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**UTILITY RATE STRUCTURES AND DISTRIBUTED THERMAL
ENERGY STORAGE: A COST/BENEFIT ANALYSIS**

Basic Research Report, October 1978—February 1979

By
Evan F. Koenig

February 1979

Work Performed Under Contract No. ET-78-S-01-3411

The George Washington University
School of Engineering and Applied Science
Washington, D. C.

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U.S. Department of Energy

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**UTILITY RATE STRUCTURES AND DISTRIBUTED
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**BASIC RESEARCH REPORT
FOR PERIOD OCTOBER 1978 - FEBRUARY 1979.**

**Evan F. Koenig
Conducted under the direction of
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February 1979

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**Prepared for the U.S. Department of Energy
Energy Storage Division
Under Special Research Support Agreement ET-78-S-01-3411**

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ABSTRACT

This paper examines three alternative methods by which electric utilities might take advantage of distributed thermal energy storage to smooth out their load profiles. These three methods are: (1) time-specific rates, (2) time invariant rates with subsidized storage, and (3) direct load controls. The optimal form of each of these policies is determined, and formulas indicating the relative desirability of each policy are developed.

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NOMENCLATURE

VARIABLES

X	--rate of electricity generation
K	--base load generating capacity
q, r, s	--rate of consumption of off-peak electricity, peak electricity, and energy from storage by an individual, respectively
p', p''	--prices of off-peak and peak electricity, respectively
p	--price of electricity, both peak and off-peak, when storage is subsidized
p^o	--price of electricity, both peak and off-peak, under a system of direct load controls without subsidization of storage
m	--measure of subsidy per unit of time, during peak hours, to storage
p^+	--marginal cost of energy obtained from storage

FUNCTIONS

$C(X, K, \bar{K})$	--cost per unit time of generating and distributing electricity at rate X , as a function of X , currently operable base load generating capacity (K), and average base load generating capacity ($\bar{K}$)
$v_1^i(q), v_2^i(r), v_3^i(s)$	--individual i 's willingness to pay for consumption of off-peak electricity, peak electricity, and energy from storage, respectively, in each case as a function of the rate of consumption
$V_1(p'), V_2(p''), V_3(p^+)$	--aggregate willingness to pay for consumption of off-peak electricity, peak electricity, and energy from storage, respectively, as a function of the corresponding marginal costs of these forms of consumption
$q^i(p'), r^i(p''), s^i(p^+)$	--individual i 's desired rate of consumption of off-peak electricity, peak electricity, and energy from storage, respectively, as a function of the corresponding marginal costs of these forms of consumption
$Q(p'), R(p''), S(p^+)$	--aggregate desired rate of consumption of off-peak electricity, peak electricity, and energy from storage, respectively, as a function of the corresponding marginal costs of these forms of consumption
$V_4(m, p')$	--aggregate willingness to pay for storage capacity and energy from storage, as a function of the subsidy given storage and the price of off-peak electricity

$s^i(m, p')$	--rate of consumption of stored energy and investment in storage equipment by individual i as a function of the subsidy given to storage and the price of off-peak electricity
$S(m, p')$	--aggregate rate of consumption of stored energy and investment in storage equipment as a function of subsidy to storage and the price of off-peak electricity
$K(p'')$	--base load generating capacity as a function of the price of electricity on-peak
$\pi(p', p'', m)$	--expected net social benefits as a function of the price of electricity off-peak, the price of electricity on-peak, and the subsidy given storage equipment

PARAMETERS

c_o, c_1	--parameters related to cost of electricity generation
n	--cost, per unit time, of base load generating capacity
a_o^i, a_1	--parameters related to individual i 's willingness to pay for off-peak consumption of electricity
b_o^i, b_1	--parameters related to individual i 's willingness to pay for on-peak consumption of electricity
$g_o^i, \bar{g}_o^i, g_1$	--parameters related to individual i 's willingness to pay for consumption of electricity from storage
A_o, B_o, G_o	--the averages, across individuals, of a_o^i, b_o^i , and g_o^i respectively
k	--a measure of the cost of a unit of storage equipment
l_1, l_2	--lengths of the off-peak and peak periods, respectively
α	--the efficiency of storage units
I	--number of customers served by the utility company

GENERAL NOMENCLATURE

When found above a variable or function, the symbol

- $\sim$ indicates the presence of a random disturbance term
- $-$ indicates the mathematical expectation has been taken
- $\wedge$ indicates optimality or an equilibrium value.

A superscript " i " indicates that the variable, function, or parameter is defined with reference to a particular individual.

ELECTRIC UTILITY RATE STRUCTURES AND DISTRIBUTED THERMAL ENERGY STORAGE: A COST-BENEFIT ANALYSIS¹

by
Evan F. Koenig²

1. Introduction and Summary³

The demand for electricity varies widely with time when the price of electricity is time invariant. These variations are to some extent random, but have a substantial systematic component. Meeting the demand for electricity as it arises is expensive for utility companies, because a large proportion of their capacity is idle much of the time. The companies could damp some of the fluctuations by investing in centralized storage technologies, but this is not necessarily the most efficient approach, and in any case, to the extent that such storage is not costless, considerable variations in marginal production costs will remain⁴

Possibly the most extensively analyzed method of dealing with the peak load problem is the introduction of time-specific pricing. Under time-specific pricing, the cost of electricity to consumers at each instant would be set equal to the marginal cost of supplying that electricity at that instant. Consequently, electricity would be more expensive at times when demand was high than at times when demand was low. Neglecting implementation costs, such a pricing system can be shown to be economically efficient⁵. That is to say, once the system was in operation, it would be impossible to make any individual better off without making some other individual worse off.

Time-specific pricing reduces fluctuations in production, hence average production costs, in two ways. First, it smooths out the time path of consumption by making it more expensive to use electricity when marginal production costs are high, less expensive when marginal production costs are low. Second, it gives consumers an incentive to install energy storage systems, allowing them to purchase energy to be used in

peak demand periods at lower, off-peak rates^{6,7}.

Implementation costs are, in fact, not negligible. In practice, time-specific rate schedules divide each day into a relatively small number of periods, with the price of electricity constant within each. A separate meter is required for each price regime, and some method for switching from one meter to another. Once the cost of this equipment is introduced into the analysis, it is no longer clear that time-specific pricing is superior to other load management techniques⁸.

Direct load controls are one alternative to time-specific pricing. Under a system of direct load controls, the price of electricity would be time invariant, and the utility would be able to shut off or limit each customer's purchases of electricity during peak hours. Demand patterns would obviously be altered under this system, even if no investment in distributed storage resulted. In fact, such investment would probably occur, since it would allow customers to more closely approximate their preferred consumption patterns than would otherwise be the case. Storage allows customers to relax the link which exists between purchases of electricity and consumption of energy.

A second alternative to time-specific electricity pricing is subsidization of storage. If customers with thermal energy devices in their homes were granted rebates on their monthly electricity bills, investment in these devices would clearly be encouraged. Installation of storage devices would, in itself, not alter the pattern of demand for electricity. Given a constant price for electricity, in fact, customers would have no incentive to use the storage devices at all. Even if used, there would be no incentive to time the charge and discharge periods so as to correspond respectively with off-peak and peak periods of electricity demand.

Subsidization of storage then, would not make economic sense unless a timing device was installed with each storage unit. This device would ensure that storage units were charged during periods of low electricity demand, and available for discharge during periods of high demand for electricity. Each customer would have an incentive to maintain the efficiency of his storage unit if he were automatically billed for the electricity needed to charge the unit to its maximum capacity.

Both of the alternatives to peak load pricing outlined above, direct load controls and subsidization of storage, do not require the installation of additional meters. Timing or switching devices are required under all three systems. As far as implementation costs are concerned then, a system of time invariant prices combined with direct load controls or subsidies to storage probably dominates peak load pricing.

What is needed, therefore, is a way to compare the disadvantage which peak load pricing has relative to other systems with respect to implementation costs, to the advantage which it has over other systems when these costs are neglected. Differences in implementation costs are fairly easy to estimate: the real difficulty is measuring the efficiency advantage of peak load pricing when metering costs and the like are ignored. This paper constitutes an attempt to show how a dollar figure can be placed upon this advantage.

The tool that economists use when studying questions of this sort is cost-benefit analysis. The notion underlying cost-benefit analysis is that a project yields net benefits to society if those who would gain from its implementation could compensate those who would lose, and still feel better off. The quantity of net social benefits is measured by the dollar amount that could then be taken from the net gainers before they would feel no better off and no worse off than they were originally. The methodology has limitations, but is probably the best guide to public policy that the discipline of economics has to offer.⁹

In general, the desirability of any load management technique will depend not simply upon its effect upon the load factor of a utility company, nor only upon the savings in production costs which result from its implementation, but also upon its effects on consumers. This paper attempts to consider these effects explicitly¹⁰. Further, a measure of uncertainty in demand and cost conditions is introduced. Because of this latter feature, the net social benefit resulting from any load management strategy will be a random variable. Attention, therefore, centers on expected net social benefits, where the word "expected" is used in a mathematical sense.

As noted above, this paper examines only three of a multitude of alternative approaches to load management. These three are (1) peak

load pricing, (2) a single price with subsidization of storage, and (3) a single price with direct load controls on those devices which in peak periods would be supplied from storage under a peak load pricing system. As will become clear, approach (3) is equivalent to approach (2) with subsidies restricted to be zero. In order to make the comparison of these systems as interesting as possible, only the best manifestations of each will be explored. For example, before comparing peak load pricing to a single price system with subsidized storage, it will be necessary to find what the peak and off-peak prices of electricity should be, given that one is going to implement peak load pricing. Similarly, it will also be necessary to determine how the price of electricity should be set and subsidies be structured, given that one is going to use the single price/subsidization approach. It will then be possible to compare the best peak load pricing system to the best single price system with subsidies to storage.

It will be shown below that the best single price system involves subsidies to storage which result in the same investment in storage by customers as results from the implementation of the best peak load pricing system. The difference between the net social benefits yielded by the two systems depends only upon the cost of base load generating capacity and the slopes of the peak and off-peak demand curves for electricity. In fact, the only storage related parameter important to any of the analysis is the efficiency of the storage technology.

On the other hand, if direct load controls are substituted for subsidies to storage in a single price system, the influence of storage related parameters is pervasive. Investment in storage falls short of the levels attained by the other load management strategies, and so do social benefits.

The formulas derived in this paper must be used with caution. They are based on a considerable number of simplifying assumptions. The exercise is useful, however, in that it isolates those parameters which are important in a discussion concerning the merits of peak-load pricing relative to other load management techniques. Further, it is hoped that the formulas will help public service commissions and utility companies obtain good initial estimates of the consequences of proposed reforms.

II. The Model

A. The Actors

There are three sets of actors which enter into the analysis. First, there is a set of customers. Each customer must first decide how much storage capacity he will invest in. This decision will depend upon the personal preferences of the consumer, the cost of storage equipment net of any subsidies which might be offered, the efficiency of available equipment, and the prices of peak and off-peak electricity. It is a long-run decision in the sense that once storage equipment of a given quantity and type has been installed, the customer will find it inordinately expensive to modify the equipment in response to day-to-day fluctuations in his situation. Only if there is a sustained shift in one of the variables or parameters which influenced his initial choice will the customer adjust the quantity or type of equipment in which he has invested.

In the short-run, with his storage capacity fixed, each customer will, in general, decide how much electricity to purchase in peak and off-peak periods, and how much to store. In the analysis which follows, it will be assumed that a particular subset of each individual's energy needs will be met out of storage during peak periods, and only out of storage. Under certain further assumptions about the structure of the load management techniques employed and the parameters of the customers decision problems, each individual will always fully charge his storage system during off-peak hours. The quantity of electricity consumed by an individual in the off-peak period will depend only upon his personal preferences and the price of electricity at that time. Energy consumed from storage will be determined completely by the size of the investment in storage equipment.

The second actor in the analysis is the electric utility company. The utility is unable to influence the timing or quantity of electricity purchases under the peak load pricing system. Under the one-price systems there will, of course, be time switches installed in people's homes. But the rules governing the operation of these switches will

here be assumed to be an integral part of system structure, fixed by the public service commission, not the utility. The utility will thus have to meet the demand for electricity as it arises.

With the quantity of electricity purchased determined solely by demand, the utility company is left with the responsibility of designing its generating plant so as to minimize the cost of supply. The mix of equipment used to supplement base load capacity will be taken as fixed, but base load capacity itself will be assumed to be subject to the utility's control. The cost of making day-to-day changes in base load capacity is prohibitive, so capacity will be selected so as to minimize long-run expected unit costs. It is shown that the optimal level of base load capacity equals peak demand for electricity less a term which is an increasing function of the cost of a unit of capacity and a decreasing function of rapidity with which costs rise once base load capacity is exceeded.

As hinted above, the final actor in the analysis is the government, perhaps in the form of a public service commission. It is the government which determines the price to be charged for electricity, and the subsidy, if any, to be given to storage. These decisions, like the utility company's concerning base load capacity and the customers' concerning investment in storage equipment, are taken to be revised only infrequently, in response to sustained shifts in conditions.

A different set of decisions concerning prices and subsidies will, of course, be optimal depending upon which load management technique is employed. Formulas for the optimal prices and subsidies will be derived in Section III. Just as the utility company must take into account the behavior of consumers when deciding upon the size of its generating plant, so must the public service commission take into account the response of both the utility company and its customers when setting the price(s) of electricity and the subsidy (if any) to storage.

B. The Customers

There are a total of i customers. Parameters, variables, and functions related to a particular individual will, except where there is no ambiguity, bear the superscript " i ". Time is divided into alternating peak and off-peak periods of fixed length. While the analysis is

quite general, the issues to be examined are most interesting in the case where the combined length of a two-period cycle is one day. The length of the off-peak period will be denoted l_1 while that of the peak period will be denoted l_2 . If time is measured in hours, l_1 and l_2 might both be twelve hours.

A distinction will be made between electricity consumption during peak periods and energy consumption during peak periods. Energy consumption during peak periods includes both energy drawn from the utility company in the form of electricity, and thermal energy, which is drawn from storage. Similarly, electricity purchases during off-peak hours may exceed electricity consumption during the same hours because some of the purchased electricity may be stored for consumption during the immediately succeeding peak period. Long-term storage is not considered.

While each individual's purchases and consumption of electricity and energy may vary from period to period over time, within any particular period the rate of purchase or consumption will be assumed constant. A typical pattern of electricity demand by an individual is illustrated in Figure 1.

Figure 1. Example of an Individual's Demand Pattern

For purposes of the analysis, each customer is completely characterized by three functions: $\tilde{v}_1^i(\cdot)$, $\tilde{v}_2^i(\cdot)$, and $\tilde{v}_3^i(\cdot)$. $\tilde{v}_1^i(q)$ represents how much money individual i is willing to pay to maintain off-peak electricity consumption at rate q . $\tilde{v}_2^i(r)$ represents i 's willingness to

pay for on-peak electricity consumption at rate r . Similarly, i is willing to pay the amount $\tilde{v}_3^i(s)$ to maintain on-peak consumption of energy from storage at rate s .

At low rates of electricity consumption, one would expect an individual's willingness to pay for electricity to be an increasing function of the quantity consumed. Thus, an individual would typically be willing to pay more for 2 KWh of electricity over a specified time period than for 1 KWh of electricity over that same time period. On the other hand, one would expect individuals to take care of their most pressing needs first, so that marginal willingness to pay declines as the rate of electricity consumption increases. Thus, one would expect an individual to be less willing to pay for an additional KWh of electricity over a specified time interval when his original rate of consumption is two KWh over that time interval, than when his original rate of consumption is only one KWh over the interval.

The most analytically convenient functional form with these properties is the quadratic. Consequently, it will be assumed that the willingness to pay functions of individual i are of the form:

$$\tilde{v}_1^i(q) = (\tilde{a}_0^i q - \frac{1}{2} a_1 q^2) \bar{z}_1, \quad (1)$$

$$\tilde{v}_2^i(r) = (\tilde{b}_0^i r - \frac{1}{2} b_1 r^2) \bar{z}_2, \quad (2)$$

$$\tilde{v}_3^i(s) = (\tilde{g}_0^i s - \frac{1}{2} g_1 s^2) \bar{z}_3, \quad (3)$$

where $\tilde{a}_0^i$, a_1 , $\tilde{b}_0^i$, b_1 , $\tilde{g}_0^i$, and g_1 are all positive, and $\bar{a}_0^i$, a_1 , $\bar{b}_1^i$, b_1 , $\bar{g}_0^i$, and g_1 are all constant. The marginal valuation placed upon a given rate of consumption is in each case subject to additive disturbances. If the weather turns cold and dark, for example, the value placed upon a small increase in the rate of electricity consumption at any given initial rate of consumption is likely to rise. The parameters a_1 , b_1 , and g_1 measure, respectively, the speed with which the marginal valuations of off-peak electricity consumption, on-peak electricity consumption, and on-peak consumption of energy from storage decline as the corresponding rate of consumption increases.

There is a close relationship between the willingness to pay function of an individual and his demand curve for the corresponding commodity. Thus, if p' denotes the price of off-peak electricity, individual

i will choose his rate of consumption so as to

$$\max_q v_1^i(q) - p'qz_1 \quad (4)$$

The rate of consumption is chosen so as to maximize the difference between what the consumer is willing to pay and the amount which he has to pay.

It follows from the first order conditions of the consumer's maximization problem that his consumption of off-peak electricity is given by

$$\tilde{q}^i(p') \equiv (\tilde{a}_0^i - p')/a_1 \quad (5)$$

Similarly, the demand curves for on-peak electricity and energy from storage are

$$\tilde{r}^i(p'') \equiv (\tilde{b}_0^i - p'')/b_1 \quad \text{and} \quad (6)$$

$$\tilde{s}^i(p^+) \equiv (\tilde{g}_0^i - p^+)/g_1 \quad (7)$$

respectively, where p'' is the price of on-peak electricity and p^+ is the marginal cost of energy from storage.

Note that these demand curves are linear and decreasing in their respective price variables. If willingness to pay is high (a_0^i , b_0^i , or g_0^i large), so is demand. Note also that the demand for each type of energy (off-peak, stored, on-peak electricity) is independent of the price of other types. In reality electricity consumption in off-peak periods probably increases as the price of on-peak electricity rises, so this treatment can only be regarded as an approximation. On the other hand, given the assumption that individuals never find it optimal to supplement storage with on-peak electricity, it is probably not unrealistic to take the demand for on-peak stored energy to be independent of the price of electricity obtained from the utility. One's demand for hot water, say is probably not much influenced by the price of the electricity used to run one's television set.

Aggregate consumption of on and off-peak electricity and on-peak energy from storage is found by simply adding up the rates of consumption of individuals. These aggregates are thus given by

$$\tilde{Q}(p') \equiv \sum_i \tilde{q}^i(p') = I(\tilde{A}_0 - p')/a_1, \quad (8)$$

$$\tilde{R}(p'') \equiv \sum_i \tilde{r}^i(p'') = l(\tilde{B}_0 - p'')/b_1, \quad \text{and} \quad (9)$$

$$\tilde{S}(p^+) \equiv \sum_i \tilde{s}^i(p^+) = l(\tilde{G}_0 - p^+)/g_1, \quad (10)$$

where $\tilde{A}_0 \equiv (\sum_i \tilde{a}_0^i)/l$, $\tilde{B}_0 \equiv (\sum_i \tilde{b}_0^i)/l$, $\tilde{G}_0 \equiv (\sum_i \tilde{g}_0^i)/l$, and l is

the total number of customers served by the utility.

It is also important to determine the amount which consumers as a whole are willing to pay to sustain their rates of consumption. Again, this involves simply adding across consumers:

$$\tilde{V}_1(p') \equiv \sum_i \tilde{v}_1^i(\tilde{q}^i(p')) = l_1 \sum_i ((\tilde{a}_0^i)^2 - (p')^2)/2a_1, \quad (11)$$

$$\tilde{V}_2(p'') \equiv \sum_i \tilde{v}_2^i(\tilde{r}^i(p'')) = l_2 \sum_i ((\tilde{b}_0^i)^2 - (p'')^2)/2b_1, \quad (12)$$

$$\tilde{V}_3(p^+) \equiv \sum_i \tilde{v}_3^i(\tilde{s}^i(p^+)) = l_2 \sum_i ((\tilde{g}_0^i)^2 - (p^+)^2)/2g_1. \quad (13)$$

$\tilde{V}_1(p')$ is the total amount customers are willing to pay for the electricity they consume in off-peak hours when the price of this electricity is p' . $\tilde{V}_2(p'')$ is the total amount customers are willing to pay for the electricity they consume in the peak period when the price of this electricity is p'' . Finally, $\tilde{V}_3(p^+)$ is the total amount customers would be willing to pay for the energy they receive from storage in peak periods were the price of this energy p^+ .

Up to this point it has been implicitly assumed that each individual is capable of obtaining as much stored energy on-peak as desired, at the price p^+ . In fact, the amount of stored energy available to a particular consumer will generally be limited by the capacity of his storage equipment, which cannot vary except over the long term. Consider the determination of $\tilde{s}^i$, now interpreted as the capacity of individual i 's storage equipment, measured in terms of its ability to supply on-peak thermal energy.

Note first that only short-term storage is considered: devices capable of meeting on-peak thermal needs with electricity consumed in the immediately preceding off-peak period. Each unit of off-peak energy put into storage is assumed to yield only α units on-peak energy, where

$0 < \alpha < 1$. Finally, the cost of a unit of storage capacity will be denoted by $k\lambda_2/(\lambda_1 + \lambda_2)$, where $k > 0$.

Suppose as before, that the prices of peak and of off-peak electricity are p'' and p' respectively. Suppose also that a subsidy of $m\lambda_2/(\lambda_1 + \lambda_2)$ dollars per unit of storage capacity per unit of time is offered to customers. The cost of fully charging a device of capacity s will then be $p's\lambda_2/\alpha$. Assuming that the individual, i , who owns the device in question uses all of the stored energy, and does not supplement it with on-peak electricity, the benefits he derives will be $\tilde{v}_3^i(s) = (\tilde{g}_0^i s - \frac{1}{2}g_1s^2)\lambda_2$. Maximizing expected benefits less costs we obtain individual i 's demand for storage capacity

$$s^i(m, p') = (\bar{g}_0^i + m - k - p'/\alpha)/g_1 \quad (14)$$

Given a storage capacity of s^i , individual i will, in fact, not find it optimal to supplement that capacity with on-peak electricity provided the marginal benefits from stored energy when the storage system is fully utilized fall short of the cost of supplementary energy. Formally the condition is $g_0^i - g_1s^i < p''$, or

$$g_0^i - \bar{g}_0^i \leq p'' + m - k - p'/\alpha. \quad (15)$$

Similarly, a storage device with capacity s^i will be fully charged during the preceding off-peak period provided expected marginal benefits exceed the cost of the electricity required to yield additional stored thermal energy on-peak. Thus, we must have $\bar{g}_0^i - g_1s^i \geq p'/\alpha$, or

$$\bar{g}_0^i - \bar{g}_0^i \geq m - k \quad (16)$$

where $\bar{g}_0^i$ is the mean of $\tilde{g}_0^i$ conditional upon information available to the customer during the off-peak period. Finally, all energy stored by individual i will be consumed unless doing so would result in negative marginal benefits. Hence all stored energy will be utilized provided $g_0^i - g_1s^i \geq 0$, or

$$g_0^i - \bar{g}_0^i \geq (m - k - p'/\alpha) \quad (17)$$

We have assumed, and will continue to assume, that each of these three inequalities is always satisfied under the optimal peak-load pricing

system. They will, in reality, not always be satisfied if $p'' - p'/\alpha$ is small, or if the variance of $\tilde{g}_0^i$ is large.

To get a formula for the aggregate investment in storage, it is necessary only to sum equation (14) across individuals. This yields

$$S(m, p') \equiv \sum_i s^i(m, p') = l(\bar{G}_0 + m - k - p'/\alpha)/g_1 \quad (18)$$

where $\bar{G}_0 = (\sum \bar{g}_0^i)/l$. Aggregate investment in storage is thus a linear function of the subsidy given a unit of storage equipment ($m l_2 / (l_1 + l_2)$), the price of a unit of equipment ($k l_2 / (l_1 + l_2)$), and the price of off-peak electricity (p'). The function is increasing in the first of these variables, decreasing in the others.

Individual i 's willingness to pay for energy obtained from storage is still given by equation (3). Summing across individuals:

$$\tilde{V}_4(m, p') \equiv \sum_i \tilde{v}_3^i(s^i(m, p')) = \sum_i l_2 \left[\tilde{g}_0^i s^i(m, p') - \frac{1}{2} g_1 s^i(m, p')^2 \right] \quad (19)$$

$\tilde{V}_4(m, p')$ is the total value which customers put upon the energy they obtain from storage when the subsidy to storage equipment is $m l_2 / (l_1 + l_2)$ and the price of off-peak power is p' . Expected benefits are thus:

$$\bar{V}_4(m, p') = l_2 \sum_i \left[(\bar{g}_0^i)^2 - (k - m + p'/\alpha)^2 \right] / 2g_1 \quad (20)$$

C. The Utility Company

The utility company will choose its base load generating plant so as to minimize the expected costs of meeting the demand for electricity as it arises. Once plant size has been determined, it will not generally change unless there is a sustained shift in cost parameters or demand. The exception is random fluctuations in capacity due to equipment failure. For present purposes, these fluctuations will be assumed to be independent of fluctuations in the demand for electricity, and their average magnitude will be assumed not to depend upon mean base load generating capacity.

There are basically three components to the utility's production costs. First, there are capacity costs. These will be assumed to take the form $n\bar{K}$, where n is the cost of a unit of capacity per unit time

and $\bar{K}$ is mean base load generating capacity. Thus a doubling of plant size is assumed to double capacity costs. Note that capacity costs are "fixed", that is, they are independent of short run fluctuations in the level of production. This is a direct consequence of the fact that capacity is also independent of these fluctuations.

Second, there is a cost component which is proportional to the rate of production. If X represents the rate of production per unit time, there is thus a cost component of the form $\tilde{c}_0 X$. Here $\tilde{c}_0$ may be thought of as the cost of the quantity of fuel needed to produce one unit of electricity when the rate of production is below capacity. As long as capacity is not exceeded, doubling the rate of production is assumed to double the rate of fuel consumption, thus doubling the total cost of the fuel consumed. Since the cost of a unit of fuel may vary over time, the utility regards it as a random variable. The fluctuations in this random variable will be assumed to be independent of those in the rate of demand and in base generating capacity.

Once base generating capacity is exceeded, fuel related costs are no longer simply proportional to the rate of output. The utility will be forced to rely on increasingly expensive types of fuels and progressively more inefficient equipment. Costs will rise faster and faster as output expands, given the level of base generating capacity. This suggests there is a third, quadratic component to the utility's cost function. In particular it suggests that when the rate of production (X) exceeds capacity ($\bar{K}$), fuel costs will take the form $\tilde{c}_0 X + \frac{1}{2} c_1 (X - \bar{K})^2$. The parameter c_1 measures the rapidity with which production costs increase at the margin as output expands past base load capacity. As before $\tilde{c}_0$ measures the price of the fuel used to generate base load electricity. The price of each of the fuels used to generate electricity when capacity is already fully utilized is taken to equal $\tilde{c}_0$ plus a time-invariant premium. As output expands, the greater the premium over the price of base load fuel which must be paid, since more expensive types of fuel are utilized.

Production costs thus contain a fixed and a proportional component when output falls short of capacity, and include a quadratic component when output exceeds capacity. If $\tilde{C}(X, \bar{K})$ represents the total cost

per unit time of producing electricity at rate X when current base load generating capacity is $\tilde{K}$ and mean base load capacity is $\bar{K}$, then

$$\tilde{C}(X, \tilde{K}, \bar{K}) = \begin{cases} \tilde{c}_0 X + n\bar{K} & \text{if } X \leq \tilde{K} \\ \tilde{c}_0 X + \frac{1}{2} c_1 (X - \tilde{K})^2 + n\bar{K} & \text{if } X \geq \tilde{K} \end{cases} \quad (21)$$

It follows that, at the margin, the cost of expanding production is given by:

$$\tilde{M}(X, \tilde{K}, \bar{K}) \equiv \frac{\delta}{\delta X} \tilde{C}(X, \tilde{K}, \bar{K}) = \begin{cases} \tilde{c}_0 & \text{if } X \leq \tilde{K} \\ \tilde{c}_0 + c_1 (X - \tilde{K}) & \text{if } X \geq \tilde{K} \end{cases} \quad (22)$$

$$(23)$$

The marginal cost curve is illustrated in Figure II for two cases: one in which $c_0 = \bar{c}_0$ and $K = \bar{K}_1$, and the other in which $c_0 = \bar{c}_0$ and $K = \bar{K}_2 > \bar{K}_1$. The diagram, in other words, illustrates the effects of an increase in base load generating capacity.

Key: $aa - \bar{M}(\cdot, \bar{K}_1, \bar{K}_1)$

$aa' - \bar{M}(\cdot, \bar{K}_2, \bar{K}_1)$

Figure II. The Effect of Increased Base Load Generating Capacity Upon the Marginal Cost Curve.

Recall that aggregate consumption of off-peak electricity is denoted by $\tilde{Q}(p')$, aggregate consumption of on-peak electricity by $\tilde{R}(p'')$, and aggregate stored energy available on-peak by $S(m, p')$. Recall also that $\frac{1}{\alpha}$ units of off-peak energy are required to yield one unit of stored energy on-peak. Total demand for off-peak electricity is $\tilde{Q}(p') + S(m, p')L_2/\alpha L_1$, while total demand for on-peak electricity is given by $\tilde{R}(p'')$. Suppose that demand for off-peak electricity can always be met by using base load generating capacity. Suppose further that the demand for on-peak electricity always exceeds base load capacity. The utility company will choose $\bar{K}$ so as to minimize expected costs, i.e., to minimize

$$\begin{aligned} E\{L_1 \tilde{C}[\tilde{Q}(p') + S(m, p')L_2/\alpha L_1, \tilde{K}, \bar{K}] + L_2 \tilde{C}[\tilde{R}(p''), \tilde{K}, \bar{K}]\} = \\ E\{L_1 \tilde{C}_0(\tilde{Q}(p') + S(m, p')L_2/\alpha L_1) + L_2 \tilde{C}_0 \tilde{R}(p'') + L_2 \frac{1}{2} c_1 (\tilde{R}(p'') - \bar{K})^2 + \\ (L_1 + L_2)n\bar{K}\} \end{aligned} \quad (24)$$

The solution to this problem is

$$\bar{K}(p'') \equiv \bar{R}(p'') - n(L_1 + L_2)/c_1 L_2 \quad (25)$$

Base capacity will thus fall short of expected peak demand by an amount which is an increasing function of the cost of additional capacity. Intuitively, each unit of base generating capacity reduces the expected costs of meeting peak demand, but the savings fall as base capacity is increased and reliance on the most expensive sources of supplementary peak power is reduced. Marginal savings fall to zero when base capacity equals base demand. The optimum is reached before this however, at the point where marginal savings equal the cost of an additional unit of base capacity. Note that the formula for the optimal level of base capacity is consistent with the assumption that on-peak demand can only be met if this capacity is supplemented.

Substituting back into (24), expected production costs over the off-peak/peak cycle are obtained as a function of p' , p'' , and m :

$$\begin{aligned} E\{L_1 \tilde{C}(\tilde{Q} + S L_2/\alpha L_1, \tilde{K}, \bar{K}) + L_2 \tilde{C}(\tilde{R}, \tilde{K}, \bar{K})\} = \\ L_1 \tilde{C}_0(\bar{Q}(p') + S(m, p')L_2/\alpha L_1) + L_2 \tilde{C}_0 \bar{R}(p'') + L_2 \frac{1}{2} c_1 \text{var}(\tilde{K}) - \\ \frac{1}{2} n^2 (L_1 + L_2)^2 / c_1 L_2 + L_2 \frac{1}{2} c_1 \text{var}(\tilde{R}(p'')) + n(L_1 + L_2)\bar{R}(p'') \end{aligned} \quad (26)$$

Note that the variance and n^2 terms are independent of p' , p'' , and m .

III. Comparing Alternative Load Management Strategies

A. Introduction

In Section IIB, formulas were derived for the behavior of customers as a function of the peak and off-peak prices of electricity and the subsidy, if any, given to storage equipment. Specifically, these formulas related to the amount of off-peak electricity consumed, the amount of off-peak electricity stored, and the demand for electricity on-peak. (See equations (8), (9), and (18).) Having established the equations governing consumer behavior, it was possible to derive measures of the aggregate value placed upon off-peak electricity consumption, stored energy, and on-peak electricity consumption by customers. (See equations (11), (12), and (19).) Again, these valuations were a function of the on- and off-peak prices of electricity and the subsidy, if any, given a unit of storage equipment.

In Section IIC, the behavior of the utility company was examined. The utility was assumed to maximize its profits (minimize its costs), taking consumer behavior as given. A formula was derived for the size of the base load generating plant resulting from this maximization. (See equation (25).) The amount of base load capacity was found to be a function of the price of on-peak electricity. Substituting this formula into the utility's cost function, expected production costs were obtained (equation (26)).

Social welfare in this paper is measured by the difference between expected benefits and expected costs. Benefits are measured by consumers' own willingness to pay for the goods they receive. Costs include the costs of production, and the costs of installing and operating storage equipment. Subsidies, if any, given to storage are benefits to those who receive them, and costs to those who pay them. In terms of the present analysis, they thus have no direct impact on social welfare. They have an indirect impact insofar as they change consumers' behavior.

The governmental or quasi-governmental body which sets the prices of on- and off-peak electricity and the amount of subsidy given storage will be assumed to do so with the goal of maximizing social welfare in view. The first step in the analysis of this section will be to determine what course the rate-setting authority should take when it is free

to charge separate prices for peak and off-peak electricity, and offer subsidies to storage. The formulas derived will thus characterize that load management strategy which is socially optimal within the class of peak-load pricing systems. This is also the strategy which dominates all other load management techniques when implementation costs are negligible. It will be seen that no subsidies to storage are offered under the optimal peak-load pricing system. The price of off-peak power should equal the expected marginal cost of producing off-peak electricity, which is independent of the rate of production in this model. The price of on-peak electricity should equal the price of off-peak electricity plus the price of base-load generating capacity. It will also equal the expected marginal cost of producing electricity on-peak.

In the second step of the analysis, the prices of peak and of off-peak electricity will be constrained to be equal. A public service commission might want to consider a one-price structure of this sort as a means of holding down metering costs. Subsidies to storage will again be allowed. The best of the pricing systems of this type will be shown to have the following characteristics: (1) the price of electricity will be a weighted average simultaneously of the expected marginal costs of peak and off-peak production, and of the electricity prices optimal under peak load pricing, (2) the price of electricity will be independent of parameters related to the demand for or technology of storage, (3) a positive subsidy will be offered to storage, such that each individual will have exactly the same storage capability as he would under the optimal peak-load pricing system, and (4) the subsidy per unit of storage capacity will be independent of those parameters related directly to the demand for storage.

The third step in the analysis will be to determine the characteristics of the optimal one-price system when subsidies are constrained to be zero. It is assumed that direct load controls are used to prevent the consumption of on-peak electricity to meet needs which under the other load management strategies would be met from storage. Thus if hot water would be supplied from storage under peak load pricing or the one-price system with subsidies to storage, then when there is a single price charged for electricity and no subsidies to storage, power to hot water heaters is presumed to be cut off during peak periods.

It is shown that when direct subsidies to storage are constrained to zero, the price of electricity should be set below what it would be if these subsidies were not so constrained. Intuitively, by holding down the price of electricity the public service commission makes it less expensive to charge storage devices with off-peak electricity, thus indirectly giving a subsidy to investment in storage. As under the one-price system with direct subsidies to storage, the price of electricity can be expressed as a weighted average of expected peak and off-peak marginal production costs, and of the peak and off-peak prices optimal under a two-price system. When direct subsidies to storage are constrained to be zero however, the weights depend upon all of the parameters related to storage technology and storage demand.

Finally, the net social benefits (neglecting implementation costs) resulting from each of the three load management strategies are compared. The peak-load pricing system of load management is shown to yield higher net social benefits than the one-price system with subsidies to storage, and the one price system with subsidies to storage is shown to yield higher net social benefits than the one-price system with direct load controls and no subsidies to storage. Neither result should be surprising. It is well known that peak load pricing is socially efficient, which is equivalent to saying that no alternative system yields higher net social benefits when implementation costs are ignored, as they have been in this analysis. Further, it is clear that within the context of a one-price system, constraining subsidies to zero limits one's flexibility. Adding constraints to a maximization problem will never yield an increased payoff.

The real purpose of the analysis of this section is thus not to derive qualitative results concerning net social benefits. It is, rather, to derive explicit formulas for the advantage which peak load pricing has over the single-price system with subsidies, and for the advantage which the single-price system with subsidies has over the single-price system without subsidies. By evaluating these formulas and comparing the resultant figures with estimates of differences in implementation costs, it should be possible to determine which of the three load management strategies is actually most desirable from a social point of view.

B. Peak Load Pricing

Under peak load pricing separate prices, p' and p'' , are charged for off-peak and peak electricity, respectively. In addition, subsidies to storage, amounting to $mL_2/(L_1+L_2)$ units of currency per unit of capacity per unit of time, are not ruled out a priori. The variables p' , p'' , and m are assumed to be chosen so as to maximize expected net social benefits.

Net social benefits have three components: (1) the value which consumers place upon the goods which they purchase less the amount they pay for those goods, (2) the amount which the utility company receives for the electricity it produces less the cost of producing that electricity, and (3) the cost to taxpayers of any subsidy given to storage. Net social benefits are the sum of the first and second components, less the third component.

The net benefits which consumers receive during one off-peak/peak cycle are given by:

$$\tilde{V}_1(p') + \tilde{V}_2(p'') + \tilde{V}_4(m, p') - L_2(k - m + p'/\alpha)S(m, p') - p'Q(p')L_1 - p''R(p'')L_2 \quad (27)$$

$\tilde{V}_1(p')$ is the value which the utility's customers place upon the electricity which they consume off-peak, denoted $Q(p')L_1$. (See equations (8) and (11).) The cost of this electricity is $p'Q(p')L_1$. $\tilde{V}_2(p'')$ is the value placed upon electricity consumed on-peak, $R(p'')L_2$. (see equations (9) and (12)). Consumers must pay $p''R(p'')L_2$ for this electricity. $\tilde{V}_4(m, p')$ is the value which the utility's customers place upon the energy they receive from storage, $S(m, p')L_2$ (see equations (18) and (19)). The amount which consumers pay for their storage equipment is $L_2(k - m)S(m, p')$. The cost of the off-peak electricity which is stored is $L_2p'S(m, p')/\alpha$.

The amount which the utility receives from its customers during one off-peak/ peak cycle is the sum of the amounts which it receives for off-peak electricity and on-peak electricity: $p'[Q(p')L_1 + S(m, p')L_2/\alpha] + p''R(p'')L_2$. The costs of producing this electricity are $L_1\tilde{C}[Q(p'), \bar{K}(p'), \bar{K}(p'')] + S(m, p')L_2/\alpha L_1\tilde{C}[\bar{K}(p'), \bar{K}(p''), \bar{K}(p'')]$, where the first term is the cost of producing the total amount of off-peak electricity purchased by consumers, and the second term is the cost of producing the

electricity purchased by consumers on-peak (see equations (21)).

Profits are thus:

$$p' [\tilde{Q}(p') L_1 + S(m, p') L_2 / \alpha] + p'' \tilde{R}(p'') L_2 - L_1 \tilde{C}[\tilde{Q}(p') + S(m, p') L_2 / \alpha L_1, \tilde{K}(p''), \bar{K}(p'')] - L_2 \tilde{C}[\tilde{R}(p''), \tilde{K}(p''), \bar{K}(p'')] \quad (28)$$

Finally, tax revenues required to subsidize storage are:

$$L_2 m S(m, p') \quad (29)$$

over each off-peak/peak cycle.

Thus net social benefits are given by:

$$\tilde{\pi}(p', p'', m) \equiv \tilde{V}_1(p') + \tilde{V}_2(p'') + \tilde{V}_4(m, p') - L_2 k S(m, p') - L_1 \tilde{C}[\tilde{Q}(p') + S(m, p') L_2 / \alpha L_1, \tilde{K}(p''), \bar{K}(p'')] - L_2 \tilde{C}[\tilde{R}(p''), \tilde{K}(p''), \bar{K}(p'')] \quad (30)$$

The task of the regulatory authorities is to find the solution to the problem $\max_{p', p'', m} \tilde{\pi}(p', p'', m)$, or

$$\max_{p', p'', m} E \{ \tilde{V}_1(p') + \tilde{V}_2(p'') + \tilde{V}_4(m, p') - L_2 k S(m, p') - L_1 \tilde{C}[\tilde{Q}(p') + S(m, p') L_2 / \alpha L_1, \tilde{K}(p''), \bar{K}(p'')] - L_2 \tilde{C}[\tilde{R}(p''), \tilde{K}(p''), \bar{K}(p'')] \} \quad (31)$$

By solving the resultant first order conditions, it can be shown that the prices of off-peak and peak electricity should be

$$\hat{p}' \equiv \bar{c}_0 \quad \text{and} \quad (32)$$

$$\hat{p}'' \equiv \bar{c}_0 + n(L_1 + L_2) / L_2 \quad (33)$$

respectively, and no subsidy should be given to storage:

$$\hat{m}_1 \equiv 0. \quad (34)$$

The optimal off-peak price is simply the expected marginal cost of production in off-peak periods. The optimal peak price is just the off-peak price plus the marginal cost of additional capacity adjusted to reflect the relative length of peak and off-peak periods. It equals the

expected marginal cost of production in peak periods.

Expected net social benefits resulting from the implementation of peak load pricing will be denoted by

$$\begin{aligned} \bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) \equiv E \{ \hat{V}_1(\hat{p}') + \hat{V}_2(\hat{p}'') - \hat{V}_4(\hat{m}_1, \hat{p}') - \\ Z_2 k S(\hat{m}_1, \hat{p}') - \\ Z_1 \hat{C} [\hat{Q}(\hat{p}') + S(\hat{m}_1, \hat{p}') Z_2 / \alpha Z_1, \hat{K}(\hat{p}''), \bar{K}(\hat{p}'')] - \\ Z_2 \hat{C} [\hat{R}(\hat{p}''), \hat{K}(\hat{p}''), \bar{K}(\hat{p}'')] \} \end{aligned} \quad (35)$$

C. One-Price System with Subsidized Storage

Suppose now that the price of electricity is to be the same on-peak as it is off-peak, and that subsidies to storage are not to be ruled out a priori. Then the regulatory authorities should choose the price of electricity, $p \equiv p' = p''$, and the subsidy to storage, $m Z_2 / (Z_1 + Z_2)$, so as to maximize net social benefits. Just as in Section IIIB, expression (30) gives a formula for these benefits. The problem facing the regulatory authority is thus $\max_{p, m} \bar{\pi}(p, p, m)$, or

$$\begin{aligned} \max_{p, m} E \{ \hat{V}_1(p) + \hat{V}_2(p) + \hat{V}_4(m, p) - Z_2 k S(m, p) - Z_1 \hat{C} [\hat{Q}(p) + \\ S(m, p) Z_2 / \alpha Z_1, \hat{K}(p), \bar{K}(p)] - Z_2 \hat{C} [\hat{R}(p), \hat{K}(p), \bar{K}(p)] \} \end{aligned} \quad (36)$$

This time the first order conditions yield:

$$\begin{aligned} \hat{p} = \left(\frac{a_1 Z_2}{a_1 Z_2 + b_1 Z_1} \right) \hat{p}'' + \left(\frac{b_1 Z_1}{a_1 Z_2 + b_1 Z_1} \right) \hat{p}' = \\ \bar{c}_0 + n(Z_1 + Z_2) a_1 / (Z_1 b_1 + Z_2 a_1) \end{aligned} \quad (37)$$

$$\hat{m}_2 = (\hat{p} - \hat{p}') / \alpha = n(Z_1 + Z_2) a_1 / \alpha (Z_1 b_1 + Z_2 a_1) \quad (38)$$

The optimal price of electricity when a single price is charged is simply a weighted average of the peak and off-peak prices which are optimal under a two-price system. Thus $\hat{p}$ is also a weighted average of

the expected peak and expected off-peak marginal cost of electricity generation. The emphasis placed upon expected peak marginal costs increases as the length of the on-peak period increases, and as the slope of the demand curve for off-peak consumption increases in magnitude. Storage equipment is a substitute for generating capacity, hence it is not surprising that the optimal subsidy to storage equipment, which is $\hat{m}_2 \hat{L}_2 / (\hat{L}_1 + \hat{L}_2) = n \hat{L}_2 a_1 / \alpha (\hat{L}_1 b_1 + \hat{L}_2 a_1)$, is an increasing function of the cost of additional generating capacity.

Expected net social benefits resulting from the implementation of the single price system with subsidies to storage will be denoted by $\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)$.

D. One-Price System with Direct Load Controls

Again suppose that there is no difference between on- and off-peak electricity prices. Suppose further that subsidy payments to those who invest in storage are to be avoided. As long as direct load controls are used to prevent the purchase of on-peak electricity to meet needs which under peak load pricing would be met from storage, the formula given for net social benefits in expression (30) still applies. The price of electricity, now denoted p^0 , should be chosen so as to maximize $\bar{\pi}(p^0, p^0, 0)$. That is, the problem facing the regulatory authority is

$$\max_{p^0} E \{ \hat{V}_1(p^0) + \hat{V}_2(p^0) + \hat{V}_4(o, p^0) - \hat{L}_2 k S(o, p^0) - \hat{L}_1 \hat{C} [\hat{Q}(p^0) + S(o, p^0) \hat{L}_2 / \alpha \hat{L}_1, \hat{K}(p^0), \bar{K}(p^0)] - \hat{L}_2 \hat{C} [\hat{K}(p^0), \hat{K}(p^0), \bar{K}(p^0)] \} \quad (39)$$

The solution to this problem takes the form

$$\hat{p}^0 = \bar{c}_0 + n(\hat{L}_1 + \hat{L}_2) a_1 \alpha^2 g_1 / (\alpha^2 g_1 \hat{L}_1 b_1 + \hat{L}_2 a_1 b_1 + \alpha^2 g_1 \hat{L}_2 a_1) = \\ [\hat{p}'(\alpha^2 g_1 \hat{L}_2 a_1) + \hat{p}''(\alpha^2 g_1 \hat{L}_1 b_1 + \hat{L}_2 a_1 b_1)] / (\alpha^2 g_1 \hat{L}_1 b_1 + \hat{L}_2 a_1 b_1 + \alpha^2 g_1 \hat{L}_2 a_1) \quad (40)$$

Expected net social benefits resulting from the implementation of the single-price system with direct load controls are $\bar{\pi}(\hat{p}^0, \hat{p}^0, 0)$.

E. Comparing the Alternatives

Consider first the implications of the alternative systems for the quantity of energy stored. Substituting $\hat{m}_1$ and $\hat{p}'$ into equations (14) and (18), the investment in storage equipment which would result from the implementation of the peak-load pricing system is

$$s^i(\hat{m}_1, \hat{p}') = (\bar{g}_0^i - k - \bar{c}_0/\alpha)/g_1 \quad (41)$$

for individual i , and

$$S(\hat{m}_1, \hat{p}') = I(\bar{G}_0 - k - \bar{c}_0/\alpha)/g_1 \quad (42)$$

in the aggregate. Now substitute $\hat{m}_2$ and $\hat{p}$ into equation (14). Clearly individual i will invest in

$$s^i(\hat{m}_2, \hat{p}) = (\bar{g}_0^i - k - \bar{c}_0/\alpha)g_1 = s^i(\hat{m}_1, \hat{p}') \quad (43)$$

units of storage capacity. Thus implementation of the one-price load management system with subsidies to storage would yield the same investment in and usage of storage as implementing of peak load pricing. With $s^i(\hat{m}_2, \hat{p}) = s^i(\hat{m}_1, \hat{p}')$ for each individual, it follows immediately that $S(\hat{m}_2, \hat{p}) = S(\hat{m}_1, \hat{p}')$.

Substitution of $\hat{p}^0$ into the equations governing investment in storage yields rather complicated expressions. It can be established, however, that

$$s^i(o, \hat{p}^0) \leq s^i(\hat{m}_1, \hat{p}') = s^i(\hat{m}_2, \hat{p}) \quad (44)$$

for all individuals i , and thus that

$$S(o, \hat{p}^0) \leq S(\hat{m}_1, \hat{p}') = S(\hat{m}_2, \hat{p}). \quad (45)$$

In other words, investment in storage under the one-price system with direct load controls but no subsidies to storage always falls short of the investment in storage resulting from the implementation of peak load pricing and the one-price system with subsidized storage.

Now consider the price of electricity. As indicated already, the

optimal prices under the one-price systems, $\hat{p}$ and $\hat{p}^0$, are weighted averages of the optimal peak and off-peak prices under the peak load pricing system, $\hat{p}''$ and $\hat{p}'$. Hence $\hat{p}' \leq \hat{p}$, $\hat{p}^0 \leq \hat{p}''$. It can be shown that, in fact, $\hat{p}^0 \leq \hat{p}$. Thus the price of electricity when storage is not subsidized should be less than the price of electricity when there is such subsidization.

$\hat{p}^0$ is not equal to $\hat{p}$ because, in the absence of a subsidy to storage, changes in $\hat{p}^0$ affect the size of the investment customers are willing to make in storage equipment. In particular, reductions in $\hat{p}^0$ result in greater investments in storage. The difference between $\hat{p}$ and $\hat{p}^0$ thus acts rather like an indirect subsidy to storage.

The formulas for $\hat{p}$, $\hat{p}'$ and $\hat{p}''$ are independent of those demand and technological parameters related to storage. For $\hat{p}'$ and $\hat{p}''$ this is true because peak load pricing allows the equation of price and expected marginal production costs at each instant, and the latter do not depend upon storage parameters. When the price of electricity is not allowed to vary with time, distortions in the demand for electricity are introduced. These distortions would affect the marginal benefits derived by customers from storage were the demand for energy from storage not independent of electricity consumption, and would affect the marginal social costs of storage were the marginal costs of electricity production in off-peak periods not independent of the quantity produced. With these two independence assumptions, however, p can be chosen to minimize the damage done by distortions in the demand for electricity, while the subsidy offered for each unit of storage capacity, $m\bar{l}_2/(\bar{l}_1+\bar{l}_2)$, is selected to optimize the investment in storage equipment. When one of these instruments, the subsidy to storage, is then eliminated, the price of electricity must be chosen with both its effects upon storage and its effects upon the demand for electricity in view. Thus parameters related to the demand and efficiency of storage have an impact upon $\hat{p}^0$, though not upon $\hat{p}$, $\hat{p}'$, or $\hat{p}''$.

How is the demand for electricity distorted under the one-price systems? Because the price of off-peak power under either of the one-price systems exceeds the price of off-peak power under peak load pricing,

too little electricity will tend to be consumed off-peak when the price of electricity is time invariant. In fact, since $\hat{p}' \leq \hat{p}^0 \leq \hat{p}$,

$$\tilde{Q}(\hat{p}') \geq \tilde{Q}(\hat{p}^0) \geq \tilde{Q}(\hat{p}). \quad (46)$$

Consumption of off-peak electricity is greatest under peak load pricing, lowest under the single-price system with subsidized storage.

On the other hand, because the price of on-peak electricity under the one-price systems falls short of its price under peak load pricing, too much electricity tends to be purchased under the one-price systems during peak hours. Since $\hat{p}'' \geq \hat{p} \geq \hat{p}^0$,

$$\tilde{R}(\hat{p}'') \leq \tilde{R}(\hat{p}) \leq \tilde{R}(\hat{p}^0) \quad (47)$$

The demand for electricity on-peak is greatest under the one-price system without subsidies to storage, lowest under peak load pricing.

It follows from this last result (see equations (25) and (47)) that base load generating capacity is greatest under the one-price system without subsidies to storage, and least under peak load pricing:

$$\bar{K}(\hat{p}'') \leq \bar{K}(\hat{p}) \leq \bar{K}(\hat{p}^0) \quad (48)$$

Finally, consider the advantage, in expected net social benefits per customer, which peak load pricing enjoys over the single-price system with subsidies to storage. Fairly complicated algebraic manipulations yield

$$[\bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) - \bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)]/1 = n^2(l_1 + l_2)^2 l_1 / 2l_2 (l_1 b_1 + l_2 a_1) \quad (49)$$

Note that this per customer advantage depends only upon the cost of capital, the relative lengths of the peak and off-peak periods, and the slopes of the demand curves for peak and off-peak electricity. By substituting estimated values of these parameters into equation (49), and comparing the resultant figure with an estimate of the difference in the cost of implementing the two load management systems, it should be

possible to determine whether the gains from implementing peak load pricing outweigh the costs.

The per customer advantage which the one-price system with subsidized storage enjoys over the one-price system with direct load controls but no subsidization of storage is given by

$$[\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2) - \bar{\pi}(\hat{p}^0, \hat{p}^0, o)]/1 =$$

$$n^2(l_1 + l_2)^2 a_1^2 l_2 / 2(l_1 b_1 + l_2 a_1)(\alpha^2 g_1 l_1 b_1 + l_2 a_1 b_1 + \alpha^2 g_1 l_2 a_1) \quad (50)$$

Combined with parameter estimates and figures for implementation costs, this formula can be used to compare the over-all desirability of the one-price system with storage to the one-price system without storage.

IV. Summary and Suggestions for Further Research

In this paper we have examined three alternative load management strategies: peak load pricing, time invariant rates with subsidized storage, and direct load controls. Ignoring differences in the relative cost of implementing the various strategies, time invariant rates with subsidized storage were found to be inferior to peak load pricing, but superior to direct load controls. Intuitively, there are inevitably distortions in the time pattern of electricity consumption when the price of electricity fails to reflect the variations in the marginal cost of producing electricity which exist between peak and off-peak periods. In particular, too little consumption occurs during off-peak hours, too much during peak hours. The latter effect leads to an expansion of base generating capacity.

Under peak load pricing the incentive to invest in storage is present in just the proper degree. Even with time invariant rates, a schedule of subsidies can be established which will result in an efficient distribution of storage equipment. When subsidies are constrained to zero, however, too little investment in storage takes place, with the result that generating capacity must be expanded beyond what it would otherwise be.

All these results could have been obtained in a framework much more general than that employed in the current paper. The payoff from our additional assumptions came in our ability to derive explicit formulas for differences in the net social benefits derived from each load management system, for the prices and/or subsidies needed to implement them, and for the generating and storage capacities associated with them. By substituting estimates of the relevant parameters into these formulas, utility companies and public service commissions should be able to obtain useful information on the long run benefits, costs, and consequences of alternative load management strategies. At the very least, some guidance may be obtained on which systems are worth investigating further.

The biggest weaknesses in the current paper are probably the assumptions relating to the independence of the demand for peak and off-peak

electricity, and those related to the extent to which storage capacity will be utilized. Simply dividing time into peak and off-peak periods and introducing peak-load pricing often results, in practice, in the formation of a demand spike in the hours just prior or just subsequent to the official peak period. To deal with this problem, and proposed solutions, analytically will require a finer division of time than has been employed heretofore. This should be fairly straightforward.

More difficult to eliminate are the assumptions that under peak load pricing storage capacity will always be fully utilized and never supplemented with peak electricity. This may make sense for appliances such as water heaters, where demand is relatively stable over time, but probably is an over-simplification with respect to, say, space heating. The problem is that the design and implementation of time invariant rate structures with subsidies for storage or load controls becomes much more complicated when it is optimal to occasionally underutilize or supplement storage devices. This suggests that the formulas derived in the current paper if anything understate the desirability of peak load pricing relative to other load management techniques. It may be possible to generalize the analysis somewhat, allowing more accurate calculations, by distinguishing between storage for different purposes.

NOTES

1. The author gratefully acknowledges the assistance of Professor Ali Cambel and Mr. George Heffernan, both of The George Washington University. Remaining errors and ambiguities are the responsibility of the author.
2. Research Assistant. Also Traveling Scholar, Harvard University.
3. More extensive treatments of many of the issues discussed in this section may be found in references [5], [7], and [8].
4. See reference [9] for a discussion of alternative centralized storage technologies; see also reference [7], p. 52.
5. The basic reference is [10]. Summaries of Steiner's analysis are contained in references [4] and [7].
6. It is the lack of such incentives rather than technological barriers which has prevented the commercialization of distributed thermal energy storage in the United States. Such storage is used extensively in some European countries. See reference [3].
7. Asbury and Mueller [4] consider the peak load problem with storage.
8. References [5] and [7] contain discussions of metering costs.
9. Harberger [6] discusses the assumptions underlying cost-benefit analysis in detail.
10. See reference [5], chapter 7 for a review of some existing cost-benefit analyses. For examples of such analyses see references [1] and [2].

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APPENDIX: DERIVATION OF EQUATIONS

Section IIB

Derivation of equation (5):

$$\max_q v_1^i(q) - p'qz_1 \quad (\text{equation (4)})$$

$$\max_q (a_0^i q - \frac{1}{2} a_1 q^2) z_1 - p'qz_1 \quad (\text{substitute from equation (1)})$$

$$(a_0^i - a_1 \hat{q}) z_1 - p'z_1 = 0 \quad (\text{first order condition})$$

$$\hat{q} = (a_0^i - p')/a_1 \quad (\text{solve for } \hat{q})$$

$$v_1^i(p') \equiv (a_0^i - p')/a_1 \quad (\text{equation (5)})$$

Derivation of equations (11), (12), and (13):

$$v_1^i(v_1^i(p')) = [a_0^i v_1^i(p') - \frac{1}{2} a_1 v_1^i(p')^2] z_1 \quad (\text{substitute from equation (1)})$$

$$v_1^i(v_1^i(p')) = z_1 a_0^i (a_0^i - p')/a_1 - \frac{1}{2} z_1 (a_0^i - p')^2/a_1 \quad (\text{substitute from equation (5)})$$

$$v_1^i(v_1^i(p')) = z_1 [(a_0^i)^2 - (p')^2]/2a_1 \quad (\text{simplify})$$

$$\bar{v}_1(p') \equiv \sum_i v_1^i(v_1^i(p')) = z_1 \sum_i [(a_0^i)^2 - (p')^2]/2a_1 \quad (\text{equation (11)})$$

Equations (12) and (13) are derived similarly.

Derivation of equation (14):

$$\max_s \{ (g_0^i s - \frac{1}{2} g_1 s^2) z_2 - s(k-m)z_2 - sp'z_2/\alpha \} \quad (\text{maximization problem})$$

$$(g_0^i - g_1 \hat{s}) z_2 - (k-m)z_2 - p'z_2/\alpha = 0 \quad (\text{first order condition})$$

$$\hat{s} = (g_0^i + m - k - p'/\alpha)/g_1 \quad (\text{solve for } \hat{s})$$

$$s^i(m, p') = (g_0^i + m - k - p'/\alpha)/g_1 \quad (\text{equation 14})$$

Derivation of equation (20):

$$\bar{v}_4(m, p') = \sum_i z_2 [g_0^i s^i(m, p') - \frac{1}{2} g_1 s^i(m, p')^2] \quad (\text{from equation (19)})$$

$$\bar{v}_4(m, p') = z_2 \sum_i g_0^i (g_0^i + m - k - p'/\alpha)/g_1 - \frac{1}{2} z_2 (g_0^i + m - k - p'/\alpha)^2/g_1 \quad (\text{substitute from equation 14})$$

$$\bar{v}_4(m, p') = z_2 \sum_i [(g_0^i)^2 - (k - m + p'/\alpha)^2]/2g_1 \quad (\text{equation (20)})$$

Section IIC

Derivation of equation (25):

$$\min_{\bar{K}} E \{ \bar{L}_1 \bar{c}_0 (\bar{Q}(p') + S(m, p') \bar{L}_2 / \alpha \bar{L}_1) + \bar{L}_2 \bar{c}_0 \bar{R}(p'') + \bar{L}_2 \frac{1}{2} c_1 (\bar{R}(p'') - \bar{K})^2 + (\bar{L}_1 + \bar{L}_2) n \bar{K} \} \quad (\text{minimization problem; see equation (24)})$$

$$\min_{\bar{K}} E \{ \bar{L}_2 \frac{1}{2} c_1 (\bar{R}(p'') - \bar{K})^2 + (\bar{L}_1 + \bar{L}_2) n \bar{K} \} \quad (\text{eliminate terms not involving } \bar{K})$$

$$\min_{\bar{K}} \bar{L}_2 \frac{1}{2} c_1 (\bar{R}(p'') - \bar{K})^2 + (\bar{L}_1 + \bar{L}_2) n \bar{K} \quad (\text{take expectation and eliminate terms not involving } \bar{K})$$

$$-\bar{L}_2 c_1 (\bar{R}(p'') - \hat{\bar{K}}) + (\bar{L}_1 + \bar{L}_2) n = 0 \quad (\text{first order condition})$$

$$\hat{\bar{K}} = \bar{R}(p'') - n(\bar{L}_1 + \bar{L}_2) / c_1 \bar{L}_2 \quad (\text{solve for } \hat{\bar{K}})$$

$$\bar{K}(p'') \equiv \bar{R}(p'') - n(\bar{L}_1 + \bar{L}_2) / c_1 \bar{L}_2 \quad (\text{equation (25)})$$

Section IIIB

Derivation of equations (32), (33), and (34):

$$\max_{p', p'', m} E \{ \bar{V}_1(p') + \bar{V}_2(p'') + \bar{V}_4(m, p') - \bar{L}_2 k S(m, p') - \bar{L}_1 \bar{C}[\bar{Q}(p') + S(m, p') \bar{L}_2 / \alpha \bar{L}_1, \bar{R}(p''), \bar{K}(p'')] - \bar{L}_2 \bar{C}[\bar{R}(p''), \bar{K}(p''), \bar{K}(p'')] \} \quad (\text{maximization problem; see equation (31)})$$

$$\max_{p', p'', m} -\bar{L}_1 I(p')^2 / 2a_1 - \bar{L}_2 I(p'')^2 / 2b_1 - \bar{L}_2 I(k-m + p'/\alpha)^2 / 2g_1 - \bar{L}_2 k S(m, p') - \bar{L}_1 \bar{c}_0 (\bar{Q}(p') + S(m, p') \bar{L}_2 / \alpha \bar{L}_1) - \bar{L}_2 \bar{c}_0 \bar{R}(p'') - n(\bar{L}_1 + \bar{L}_2) \bar{R}(p'') \quad (\text{substitute from equations (11), (12), (19), and (26); eliminate terms not involving } p', p'', \text{ or } m)$$

The resultant first order conditions are:

$$-\bar{L}_1 I \hat{p}' / a_1 - \bar{L}_2 I(k - \hat{m} + \hat{p}' / \alpha) / g_1 \alpha - \bar{L}_2 k \frac{\partial}{\partial p'} S(\hat{m}, \hat{p}') - \bar{L}_1 \bar{c}_0 \left(\frac{\partial}{\partial p'} \bar{Q}(\hat{p}') + \frac{\partial}{\partial p'} S(\hat{m}, \hat{p}') \bar{L}_2 / \alpha \bar{L}_1 \right) = 0,$$

$$-\bar{L}_2 I \hat{p}'' / b_1 - \bar{L}_2 \bar{c}_0 \frac{\partial}{\partial p''} \bar{R}(\hat{p}'') - n(\bar{L}_1 + \bar{L}_2) \frac{\partial}{\partial p''} \bar{R}(\hat{p}'') = 0, \text{ and}$$

$$I \bar{L}_2 (k - \hat{m} + \hat{p}' / \alpha) / g_1 - \bar{L}_2 k \frac{\partial}{\partial m} S(\hat{m}, \hat{p}') - \bar{L}_2 \bar{c}_0 \frac{\partial}{\partial m} S(\hat{m}, \hat{p}') / \alpha = 0$$

Note that:

$$\frac{\partial}{\partial p'} S(m, p') = \frac{\partial}{\partial p'} I(\bar{G}_0 + m - k - p' / \alpha) / g_1 = -I / \alpha g_1 \quad (\text{see equation (18)})$$

$$\frac{\partial}{\partial p'} \bar{Q}(p') = \frac{\partial}{\partial p'} I(\bar{A}_0 - p')/a_1 = -I/a_1 \quad (\text{see equation (8)})$$

$$\frac{\partial}{\partial p''} \bar{R}(p'') = \frac{\partial}{\partial p''} I(\bar{B}_0 - p'')/b_1 = -I/b_1 \quad (\text{see equation (9)})$$

$$\frac{\partial}{\partial m} S(m, p') = \frac{\partial}{\partial m} I(\bar{G}_0 + m - k - p'/\alpha)/g_1 = I/g_1 \quad (\text{see equation (18)})$$

Substitute into the first order conditions:

$$-I\bar{L}_1 \hat{p}'/a_1 - I\bar{L}_2 (k - \hat{m} + \hat{p}'/\alpha)/g_1 + I\bar{L}_2 k/\alpha g_1 + \bar{L}_1 \bar{c}_0 (I/a_1 + I\bar{L}_2/\alpha^2 \bar{L}_1 g_1) = 0$$

$$-I\bar{L}_2 \hat{p}''/b_1 + I\bar{L}_2 \bar{c}_0/b_1 + n(\bar{L}_1 + \bar{L}_2)I/b_1 = 0$$

$$I\bar{L}_2 (k - \hat{m} + \hat{p}'/\alpha)/g_1 - I\bar{L}_2 k/g_1 - I\bar{L}_2 \bar{c}_0/\alpha g_1 = 0$$

Simplify:

$$(\bar{c}_0 - \hat{p}')(\bar{L}_1 \alpha^2 g_1 + \bar{L}_2 a_1) + \hat{m} \bar{L}_2 a_1 \alpha = 0$$

$$-\hat{p}'' \bar{L}_2 + \bar{c}_0 \bar{L}_2 + n(\bar{L}_1 + \bar{L}_2) = 0$$

$$\hat{p}' - \bar{c}_0 - \alpha \hat{m} = 0$$

Solve for $\hat{p}'$, $\hat{p}''$, and $\hat{m}_1$:

$$\hat{p}' = \bar{c}_0 \quad (\text{equation (32)})$$

$$\hat{p}'' = \bar{c}_0 + n(\bar{L}_1 + \bar{L}_2)/\bar{L}_2 \quad (\text{equation (33)})$$

$$\hat{m}_1 = 0 \quad (\text{equation (34)})$$

Section IIIC

Derivation of equations (37) and (38):

$$\begin{aligned} \max_{p, m} E\{\bar{V}_1(p) + \bar{V}_2(p) + \bar{V}_4(m, p) - \bar{L}_2 k S(m, p) - \bar{L}_1 \bar{c}[\bar{Q}(p) + \\ S(m, p) \bar{L}_2/\alpha \bar{L}_1, \bar{K}(p), \bar{K}(p)] - \bar{L}_2 \bar{c}[\bar{R}(p), \bar{K}(p), \bar{K}(p)]\} \\ (\text{maximization problem; see equation (36)}) \end{aligned}$$

$$\begin{aligned} \max_{p, m} - \bar{L}_1 I p^2/2a_1 - \bar{L}_2 I p^2/2b_1 - \bar{L}_2 I (k - m + p/\alpha)^2/2g_1 - \bar{L}_2 k S(m, p) - \\ \bar{L}_1 \bar{c}_0 (\bar{Q}(p) + S(m, p) \bar{L}_2/\alpha \bar{L}_1) - \bar{L}_2 \bar{c}_0 \bar{R}(p) - n(\bar{L}_1 + \bar{L}_2) \bar{R}(p) \\ (\text{substitute from equations (11), (12), (19),} \\ \text{and (26); eliminate terms not involving } p \text{ or } m) \end{aligned}$$

The resultant first order conditions are:

$$\begin{aligned}
& -\lambda_1 \hat{p}/a_1 - \lambda_2 \hat{p}/b_1 - \lambda_2 I(k - \hat{m} + \hat{p}/\alpha)/g_1 \alpha - \lambda_2 k \frac{\partial}{\partial p} S(\hat{m}, \hat{p}) \\
& -\lambda_1 \bar{c}_0 \left(\frac{\partial}{\partial p} \bar{Q}(\hat{p}) + \frac{\partial}{\partial p} S(\hat{m}, \hat{p}) \lambda_2 / \alpha \lambda_1 \right) - \lambda_2 \bar{c}_0 \frac{\partial}{\partial p} \bar{R}(\hat{p}') - \\
& n(\lambda_1 + \lambda_2) \frac{\partial}{\partial p} \bar{R}(\hat{p}) = 0
\end{aligned}$$

$$I\lambda_2(k - \hat{m} + \hat{p}/\alpha)/g_1 - \lambda_2 k \frac{\partial}{\partial m} S(\hat{m}, \hat{p}) - \lambda_2 \bar{c}_0 \frac{\partial}{\partial m} S(\hat{m}, \hat{p})/\alpha = 0$$

Substitute:

$$\begin{aligned}
& -\lambda_1 \hat{p}/a_1 - \lambda_2 \hat{p}/b_1 - \lambda_2 I(k - \hat{m} + \hat{p}/\alpha)/g_1 \alpha + \lambda_2 Ik/\alpha g_1 + \\
& \lambda_1 \bar{c}_0 (1/a_1 + \lambda_2/\alpha^2 \lambda_1 g_1) + \lambda_2 \bar{c}_0/b_1 + (\lambda_1 + \lambda_2) I n/b_1 = 0 \\
& \text{(see equations (8), (9), and (18))}
\end{aligned}$$

$$\lambda_2 I(k - \hat{m} + \hat{p}/\alpha)g_1 - \lambda_2 Ik/g_1 - \lambda_2 \bar{c}_0/\alpha g_1 = 0 \quad \text{(see equation (18))}$$

Simplify:

$$\begin{aligned}
& (\bar{c}_0 - \hat{p})(\lambda_1 \alpha^2 g_1 b_1 + \lambda_2 a_1 b_1 + \lambda_2 a_1 \alpha^2 g_1) + \hat{m} \lambda_2 a_1 b_1 \alpha + \\
& n(\lambda_1 + \lambda_2) a_1 \alpha^2 g_1 = 0
\end{aligned}$$

$$\hat{p} - \bar{c}_0 - \alpha \hat{m} = 0$$

Solve for $\hat{p}$ and $\hat{m}_2$:

$$\hat{p} = \bar{c}_0 + n(\lambda_1 + \lambda_2) a_1 / (\lambda_1 b_1 + \lambda_2 a_1) \quad \text{(equation (37))}$$

$$\hat{m}_2 = n(\lambda_1 + \lambda_2) a_1 / (\lambda_1 b_1 + \lambda_2 a_1) \alpha \quad \text{(equation (38))}$$

Note that:

$$\begin{aligned}
\hat{p} &= \left(\frac{\lambda_2 a_1}{\lambda_1 b_1 + \lambda_2 a_1} \right) (\bar{c}_0 + n(\lambda_1 + \lambda_2)/\lambda_2) + \left(\frac{\lambda_1 b_1}{\lambda_1 b_1 + \lambda_2 a_1} \right) \bar{c}_0 \\
&= \left(\frac{\lambda_2 a_1}{\lambda_1 b_1 + \lambda_2 a_1} \right) \hat{p}'' + \left(\frac{\lambda_1 b_1}{\lambda_1 b_1 + \lambda_2 a_1} \right) \hat{p}' \quad \text{(equation (37))}
\end{aligned}$$

Section IIID

Derivation of equation (40):

$$\max_{p^0} E\{\hat{V}_1(p^0) + \hat{V}_2(p^0) + \hat{V}_4(0, p^0) - \lambda_2 k S(0, p^0) - \lambda_1 \hat{c}[\hat{Q}(p^0) + S(0, p^0) \lambda_2 / \alpha \lambda_1, \hat{K}(p^0), \bar{K}(p^0)] - \lambda_2 \hat{c}[\hat{R}(p^0), \hat{K}(p^0), \bar{K}(p^0)]\}$$

(maximization problem; see equation (39))

$$\begin{aligned}
\max_{p^0} & -\lambda_1 I(p^0)^2/2a_1 - \lambda_2 I(p^0)^2/2b_1 - \lambda_2 I(k + p^0/\alpha)^2/2g_1 - \lambda_2 k S(0, p^0) - \\
& \lambda_1 \bar{c}_0 (\bar{Q}(p^0) + S(0, p^0) \lambda_2 / \alpha \lambda_1) - \lambda_2 \bar{c}_0 \bar{R}(p^0) - n(\lambda_1 + \lambda_2) \bar{R}(p^0)
\end{aligned}$$

(substitute from equations (11), (12), (19), and (26); eliminate terms not involving p)

The resultant first order condition is:

$$\begin{aligned} & -\mathcal{L}_1 \hat{p}^0 / a_1 - \mathcal{L}_2 \hat{p}^0 / b_1 - \mathcal{L}_2 I(k + \hat{p}^0 / \alpha) / g_1 \alpha - \mathcal{L}_2 k \frac{\partial}{\partial p} S(0, \hat{p}^0) - \\ & \mathcal{L}_1 \bar{c}_0 \left(\frac{\partial}{\partial p} \bar{Q}(\hat{p}^0) + \frac{\partial}{\partial p} S(0, \hat{p}^0) \mathcal{L}_2 / \alpha \mathcal{L}_1 \right) - \mathcal{L}_2 \bar{c}_0 \frac{\partial}{\partial p} \bar{R}(\hat{p}^0) - \\ & n(\mathcal{L}_1 + \mathcal{L}_2) \frac{\partial}{\partial p} \bar{R}(\hat{p}^0) = 0 \end{aligned}$$

Substitute and simplify:

$$(\bar{c}_0 - \hat{p}^0) (\mathcal{L}_1 \alpha^2 g_1 b_1 + \mathcal{L}_2 a_1 b_1 + \mathcal{L}_2 a_1 \alpha^2 g_1) + n(\mathcal{L}_1 + \mathcal{L}_2) a_1 \alpha^2 g_1 = 0$$

(see equations (8), (9), and (18))

Solve for $\hat{p}^0$:

$$\hat{p}^0 = \bar{c}_0 + n(\mathcal{L}_1 + \mathcal{L}_2) a_1 \alpha^2 g_1 / (\alpha^2 g_1 \mathcal{L}_1 b_1 + \mathcal{L}_2 a_1 b_1 + \alpha^2 g_1 \mathcal{L}_2 a_1)$$

(equation (40))

Note that:

$$\hat{p}^0 = \left(\frac{\alpha^2 g_1 \mathcal{L}_2 a_1}{\alpha^2 g_1 \mathcal{L}_1 b_1 + \mathcal{L}_2 a_1 b_1 + \alpha^2 g_1 \mathcal{L}_2 a_1} \right) \hat{p}'' + \left(\frac{\alpha^2 g_1 \mathcal{L}_1 b_1 + \mathcal{L}_2 a_1 b_1}{\alpha^2 g_1 \mathcal{L}_1 b_1 + \mathcal{L}_2 a_1 b_1 + \alpha^2 g_1 \mathcal{L}_2 a_1} \right) \hat{p},$$

$< \hat{p}.$

Section III E

Derivation of equation (49):

$$\begin{aligned} & [\bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) - \bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)] / I = E\{[\hat{V}_1(\hat{p}') - \hat{V}_1(\hat{p})] + [\hat{V}_2(\hat{p}'') - \hat{V}_2(\hat{p})] + \\ & [\hat{V}_4(\hat{m}_1, \hat{p}') - \hat{V}_4(\hat{m}_2, \hat{p})] - \mathcal{L}_2 k[S(\hat{m}_1, \hat{p}') - S(\hat{m}_2, \hat{p})] - \mathcal{L}_1 [\hat{C}[\hat{Q}(\hat{p}') + \\ & S(\hat{m}_1, \hat{p}') \mathcal{L}_2 / \alpha \mathcal{L}_1, \hat{K}(\hat{p}''), \bar{K}(\hat{p}'')] - \hat{C}[\hat{Q}(\hat{p}) + S(\hat{m}_2, \hat{p}) \mathcal{L}_2 / \alpha \mathcal{L}_1, \hat{K}(\hat{p}), \bar{K}(\hat{p})]] - \\ & \mathcal{L}_2 [\hat{C}[\hat{R}(\hat{p}''), \hat{K}(\hat{p}''), \bar{K}(\hat{p}'')] - \hat{C}[\hat{R}(\hat{p}), \hat{K}(\hat{p}), \bar{K}(\hat{p})]]\} / I \end{aligned}$$

(see equations (31) and (36))

Substitute:

$$\begin{aligned} & [\bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) - \bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)] / I = \mathcal{L}_1 (\hat{p}^2 - (\hat{p}')^2) / 2a_1 + \mathcal{L}_2 (\hat{p}^2 - (\hat{p}'')^2) / 2b_1 + \\ & \mathcal{L}_2 [(k - \hat{m}_2 + \hat{p} / \alpha)^2 - (k - \hat{m}_1 + \hat{p}' / \alpha)^2] / 2g_1 + \mathcal{L}_2 k[S(\hat{m}_2, \hat{p}) - S(\hat{m}_1, \hat{p}')] / I + \\ & \mathcal{L}_1 \bar{c}_0 (\bar{Q}(\hat{p}) - \bar{Q}(\hat{p}')) / I + \mathcal{L}_2 \bar{c}_0 (S(\hat{m}_2, \hat{p}) - S(\hat{m}_1, \hat{p}')) / I \alpha + \\ & \mathcal{L}_2 \bar{c}_0 (\bar{R}(\hat{p}) - \bar{R}(\hat{p}'')) / I + n(\mathcal{L}_1 + \mathcal{L}_2) (\bar{R}(\hat{p}) - \bar{R}(\hat{p}'')) / I \end{aligned}$$

(see equations (11), (12), (19), and (26))

But $k - \hat{m}_2 + \hat{p} / \alpha = k - \hat{m}_1 + \hat{p}' / \alpha$ and $S(\hat{m}_2, \hat{p}) - S(\hat{m}_1, \hat{p}')$, so

$$\begin{aligned} & [\bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) - \bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)] / I = \mathcal{L}_1 (\hat{p}^2 - (\hat{p}')^2) / 2a_1 + \mathcal{L}_2 (\hat{p}^2 - (\hat{p}'')^2) / 2b_1 + \\ & \mathcal{L}_1 \bar{c}_0 (\bar{Q}(\hat{p}) - \bar{Q}(\hat{p}')) / I + \mathcal{L}_2 \bar{c}_0 (\bar{R}(\hat{p}) - \bar{R}(\hat{p}'')) / I + n(\mathcal{L}_1 + \mathcal{L}_2) (\bar{R}(\hat{p}) - \bar{R}(\hat{p}'')) / I \end{aligned}$$

Substitute for $\bar{Q}(\hat{p}) - \bar{Q}(\hat{p}')$ and $\bar{R}(\hat{p}) - \bar{R}(\hat{p}'')$:

$$[\bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) - \bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)]/I = \mathcal{L}_1(\hat{p}^2 - (\hat{p}')^2)/2a_1 + \mathcal{L}_2(\hat{p}^2 - (\hat{p}'')^2)/2b_1 + \mathcal{L}_1\bar{c}_0(\hat{p}' - \hat{p})/a_1 + (\mathcal{L}_2\bar{c}_0 + n(\mathcal{L}_1 + \mathcal{L}_2))(\hat{p}'' - \hat{p})/b_1$$

(see equations (8) and (9))

Rearranging terms:

$$[\bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) - \bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)]/I = \mathcal{L}_1(\hat{p} - \hat{p}')^2/2a_1 + \mathcal{L}_2(\hat{p} - \hat{p}'')^2/2b_1$$

(see equations (32) and (33))

Substitute for $\hat{p} - \hat{p}'$ and $\hat{p} - \hat{p}''$, and simplify:

$$[\bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) - \bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)]/I = n^2(\mathcal{L}_1 + \mathcal{L}_2)^2 \mathcal{L}_1^2 \mathcal{L}_2^2 b_1 / 2\mathcal{L}_2(\mathcal{L}_1 b_1 + \mathcal{L}_2 a_1)^2 + \mathcal{L}_1 n^2(\mathcal{L}_1 + \mathcal{L}_2)^2 a_1 / 2(\mathcal{L}_1 b_1 + \mathcal{L}_2 a_1)^2$$

$$[\bar{\pi}(\hat{p}', \hat{p}'', \hat{m}_1) - \bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2)]/I = n^2(\mathcal{L}_1 + \mathcal{L}_2)^2 \mathcal{L}_1 / 2\mathcal{L}_2(\mathcal{L}_1 b_1 + \mathcal{L}_2 a_1)$$

(equation (49))

Derivation of equation (50):

$$[\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2) - \bar{\pi}(\hat{p}^0, \hat{p}^0, 0)]/I = E[\hat{\mathcal{V}}_1(\hat{p}) - \hat{\mathcal{V}}_1(\hat{p}^0)] + [\hat{\mathcal{V}}_2(\hat{p}) - \hat{\mathcal{V}}_2(\hat{p}^0)] + [\hat{\mathcal{V}}_4(\hat{m}_2, \hat{p}) - \hat{\mathcal{V}}_4(0, \hat{p}^0)] - \mathcal{L}_2 k[S(\hat{m}_2, \hat{p}) - S(0, \hat{p}^0)] - \mathcal{L}_1[\hat{\mathcal{C}}[\hat{\mathcal{Q}}(\hat{p}) + S(\hat{m}_2, \hat{p})\mathcal{L}_2/\alpha\mathcal{L}_1, \hat{\mathcal{K}}(\hat{p}), \bar{\mathcal{K}}(\hat{p})] - \hat{\mathcal{C}}[\hat{\mathcal{Q}}(\hat{p}^0) + S(0, \hat{p}^0)\mathcal{L}_2/\alpha\mathcal{L}_1, \hat{\mathcal{K}}(\hat{p}^0), \bar{\mathcal{K}}(\hat{p}^0)]] - \mathcal{L}_2[\hat{\mathcal{C}}[\hat{\mathcal{R}}(\hat{p}), \hat{\mathcal{K}}(\hat{p}), \bar{\mathcal{K}}(\hat{p})] - \hat{\mathcal{C}}[\hat{\mathcal{R}}(\hat{p}^0), \hat{\mathcal{K}}(\hat{p}^0), \bar{\mathcal{K}}(\hat{p}^0)]]/I$$

(see equations (36) and (39))

Substitute:

$$[\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2) - \bar{\pi}(\hat{p}^0, \hat{p}^0, 0)]/I = \mathcal{L}_1((\hat{p}^0)^2 - \hat{p}^2)/2a_1 + \mathcal{L}_2((\hat{p}^0)^2 - \hat{p}^2)/2b_1 + \mathcal{L}_2[(k + \hat{p}^0/\alpha)^2 - (k - \hat{m}_2 + \hat{p}/\alpha)^2]/2g_1 + \mathcal{L}_2 k[S(0, \hat{p}^0) - S(\hat{m}_2, \hat{p})]/I + \mathcal{L}_1\bar{c}_0(\bar{\mathcal{Q}}(\hat{p}^0) - \bar{\mathcal{Q}}(\hat{p}))/I + \mathcal{L}_2\bar{c}_0(S(0, \hat{p}^0) - S(\hat{m}_2, \hat{p}))/I\alpha + \mathcal{L}_2\bar{c}_0(\bar{\mathcal{R}}(\hat{p}^0) - \bar{\mathcal{R}}(\hat{p}))/I + n(\mathcal{L}_1 + \mathcal{L}_2)(\bar{\mathcal{R}}(\hat{p}^0) - \bar{\mathcal{R}}(\hat{p}))/I$$

(see equations (11), (12), (19) and (26))

Substitute for $\hat{m}_2 + \hat{p}/\alpha$, $S(0, \hat{p}^0) - S(\hat{m}_2, \hat{p})$, $\bar{\mathcal{Q}}(\hat{p}^0) - \bar{\mathcal{Q}}(\hat{p})$, and $\bar{\mathcal{R}}(\hat{p}^0) - \bar{\mathcal{R}}(\hat{p})$:

$$[\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2) - \bar{\pi}(\hat{p}^0, \hat{p}^0, 0)]/I = \mathcal{L}_1((\hat{p}^0)^2 - \hat{p}^2)/2a_1 + \mathcal{L}_2((\hat{p}^0)^2 - \hat{p}^2)/2b_1 + \mathcal{L}_2[(k + \hat{p}^0/\alpha)^2 - (k + \bar{c}_0/\alpha)^2]/2g_1 + \mathcal{L}_2 k[(k + \bar{c}_0/\alpha) - (k + \hat{p}^0/\alpha)]/g_1 + \mathcal{L}_1\bar{c}_0(\hat{p} - \hat{p}^0)/a_1 + \mathcal{L}_2\bar{c}_0[(k + \bar{c}_0/\alpha) - (k + \hat{p}^0/\alpha)]/\alpha g_1 + (\mathcal{L}_2\bar{c}_0 + n(\mathcal{L}_1 + \mathcal{L}_2))(\hat{p} - \hat{p}^0)/b_1$$

(see equations (8), (9), (37), and (38))

Rearrange terms:

$$\begin{aligned} [\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2) - \bar{\pi}(\hat{p}^0, \hat{p}^0, 0)]/I &= (\hat{p}^0 - \hat{p})(\hat{p}^0 + \hat{p} - 2\bar{c}_0)l_1/2a_1 + \\ &(\hat{p}^0 - \hat{p})[\hat{p}^0 + \hat{p} - 2(\bar{c}_0 + n(l_1 + l_2)/l_2)]l_2/2b_1 + \\ &[(k + \hat{p}^0/\alpha) - (k + \bar{c}_0/\alpha)][(k + \hat{p}^0/\alpha) + (k + \bar{c}_0/\alpha) - 2(k + \bar{c}_0/\alpha)]l_2/2g_1 \end{aligned}$$

Substitute and simplify:

$$\begin{aligned} [\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2) - \bar{\pi}(\hat{p}^0, \hat{p}^0, 0)]/I &= (\hat{p}^0 - \hat{p})(\hat{p}^0 + \hat{p} - 2\hat{p}')l_1/2a_1 + \\ &(\hat{p}^0 - \hat{p})(\hat{p}^0 + \hat{p} - 2\hat{p}'')l_2/2b_1 + (\hat{p}^0 - \hat{p}')^2 l_2/2g_1 \alpha^2 \end{aligned}$$

(see equations (32) and (33))

Now:

$$\begin{aligned} \hat{p}^0 - \hat{p} &= -n(l_1 + l_2)a_1^2 l_2 b_1 / (\alpha^2 g_1 l_1 b_1 + l_2 a_1 b_1 + \alpha^2 g_1 l_2 a_1)(l_1 b_1 + l_2 a_1) \\ (\hat{p}^0 - \hat{p}')^2 &= n^2(l_1 + l_2)^2 a_1^2 \alpha^4 g_1^2 / (\alpha^2 g_1 l_1 b_1 + l_2 a_1 b_1 + \alpha^2 g_1 l_2 a_1)^2 \\ \hat{p}^0 + \hat{p} - 2\hat{p}' &= n(l_1 + l_2)a_1(2\alpha^2 g_1 l_1 b_1 + l_2 a_1 b_1 + 2\alpha^2 g_1 l_2 a_1) / (l_1 b_1 + l_2 a_1) \\ &\quad (\alpha^2 g_1 l_1 b_1 + l_2 a_1 b_1 + \alpha^2 g_1 l_2 a_1) \\ \hat{p}^0 + \hat{p} - 2\hat{p}'' &= -n(l_1 + l_2)[l_1 b_1(2\alpha^2 g_1 l_1 b_1 + l_2 a_1 b_1 + 2\alpha^2 g_1 l_2 a_1) + \\ &\quad l_2 a_1 b_1(l_1 b_1 + l_2 a_1)] / l_2(l_1 b_1 + l_2 a_1) (\alpha^2 g_1 l_1 b_1 + \\ &\quad l_2 a_1 b_1 + \alpha^2 g_1 l_2 a_1) \end{aligned}$$

Substitute and simplify:

$$\begin{aligned} [\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2) - \bar{\pi}(\hat{p}^0, \hat{p}^0, 0)]/I &= -(\hat{p}^0 - \hat{p}) \cdot n(l_1 + l_2)l_2 a_1 / 2(\alpha^2 g_1 l_1 b_1 + \\ &l_2 a_1 b_1 + \alpha^2 g_1 l_2 a_1) + n^2(l_1 + l_2)^2 a_1^2 \alpha^2 g_1 l_2 / 2(\alpha^2 g_1 l_1 b_1 + l_2 a_1 b_1 + \\ &\alpha^2 g_1 l_2 a_1)^2 \end{aligned}$$

$$\begin{aligned} [\bar{\pi}(\hat{p}, \hat{p}, \hat{m}_2) - \bar{\pi}(\hat{p}^0, \hat{p}^0, 0)]/I &= n^2(l_1 + l_2)^2 a_1^2 l_2 / 2(\alpha^2 g_1 l_1 b_1 + \\ &l_2 a_1 b_1 + \alpha^2 g_1 l_2 a_1)(l_1 b_1 + l_2 a_1) \quad (\text{equation (50)}) \end{aligned}$$