to be as high as ± 500 m/sec. It follows then that a velocity being assumed too high leads to a lower accuracy of the source position than a velocity being assumed too low, provided the absolute value of the velocity deviation is equal for both cases. The influence of the velocity deviation is higher than the influence of the time error.

As the use of underground cavities for storage of nuclear waste accelerates, the monitoring of rock noise for stability takes on added importance. RBM techniques appear to provide the only protected means for such monitoring.

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APPENDIX IV

Theoretical Study of a Well Test in a Nonhomogeneous Aquifer

IV-1.0 THEORETICAL STUDY OF A WELL TEXT IN A NONHOMOGENEOUS AQUIFERIV-1.1 Purpose and Scope

Hydraulic conductivity is a critical parameter in current predictive models of mass transport and thus will likely be a critical parameter in any performance model used for repository evaluations. Hydraulic conductivity is normally determined on the basis of water yield rather than mass transport. It is possible that further development of mass transport theory supported by laboratory and field studies will result in a hydraulic conductivity (or similar parameter) which specifically relates to mass transport rather than yield.

Uncertainty in hydraulic conductivity stems from three basic sources:

- Uncertainty in defining the natural variability of hydraulic conductivity within the porous media
- Uncertainty in testing techniques, including the method of data analysis and parameter derivation
- Uncertainty in the performance models, including the uncertainty in using measured values of hydraulic conductivity as a model parameter.

This study primarily addresses the second source of uncertainty listed above. It is assumed that the geometry and aquifer properties are entirely defined so that no uncertainty exists because of natural variability. This study does not consider the actual performance models and their uncertainty since these models are not yet fully developed. Eventually, consideration of parameter uncertainty in the performance models will be necessary. The parameters needing consideration will be dependent

upon the performance models chosen. The way in which parameters are derived will also be dependent upon the performance models. For instance, a performance model utilizing a statistical distribution of parameter values may require different exploratory and analytical techniques than a model utilizing "average" parameter values.

Pump tests consisting of an abstraction well and one or more observation wells or piezometers are generally considered to be the most effective means of determining the areal aquifer characteristics of hydraulic conductivity and specific storage. This is because single well pump tests, packer tests, falling head tests, and other single-hole testing techniques measure the aquifer characteristics of only a small area about the hole; thus, single-hole tests can be significantly influenced by local geologic anomalies, mud cake in the hole, fracturing caused by drilling, well bore storage, and other factors. Multiple-well tests are much less influenced by these local effects.

Multiple-well pump tests are commonly analyzed using the Theis type curve method or an adaptation of it for specific conditions such as constant head or impermeable boundaries, aquifer leakance from confining layers, delayed yield, and others. A summary of many available methods of analysis is given in Lohman (1972). These analytic methods are valid for a homogeneous isotropic aquifer and, by using a type curve technique devised by Papadapoulos (1965), can be extended to the anisotropic case.

The objective of this study is to look at the applicability of current analytical techniques applied to a pump test in a nonhomogeneous aquifer and the resulting uncertainty. The pump test is modeled by Golder Associates' finite element AFPM computer program. Drawdown caused by a single abstraction well is computed at various chosen locations for a

pumping duration of 100 hours. The resulting time drawdown data are plotted on log-log graph paper, and the hydraulic conductivity and specific storage are determined using the Theis type curve method.

One homogeneous case is modeled to establish that the computer model accurately predicts drawdown as calculated using the Theis equation. Two nonhomogeneous cases are considered: one in which three discrete values of hydraulic conductivity are distributed in bands such that the conductivity of each band varies from that of the two adjacent bands and another in which the three hydraulic conductivity values are randomly distributed into blocks. The specific storage was held constant over the entire aquifer in order to reduce the number of variables.

Because hydraulic conductivity is a critical parameter in current predictive models of mass transport within an aquifer, it is important to define the distribution of conductivity values as accurately as possible. This task may be very difficult or even impossible using current testing techniques. If an aquifer is nonhomogeneous, it may be possible to model mass transport by using an "average" hydraulic conductivity and incorporating the effects of nonhomogeneity by accurately measuring the aquifer dispersivity over a large area. Another approach may be to statistically define the distribution of hydraulic conductivities and incorporate this distribution into a Monte Carlo or other statistical routine which can be used in the performance model. Field testing of mass transport in nonhomogeneous aquifers for a variety of different geologic and hydrologic environments will be necessary to gain confidence in using any predictive models.

IV-2.0 PROCEDURE

IV-2.1 Finite Element Mesh

The finite element mesh used in the study is shown in Figures IV-1A and IV-1B. It consists of 107 nodes and 83 elements. The aquifer is assumed to be symmetric about the x and y axes; thus only one-quarter of the aquifer area around the well is considered. The boundaries along the x and y axes are considered impermeable since they fall on streamlines. The mesh is extended to 1600 meters in both the x and y directions with the abstraction well placed at the origin. The effects of pumping for 100 hours are minimal at 1600 meters from the well; thus the remaining two boundaries are set impermeable.

Most elements in the mesh have dimensions of 200 meters by 200 meters. Elements near the abstraction well become progressively smaller in order to model the steeper portion of the drawdown curve. Much of the mesh is generated internally by the AFPM program, which significantly reduces the amount of input data.

The aquifer consists of a 1 meter thick layer confined above and below by impermeable units. The initial head is set at zero elevation, which allows direct output of aquifer heads in terms of drawdown. Because the aquifer is of unit thickness (1 meter), hydraulic conductivity and specific storage are equivalent to transmissivity and storage coefficient, respectively.

IV-2.2 Program Operation

The time steps for which model heads are computed are progressively lengthened from time equals 0 hours to time equals 100 hours in

(reduced from original scale)

TYPE OF ANALYSIS: WELL TEST IN HOMOGENEOUS AQUIFER, RUN 1
 FUNCTION: CONTINUED & MESH ONLY
 DATE: 05/14/79
 MATERIAL: TYPE 1 SELECTED & RUN

LEGEND: A - D
 SCALE: 1.33E+02 H. / INCH

25/05/79 14:41.07
 INTERVAL: G.
 SCALE: 1.33E+02 H. / INCH

FINITE ELEMENT MESH TOTAL AREA Figure IV-1A

FINITE ELEMENT MESH NEAR ABSTRACTION WELL

Figure IV-18

(reduced from original scale)

accordance with the standard transient response to a pumping well. The various time steps used in the program are listed below.

<u>Time (Hours)</u>	<u>Number of Time Steps</u>
0 to 0.5	5
0.5 to 2.5	4
2.5 to 10.0	3
10 to 50	4
50 to 100	2

Nodal heads are computed and can be outputted for the time steps listed above. Intermediate time steps are generated internally by the computer program between each of the specified time steps. The number of intermediate steps is controlled by an input parameter specifying the maximum allowable error between actual and computed drawdowns. For this study, a 2 percent error was allowed.

A program option called history tracking was used to record drawdown at 10 observation points (i.e., piezometers, observation wells) within the mesh for each specified time step. These observation points are specified by x and y coordinates and can lie within an element or at a node. The use of history tracking eliminates the need of printing out complete nodal head data for each time step. The time-drawdown histories of the 10 observation points are used to produce the plots for the Theis analysis.

The program also produces line printer plots of head contours at the end of any specified time step. These plots show the configuration of the computed drawdown curves.

IV-2.3 Theis Analysis

Hydraulic conductivity and specific storage can be derived from drawdown data measured in wells or piezometers during a pump test by using a curve matching technique based upon the Theis equation expressed as:

[IV-1]

$$s = \frac{Q}{4\pi m K} W(u)$$

where:

s = drawdown

Q = pumping rate

m = aquifer thickness

K = hydraulic conductivity

W(u) = well function

The well function, W(u), is found by evaluating the following integral for specific values of the parameter, u:

$$W(u) = \int_u^{\infty} \frac{e^{-u}}{u} du \quad [IV-2]$$

with u defined by:

$$u = \frac{r^2 S_s}{4Kt} \quad [IV-3]$$

where:

r = radial distance from pumping well

S_s = specific storage

K = hydraulic conductivity

t = time of drawdown measurement

Corresponding values of u and $W(u)$ evaluated by equation IV-2 are plotted on log-log graph paper to give the type curve in Figure IV-2. The parameters K and S_s are found by plotting time-drawdown data on log-log paper to the same scale as the type curve, superimposing the two curves until they match, keeping the axes parallel to one another, and picking a match point. Values of u and $W(u)$ at the match point are read from the type curve and values of s and t at the match point are read from the time-drawdown plot. The values of s and $W(u)$ are then substituted into equation IV-1 to find K . The value S_s is found by substituting u , t , and the derived value at K into equation IV-3.

The basic Theis analysis is valid for a homogeneous, isotropic aquifer of infinite areal extent. This same basic curve matching technique has been extended to other aquifer boundary conditions and the anisotropic case, but is not considered valid for the nonhomogeneous case. In most pump tests, the knowledge of the aquifer is not sufficient to evaluate whether the assumption of homogeneity is valid. Generally, the aquifer is assumed to be homogeneous, and the parameters derived from the Theis analysis are considered to be representative of the aquifer. As mentioned previously, this assumption results in uncertainty in the value(s) of hydraulic conductivity used in the repository performance model.

THEIS TYPE CURVE

Figure IV-2

Project No. _____ Reviewed _____ Date _____

CASE 1 - HOMOGENEOUS ISOTROPIC AQUIFER
THEORETICAL AND COMPUTED DRAWDOWNS AT RADIUS 50m

Figure IV-3

IV-3.0 RESULTS

Several different cases of aquifer nonhomogeneity are considered in this study. For each case, time-drawdown data computed at various locations are analysed using the Theis equation. The values of K and S_s derived from the Theis analysis are then compared to the actual values of K and S_s assigned in the computer model. In this way, it is possible to study the response of a nonhomogeneous aquifer during a pump test and to examine the uncertainty involved in using K and S_s values derived from pump tests in cases where the aquifer nonhomogeneity is well defined.

In field situations it is seldom if ever possible to completely define the nonhomogeneities within an aquifer. Thus, there is uncertainty resulting from the natural variability. The use of a computer model of a hypothetical aquifer avoids this problem and is able to focus on the uncertainty caused by the aquifer testing and parameter derivation technique. There is some uncertainty in how closely the computer model predicts the aquifer response to the abstraction well. This uncertainty can be checked by comparing computer solutions of simple cases with analytical solutions. The uncertainty should be small if an appropriate model is used.

IV-3.1 Case 1: Homogeneous Aquifer

The first case is of a homogeneous aquifer with $K = 3.6 \times 10^{-3}$ m/hr. and $S_s = 1 \times 10^{-6} \text{m}^{-1}$. This case was run to check the computed drawdowns against the theoretical drawdowns predicted by the Theis equation. Computed and theoretical drawdowns at radii of 50 m and 400 m are plotted in Figures IV-3 and IV-4, respectively. A close agreement between computed and theoretical drawdowns is obtained indicating the

CASE 1 - HOMOGENEOUS ISOTROPIC AQUIFER
THEORETICAL AND COMPUTED DRAWDOWNS AT RADIUS 400m

Figure IV-4

validity of using the computer model to predict drawdowns as determined by the Theis equation. Contours of drawdown after 100 hours are shown in Figure IV-5.

IV-3.2 Case 2: Nonhomogeneous Aquifer, Banded K

The second case uses three values of hydraulic conductivity distributed in bands with dimensions of 200 m in the y direction and 1600 m in the x direction, as shown in Figure IV-6. The three hydraulic conductivities assigned to the mesh are 1.8×10^{-2} m/hr, 3.6×10^{-3} m/hr, and 7.2×10^{-4} m/hr. The high and low values are five times and one-fifth of the intermediate value, respectively. The geometric mean value of conductivity is 3.6×10^{-3} m/hr. The specific storage is set to a constant value of 1×10^{-6} m⁻¹.

The drawdown around the well after 100 hours of pumping is contoured in Figure IV-7. The drawdown near the abstraction well is relatively symmetric, but it becomes asymmetric as it progresses outward from the well into the bands of different hydraulic conductivity. The low conductivity zone acts as a partial barrier to flow in the y direction, while the high conductivity zone acts as a conduit for water flow along the x direction. This results in a more rapid spread in drawdown in the x direction than in the y direction. Some of the minor irregularities in Figure IV-7 and in all the computer plots are the result of interpolation of heads between nodal points in the relatively coarse finite element mesh. These small-scale irregularities do not affect the overall configuration of the drawdown cones to a significant degree.

The observation points for which history tracks of drawdown were computed are shown in Figure IV-6. They are referred to in terms of their

Figure IV-5

DISTRIBUTION OF HYDRAULIC CONDUCTIVITY AND RESULTS OF
 RUN 2 — NONHOMOGENEOUS AQUIFER, BANDED K

Figure IV-6

- Abstraction Well (at origin)
- Point with history track and calculated hydraulic conductivity
 .0050
- Supplementary point and calculated hydraulic conductivity
 .0050

Zones of hydraulic conductivity used in the model were:

- 3.6×10^{-3} m/hr
- 1.8×10^{-2} m/hr
- ▤ 7.2×10^{-4} m/hr

COMPUTED DRAWDOWN (METERS) FOR CASE 2 BANDED K

Figure IV-7

(reduced from original scale)

TITLE OF ANALYSIS: SHELL TEST IN NONHOMOGENEOUS AQUIFER, RUN 2
 FUNCTION CONTOURED: POTENTIAL
 PLOT ORIGIN (X) IS AT X = 0.
 MATHEMATICAL TYPES SELECTED: ALL
 LEGEND: R = 2.150E+02 R = 0.
 X SCALE = 1.33E+02 M./INCH Y SCALE = 1.33E+02 M./INCH
 25/05/79 10:17:40
 INTERVAL = 5.00E+03
 Y SCALE = 1.33E+02 M./INCH

x-y coordinates; thus, the observation point with $x = 50$ and $y = 0$ is $P(50,0)$. The plotted time-drawdown data at $P(50,0)$ are given in Figure IV-8, along with the "best fit" of the type curve through the plotted points. The calculated hydraulic conductivity is 4.2×10^{-3} m/hr, which is near the actual value of 3.6×10^{-3} m/hr for that particular region of the aquifer. The calculated specific storage is $1.01 \times 10^{-6} m^{-1}$, which is equivalent to the actual value.

Another observation point, $P(0,350)$, is plotted in Figure IV-9 and yields a calculated hydraulic conductivity of 5.5×10^{-3} m/hr, which is substantially lower than the actual value of 1.8×10^{-2} m/hr. The calculated specific storage is $1 \times 10^{-6} m^{-1}$, which is equivalent to the actual value.

Figure IV-6 shows the computed and actual values of hydraulic conductivity at the 10 observation points for which drawdown histories were obtained in Case 2. Several supplementary observation points at which hydraulic conductivities were calculated from nodal head data are also included. The values of hydraulic conductivity determined by the Theis analysis are listed in Table IV-1 along with the actual values assigned in the model. In Figure IV-10, the actual values are plotted against the calculated values. The agreement between them is poor.

The geometric mean of the hydraulic conductivities calculated by the Theis analysis at history track points is 0.0049 m/hr, with one standard deviation from the mean ranging by a factor of 1.18. The geometric mean of the same 10 observation points calculated using their actual conductivities as assigned in the model is 0.0036 m/hr, with a factor of 2.92 defining one standard deviation. Thus, the geometric mean of the calcu-

TABLE IV-2
RESULTS OF CASES 3 AND 3A - RANDOM K

LOCATION	RADIUS	MODEL PARAMETERS		CASE 3-CALCULATED PARAM. (Theis Analysis)		CASE 3A-CALCULATED PARAM. (Theis Analysis)	
		K(m/hr)	S (m)	K(m/hr)	S (m)	K(m/hr)	S (m)
P(500,0)	500	1.8×10^{-2}	1.0×10^{-6}	6.8×10^{-3}	9.0×10^{-7}	3.3×10^{-3}	1.3×10^{-6}
P(900,0)	900	3.6×10^{-3}	1.0×10^{-6}	7.3×10^{-3}	6.9×10^{-7}	3.4×10^{-3}	7.3×10^{-7}
P(0,50)	50	$*3.6 \times 10^{-3}$ 7.2×10^{-4}	1.0×10^{-6}	4.0×10^{-3}	9.6×10^{-7}	7.8×10^{-4}	9.3×10^{-7}
P(0,350)	350	1.8×10^{-2}	1.0×10^{-6}	5.6×10^{-3}	9.2×10^{-7}	3.0×10^{-3}	1.6×10^{-5}
P(0,500)	500	3.6×10^{-3}	1.0×10^{-6}	8.3×10^{-3}	8.8×10^{-7}	4.6×10^{-3}	1.4×10^{-6}
P(0,900)	900	1.8×10^{-2}	1.0×10^{-6}	8.2×10^{-3}	1.2×10^{-6}	4.4×10^{-3}	1.2×10^{-6}
P(354,354)	500	7.2×10^{-4}	1.0×10^{-6}	4.2×10^{-3}	8.8×10^{-7}	2.7×10^{-3}	1.3×10^{-6}
P(700,912)	1150	7.2×10^{-4}	1.0×10^{-6}	5.0×10^{-3}	1.1×10^{-6}	3.4×10^{-3}	1.1×10^{-6}
P(900,716)	1150	1.8×10^{-2}	1.0×10^{-6}	6.1×10^{-3}	9.2×10^{-7}	7.3×10^{-3}	1.9×10^{-6}
P(300,1110)	1150	3.6×10^{-3}	1.0×10^{-6}	6.4×10^{-3}	1.1×10^{-6}	3.9×10^{-3}	1.1×10^{-6}

* Top number is value used in model in Case 3; bottom number is value in Case 3A.

COMPARISON BETWEEN CONDUCTIVITY USED IN MODEL AND
CONDUCTIVITY COMPUTED USING THEIS EQUATION
CASE 2

Figure IV-10

lated values is about 27 percent greater than the geometric mean of the actual values, but the variability of the actual values of conductivity is much greater than the variability of the values based upon the Theis analysis. The calculated specific storage values are generally near the actual value.

The results indicate that where aquifer nonhomogeneity such as that modeled in Case 2 is present pump tests can yield misleading values of point hydraulic conductivity, although the overall average value may be predicted reasonably well. Some indication of aquifer nonhomogeneity and/or anisotropy is given by the drawdown contours. Contours around an abstraction well which are elliptic in shape are generally considered to be the result of anisotropy (see Section IV-3.2.1).

It is reasonable to assume that the calculated values of hydraulic conductivity reflect some kind of average conductivity between the abstraction well and the point at which the drawdown is measured. Where flow is perpendicular to zones of alternating conductivity, the average conductivity can be calculated by the equation.

$$K_{avg} (1) = \frac{D_{tot}}{n \sum_{i=1} \frac{D_i}{K_i}} \quad [IV-4]$$

where:

$K_{avg} (1)$ = average hydraulic conductivity (for flow perpendicular to the zones)

D_{tot} = total distance of flow

K_i = hydraulic conductivity of zone i

D_i = distance of flow in zone i

n = total number of zones through which flow occurs.

Values of hydraulic conductivity calculated using equation IV-4 are given in Table IV-1. The distance D_{tot} is assumed to be the radial distance of the observation point from the abstraction well. The conductivities calculated by equation IV-4 have a geometric mean of 0.0039 m/hr and a factor of 1.23 defining one standard deviation. The mean of the calculated values is close to the actual mean, but the variability is significantly less than the actual variability. Figure IV-11 shows that the conductivities calculated by equation IV-4 generally agree better with the values derived from the Theis analysis than do the actual point values (see Figure IV-10), but that there is still a significant difference.

When flow is parallel to zones of different conductivity, an average value of hydraulic conductivity, $K_{avg}(11)$, can be found by the equation:

$$K_{avg} (11) = \frac{\sum_{i=1}^n D_i K_i}{D_{tot}} \quad [IV-5]$$

where the parameters are as previously defined.

Neither of the two equations above is strictly applicable to Case 2, since flow within the aquifer is neither completely parallel nor completely perpendicular to the different conductivity zones.

COMPARISON BETWEEN CONDUCTIVITY CALCULATED USING
EQUATION IV-4 AND CONDUCTIVITY USING THEIS EQUATION
CASE 2

Figure IV-11

IV-3.2.1 Case 2A: Homogeneous Anisotropic Aquifer

The drawdown contours computed in Case 2 are similar to contours expected in an anisotropic aquifer. To make a comparison between Case 2 and an anisotropic case, equations IV-4 and IV-5 were used to compute effective hydraulic conductivities in the y and x direction, and the resulting anisotropic conductivities were used in the model. The effective hydraulic conductivity in the y direction is 2.00×10^{-3} m/hr, and that in the x direction is 6.48×10^{-3} m/hr. These conductivities result in the drawdown contours plotted in Figure IV-12.

The contours in Figure IV-12 are elliptic in shape with the abstraction well at the center. They are much smoother and more regular than the contours for Case 2 (Figure IV-9). The most notable difference between the two contour plots is along the y axis. Drawdown in Case 2 decreases rapidly where a zone of low conductivity at y equals 600 m is encountered as reflected by the closely spaced contours. In Case 2A, the contours become more widely spaced with distance from the origin as is expected in an anisotropic aquifer.

The key to being able to distinguish between Case 2 and Case 2A in a field situation is to have a sufficient number of piezometers to accurately define the drawdown. A lack of adequate piezometers could lead to interpreting Case 2 as an anisotropic aquifer. Such an interpretation could have a significant effect upon predictions of mass transport within the porous media.

IV-3.3 Case 3: Nonhomogeneous Aquifer, Random K

Case 3 uses the same three values of hydraulic conductivity as used in the previous cases, but distributes them randomly in regions 200 m by 200 m in area. The random distribution is achieved by the Monte Carlo

Figure IV-12

method. Each of the three hydraulic conductivities is assigned to one of three sets of numbers as shown below.

<u>Numbers</u>	<u>K (m/hr)</u>
0-33	3.6×10^{-3}
34-67	7.2×10^{-4}
68-100	1.8×10^{-2}

A random number read from a table of random numbers between 0 and 100 is then assigned to each 200 m by 200 m region in the mesh, and the corresponding value of hydraulic conductivity is assigned to all elements within the region. In this manner, 64 regions of conductivity within the mesh were generated as shown in Figure IV-13.

Time-drawdown data were computed for the 10 observation points in Figure IV-13 and were analyzed using the Theis method to obtain the conductivities shown. Figures IV-14 and IV-15 show the time-drawdown data at points P(0,50) and P(354,354), respectively, and also show the best fit of the Theis curve through the data. The curve fits the data very well except at large times. At large time, the data in Figure IV-14 begin to fall below the type curve, which indicates a source of recharge. This is probably the effect of a region of higher hydraulic conductivity near P(0,50). Figure IV-15 shows computed data deviating above the type curve at large times, which indicates either the impermeable boundary at the outer limit of the mesh or, possibly, a region of low conductivity within the mesh.

The calculated values of hydraulic conductivity and specific storage along with the actual values used in the model are given in Table IV-2, as well as in Figure IV-13. The computed values do not reflect the actual values assigned in the model to the regions in which the 10 observation points were located, but instead vary over a much smaller range

TABLE IV-1
RESULTS OF CASE 2 - BANDED K

LOCATION	RADIUS	MODEL PARAMETERS		CALCULATED PARAMETERS (Theis Analysis)		CALCULATED PARAMETERS (Eqn IV-4)
		K(m/hr)	S (m)	K(m/hr)	S (m)	K(m/hr)
P(50,0)	50	3.6×10^{-3}	1.0×10^{-6}	4.2×10^{-3}	1.0×10^{-6}	3.6×10^{-3}
P(350,0)	350	3.6×10^{-3}	1.0×10^{-6}	5.0×10^{-3}	1.1×10^{-6}	3.6×10^{-3}
P(700,0)	700	3.6×10^{-3}	1.0×10^{-6}	4.3×10^{-3}	9.4×10^{-7}	3.6×10^{-3}
P(0,50)	50	3.6×10^{-3}	1.0×10^{-6}	4.1×10^{-3}	9.4×10^{-7}	3.6×10^{-3}
P(0,350)	350	1.8×10^{-2}	1.0×10^{-6}	5.5×10^{-3}	1.0×10^{-6}	5.5×10^{-3}
P(0,700)	700	7.2×10^{-4}	1.0×10^{-6}	6.1×10^{-3}	9.9×10^{-7}	3.4×10^{-3}
P(35,35)	50	3.6×10^{-3}	1.0×10^{-6}	4.2×10^{-3}	8.1×10^{-6}	3.6×10^{-3}
P(248,248)	350	1.8×10^{-2}	1.0×10^{-6}	6.1×10^{-3}	1.0×10^{-6}	4.2×10^{-3}
P(495,495)	700	3.6×10^{-3}	1.0×10^{-6}	4.2×10^{-3}	7.6×10^{-7}	5.4×10^{-3}
P(707,707)	1000	7.2×10^{-4}	1.0×10^{-6}	5.5×10^{-3}	1.1×10^{-6}	2.8×10^{-3}
*N(130,0)	130	3.6×10^{-3}	1.0×10^{-6}	4.2×10^{-3}	1.1×10^{-6}	3.6×10^{-3}
*N(400,200)	447	3.6×10^{-3}	1.0×10^{-6}	7.8×10^{-3}	8.5×10^{-7}	3.6×10^{-3}
*N(0,200)	200	3.6×10^{-3}	1.0×10^{-6}	7.3×10^{-3}	2.0×10^{-6}	3.6×10^{-3}
*N(0,400)	400	1.8×10^{-2}	1.0×10^{-6}	5.0×10^{-3}	7.8×10^{-7}	6.1×10^{-3}
*N(0,600)	600	3.6×10^{-3}	1.0×10^{-6}	3.9×10^{-3}	8.3×10^{-7}	4.9×10^{-3}

*Signifies supplementary point whose parameters were calculated using nodal head output rather than history tracks.

DISTRIBUTION OF HYDRAULIC CONDUCTIVITY AND RESULTS OF
 RUN 3 — NONHOMOGENEOUS AQUIFER, RANDOM K

Figure IV-13

- Abstraction Well (at origin)
- Point with history track and calculated hydraulic conductivity
 .0050

Zones of hydraulic conductivity used in the model were

- 3.6×10^{-3} m/hr
- 1.8×10^{-2} m/hr
- 7.2×10^{-4} m/hr

THEIS ANALYSIS OF POINT P(0,50) FOR RUN 3

Figure IV-14

than do the actual values. The geometric mean of the actual values at the 10 observation points is 5×10^{-3} m/hr, and one standard deviation from the mean varies by a factor of 3.56. The geometric mean of the computed values is 6×10^{-3} m/hr with one standard deviation varying by a factor of 1.29. Thus, although the mean values of both the actual and computed hydraulic conductivities are nearly the same (17 percent difference), the variation in the actual values is much greater than the variation in the computed values. This indicates that pump tests tend to smooth out major variations in hydraulic conductivity in a nonhomogeneous aquifer and that this analysis of the results yields calculated hydraulic conductivities ranging slightly above the geometric mean.

The drawdown after 100 hours, contoured in Figure IV-16, is irregular due to the aquifer nonhomogeneity. (Small-scale irregularities are due to model error because of mesh coarseness and interpolation.) The irregular shape of the drawdown cone would probably be hard to define in a normal pump test, since only several holes surrounding the abstraction well are generally monitored in most tests. Based upon only several drawdown measurements, the drawdown cone could be erroneously assumed to be much more regular.

IV-3.3.1 Case 3A: Low K Region Near Well

Case 3A is equivalent to Case 3, except that the region around the abstraction well is given a hydraulic conductivity of 7.8×10^{-4} m/hr rather than 3.6×10^{-3} m/hr. The resulting conductivities, as determined from this analysis, are shown in Figure IV-17 and listed in Table IV-2. The geometric mean of conductivity dropped from 6.0×10^{-3} m/hr in Case 3 to 3.3×10^{-3} m/hr in Case 3A. Thus, decreasing the

COMPUTED DRAWDOWN (METERS) FOR CASE 3 RANDOM K

Figure IV-16

(reduced from original scale)

TITLE OF ANALYSIS: TEST IN NONHOMOGENEOUS ROUFEER, RUN 3
 FUNCTION CONTOURED: POTENTIAL
 LEGEND: A = 2.100E-02 D = 0.
 0.0 ORIGIN 1.1 IS AT X = 0.
 MATERIAL TYPES SELECTED: ALL
 DATE: 04/05/79 09:02:32
 INTERVAL: 5.00E+00
 SCALE: 1.33E-02 M./INCH
 SCALE: 1.33E-02 M./INCH

DISTRIBUTION OF HYDRAULIC CONDUCTIVITY AND RESULTS OF
 RUN 3A — NONHOMOGENEOUS AQUIFER, RANDOM K

Figure IV-17

- Abstraction Well (at origin)
- Point with history track and calculated hydraulic conductivity

Zones of hydraulic conductivity used in the model were

- 3.6×10^{-3} m/hr
- 1.8×10^{-2} m/hr
- 7.2×10^{-4} m/hr

conductivity of the region around the abstraction well by a factor of 5 decreased the geometric mean conductivity by a factor of approximately 2. The factor describing one standard deviation from the mean increased from 1.29 in Case 3 to 1.78 in Case 3A. Most of the increase in variation can be attributed to the observation point P(0,50), which was located in the same low conductivity zone as the well. Disregarding P(0,50) the geometric mean of the other nine observation points is 3.8×10^{-3} m/hr and the standard deviation factor is 1.34.

The contours of drawdown calculated in Case 3A are given in Figure IV-18. Because of the low conductivity of the region near the pumping well, the cone of depression is very steep near the well and then flattens out away from the well. As in Case 3, the shape of the drawdown cone is somewhat irregular.

COMPUTED DRAWDOWN (METERS) FOR CASE 3A
RANDOM K

Figure IV-18

IV-4.0 CONCLUSIONS

Although the nonhomogeneous cases considered in this study covered only several simple and rather idealized situations, some interesting conclusions can be made.

- The Theis analysis of time-drawdown data is not valid for nonhomogeneous aquifers. This is expected, since the basic theory used to derive the Theis solution assumes a homogeneous aquifer.
- Time-drawdown data collected in observation wells or piezometers surrounding an abstraction well generally can be fitted to the Theis type curve. The resulting estimates of hydraulic conductivity do not necessarily represent the conductivity of the aquifer in the region immediately around the observation well or piezometer. Instead, they appear to be representative of an "average" value of conductivity for a large portion of the aquifer.
- The variation in calculated hydraulic conductivities about the "average" value can be much less than the variation in actual values of conductivity. Thus, the calculated values do not represent the true distribution of hydraulic conductivity. The applicability of using the "average" value will depend upon the performance model being used.
- Single-hole tests rather than multiple-hole tests may give a much better indication of the distribution of point values of hydraulic conductivity within a nonhomogeneous aquifer.
- Multiple-hole tests usually include a limited number of observation points at which drawdown data can be collected. Contours of drawdown based upon a limited number of measured drawdowns can be misleading because contouring of only a few observation points will not adequately define the irregularities in the drawdown cone. Such irregularities may be important to interpreting the aquifer characteristics. (Note that some of the irregularities apparent in the computer plots are due to the coarseness of the mesh and interpolation of heads between nodes. The irregularities referred to above are major deviations in the shape of the drawdown cone, which could be missed if a limited number of data points are available.)

Appendix IV - References

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APPENDIX V

Basic Statistical and Probability Theory
that may be used in the Derivation of Parameters

V-1.0 BASIC STATISTICAL AND PROBABILITY THEORY THAT MAY
BE USED IN THE DERIVATION OF INPUT PARAMETERS

V-1.1 Introduction

The purpose of this appendix is two-fold. The primary purpose is to describe basic techniques available for handling and reducing data from geological, geotechnical, and hydrogeological tests. In addition, this appendix clarifies the issue of uncertainty in the derivation of parameters to be used as input to performance evaluation models, and provides techniques for handling that uncertainty.

V-2.0 DATA REDUCTIONV-2.1 Graphical Displays

A useful first step when faced with a set of data representing results of some test is to organize it into graphical form. The simplest graphical representation is a bar chart known as a histogram* (see Figure V-1a). This chart represents the number of data points observed in any arbitrarily selected interval over the range of data.

Some care must be taken in the choice of interval width, since varying its width can often significantly affect the shape of the histogram.

If the ordinate scale of the histogram is divided by the total number of data points, an alternate form, called the frequency distribution, results (see right-hand ordinate Figure V-1a).

The cumulative frequency distribution, another useful graphical representation of data, is obtained from the frequency distribution by calculating the successive partial sums of frequencies up to each interval division point. These points are then plotted and connected by straight lines to form a monotonic function from zero to unity (see Figure V-1b).

The cumulative frequency distribution has the advantage of being able to display every data point. This results in an accurate representation of the data without the distortion inherent in histogram representations.

* Note: Underlined words are defined in the glossary at the end of this appendix.

GRAPHICAL DISPLAYS OF DATA

Figure V-1

(a) HISTOGRAM AND FREQUENCY DISTRIBUTION OF DATA

(b) CUMULATIVE FREQUENCY DISTRIBUTION OF DATA

V-2.2 Numerical Summaries

Given a set of data which are the result of some test or experiment, there are several easily obtained numbers which help one to quickly visualize the results. The most helpful number is the average or arithmetic mean. If the result of each of n tests is x_i , then the sample mean, $\bar{x}$, is as follows:

$$\bar{x} = \frac{\text{sum of } x}{\text{number of tests}} = \frac{1}{n} \sum_{i=1}^n x \quad [V-1]$$

The arithmetic mean is often interpreted as a typical value or central value of the data (see Figure V-2). Other measures of the central tendency of a data set includes the mode (the most frequently occurring value in the data set) and the median (the middle value in a numerically ordered list).

It is also desirable to be able to summarize in a single number something about the variability of the data. The simplest measure is the range, which is the difference between the maximum and minimum value observed. It has the advantage of being easy to calculate, but suffers from several disadvantages. It places too much emphasis on the extremes, which may be due to experimental error. The range between minimum and maximum values gives little idea of the variability of the data. Also, the range is sensitive to the number of data points.

A more satisfactory measure of dispersion is found in the sample variance. It is analogous to the moment of inertia in that it deals with the squares of distance from a center of gravity, i.e., the mean. The sample variance, S^2 , is defined:

$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad [V-2]$$

CHANGES IN DATA FREQUENCY DISTRIBUTIONS REFLECTING VARIOUS VALUES OF NUMERICAL SUMMARIES Figure V-2

(a) data with mean $\bar{X}_1$ and standard deviation S_1

(b) data with standard deviation S_1 and mean $\bar{X}_2$, where $\bar{X}_2 > \bar{X}_1$

(c) data with mean $\bar{X}_2$ and standard deviation S_2 , where $S_2 > S_1$

(d) data with different coefficients of skewness

(After Benjamin and Cornell)

To eliminate dependence on sample size (n), the squared distances are divided by n to yield an average squared deviation. (Note: there are sound reasons for dividing by $n-1$ instead of by n ; however, for the purposes of this appendix, the intuitively more satisfactory form n will be used.)

The positive square root of the sample variance, S , is known as the standard deviation. It is analogous to the radius of gyration of a structural cross section. This number has the same units as the original data and, next to the mean, it conveys more useful information than any other single number which can be computed from the set of data. Roughly speaking, the smaller the standard deviation of the sample, the more clustered about the sample mean is the data and the less frequent are large variations from the average value (compare figures V-2b and V-2c). For example, for the normal distribution (see Section V-4.4.2) 68 percent of the data will lie within one standard deviation of the mean (95 percent within two standard deviations).

When comparing the relative dispersion of more than one kind of data, it is convenient to have a dimensionless description such as the sample coefficient of variation. This value V is defined as the ratio of the sample standard deviation to the sample mean.

$$V = \frac{S}{\bar{x}} \quad [V-3]$$

One other numerical summary of observed data is simply a logical extension of the reasoning leading to the formula for the sample variance. Where the variance was the second moment about the mean, so the sample coefficient of skewness is related to the third moment about the mean. To

make the coefficient dimensionless, the moment is divided by the cube of the sample deviation. The coefficient of skewness g_1 provides a measure of the degree of a symmetry about the mean of the data:

$$g_1 = \frac{(\sqrt{n}) \sum_{i=1}^n (x_i - \bar{x})^3}{s^3} \quad [V-4]$$

The coefficient is positive for histograms skewed towards higher data values (i.e., with longer tails towards the higher values) and negative for those skewed towards the lower values (see Figure V-2d). Zero skewness results from symmetry, but does not necessarily imply it.

V-3.0 ELEMENTS OF PROBABILITY THEORY

V-3.1 General

A first step in the solution of an engineering problem is the formulation of a set of mathematical models - that is, descriptions of real situations in a simplified, idealized form suitable for computation. Frequently these models are deterministic: a single number describes each independent variable (input parameter), and a formula predicts a specific value for the dependent variable (output parameter). When a range of input parameter values, perhaps due to spatial variability or measurement error, is to be considered explicitly, the models derived are probabilistic and must be analyzed according to the rules of probability theory. Probabilistic input parameters, when used in analyses which explicitly recognize their probable nature, produce probabilistic output parameters.

V-3.2 Variables and Distributions

When a problem is modeled probabilistically (stochastically), the independent variable (input parameter) is referred to as a random variable. It is a numerical variable whose specific value cannot be represented as a single value. The behavior of a random variable is described by its probability law, which in turn may be characterized in a number of ways. The most common way is through the probability distribution of the random variable.

When the number of values that a random variable can assume is restricted to discrete values (e.g., the values 0, 1000, 2000, 3000, and 4000), its probability law is usually presented in the form of a probability mass function, or PMF (see Figure V-3a). If the random variable can assume any value within a given range (i.e., it is continuous), its pro-

DISCRETE AND CONTINUOUS RANDOM VARIABLE DISTRIBUTIONS

Figure V-3

DISCRETE RANDOM VARIABLE DISTRIBUTION

(a) Probability mass function (PMF)

(b) Cumulative distribution function (CDF)

CONTINUOUS RANDOM VARIABLE DISTRIBUTION

(c) Probability density function (PDF)

(d) Cumulative distribution function (CDF)

(After Benjamin and Cornell)

bability law is defined by a function $f_X(x)$ and presented in the form of a probability density function, or PDF (see Figure V-3c). In either case, an equivalent means of describing the probability distribution of a random variable is the cumulative distribution function, or CDF, which is simply the probability that the random variable will take on a value equal to or less than the stated value (see figures V-3b and V-3d).

Although they may be similar in appearance, the distinction between histograms and density functions and the distinction between cumulative frequency distributions and cumulative distribution functions must be well understood. The first are representations of observed empirical data; the second are descriptions of the probability laws which govern the behavior of a mathematical variable. The first are an aid to the choice of the second. Given the first, the engineer might choose to assume that it perfectly describes the mathematical model representing the probability law governing the behavior of the random variable. In this case, the two would be identical. It is possible, however, that the engineer might choose an alternate mathematical model to represent the probability law. The motive for the choice might be experience, which indicates that a certain model has historically proven to be a reliable description of the probability law or that, without sacrificing accuracy, one model is mathematically simpler to use than another. There are many reasons why one might choose different mathematical models describing the probability laws governing the behavior of random variables. Unfortunately, it is not possible to state in general which is the "best" mathematical model of the physical phenomenon.

V-3.3 Moments and Expected Values

Once a probability law has been selected, it is advantageous to define more concise descriptors (parameters) of the PDF or PMF. These are the familiar mean, variance, standard deviation, and coefficient of variation. They are defined as sums of the possible values of the random variable or as sums of the squared deviations from the mean, weighted by their probabilities of occurrence. This is analogous to the method used when these values were calculated for empirical data previously. It is sufficient for the purposes of this paper to say that these descriptors can easily be found and are extremely valuable.

V-3.4 Jointly Distributed Random Variables

Up to this point, only independent random variables have been discussed. It is sometimes the case that two (or more) random variables may be dependent. That is, knowing the value of one of them helps in determining the other. This can be of extreme importance in the case of two variables which are dependent, one of which is difficult to measure. By testing the more easily measured and knowing the dependence between them, a reliable estimate of the other can possibly be made. Knowledge of dependence between variable can also help avoid unrealistic combinations of random variables as input to a problem model (performance model).

By means of a statistical test known as correlation analysis, deductions can be made about possible dependency between variables. Care must be taken in interpretation of correlation studies. It is possible that spurious correlation may invite unwarranted conclusions of a cause-and-effect relationship between variables when, in fact, none exists. For example, a third factor might be the underlying cause for an apparent correlation.

An extension of these ideas forms the underlying basis for a fairly recent geostatistical ore reserve method developed in France in the early 1960's by G. Matheron (Matheron, 1971). The mathematical basis for the method is called the "theory of regionalized variables". Basically, the method assumes that data obtained from drill holes are, in fact, correlated to each other spatially. By measuring the correlation between data along various lines through space and for different distances along those lines, information is derived and expressed in the form of variograms which greatly enhance the process of interpolation between the drill holes. This method is used extensively in orebody evaluation, where much data as well as assay values or orebody thicknesses are available from a relatively small volume of ground. By describing the spatial correlation between sample points, variograms can be used in evaluating the adequacy of a sampling system (e.g., how many tests and on what spacing are the tests required to adequately describe the natural system).

The application of variogram analysis to the manipulation of data from geotechnical or hydrogeological tests is relatively new and untried. The major problems associated with application of the technique to repository performance evaluation, especially during the early stages of program, will result from the following factors:

- Inadequate number of tests
- Inadequate range of spacings of tests
- Inadequate spatial correlation between adjacent test results.

Variograms, once produced, may be used in assigning parametric input to two, quasi-three, or three-dimensional performance models by means of the orebody modeling technique called "kriging" (see, e.g. David, 1977).

The flexibility and benefits associated with kriging certainly justify further study of the application of geostatistical methods to performance evaluation modeling.

V-4.0 COMMON PROBABILITY DISTRIBUTIONSV-4.1 Discussion

In the application of probability theory, certain models of probability laws appear time and time again. These distributions have proven to reliably duplicate various natural phenomena and often are selected for just such a reason. In such situations, the choice among common distributions is usually based on a comparison between the shape of a histogram of data and the shape of the PDF or PMF of the mathematical distribution.

Such an empirical path is often the only one available to the engineer, but it should be remembered that in these cases the subsequent conclusions, being based on the adopted distribution, are usually dependent to a certain degree on its properties. In order to make the conclusions as accurate and as useful as possible and in order to justify, when necessary, extrapolation beyond available data, it is preferable that the choice of distribution be based whenever possible on an understanding of how the physical situation might have given rise to the distribution.

V-4.2 Distributions from Simple Discrete Random Trials

V-4.2.1 The Bernoulli Distribution: The Single Trial

This is the simplest type of experiment, one where the outcome is either a "success" or a "failure." Examples include winning or losing a race, head or tails in the flip of a coin, etc. If one outcome of the experiment has the probability of "success" of p , then the other has the probability of "failure" of $(1-p)$. The PMF describes a random variable with mean p and variance $(1-p)p$.

V-4.2.2 The Binomial Distribution: Repeated Trials

A sequence of simple Bernoulli experiments, when the outcomes of the experiments are mutually independent and when the probability of success remains unchanged, are called Bernoulli trials. The PMF of the number of successes in n number of trials is known as the binomial distribution and has the notation $B(n,p)$, where n is the number of experiments and p is the probability of "success." The PMF shown in Figure V-4(a) is described by the following:

$$P_Y(y) = \binom{n}{p} p^y (1-p)^{n-y} \quad [V-5]$$

with mean np and variance $np(1-p)$.

V-4.2.3 The Geometric and Negative Binomial Distributions: Repeated Trials

Rather than the question, "How many successes will occur in a fixed number of Bernoulli trials?" the alternative question, "At what trial will the first success occur?" may be asked. This probability law is known as the geometric distribution with notation $G(p)$. The PMF shown in Figure V-4(b) is defined by the following:

$$P(N=n) = (1-p)^{n-1} p \quad [V-6]$$

where N = the number of trials to the first success with a mean $1/p$ and variance $(1-p)/p^2$

A similar question--"On what trial will the K th success occur?"--leads to the negative binomial distribution (Pascal) and is denoted by NB

EXAMPLES OF BINOMIAL AND GEOMETRIC DISTRIBUTIONS

Figure V-4

(a) PMF of BINOMIAL DISTRIBUTION $B(n, p)$

(b) PMF of GEOMETRIC DISTRIBUTION $G(p)$

(After Benjamin and Cornell)

(K, p) . The PMF of how many trials, w , will occur before the K th success is described by the following:

$$P_{w_k}(w) = \binom{w-1}{k-1} (1-p)^{w-k} p^k \quad [V-7]$$

with mean k/p and variance $k(1-p)p^2$.

V-4.3 Distributions from Random Occurrences

V-4.3.1 Discussion

In many distributions it is not possible to identify individual discrete trials at which events (successes) might occur, but it is known that the number of trials is many. The following distributions arise out of consideration of situations where the number of trials is infinitely large. Examples include situations where events can occur at any instant over an interval of time or at any location along the length of a line or on the area of a surface.

V-4.3.2 The Poisson Distribution: Counting Events

The basic mechanism from which the Poisson process arises--namely independent incidents occurring along a continuous axis (time or space) with a constant average rate of occurrence--suggests why it is often referred to as the "distribution of random events." It has been successfully employed to describe such diverse problems as occurrences of storms, major floods, overloads on structures, and flaws in materials, as well as other "random" events. If several necessary conditions are met (these are beyond the scope of this paper), then for an average arrival rate, for

any interval of time of length, t , the number of events X which occur is described by the following:

$$P_x(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad [V-8]$$

Where $\lambda = \lambda t$ (see Figure V-5) and has a mean and variance both equal to λ .

V-4.3.3 The Exponential Distribution: Time Between Events

The distribution of time T between random, independent incidents has an exponential distribution with parameter λ and is described by the following:

$$f_T(t) = \lambda e^{-\lambda t} \quad [V-9]$$

With mean $1/\lambda$ and variance $1/\lambda^2$ (see Figure V-6a).

In the previous section λ was found to be the average number of events per unit time, while here $1/\lambda$ is the average time between arrivals. The exponential density function is denoted by $\text{Exp}(\lambda)$.

V-4.3.4 The Gamma Distribution: Time to the K th Event

The time S_k between k incidents has a gamma distribution and is described by the following:

$$F_{S_k}(s) = \frac{\lambda (\lambda s)^{k-1} e^{-\lambda s}}{(k-1)!} \quad [V-10]$$

With mean k/λ and variance k/λ^2 .

EXAMPLE OF POISSON DISTRIBUTION

Figure V-5

PMF of POISSON DISTRIBUTION $P(\nu)$

(After Benjamin and Cornell)

EXAMPLE OF EXPONENTIAL AND GAMMA DISTRIBUTION

Figure V-6

(a) PDF of EXPONENTIAL DISTRIBUTION $EX(\lambda)$

(b) PDF of GAMMA DISTRIBUTIONS $G(k, \lambda)$

(After Benjamin and Cornell)

The shape of the gamma density function denoted by $G(k, \lambda)$ (see Figure V-6b) indicates why it is widely used in engineering applications. Like observed data from many phenomena, it is limited to positive values and is skewed to the right. Notice that λ can be interpreted as a scaling parameter and k as a shape parameter of the distribution.

V-4.4 Distributions from Limited Cases

V-4.4.1 Discussion

The single most important class of distributions is that in which the distribution arises as a limit to an argument about the relationship between the phenomenon of interest and its many "causes." The uncertainty in a physical variable may be the result of the combined effects of many contributing causes, each difficult to isolate and observe. In several important cases, if something is known about the mechanism by which the individual causes affect the variable of interest, a distribution can be determined for the latter variable without studying in detail the individual effects. Three of these cases will be mentioned.

V-4.4.2 The Normal Distribution: The Distribution of Sums

The normal (or Gaussian) distribution is one of the most important and widely used distributions in engineering. This is the result of one of the most important theorems of probability--the Central Limit Theorem. It states that "under very general conditions, as the number of variables in the sum become large, the distribution of the sum of random variables will approach the normal distribution." As an example, suppose one is trying to determine the total thickness of four aquifers in order to serve as input to the performance model. Assume that each aquifer has a certain thick-

ness, y , with an error or ± 10 feet. Furthermore, assume that there is an equal chance of the error being anywhere between $+10$ feet and -10 distribution (see Figure V-7a). If two aquifer thicknesses are summed, the distribution of error in their sum is indicated by Figure V-7b; likewise if a third and a fourth are added. Each time the distribution of error gets closer and closer to the normal distribution (the dotted line in Figure V-7). There are cases in which the theorem does not hold (if all the variables are highly dependent), but since the random variation in many phenomena arise from a number of additive variations, it is not surprising that histograms approximating this distribution are frequently observed in nature.

The normal distribution is especially attractive for modelling purposes because of the ease with which normally distributed random variables can be manipulated. For example, normally distributed input parameters with easily calculated movements. In addition, the parameters of the normal distribution, and are simply mean and standard deviation which can be derived from experimental data.

The PDF of the normal distribution, which is symmetrical, is described by the following:

$$f_X(x) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - m_X}{\sigma_X} \right)^2 \right] \quad [V-11]$$

With mean m and variance σ_X^2 (see Figure V-8); its notation is $N(m, \sigma^2)$

V-4.4.3 The Lognormal Distribution: The Distribution of Products

The normal distribution arose from the sum of many small effects. Likewise, the lognormal distribution is the result of a multiplicative

DISTRIBUTIONS OF ERROR IN THE TOTAL THICKNESS OF ONE, TWO, THREE, OR FOUR AQUIFERS

Figure V-7

(a) Error probability distribution
in total thickness of
one aquifer

(b) Error distribution of sum of
two aquifers

(c) Error distribution of sum of
three aquifers

(d) Error distribution of sum of
four aquifers

(After Benjamin and Cornell)

PDF OF VARIOUS
NORMAL DENSITY FUNCTIONS

Figure V-8

PDF of NORMAL DENSITY FUNCTIONS

(After Benjamin and Cornell)

mechanism acting on a number of factors. An example of such a mechanism occurs in breakage processes such as the crushing of aggregate or transport of sediments in streams. Simply stated, the variable, x , is said to be lognormally distributed if $\log_e x$ is normally distributed. The PDF of the lognormal distribution is identical to the normal distribution, except that x is replaced with $\log_e x$.

This distribution is often used when the random variable varies by orders of magnitude. The skewed quality of the distribution, not uncommon in many kinds of data, plus the fact that the distribution avoids the non-zero probability associated with the normal model, have combined to make it a commonly used model in engineering. (See Figure V-9). In addition, the lognormal distribution is attractive for modelling purposes because, like the normal distribution, the lognormal distribution regenerates itself. The product of lognormal random variables is a lognormal r.v. with mean and standard deviation which can be calculated from input parameters means and standard deviation.

V-4.4.4 The Extreme Value Distributions: The Distribution of Extremes

This distribution helps to predict the maximum or minimum value of a number of random variables. If the proper conditions are met, then the shape of the distribution is relatively insensitive to the exact shapes of the contributing distributions. In this sense, this situation is not unlike those already encountered in the previous two sections on the normal and lognormal distributions. When dealing with extreme values, however, no single limiting distribution exists. There are, however, various examples which can be found in more complete texts dealing with statistics.

LOGNORMAL DISTRIBUTIONS
SHOWING INFLUENCE OF $\sigma_{\ln x}$

Figure V-9

(After Benjamin and Cornell)

V-4.5 Additional Common Distributions

There are numerous other distributions commonly used in engineering. These include the uniform distribution, which assigns an equal probability to all outcomes; the beta distribution, which is capable of assuming a large number of shapes; and the normal related distributions such as the Chi-square, Chi, t, and F, which are most often encountered in statistical applications.

Another distribution often used is a non-functional description of the PDF or CDF. Certain points of interest on the distribution are exactly defined: for example, the 0, 5, 50, and 100 percent probabilities. The points are then connected by straight lines. The result is a series of straight line segments closely approximating the proposed shape of the stochastic distribution. This distribution, known here as the "discretized distribution," offers several advantages. It is easily fitted to almost any set of data. It also permits segments of the distribution of particular interest (such as the tails) to be described in detail, while those of less interest can be less rigorously defined. These properties of the "discretized distribution" result in its often being preferred to more rigorous mathematical descriptions of stochastic distributions.

The ultimate choice as to which distribution to use will depend on the natural process which the distribution is intended to describe, as well as how the distribution will be used in the final analysis of the problem.

V-5.0 PROBABILITY DISTRIBUTIONS AND OBSERVED DATA

V-5.1 Estimation of Model Parameters

V-5.1.1 Method of Moments

Once a mathematical distribution describing the probability law governing a random variable is chosen, the parameter(s) which define that distribution must be determined. One way to do this is the method of moments. For a normal distribution, for example, it is essentially calculating the mean and variance from the observed data and using them as estimators of the distribution parameters. The more data collected, the closer these estimates should come to the "true" values of the distribution parameters. Based on the observed data, an interval can be defined within which a certain degree of confidence can be expressed that the "real" value of the parameter will be within it. This is known as the "confidence interval" and helps to quantify the magnitude of the uncertainty associated with that distribution parameter.

V-5.1.2 The Method of Maximum Likelihood

The method of maximum likelihood is a widely used rule for estimating the distribution parameter(s). Assuming a probability distribution, the data are assumed to fit that distribution. By varying the value(s) of the distribution parameter(s), relative likelihoods for the specific data set are obtained for the assumed parameter(s). By maximizing the relative likelihood, the "best" estimate of the distribution parameter(s) can be chosen. As an added benefit, the method also produces an approximation of the probability distribution of the distribution parameter(s) from which approximate confidence intervals may be constructed.

V-5.2 Verification of Distribution

V-5.2.1 Comparing Shapes: Histograms and Probability Paper

One of the easiest and most direct ways of verifying a chosen distribution is to compare the shapes of the PDF or CDF of the distribution to the corresponding histogram or cumulative frequency polygon. The comparison of cumulative curves offer distinct advantages. As has been mentioned previously, the shape of the histogram can be altered by selecting different interval widths. If too wide an interval is chosen, the data are "smeared" as a result of the wide, flat top of each interval. If too short an interval is chosen, the resulting curve is very irregular and discontinuous. These problems are avoided by the use of the cumulative curves, since every data point can be used if desired to construct the curve. This results in a relatively smooth curve which can show the effect of each data point.

In practice, both the plotting and comparison of cumulative curves can be simplified by scale changes; that is, by special plotting paper. By using the appropriate paper for the distribution of interest, the cumulative curve should be reduced to a relatively straight line.

The quality of "fit" of the proposed distribution to the data is determined by how "straight" the cumulative curve of the data plots on the chosen plotting paper. A relative "straight" line will indicate that the data appear to conform to the selected distribution. It should be remembered that wide excursions from the cumulative frequency polygon are very likely to occur in the "tails" of the distribution. These are most likely due to distortion caused by the probability paper's scale and can seldom be interpreted as reason for rejection of the proposed distribution.

V-5.2.2 "Goodness-of-fit" Significance Tests

There are various quantitative methods that can be used to help decide how well the chosen mathematical distribution fits the data. The most popular is known as the Chi square "goodness-of-fit" test. The purpose of the test is solely to answer the question of whether the histogram deviates a statistically significant amount from the distribution prediction. A second quantitative "goodness-of-fit" test, the Kolmogorov-Smirnov test, is based on deviations between the hypothesized cumulative distribution and the observed cumulative histogram. Although these "goodness-of-fit" tests have certain limitations there is value and convenience in their application if good judgement is applied in their interpretation.

V-6.0 SUBJECTIVE PROBABILITY: BAYES' THEORYV-6.1 Discussion

The previous sections of this appendix have been concerned with probability distributions and statistical analyses of these distributions. The emphasis has been on finding a realistic description of a random natural phenomenon. Ultimately, however, the goal of the designer is to make a decision, often with incomplete and imprecise data.

There has been little discussion of the influence that the particular situation or the analysts have on the description of the random variable. It has been assumed so far that, given the same information, the decision makers would arrive at substantially the same description of the random variables. In the preceding section, methods were presented which could be used to estimate the parameter(s) of a probability distribution, though these "estimates" would vary depending on the data available concerning the random variable. Two or more probability distributions may reasonably fit the observed data. It is not clear, then, whether the results of applied probability and statistics is necessarily influenced by the situation as well as by the analyst.

Over the past few years, a new approach to applied probability and statistics has been developed. This new philosophy recognizes not only that the ultimate use of probabilistic methods is decision making, but also that the individual subjective elements of the analysis are inseparable from the more objective elements. This new approach, Bayesian statistical decision theory, provides a mathematical model for making engineering decisions in the face of uncertainty. It derives its name from Thomas Bayes, an English mathematician who introduced the equation now used to relate certain probabilities in the decision model.

from Thomas Bayes, an English mathematician who introduced the equation now used to relate certain probabilities in the decision model.

V-6.2 Illustration of Bayes' Theorem

Perhaps the easiest way to explain Bayes' theorem is by example.

Suppose there are 1000 jars from which one jar must be selected. In 800 of the jars (type θ_1) there are 4 red balls and 6 black balls. In the other 200 (type θ_2) there are 9 red balls and 1 black ball (see Figure V-10). After selecting a jar, one ball is "sampled" from that jar. The chosen ball is red. The question now is, "What are the chances that the chosen jar is of type θ_1 ?" It is by use of Bayes' theorem that this question can be answered. According to the theorem:

$$P(\theta_1/R) = \frac{P(R/\theta_1) P(\theta_1)}{P(R/\theta_1) P(\theta_1) + P(R/\theta_2) P(\theta_2)} \quad [V-12]$$

$P(\theta_1/R)$ is read as "probability of θ_1 , given that R has occurred." As shown in Figure V-10, $P(\theta_1/R)$ or "the chance of the jar being type θ_1 , given that a red ball was chosen from it" is .64. Prior to the selection of the red ball, the probability of the selected jar being type θ_1 was .8 (800/1000), which is known as the "prior probability"; .64 is known as the "posterior probability." The above formula can then be reworded as follows:

$$\begin{matrix} \text{(posterior probability)} \\ \text{of } \theta_1, \text{ given the sample} \end{matrix} = \begin{matrix} \text{(normalizing)} \\ \text{(constant)} \end{matrix} \begin{matrix} \text{(likelihood of the)} \\ \text{sample given } \theta_1 \end{matrix} \begin{matrix} \text{(prior probability)} \\ \text{of } \theta_1 \end{matrix} \quad [V-13]$$

EXAMPLE OF APPLICATION OF BAYES' THEOREM

Figure V-10

$$P(R|\theta_1) = \frac{4}{10} = .4$$

$$P(\theta_1) = \frac{800}{1000} = .8$$

$$P(R|\theta_2) = \frac{9}{10} = .9$$

$$P(\theta_2) = \frac{200}{1000} = .2$$

$$\begin{aligned} \therefore P(\theta_1|R) &= \frac{P(R|\theta_1) \cdot P(\theta_1)}{P(R|\theta_1) \cdot P(\theta_1) + P(R|\theta_2) \cdot P(\theta_2)} \\ &= \frac{(.4)(.8)}{(.4)(.8) + (.9)(.2)} \\ &= .64 \end{aligned}$$

NOTE: $P(R|\theta_1)$ is read "Probability that a red ball will be chosen given that the jar is of type θ_1 ."

V-6.3 Probability Assignments

The above example, using red and black balls of known quantities, is the oldest and most popular view of probability measure. That is, the probability measures are assigned as a result of precise knowledge of the relative frequencies of the various outcomes. This is the classical concept of probability and its practitioners are known as "frequentists."

Unfortunately, the world and its problems do not consist entirely of jars filled with known quantities of red and black balls. There are seldom situations in which precise knowledge of all information is known. Many times information regarding the situation has not or cannot be obtained. Can a decision be made without precise information? This is a meaningless question. To not make a decision is, in effect, to make a decision. It is a decision to do nothing. The question should be, "Is it better to make a decision with partial information than to do nothing?" It is at this point that Bayesian statistical decision theory becomes useful.

In order to answer the above question, a more liberal interpretation of $P(A)$ [the probability of event A] is required. For example, the probabilities assigned by a person to the possible outcomes of an experiment can also be thought of as a set of weights which express that individual's measure of the relative likelihood of the outcomes. That is, the probability of an event might be simply a subjective measure of the "degree of belief" a person has in a judgment or prediction. Notice that if repetitions are involved, the notions of relative frequencies and degree of belief should be compatible. This interpretation of probability is an intellectual concept rather than a physical property also can serve as a basis for probability theory.

Like its interpretations, the sources of the probability measure to be assigned to the value of the parameters are also varied. The values may actually be the result of frequent observations. In this case, "classical" statistics are adequate. Reflecting the second interpretation, however, the probability measure may be assigned in a more subjective manner. For example, calling upon past experience in similar situations, a knowledge of local geology and data collected from similar sites, a geologist might state "odds" that each of several different situations might be found at a certain location. These "odds" should include any uncertainties associated with his data, experience, or interpretation of the geology. These kinds of "degree-of-belief" estimates of probability are valuable tools in the decision-making process.

V-6.4 Further Discussion of Prior and Posterior Probabilities

In the example demonstrating Bayes' theorem, the concept of prior and posterior probability distributions was introduced. The prior distribution is the initial probability distribution. It reflects not only the natural variability of the parameter, but also all the uncertainties associated with it. The more variable the parameter and the greater the uncertainty, the more "spread" or "flat" the distribution will be. The less variable the parameter and the greater the certainty, the more "peaked" will be the distribution. Receipt of new information or data (represented by the selection of a red ball in the illustrated example of Bayes' theorem) can also be visualized as a probability distribution. It can be "flat" or "peaked," depending on the "quality" of the information.

The posterior distribution should be recognized as the product of two functions: one representing prior information, the other carrying new

sample information as to the relative likelihoods of values of the unknown parameter. In Figure V-11, a number of sketches representing situations ranging from that where the prior information is well defined and the sample information minimal to that where the prior information is vague and the sample information, owing to the large sample, is dominant. Notice that the sampling always leads to a sharpened posterior distribution. This indicates a reduction in the variance of the parameter and hence in the uncertainty in the value of the unknown parameter. In those cases where one source of information is very weak, the shape of the posterior distribution is simply that of the other (or dominant) function.

PRIOR VERSUS SAMPLE INFORMATION RELATIONSHIPS Figure V-11

----- Prior distribution
———— Posterior distribution
----- Sample distribution

(a) Strong prior and relatively small sample

(b) Prior and sample information of comparable weight

(c) Vague prior and relatively large sample

(After Benjamin and Cornell)

V-7.0 APPLICATION OF BAYES' THEOREM TO DERIVATION
OF INPUT PARAMETERS FOR PERFORMANCE MODELS

V-7.1 Discussion

The problem of determining values for the parameters which will be input to the performance model is one which must be handled, at least in part, by stochastic methods. The critical parameters must be described by appropriate probability distributions. Because of constraints on the extent of site investigation, a certain amount of subjective judgment must be used to supplement the data collected. Bayes' theorem offers a means of including this information in a meaningful way.

V-7.2 Explanation of Parameter Assessment Process

Figure V-12 shows the general process which would be followed in the determination of the probability distributions of the input parameters of the performance model. Initially, information would be collected from sites with similar characteristics (generic data base). Incorporating uncertainties associated with this information and its interpretation, the initial (or prior) probability distribution would be developed. This includes determining the probability distribution to be used to model the input parameter and the parameter(s) (descriptor[s]) necessary to fully define that distribution. Next, data is collected from measurements (on site or off), and sample probability distributions (including any associated uncertainties) reflecting these data are developed. Constant care must be exercised throughout this process to eliminate "faulty data" to the extent possible. The data should be examined for obvious error, bias, or correlation with other input parameters, and appropriate corrective action should be taken. Collected data should be checked, if possible, for "fit"

PERFORMANCE MODEL PARAMETER ASSESSMENT PROCESS

Figure V-12

NOTES

- 1 This will assist in a choice of the particular stochastic distribution best suited to the parameter and the values of the descriptors necessary to totally define it.
- 2 Choose particular stochastic model for parameter, e.g., discrete uniform, beta, lognormal, t, etc.
- 3 e.g., mean, range, standard deviation, etc.
- 4 e.g., different stratigraphy, changes with depth, etc.
- 5 e.g., discard or ignore probable erroneous data points, rearrange data base as indicated by correlation analysis, etc.
- 6 i.e., change from lognormal to t distribution, etc.
- 7 e.g., use of Bayes theorem, informed engineering judgment, etc.

with the prior probability distribution. By means of Bayes' theorem, a posterior probability distribution would be developed. This new probability distribution would then become the prior distribution should more information be received or measurements taken. The process would be repeated as often as required.

V-7.3 A Work About Uncertainty

A probability distribution which describes a parameter can be visualized as being the total of two individual parts. The first part is the real variability of the measured parameter. For example, if 100 blocks of salt are taken from the same location and are tested by a process which gives the "true" strength, it is unlikely that each will give the same strength. This is due to the natural differences inherent on a microscopic scale between any two "homogeneous" samples. The probability distribution reflecting the "true" data would show only the variability of the parameter. If the test performed on these 100 blocks of salt had not given the "true" strength of salt (as no test actually does) but rather gave a strength that was within 10 percent of the strength, then uncertainty would be included in a probability distribution developed from the results of this "imperfect" test. If the two distributions could be compared, they would appear similar except that the "imperfect" distribution would be "spread" more, reflecting the added uncertainty associated with the "imperfect" test.

There are several sources of uncertainty as related to the parameter assessment process. The first has already been mentioned, indirectly, in the preceding example. These are known as parametric uncertainties. These relate to any uncertainties associated with the measurement of the

parameters. Identified bias can be accounted for, provided its degree is known. Parametric uncertainty mainly includes unidentified bias and reflects the "representativeness" of the probability distribution. In other words, how closely the measured value of the parameter is to the value actually realized "in situ."

The second uncertainty associated with the parameter assessment process is the modeling uncertainty. From limited data, some probability distribution must be chosen to describe the behavior of the parameter. How well does the chosen distribution come to really modeling the actual parameter values? Based on limited data appropriately corrected for parametric uncertainties, there is still doubt about how well the chosen probability distribution "fits" the data. This may be particularly true at the "tails" of the distribution, where statistical theory has many of its problems in predicting events.

The third uncertainty is residual uncertainty. This relates to the enormous range of "real world" conditions, processes, and possibilities (both identified and unforeseen) which are not included by the first two types of uncertainties. This could include, for example, possible changes in parameter values brought about by the effect of the repository.

Both modeling uncertainty and residual uncertainty not only apply to the parameter determination, but also the performance model itself. These performance model uncertainties can be of such magnitude that further efforts to reduce parameter uncertainty (including all three types) are of little further use. Attention should always be directed towards the areas of uncertainty, which will result in the greatest overall reduction in uncertainty. It is quite possible that the areas of uncertainty associated with the performance model will far outweigh the importance of any further reduction in individual parameter uncertainty.

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GLOSSARY

Arithmetic Mean: the average of a set of data.

Coefficient of Skewness: a dimensionless number related to the third moment about the mean of a data set which gives an indication of the type and amount of asymmetry of those data.

Correlation Analysis: a statistical test to determine if there exists any dependency between two random variables.

Cumulative Distribution Function (CDF): a mathematical description of the probability law which gives the probability of occurrence of any random variable assuming a value equal to, or less than, any particular value.

Cumulative Frequency Distribution: a graphical display derived from a frequency distribution representing the partial sums of frequencies up to each interval division point.

Frequency Distribution: a histogram with its ordinate scale divided by the total number of data points.

Histogram: a bar chart representing the number of observations in each arbitrarily selected interval over the range of data.

Median: the middle value of a numerically ordered list of data.

Mode: the most frequently occurring value in a data set.

Parameter: any of a set of physical properties whose values determine the characteristics or behavior of a system.

Probability Density Function (PDF): a mathematical description of the probability law which gives the probability of occurrence of any value.

Probability Distribution: a relationship which describes the probability of occurrence of a particular outcome of an experiment.

Probability Mass Function (PMF): a mathematical description of the probability law which gives the probability of occurrence of any discrete value.

Probability Measure: a number which describes the "chance" of a particular outcome to an experiment or test.

Random Variable: a numerical value whose specific value cannot be predicted with certainty before an experiment.

Range: the difference between the maximum and minimum value observed.

Standard Deviation: the positive square root of the variance.

Uncertainty: the degree of doubt associated with a proposed value of a parameter, as compared with its "actual" value.

Variability: the natural, inherent range of "actual" values of any parameter within a homogeneous medium.

Variance: the average squared deviation of data from the mean.

Appendix V - References

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